Update of HPQCD $B_c \to J/\psi$ Form Factors

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41st Lattice Conference
Tuesday 30 July 2024

Form factor definitions

For $B_c \to J/\psi$, the non-zero matrix elements are parameterised in terms of local hadronic form factors. For the vector and axial-vector these are given by

$$\langle J/\psi(p',\lambda)|\bar{c}\gamma^\mu b|B_c^-(p)\rangle = \frac{2i\,V(q^2)}{M_{B_c}+M_{J/\psi}}\varepsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu(p',\lambda)p'_\rho p_\sigma$$

$$\langle J/\psi(p',\lambda)|\bar{c}\gamma^\mu\gamma^5 b|B_c^-(p)\rangle = A_0(q^2)2M_{J/\psi}\frac{\epsilon^*(p',\lambda)\cdot q}{q^2}q^\mu$$

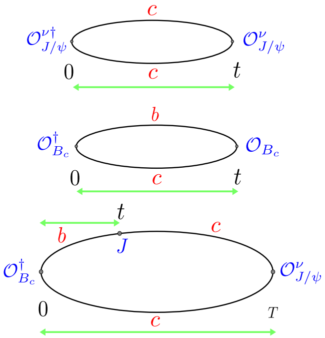
$$+\,A_1(q^2)\left[\epsilon^{*\mu}(p',\lambda) - \frac{\epsilon^*(p',\lambda)\cdot q}{q^2}q^\mu\right](M_{B_c}+M_{J/\psi})$$

$$-\,A_2(q^2)\frac{\epsilon^*(p',\lambda)\cdot q}{M_{B_c}+M_{J/\psi}}\left[p^\mu + p'^\mu - \frac{M_{B_c}^2 - M_{J/\psi}^2}{q^2}q^\mu\right]$$

Previous HPQCD calculation: 2007.06957

- 2+1+1 MILC HISQ gluon configurations, $a = 0.09\text{fm},\ 0.06\text{fm},\ 0.045\text{fm}$
- Physical pions on $a = 0.09\text{fm}$ ensemble
- HISQ charm and heavy valence quarks, with $1.3am_c < am_h < 0.9am_b$

Matrix elements extracted from fits to two-point and three-point functions



$$\rightarrow \sum_n^{N_{\text{exp}}} |A_n^{J/\psi}|^2 e^{-E_n^{J/\psi} t} + (-1)^t ...$$

$$\rightarrow \sum_n^{N_{\text{exp}}} |A_n^{B_c}|^2 e^{-E_n^{B_c} t} + (-1)^t ...$$

$$\rightarrow \sum_{n,m}^{N_{\text{exp}}} A_n^{J/\psi} A_m^{B\dagger} V_{nm} e^{-tE_m^{B_c} - (T-t) E_n^{J/\psi}} + (-1)^t ... + (-1)^T ... + (-1)^{T-t} ...$$

where the oscillating contributions come from our use of staggered quarks.

$$V_{00} \sim \langle J/\psi(p', \lambda) | J | B_c^-(p) \rangle \rightarrow F(am_c, am_h, q^2, M_{\eta_h}, M_\pi^2)$$

Previous HPQCD calculation: 2007.06957

Form factors then extrapolated to physical continuum using fit to pseudo-BGL parameterisation, excluding outer functions

$$F(q^2) = \frac{1}{\cancel{\phi(q^2)} P(q^2)} \sum_{n=0}^{3} a_n z^n \mathcal{N}_n$$

where

$$P(q^2) = \prod_{M_{\text{pole}}} z(q^2, M_{\text{pole}}^2)$$

$$a_n = \sum_{j,k,l=0}^{3} b_n^{jkl} \Delta_h^{(j)} \left(\frac{am_c^{\text{val}}}{\pi}\right)^{2k} \left(\frac{am_h^{\text{val}}}{\pi}\right)^{2l}$$

with $\Delta_h^{(0)} = 1$ and

$$\Delta_h^{(j\neq 0)} = \left(\frac{2\Lambda}{M_{\eta_h}}\right)^j - \left(\frac{2\Lambda}{M_{\eta_b}^{\text{phys}}}\right)^j$$

- Omission of outer functions, which depend on z and m_h, makes choice of priors unclear.
- Ideally include outer functions which depend on susceptibilities, χ.
- For $b \to c$, susceptibilities are well known perturbatively, but for $b \to \text{light}$ nonperturbative condensate effects are important
- we compute $\bar{b}c$ susceptibilities nonperturbatively on lattice before moving on to $\bar{b}s$ and $\bar{b}u/\bar{b}d$

$\bar{b}c$ Susceptibilities: 2405.01390

Susceptibilities are defined in terms of subtracted polarisation tensors for current j^δ_μ,

$$(-q^2 g_{\mu\nu} + q_\mu q_\nu)\Pi^\delta(q^2) + q_\mu q_\nu \Pi^\delta_L(q^2) = i \int dx e^{iqx} \langle 0| T j^\delta_\mu(x) j^{\delta\dagger}_\nu(0)|0\rangle,$$

e.g.

$$\chi_{1^+}(q^2 = 0) \equiv \frac{1}{2}\frac{\partial^2}{\partial^2 q^2}\left(q^2 \Pi^A(q^2)\right)\Big|_{q^2=0},$$

Dispersive parameterisations like BGL use the optical theorem to write the imaginary part of the polarisation tensor as

$$\mathrm{Im}\Pi^\delta_{\mu\nu}(q^2) = \frac{1}{2}\sum_X (2\pi)^4 \delta(p_X - q)\langle 0|j^\delta_\mu|X(q)\rangle\langle X(q)|j^{\delta\dagger}_\nu|0\rangle.$$

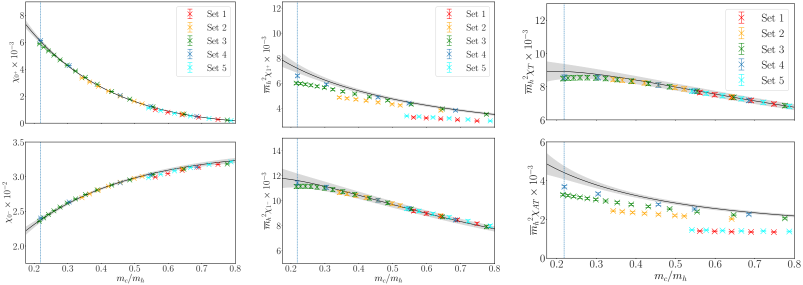
and crossing symmetry to relate the $0 + J \to 2$ matrix elements to the $1 + J \to 1$ hadronic form factors F, arriving at an expression of the form:

$$\chi_{J^P} \geq \int_{t_X}^{\infty} dq^2 \lambda(q^2)|F(q^2)|^2$$

$$\chi_{J^P} \geq \frac{1}{2\pi i}\int_{\mathcal{C}_\alpha} \frac{dz}{z}|B(z)\phi(z)F(z)|^2.$$

$\bar{b}c$ Susceptibilities: 2405.01390

- Compute χ^Γ directly on the lattice (e.g. Martinelli et al. 2105.07851) from time moments of Euclidean correlation functions



HPQCD 2405.01390

- We will use these susceptibilities to perform a fully dispersively bounded fit to new data for $B_c \to J/\psi$

$B_c \to J/\psi$ Update: Dispersive Fit

$t_X = (M_{B_c} + M_{J/\psi})^2$ is above 2-particle threshold $t_+ = (M_{D^*} + M_B)^2$.
Following 2305.06301, we use orthonormal polynomials on unit circle,

$$\int_{-\alpha(t_X)}^{\alpha(t_X)} d\theta p_n(e^{i\theta}) p_m(e^{-i\theta}) = \delta_{nm}$$

The dispersive parameterisation is then

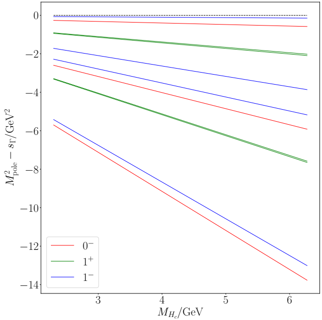
$$F^Y(z) = \frac{1}{P^Y(z)\phi^Y(z)} \sum_n a_n^Y p_n(z)$$

where $P^Y(z) = \prod_i z(q^2, t_+, M^2_{\mathrm{pole},i})$ are Blaschke factors and ϕ are outer functions, which depend on the susceptibilities, χ, and are analytic on the open unit disk in z.

To determine $M_{\mathrm{pole},i}$ we use the lattice H_c masses, extracted from correlator fits, and the physical-continuum splittings between M_{B_c} and the $\bar{b}c$ poles,

$$M_{\mathrm{pole},i} = M_{H_c}^{\mathrm{latt}} + \Delta_i^{\mathrm{phys}}.$$

With this prescription, none of the poles cross threshold



$B_c \to J/\psi$ Update: Dispersive Fit

We expand the a_n appearing in the physical continuum parameterisation,

$$F^Y(z) = \frac{1}{P^Y(z)\phi^Y(z)} \sum_n a_n^Y p_n(z),$$

to allow for dependence on m_h and π masses:

$$a_n^Y = \alpha_n^Y \times \Big(1 + \sum_{j\neq 0}^{3} b_n^{Y,j} \Delta_h^{(j)} + \delta_\chi \sum_{j=0}^{3} \tilde{b}_n^{Y,j} \Delta_h^{(j)}\Big) \mathcal{N}_n$$

$$\delta_\chi = \left(\frac{M_\pi}{\Lambda_\chi}\right)^2 - \left(\frac{M_\pi^{\text{phys}}}{\Lambda_\chi}\right)^2, \quad \Delta_h^{(j\neq 0)} = \left(\frac{\Lambda}{M_H}\right)^j - \left(\frac{\Lambda}{M_B^{\text{phys}}}\right)^j$$

The dispersive bounds then allow us to take priors of 0 ± 1 for each α_n^Y, where we also truncate the sum at $\mathcal{O}(z^3)$.

HPQCD 2304.03137

Discretisation effects included at the level of matrix elements as

$$J^{\nu,\Gamma(s)}_{\rm latt} = J^{\nu,\Gamma(s)}_{\rm phys} + \sum_{j,n=0}^{3}\sum_{k,l\neq 0}^{3} c_n^{(\nu,\Gamma),jkl}\Delta_h^{(j)}(w-1)^n\left(\frac{am_c^{\rm val}}{\pi}\right)^{2k}\left(\frac{am_h^{\rm val}}{\pi}\right)^{2l} + \sum_{j,n=0}^{3}\sum_{k,l\neq 0}^{3} \tilde{c}_n^{(\nu,\Gamma)(s),jkl}\Delta_h^{(j)}(w-1)^n\left(\frac{am_c^{\rm val}}{\pi}\right)^{2k}\left(\frac{am_h^{\rm val}}{\pi}\right)^{2l}\delta_\chi^{(s)}$$

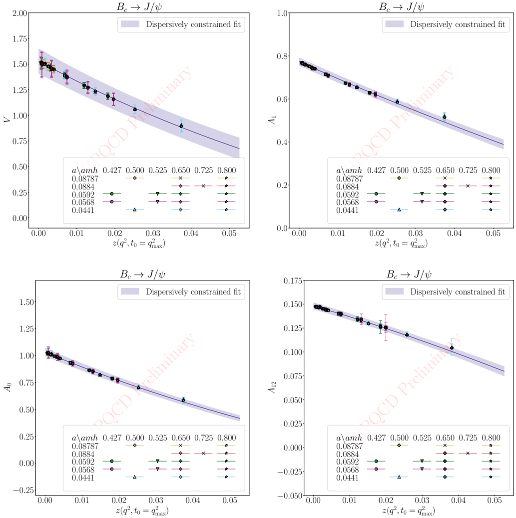
This is important for the noisy form factors, such as A_2, where we have

$$A_2 \sim \frac{J - J'}{p^2/M^2_{J/\psi}}$$

where J and J' are $\mathcal{O}(1)$ matrix elements.

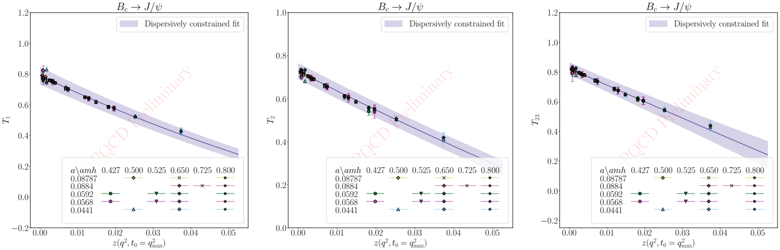
$B_c \to J/\psi$ Update: Results

Update includes increase in time sources on each ensemble and additional $a = 0.06\text{fm}$ ensemble with physical pions. $a = 0.03\text{fm}$ ensemble also in progress.



Data points here are corrected to better reflect fit quality:
$F^{\rm corrected} = F^{\rm data} + (F^{\rm phys} - F^{\rm fit})$

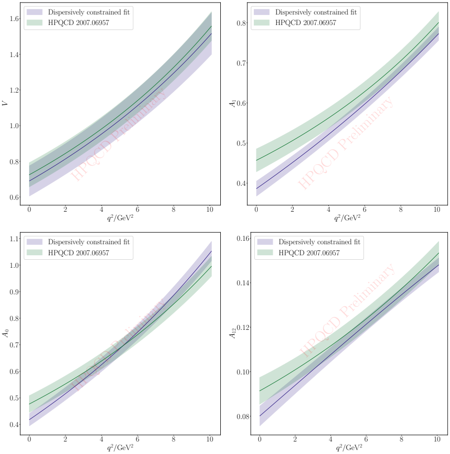
$B_c \to J/\psi$ Update: Preliminary Dispersive Fit



Also have good control over the heavy-HISQ extrapolation for the tensor.

Comparison to 2007.06957

$\times 2$ reduction in uncertainty for $1^{\pm}$ FFs vs 2007.06957, some change in shape.



Shift in $R(J/\psi)$:

$$0.2582(38) \to 0.2674(31) \text{ (prelim.)}$$

and F_L :

$$0.4416(92) \to 0.4510(88) \text{ (prelim.)}$$

Seems to go against arguments that $R(J/\psi)$ and F_L should move move in opposite directions

Conclusions and outlook

- Update of $B_c \to J/\psi$ form factors near completion, with large reduction in uncertainty for 1^+ currents

- $a = 0.03\mathrm{fm}$ ensemble in progress and stability analysis still to do

- $\bar{b}c$ susceptibilities now computed (2405.01390), $\bar{b}s$ and $\bar{b}l$ susceptibilities now underway

- Outer functions should be included!

Thanks for listening!