Nulceon TMDPDFs within the twisted mass formulation of lattice QCD

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TMDPDF definition

Transverse momentum dependent parton distribution functions (TMDPDFs) provide the distribution of momentum fraction of partons in a transverse plane to the direction of the hadron momentum.

Consider SIDIS

$$e^-(l) + p(P) \to e^-(l') + h(P_h) + X$$

cross-section from the parton model

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\ \mathrm{d}y\ \mathrm{d}z_h\ \mathrm{d}^2\mathbf{P}_{hT}} = \sum_i \hat{\sigma}_{ii}^{\mathrm{TMD}}(Q,x,y) \int \mathrm{d}^2\mathbf{p}_T\ \mathrm{d}^2\mathbf{k}_T \delta^{(2)}\left(\mathbf{P}_{hT} - z_h\mathbf{k}_T - \mathbf{p}_T\right)$$
$$\times f_{i/p}\left(x,\mathbf{k}_T\right) D_{h/i}\left(z_h,\mathbf{p}_T\right)\left[1 + O\left(\frac{P_{hT}^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)\right]$$

$f_{i/p}$ is TMDPDF of parton i within hadron p

[Boussarie *et al.* arxiv:2304.03302]

LaMET

parton distributions are defined on light-cone $\implies$ not possible to compute on Euclidean lattice

(one) **Solution:** Large momentum effective theory [Ji PRL 110 262002]

define a quasi-observable $\tilde{f}$ at large momentum boost P^z and expand the momentum dependence to match it to the physical observable f, assuming $P^z >> \Lambda_{\mathrm{QCD}}$

$$\tilde{f}\left(x, \mu^2, P^z\right) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) f\left(y, \mu^2\right) + \mathcal{O}\left(\Lambda^2/\left(P^z\right)^2, M^2/\left(P^z\right)^2\right)$$

at large enough P^z, the expansion converges

the matching factor Z is calculable in perturbation theory

TMDPDFs on the lattice

$$f^{TMD}(x,b,\mu,\zeta) = H_f(\frac{\zeta_z}{\mu^2})\, e^{-\ln\left(\frac{\zeta_z}{\zeta}\right)K(b,\mu)}\, S_r^{\frac{1}{2}}(b,\mu)\, \tilde{f}(x,b,\mu,\zeta_z) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b^2\zeta_z}\right)$$

- $\tilde{f}(x,b,\mu,\zeta_z)$ - quasi-TMDPDF
- $S_r^{\frac{1}{2}}(b,\mu)$ - reduced soft function
- $K(b,\mu)$ - Collins-Soper kernel
- $H_f(\zeta_z/\mu^2)$ - matching kernel for TMDPDFs
- $\zeta_z = (2zP^z)^2$ - Collins-Soper scale for quasi-TMDPDF
- $\zeta = 2(xP^+)^2 e^{2y_\eta}$ - Collins-Soper scale for quasi-TMDPDF

[Ji *et al.* RMP 93 035005]

Lattice setup

we use $N_f = 2+1+1$ clover improved twisted mass fermion ensembles generated by the Extended Twisted Mass Collaboration (ETMC)

name	$L^3 \times T/a^4$	a [fm]	$a\mu_l$	M_π [MeV]	$M_\pi L$	N_{conf}
cA211.53.24	$24^3 \times 48$	0.093	0.0053	350	4	600

gauge links are APE smeared and we apply momentum smearing to the propagators

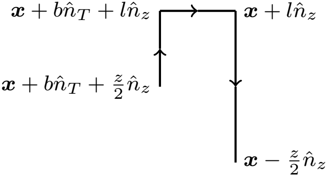
5 steps of stout smearing has been applied to all the gauge links used in the construction of the staple shaped Wilson line

quasi-TMDPDF

$$f^{TMD}(x,b,\mu,\zeta) = H_f\left(\frac{\zeta_z}{\mu^2}\right) e^{-\ln\left(\frac{\zeta_z}{\zeta}\right)K(b,\mu)}\, S_r^{\frac{1}{2}}(b,\mu)\, \tilde{f}(x,b,\mu,\zeta_z) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b^2\zeta_z}\right)$$

defined in terms of the quasi-beam function

$$\tilde{f}\,(x,b,\mu,\zeta_z) = \int_{-\infty}^{+\infty} \frac{P^z dz}{2\pi} e^{ixzP^z} B\,(z,b,\mu,P^z)$$



where

$$B_\Gamma\,(z,b,\mu,P^z) = \lim_{l\to\infty} \frac{\left\langle H\,(P^z)\left|\mathcal{O}_\Gamma(0,\vec{0},b,l,z)\right|H\,(P^z)\right\rangle}{\sqrt{Z_E(2|l|,|b|)}}$$

$\mathcal{O}_\Gamma$ is a staple-shaped Wilson line quark-bilinear operator

$$\mathcal{O}_\Gamma(t,\boldsymbol{x},b,l,z) = \bar{q}\left(t,\boldsymbol{x} + b\hat{n}_T + \frac{z}{2}\hat{n}_z\right)\Gamma\mathcal{W}_{\text{staple}}\,(\boldsymbol{x},b,l,z)q\left(t,\boldsymbol{x} - \frac{z}{2}\hat{n}_z\right)$$

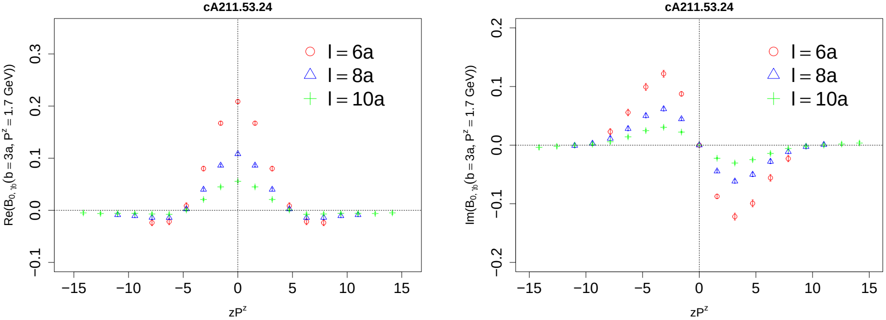
[Ji *et al.* RMP 93 035005]

quasi-beam function

define an "unsubtracted" quasi-beam function ($B_{0,\Gamma}$), where we do not take the ratio with the rectangular Wilson loop Z_E

can be obtained on the lattice from the ratio of a 3-point and a 2-point function

$$\frac{\mathcal{P}_{\alpha\beta}\sum_{\boldsymbol{x},\boldsymbol{y}} e^{-iP^z x_z}\langle N_\alpha(t_s,\boldsymbol{x})\mathcal{O}_\Gamma(\tau,\boldsymbol{y},b,l,z)\bar{N}_\beta(0,\mathbf{0})\rangle}{\mathcal{P}_{\alpha\beta}\sum_{\boldsymbol{x}} e^{-iP^z x_z}\langle N_\alpha(t_s,\boldsymbol{x})\bar{N}_\beta(0,\mathbf{0})\rangle}$$



$b = 0.27$ fm, $P^z = 1.7$ GeV

Renormalization

In [Alexandrou *et al.* PRD 108 114503] we showed that for unpolarized TMDPDF ($\Gamma = \gamma_0$) the minimal set of operators that are allowed to mix are

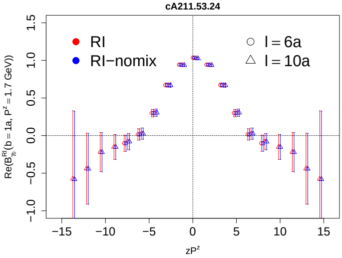
$$\{\gamma_0, \gamma_0\gamma_2, \gamma_0\gamma_3, \gamma_5\gamma_1\}$$

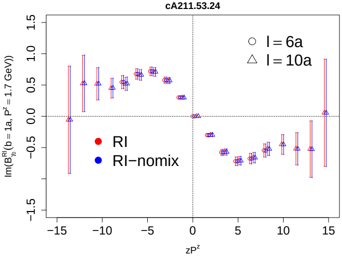
and found using RI/MOM

$$\frac{Z^{\mathrm{RI}}_{\Gamma\Gamma'}(b,l,z,\mu_0;1/a)}{Z^{\mathrm{RI}}_q(\mu_0;1/a)} \times \frac{1}{12}\,\mathrm{Tr}\left[\frac{\Lambda_0^{\Gamma}(b,l,z,p;1/a)\Gamma'}{e^{ip^z z + ip_\perp b}}\right]\Bigg|_{p=\mu_0} = 1$$

that mixing has negligible effect

$b = 0.09$ fm, $P^z = 1.7$ GeV





Renormalization

since mixing has no effect, we use the rectangular Wilson loop to cancel the pinch-pole singularity associated with l, cusp divergences and divergences arising from the length of the Wilson line.

the remaining UV divergences can be cancelled by a multiplicative factor

Short distance ratio (SDR)

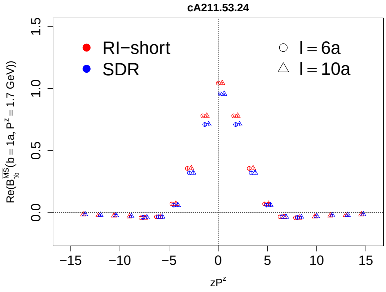
$$Z_{SDR} = \frac{1}{B_\Gamma(z = z_0, b = b_0, P^z = 0)}$$

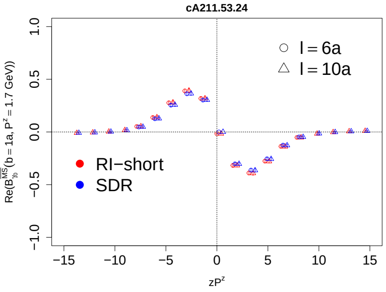
[Zhang *et al.* PRL 129 082002]

Short distance RI (RI-short)

$$Z_{RI-short} = Z^{RI}_{\gamma_0\gamma_0}(z_0, b_0, l, \mu_0) \times \sqrt{Z_E(2l, b_0)}$$

[Alexandrou *et al.* PRD 108 114503]



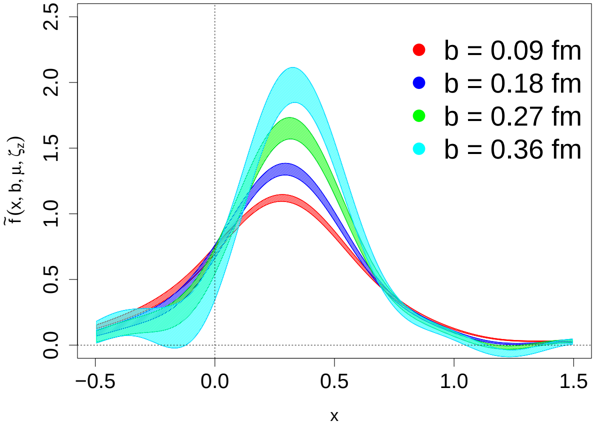


$b = 0.09$ fm, $P^z = 1.7$ GeV

quasi-TMDPDF

we perform a discrete Fourier transformation to obtain the quasi-TMDPDF in the momentum space

$$\tilde{f}\left(x, b, \mu, \zeta_z\right) = \sum_{-z_{\max}}^{+z_{\max}} \frac{P^z}{2\pi} e^{ixzP^z} B\left(z, b, \mu, P^z\right)$$



Collins-Soper kernel

$$f^{TMD}(x,b,\mu,\zeta) = H_f(\frac{\zeta_z}{\mu^2})\, e^{-\ln\left(\frac{\zeta_z}{\zeta}\right) K(b,\mu)}\, S_r^{\frac{1}{2}}(b,\mu)\, \tilde{f}(x,b,\mu,\zeta_z) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b^2\zeta_z}\right)$$

the Collins-Soper kernel describes the evolution of TMDPDFs, it can be obtained by taking ratios of TMDPDFs at different Collins-Soper scales (momentum boosts)

in this work, we calculate it from ratios of quasi-TMDWFs

$$K(b,\mu) = \frac{1}{\frac{1}{2}\ln\left(\frac{\zeta_{z_1}}{\zeta_{z_2}}\right)} \ln\left(\frac{\tilde{\Psi}_{\bar{q}q}\left(x,b,\mu,\zeta_{z_1},\bar{\zeta}_{z1}\right)}{\tilde{\Psi}_{\bar{q}q}\left(x,b,\mu,\zeta_{z_2},\bar{\zeta}_{z2}\right)}\right)$$

where, the quasi-TMDWF is given by

$$\tilde{\psi}_{\bar{q}q}\left(z,b,\mu,P^z\right) = Z_\Gamma(\mu,P^z) \lim_{l\to\infty} \frac{\langle\Omega|\,\mathcal{O}_\Gamma(0,\overrightarrow{0},b,l,z)\,|\pi(P^z)\rangle}{\sqrt{Z_E(2|l|,|b|)}}$$

$\mathcal{O}_\Gamma$ is the staple-shaped Wilson line quark bilinear operator as before

quasi-TMDWF

is obtained from the 2 -point correlator

$$\sum_{\boldsymbol{x}} e^{-iP^z x_z} \langle \mathcal{O}_\Gamma(x,b,l,z)\mathcal{O}_\pi^\dagger(0,P^z)\rangle$$

where

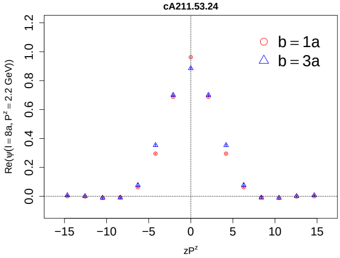
$$\mathcal{O}_\pi^\dagger(t,P^z) = \sum_{\boldsymbol{x},\boldsymbol{y}} \bar{u}(t,\boldsymbol{x})\gamma_5 d(t,\boldsymbol{y}) e^{i\frac{P^z}{2}(y_z - x_z)}$$

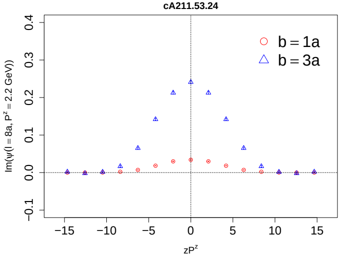
are built from Coulomb-gauge-fixed wall sources.

we use a heavier pion mass of ~ 640 MeV

[Chu *et al.* PRD 106 034509]

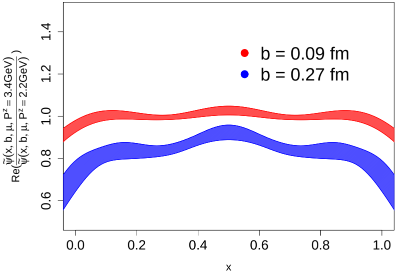
$l = 0.72$ fm, $P^z = 2.2$ GeV

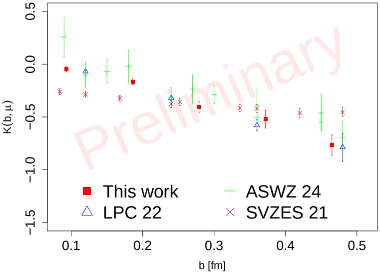




Collins-Soper kernel

$$\frac{\tilde{\Psi}_{\bar{q}q}\left(x, b, \mu, \zeta_{z_1}, \bar{\zeta}_{z1}\right)}{\tilde{\Psi}_{\bar{q}q}\left(x, b, \mu, \zeta_{z_2}, \bar{\zeta}_{z2}\right)}$$





$$K(b,\mu) = \frac{1}{\frac{1}{2}\ln\left(\frac{\zeta_{z_1}}{\zeta_{z_2}}\right)} \ln\left(\frac{\tilde{\Psi}_{\bar{q}q}\left(x, b, \mu, \zeta_{z_1}, \bar{\zeta}_{z1}\right)}{\tilde{\Psi}_{\bar{q}q}\left(x, b, \mu, \zeta_{z_2}, \bar{\zeta}_{z2}\right)}\right)$$

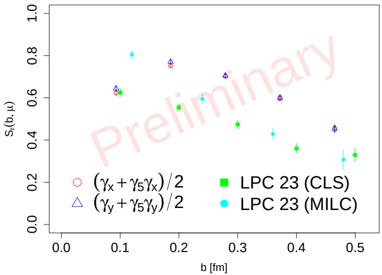
[Chu *et al.* PRD 106 034509, Avkhadiev *et al.* PRL 132 231901, Schlemmer *et al.* JHEP 08 004]

Soft function

$$f^{TMD}(x,b,\mu,\zeta) = H_f(\frac{\zeta_z}{\mu^2})\, e^{-\ln\left(\frac{\zeta_z}{\zeta}\right)K(b,\mu)}\, S_r^{\frac{1}{2}}(b,\mu)\, \tilde{f}(x,b,\mu,\zeta_z) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b^2\zeta_z}\right)$$

soft function is required to cancel the rapidity divergences coming from soft gluon emissions

$$S_r(b,\mu) = \frac{\langle \pi(P')\,|\bar{q}_2(b)\Gamma q_2(b)\bar{q}_1(0)\Gamma q_1(0)|\,\pi(P)\rangle}{\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' H_{F_\Gamma}(x,x',P,P',\mu)\,\tilde{\Psi}^\dagger_{\bar{q}q}(x',b,\mu,P')\,\tilde{\Psi}_{\bar{q}q}(x,b,\mu,P)}$$



following [Li *et al.* PRL 128 062002] we use $\gamma_\perp + \gamma_5\gamma_\perp$ combination to reduce higher twist effects

[Chu *et al.* JHEP 08 172]

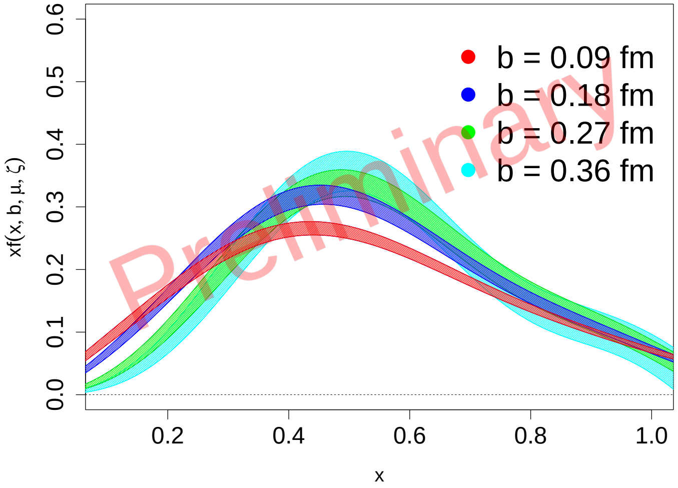
Matching to physical TMDPDF

$$f^{TMD}(x,b,\mu,\zeta) = \tilde{f}(x,b,\mu,\zeta_z)$$
$$\times \exp\left(-\frac{\ln(\zeta_z/\zeta)\ln\left[\tilde{\Psi}\left(x,b,\mu,\zeta_{z1}\bar{\zeta}_{z1}\right)/\tilde{\Psi}\left(x,b,\mu,\zeta_{z_1},\bar{\zeta}_{zz}\right)\right]}{\frac{1}{2}\ln\left(\zeta_{z_1}/\zeta_{z_2}\right)}\right)\left(\frac{F_\Gamma\left(b,P,P',\mu\right)}{N_\Gamma J_1^2}\right)^{1/2}$$
$$\times\left\{1-\frac{\alpha_s}{4\pi}C_F\left[-4+\frac{\pi^2}{6}+2\ln\left(\frac{\zeta_z}{\mu^2}\right)-\ln^2\left(\frac{\zeta_z}{\mu^2}\right)-\ln\left(\frac{\zeta_z}{\zeta}\right)\left(4-\ln\left(\frac{\zeta_{z1}\bar{\zeta}_{z1}\zeta_{zz}\bar{\zeta}_{z2}}{\mu^4}\right)\mp 4i\pi\right)\right.\right.$$
$$\left.\left.+\frac{1}{2}\left(h_0^\Gamma+h_1^\Gamma\left(\ln\left(\frac{16\left(P^z\right)^4}{\mu^4}\right)+2\frac{J_2+J_3}{J_1}\right)+2\left(\frac{J_4+J_5}{J_1}-\frac{J_2^2+J_3^2}{J_1^2}\right)\right)\right]+\mathcal{O}(\alpha_s^2)\right\}$$

$$J_1=\int_{-\infty}^{\infty}dx\tilde{\Psi}\left(x,b,P^z\right) \qquad J_2=\int_{-\infty}^{\infty}dx\tilde{\Psi}\left(x,b,P^z\right)\ln|x|$$
$$J_3=\int_{-\infty}^{\infty}dx\tilde{\Psi}\left(x,b,P^z\right)\ln|1-x| \qquad J_4=\int_{-\infty}^{\infty}dx\tilde{\Psi}\left(x,b,P^z\right)\ln^2|x|$$
$$J_5=\int_{-\infty}^{\infty}dx\tilde{\Psi}\left(x,b,P^z\right)\ln^2|1-x|$$

[Ji *et al.* RMP 93 035005, Deng *et al.* JHEP 09 046, Chu *et al.* PRD 106 034509, Avkhadiev *et al.* PRL 132 231901, Chu *et al.* JHEP 08 172]

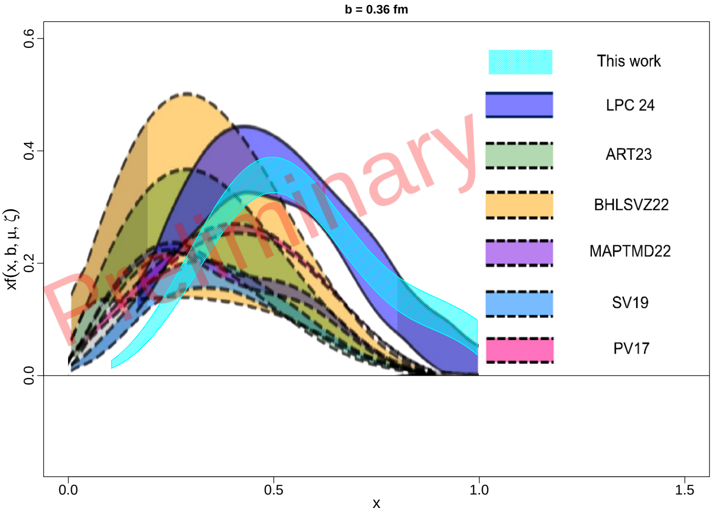
physical TMDPDF



physical TMDPDF

we compare with a recent first calculation of the unpolarized TMDPDF by LPC

[He *et al.* PRD 109 114513]



Summary and outlook

- We computed the quasi-TMDPDF, Collins-Soper kernel, reduced soft function non-perturbatively.
- Performed a matching to the physical TMDPDF including $\mathcal{O}(\alpha_s)$ corrections.
- Calculation on a physical point ensemble is currently in progress.
- A systematic study of the discretization effects and continuum extrapolation will be performed in future.