Heavy-light Meson Decay Constants and Hyperfine Splittings with the Heavy-HISQ Method

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***speaker**

What We Are Researching

Heavy-light (Vector) Mesons

- Vector and tensor decay constants

$$\text{Vector:} \quad \left\langle 0 \middle| V_\mu^{(s)} \middle| H_{(s)}^*(p) \right\rangle \equiv M_{H_{(s)}^*} f_{H_{(s)}^*} \epsilon_\mu(p),$$

$$\text{Tensor:} \quad \left\langle 0 \middle| Z_T^{\overline{\text{MS}}} T_{\alpha\beta}^{(s)} \middle| H_{(s)}^*(p) \right\rangle \equiv i f_{H_{(s)}^*}^T (\epsilon_\alpha p_\beta - \epsilon_\beta p_\alpha),$$

with $V_\mu^{(s)} = \bar{q}\gamma_\mu h; \quad T_{\alpha\beta}^{(s)} = \bar{q}\sigma_{\alpha\beta} h; \quad h = b, c; \quad \bar{q} = \bar{u}/\bar{d}, \bar{s}$

- Hyperfine splittings: $\Delta_{H_{(s)}^* - H_{(s)}} = M_{H_{(s)}^*} - M_{H_{(s)}}$

Why Are We Interested?

Precision tests of the Standard Model

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Decay constants:

- Can study CKM matrix elements (e.g. $V_{cs},\ V_{cd}$)
- Appear in dispersive parametrisations of form factors
- Sensitivity to potential New Physics
  - FCNCs (e.g. $b \to s,\ b \to d$) — suppressed in SM
  - Can constrain high-scale contributions to interactions via WET effective Wilson coefficients

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Hyperfine splittings:

- Easy access on the lattice
- Another precision observable for test against experiment
- Important phenomenological input in, e.g., HQET calculations

Lattice Calculation

Set-up Details

- 2nd-generation MILC $n_f = 2 + 1 + 1$
- Highly Improved Staggered Quark (HISQ) action
- 5 lattice spacings across 10 ensembles: $0.15\,\mathrm{fm}$ down to $0.045\,\mathrm{fm}$
- Physical pions on 5 ensembles ($0.15\,\mathrm{fm}$ – $0.06\,\mathrm{fm}$); unphysically heavy pions ($m_l = m_s/5$) on the other 5
- Strange valence tuned using η_s [1303.1670]
- Charm valence tuned using η_c — pure QCD, connected diagram value [2305.06231]
- Heavy-HISQ method: $m_c \leq m_h \leq m_b$

Lattice Current Renormalisation

Vector and tensor renormalisation factors previously calculated by members of HPQCD

- Vector renormalisation, Z_V, calculated in 1909.00756
  - Non-perturbative, in RI-SMOM scheme
- Scale-dependent tensor renormalisation, $Z_T^c(\mu', \mu)$, calculated in 2008.02024
  - Non-perturbative, matched to $\overline{\text{MS}}$ via RI-SMOM at scale μ'

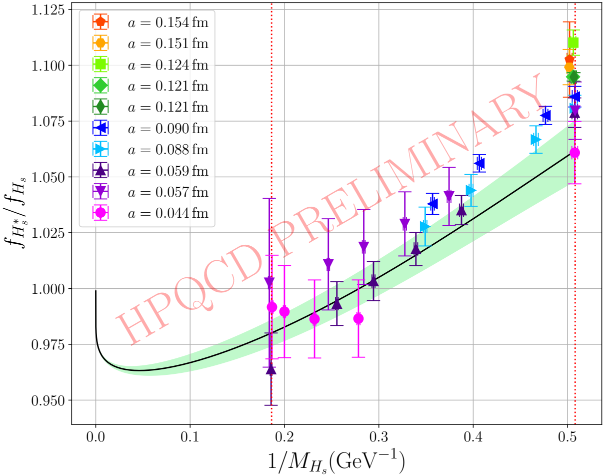
Results

Introduction to Ratios

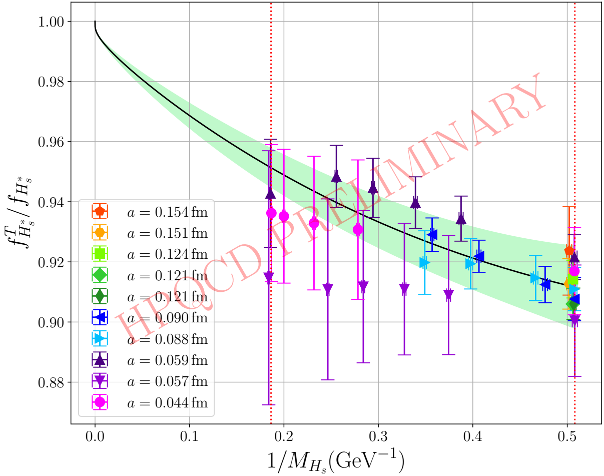
- $V/P,\ T/V$ for both $H^{(*)}$ and $H_s^{(*)}$
- HQET-inspired functional form
- Correlations provide higher precision

$$\frac{f^{(T)}_{H^*_{(s)}}(\mu)}{f_{H^{(*)}_{(s)}}} = 1 + \sum_{n=1}^{4} c_n \left(\frac{\alpha_s(\hat{m}_h)}{\pi}\right)^n + \mathcal{N}_{(s)} \sum_{i,j,k,l=0}^{3} c'_{ijkl} \left(\frac{\Lambda_{\text{QCD}}}{M_{H_s}}\right)^i (a\Lambda_{\text{QCD}})^{2j} \left(\frac{am_h}{\pi}\right)^{2k} \left(\frac{M_{\pi(K)}}{\Lambda_\chi}\right)^{2l}$$

High-Precision Ratios of Decay Constants



High-Precision Ratios of Decay Constants



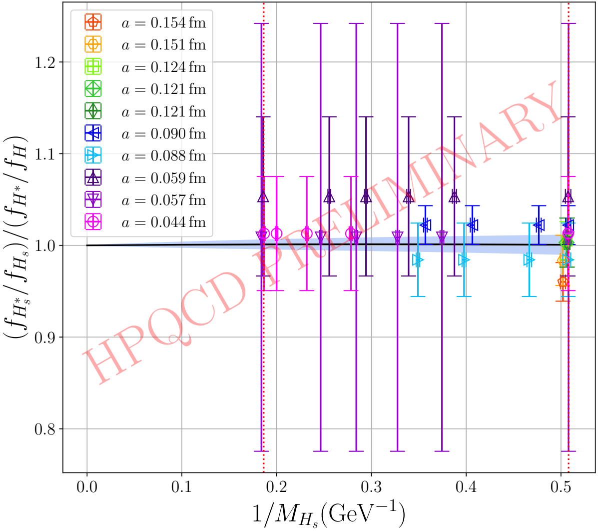
Introduction to Double Ratios

- $H_s^{(*)}$-to-$H^{(*)}$ ratios of V/P, T/V ratios
- Can combine with $H_s^{(*)}$ ratios to obtain higher-precision $H^{(*)}$ ratios
- X is $\left(f_{H_s^*}/f_{H_s}\right) \Big/ \left(f_{H^*}/f_H\right)$ or $\left(f^T_{H_s^*}/f_{H_s^*}\right) \Big/ \left(f^T_{H^*}/f_{H^*}\right)$:

$$X = 1 + \mathcal{N}_s \left(\frac{M_K^2 - M_\pi^2}{\Lambda_\chi^2}\right) \sum_{i,j=0}^{3} c_{ij} \left(\frac{\Lambda_{\text{QCD}}}{M_{H_s}}\right)^i (a\Lambda_{\text{QCD}})^{2j} \,,$$

with $c_{00} = 0$ to ensure the correct heavy-quark limit.

Double Ratios (i.e., ratios of ratios)



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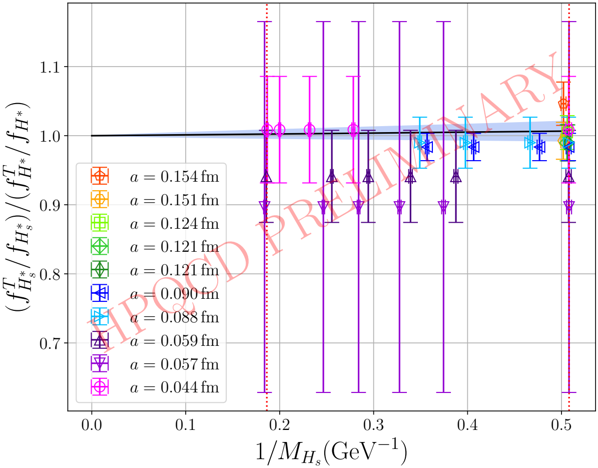


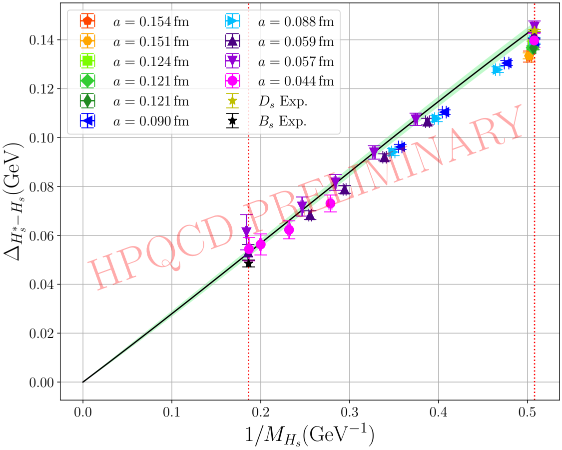
Introduction to Hyperfine Splittings and Ratio

- Hyperfine splittings: $c_{000l} = 0 \ \forall l$
- Ratio: c_{00} not constrained to 0
- Experimental data points (from PDG) on upcoming plots

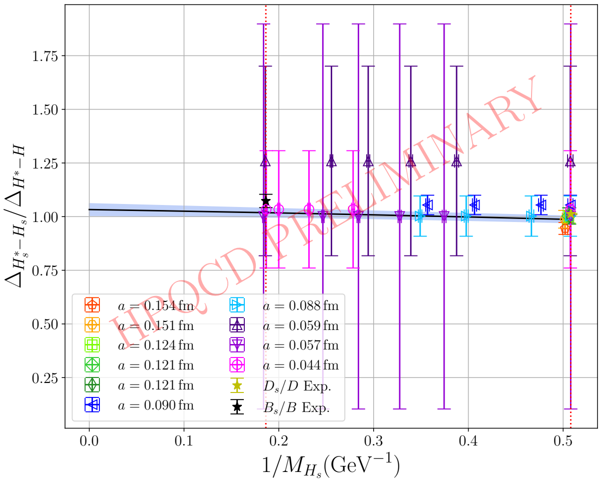
$$\Delta_{H^*_{(s)} - H_{(s)}} = \mathcal{N}_{(s)} \sum_{i,j,k,l=0}^{3} c_{ijkl} \left(\frac{\Lambda_{\text{QCD}}}{M_{H_s}} \right)^i (a\Lambda_{\text{QCD}})^{2j} \left(\frac{am_h}{\pi} \right)^{2k} \left(\frac{M_{\pi(K)}}{\Lambda_\chi} \right)^{2l}$$

$$\frac{\Delta_{H^*_s - H_s}}{\Delta_{H^* - H}} = 1 + \mathcal{N}_s \left(\frac{M_K^2 - M_\pi^2}{\Lambda_\chi^2} \right) \sum_{i,j=0}^{3} c_{ij} \left(\frac{\Lambda_{\text{QCD}}}{M_{H_s}} \right)^i (a\Lambda_{\text{QCD}})^{2j}$$

Hyperfine Splittings



Hyperfine Splitting Ratio



PRELIMINARY

Decay constant ratios:

$$\frac{f_{D^*}}{f_D} = 1.061(17), \frac{f^T_{D^*}}{f_{D^*}} = 0.906(19), \frac{f_{B^*}}{f_B} = 0.9790(72), \frac{f^T_{B^*}}{f_{B^*}} = 0.9491(85)$$

$$\frac{f_{D_s^*}}{f_{D_s}} = 1.062(13), \frac{f^T_{D_s^*}}{f_{D_s^*}} = 0.912(14), \frac{f_{B_s^*}}{f_{B_s}} = 0.9801(58), \frac{f^T_{B_s^*}}{f_{B_s^*}} = 0.9513(64)$$

Hyperfine splittings (GeV):

$$\Delta_{D^*-D} = 0.1468(32), \quad \Delta_{B^*-B} = 0.0519(17)$$

$$\Delta_{D_s^*-D_s} = 0.1449(21), \quad \Delta_{B_s^*-B_s} = 0.0528(14)$$

Thank you for your time and attention.

If you'd like to know more, speak to me or email
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Questions?

Backup Slides

Backup – Example Showing Effective Wilson Coefficients

$$\Gamma(B_s^* \to \ell^+\ell^-) = \frac{G_F^2 |\lambda_{ts}|^2 \alpha_{\text{em}}^2}{96\pi^3} M_{B_s^*} f_{B_s^*}^2 \times \left(\left| C_9^{\text{eff}} + 2\frac{m_b f_{B_s^*}^T}{M_{B_s^*} f_{B_s^*}} C_7^{\text{eff}} \right|^2 + |C_{10}|^2 \right)$$

Backup – LO HQET

$$c_1^{V/P} = -2/3, \quad c_1^{T/V} = 0$$

$c_2^R,\ c_3^R$ are non-trivial functions of $x = m_c^{\text{sea}}/m_h^{\text{pole}}$ and its logarithms – calculated using 0911.3356

Backup – Experimental Hyperfine Splitting Values

$$\Delta_{D^*-D} = 0.142014(30), \quad \Delta_{B^*-B} = 0.04518(20)$$
$$\Delta_{D_s^*-D_s} = 0.1438(4), \quad \Delta_{B_s^*-B_s} = 0.0485(14)$$