

LCDAs moments of meson

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Outline

- **Motivation and Formula**
- **Results for first moments**
- **Results for second moments**
- **Summary**

Motivation

Light-cone distribution amplitudes (LCDAs) plays an important role in the hardon hard exclusive reactions.

There are two ways to extract information of LCDA on lattice:

1. Calculate the quasi-DA on the lattice and match it to the LCDA through LaMET.
2. Calculate moments using local operator on lattice and reconstruct the LCDA.

$$\xi_2^\pi = 0.300(41), \quad \xi_2^K = 0.258(32)$$

Gegenbauer moments	a_1	a_2	a_3	a_4
π	—	0.258(70)(52)	—	0.122(46)(31)
K	-0.108(14)(51)	0.170(14)(44)	-0.043(06)(22)	0.073(08)(21)

Results from J. Hua et al. (Lattice Parton Collaboration), Phys. Rev. Lett. 129, 132001 (2022).

M	RI'	order	$\langle \xi^2 \rangle_M$	a_2^M
π	SMOM	N ³ LO	0.240 ⁺⁶ ₋₆ (2) _r (3) _a (2) _m	0.116 ⁺¹⁶ ₋₁₇ (4) _r (9) _a (5) _m
π	SMOM	NNLO	0.234 ⁺⁶ ₋₆ (4) _r (4) _a (2) _m	0.101 ⁺¹⁷ ₋₁₇ (12) _r (10) _a (5) _m
π	SMOM	NLO	0.227 ⁺⁶ ₋₆ (5) _r (5) _a (2) _m	0.078 ⁺¹⁸ ₋₁₉ (16) _r (13) _a (5) _m

Results from Bali, G.S., Braun, V.M., Bürger, S. *et al.* J. High Energ. Phys. **2019**, 65 (2019).

Formula

LCDA definition: $\langle \Omega | \bar{q}_2(z_2 n_\mu) e^{i \int_{z_2}^{z_1} A_\mu(x n_\mu) dx} \not{n} \gamma_5 q_1(z_1 n_\mu) | M(\vec{p}) \rangle = i f_M p \cdot n \int_{-1}^1 dx e^{-i(z_1 x + z_2(1-x))p \cdot n} \phi_M(x, \mu^2)$

Moments definition

$$\xi = x - (1 - x) = 2x - 1$$

$$\langle \xi^n \rangle_M(\mu^2) = \int_{-1}^1 dx (2x - 1)^n \phi_M(x, \mu^2)$$

The moments could be extracted from hadron matrix elements of twist-2 operators:

$$M_{\rho\mu_1 \dots \mu_{k+l}}^{(k,l)} = \bar{q}_2(x) \overleftarrow{D}_{(\mu_1} \dots \overleftarrow{D}_{\mu_k} \overrightarrow{D}_{\mu_{k+1}} \dots \overrightarrow{D}_{\mu_{k+l}} \gamma_\rho) \gamma_5 q_1(x)$$

For first moments:

$$O_{\rho\mu}^-(x) = \bar{q}_2(x) \left[\overleftarrow{D}_{(\mu} - \overrightarrow{D}_{\mu)} \right] \gamma_\rho \gamma_5 q_1(x),$$

$$\langle \Omega | O_{\rho\mu}^- | M(\vec{p}) \rangle = i f_M p_{(\rho} p_{\mu)} \langle \xi \rangle$$

For second moments:

$$O_{\rho\mu\nu}^\pm(x) = \bar{q}_2(x) \left[\overleftarrow{D}_{(\mu} \overleftarrow{D}_{\nu)} \pm 2 \overleftarrow{D}_{(\mu} \overrightarrow{D}_{\nu)} + \overrightarrow{D}_{(\mu} \overrightarrow{D}_{\nu)} \right] \gamma_\rho \gamma_5 q_1(x),$$

$$\langle \Omega | O_{\rho\mu\nu}^- | M(\vec{p}) \rangle = i f_M p_{(\rho} p_{\mu} p_{\nu)} \langle \xi^2 \rangle$$

$$\langle \Omega | O_{\rho\mu\nu}^+ | M(\vec{p}) \rangle = i f_M p_{(\rho} p_{\mu} p_{\nu)} \langle 1^2 \rangle$$

The $O_{\rho\mu\nu}^-$ and $O_{\rho\mu\nu}^+$ would mixing under the renormalization.

$$Z_q^{-1} \sum_{m''}^M Z_{mm''} \sum_i^d \text{Tr} \left[\Lambda_{m''}^{(i)}(p_1, p_2) \Lambda_{m', \text{tree}}^{(i)}(p_1, p_2) \right] = \sum_i^d \text{Tr} \left[\Lambda_{m, \text{tree}}^{(i)}(p_1, p_2) \Lambda_{m', \text{tree}}^{(i)}(p_1, p_2) \right] \Big|_{p_1^2 = p_2^2 = (p_1 - p_2)^2}$$

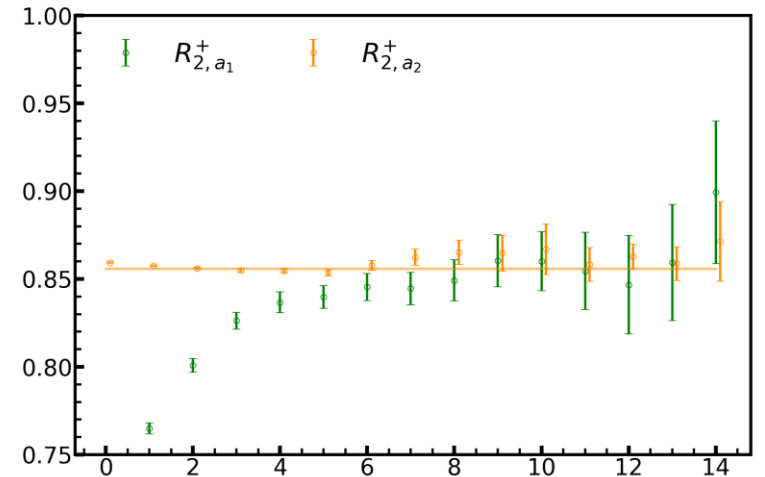
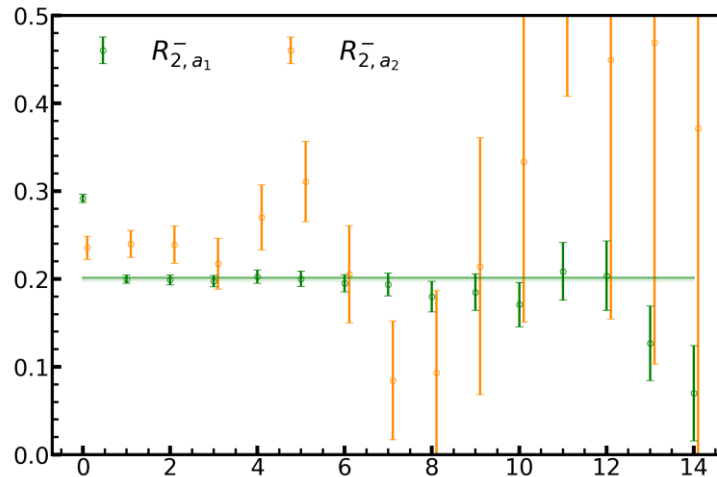
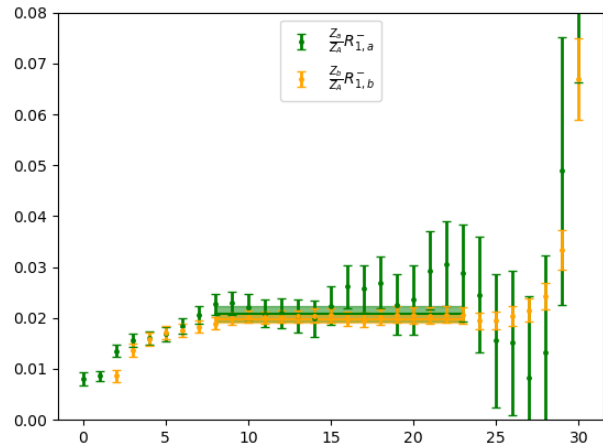
Pseudo scalar meson

For first moments:

$$R_{1,a}^- = -\frac{i}{3} \sum_{i=1}^3 \frac{1}{p_i} \frac{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle \mathcal{O}_{4i}^-(\vec{x}, t) P(\vec{0}, 0) \rangle}{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle A_4(\vec{x}, t) P(\vec{0}, 0) \rangle} \quad R_{2,b}^- = -\frac{4E}{3E^2 + p^2} \frac{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle \mathcal{O}_{44}^-(\vec{x}, t) P(\vec{0}, 0) \rangle}{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle A_4(\vec{x}, t) P(\vec{0}, 0) \rangle}$$

For second moments:

$$R_{2,a_1}^\pm = -\frac{1}{3} \sum_{i \neq j}^3 \frac{1}{p_i p_j} \frac{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle \mathcal{O}_{4ij}^\pm(\vec{x}, t) P(\vec{0}, 0) \rangle}{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle A_4(\vec{x}, t) P(\vec{0}, 0) \rangle} \quad R_{2,a_2}^\pm = -\frac{1}{3} \sum_{i=1}^3 \frac{p_i}{p_1 p_2 p_3} \frac{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle \mathcal{O}_{123}^\pm(\vec{x}, t) P(\vec{0}, 0) \rangle}{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle A_i(\vec{x}, t) P(\vec{0}, 0) \rangle}$$



$a=0.12\text{fm}$ using gaussian point source

Vector meson

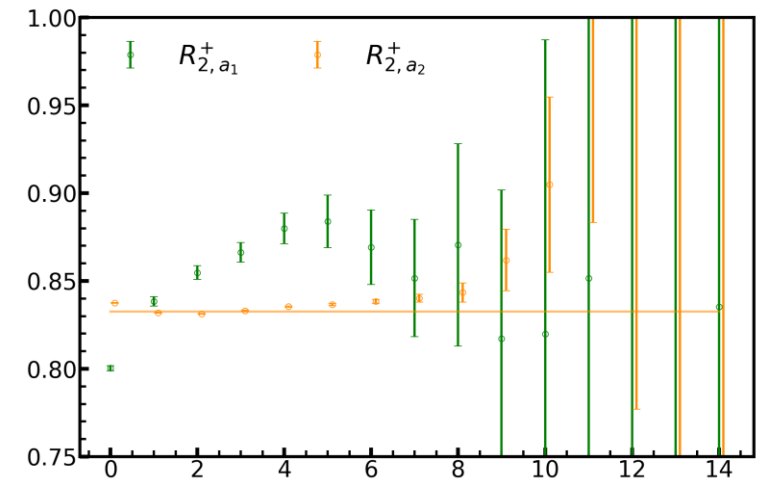
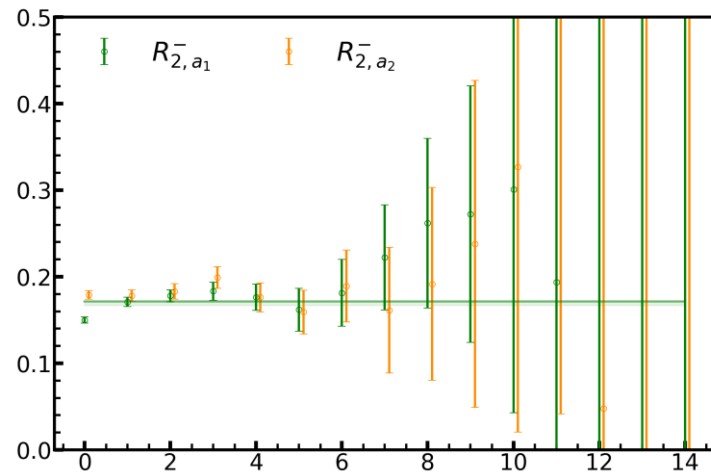
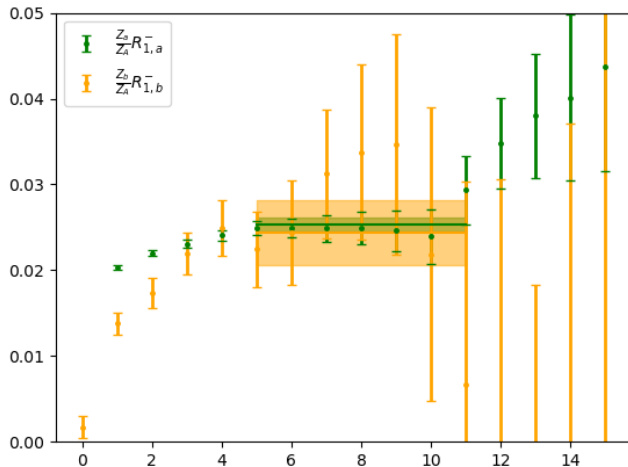
For first moments:

$$R_{1,a}^- = \frac{2}{3} \sum_{i=1}^3 \frac{1}{E} \frac{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle \mathcal{O}_{4i}^-(\vec{x}, t) V_i(\vec{0}, 0) \rangle}{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle V_i(\vec{x}, t) V_i(\vec{0}, 0) \rangle} \quad R_{2,b}^- = -\frac{4E}{3E^2 + p^2} \sum_{i=1}^3 \frac{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle \mathcal{O}_{44}^-(\vec{x}, t) V_i(\vec{0}, 0) \rangle}{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle V_4(\vec{p}, t) V_i(\vec{0}, 0) \rangle}$$

$$\begin{aligned} \langle V_\rho V_\sigma \rangle &\propto g_{\rho\sigma} - \frac{p_\rho p_\sigma}{M_V^2} \\ \langle \mathcal{O}_{\mu\rho}^- V_\sigma \rangle &\propto p_{(\mu} (g_{\rho)\sigma} - \frac{p_\rho p_\sigma}{M_V^2}) \\ \langle \mathcal{O}_{\mu\nu\rho}^- V_\sigma \rangle &\propto p_{(\mu} p_{\nu} (g_{\rho)\sigma} - \frac{p_\rho p_\sigma}{M_V^2}) \end{aligned}$$

For second moments:

$$R_{2,a_1}^\pm = -\frac{1}{3} \sum_{i \neq j}^3 \frac{1}{p_i p_j} \frac{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle \mathcal{O}_{4ij}^\pm(\vec{x}, t) V_k(0,0) \rangle}{\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle V_4(\vec{x}, t) V_k(0,0) \rangle} \quad R_{2,a_2}^\pm = -\frac{1}{3} \sum_{i=1}^3 \frac{p_i}{p_1 p_2 p_3} \frac{\sum_{j=1}^3 p_j \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle \mathcal{O}_{123}^\pm(\vec{x}, t) V_j(\vec{0}, 0) \rangle}{\sum_{j=1}^3 p_j \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle V_i(\vec{x}, t) V_j(\vec{0}, 0) \rangle}$$



$a=0.12\text{fm}$ using gaussian point source

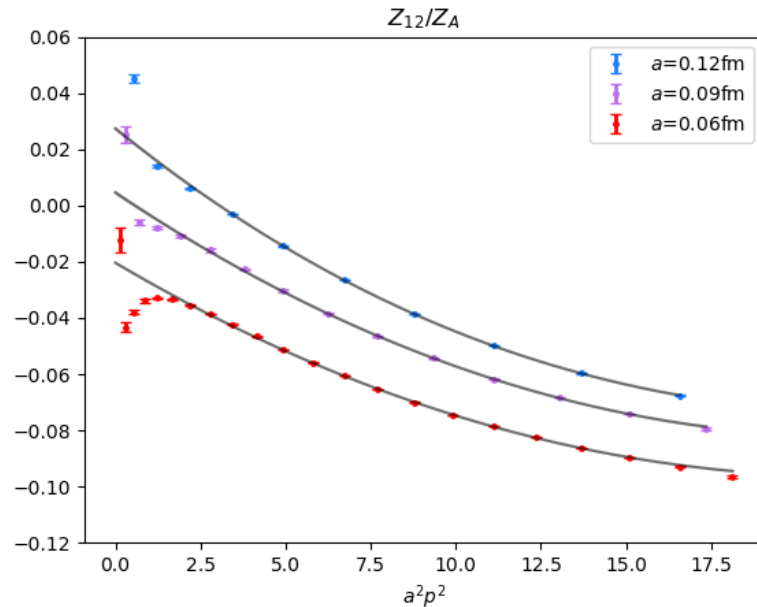
Renormalization

The $\mathcal{O}_{\rho\mu\nu}^-$ and $\mathcal{O}_{\rho\mu\nu}^+$ would mixing under the renormalization.

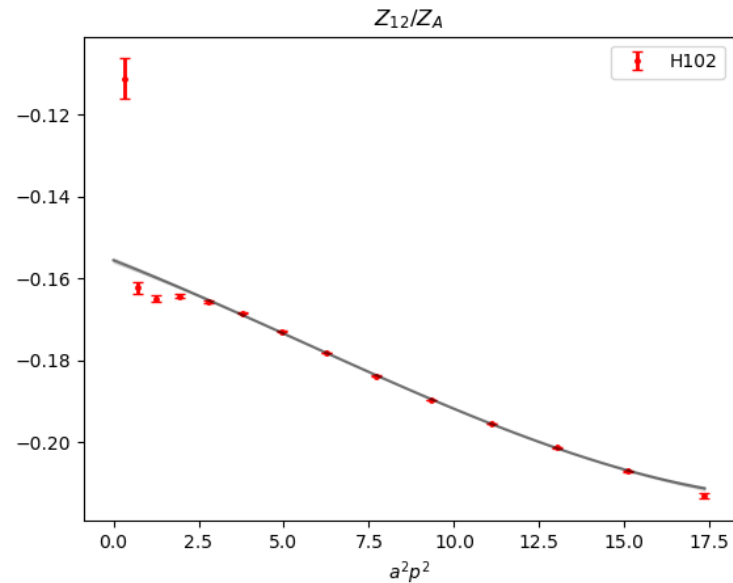
$$Z_q^{-1} \sum_{m''}^M Z_{mm''} \sum_i^d \text{Tr} \left[\Lambda_{m''}^{(i)}(p_1, p_2) \Lambda_{m', \text{tree}}^{(i)}(p_1, p_2) \right] = \sum_i^d \text{Tr} \left[\Lambda_{m, \text{tree}}^{(i)}(p_1, p_2) \Lambda_{m', \text{tree}}^{(i)}(p_1, p_2) \right] \Big|_{p_1^2 = p_2^2 = (p_1 - p_2)^2}$$

$$\begin{cases} \mathcal{O}_{\rho\mu\nu}^{-(R)} = Z_{11} \mathcal{O}_{\rho\mu\nu}^{-(B)} + Z_{12} \mathcal{O}_{\rho\mu\nu}^{+(B)} \\ \mathcal{O}_{\rho\mu\nu}^{+(R)} = Z_{22} \mathcal{O}_{\rho\mu\nu}^{+(B)} \end{cases}$$

using 1step-HYP smearing clover action



using normal clover action



smearing can help reduce the mixing effect.

Lattice set up

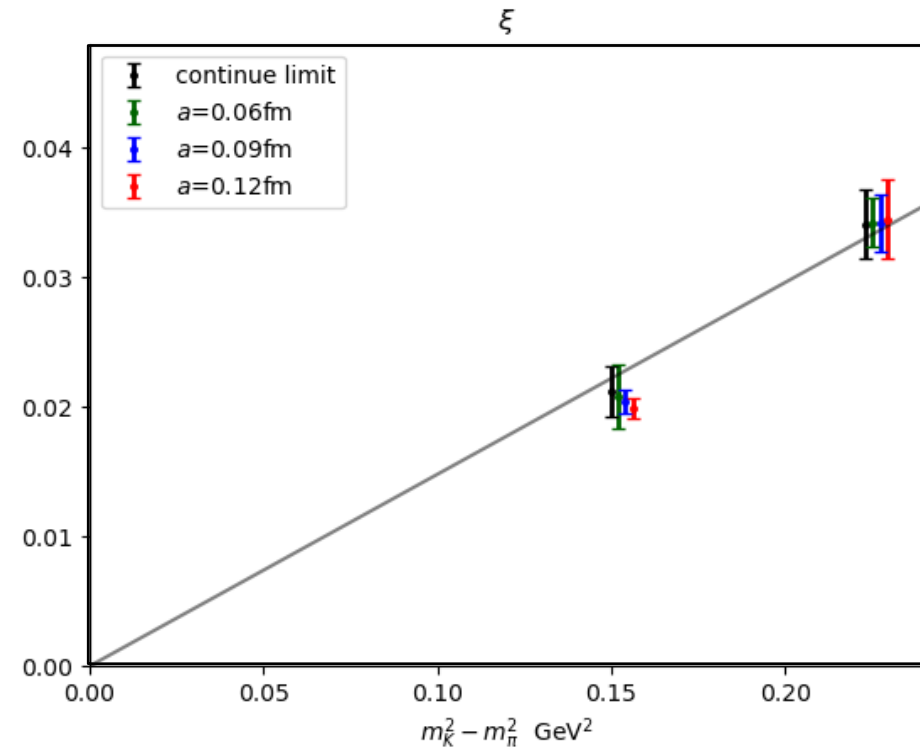
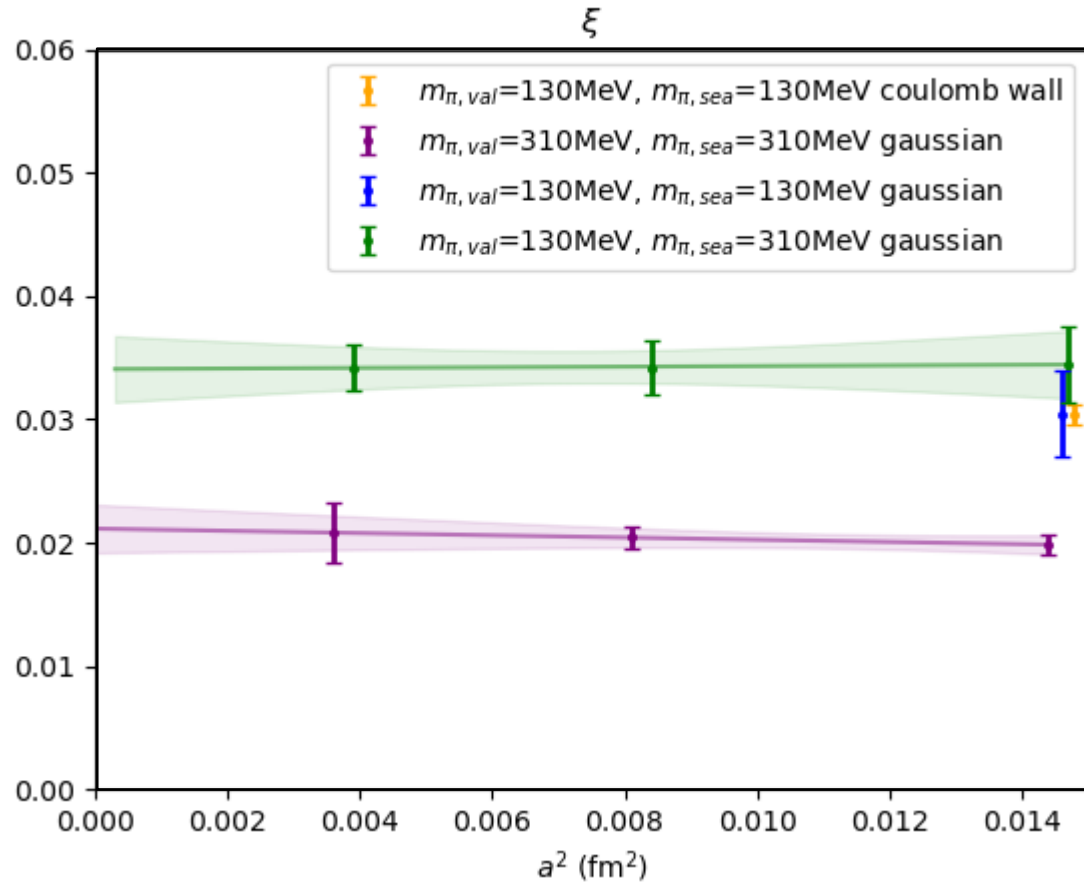
Fermion action use the 1step-HYP smearing clover action on MILC ensembles

	a12m310		a12m130	a09m310		a06m310	
$L^3 \times T$	$24^3 \times 64$		$48^3 \times 64$	$32^3 \times 96$		$48^3 \times 144$	
Lattice spacing	~ 0.12fm		~ 0.12fm	~ 0.09fm		~ 0.06fm	
m_π	~310MeV		~130MeV	~310MeV		~310MeV	
c_{sw}	1.0508		1.0508	1.0424		1.03493	
am_l	-0.0695	-0.0785	-0.0785	-0.0514	-0.0580	-0.0398	-0.0439
am_s	-0.0312	-0.0191	-0.0191	-0.0227	-0.0174	-0.0242	-0.0191

Hardon matrix elements: Coulomb wall source and gaussian point source.

Renormalization: Volume source with $p = \frac{2\pi}{n_s}(n, n, 0, 0)$ and $p = \frac{2\pi}{n_s}(n, 0, n, 0)$ ($n = 2, 3 \dots \frac{n_s}{2} - 1$)

K First Moments

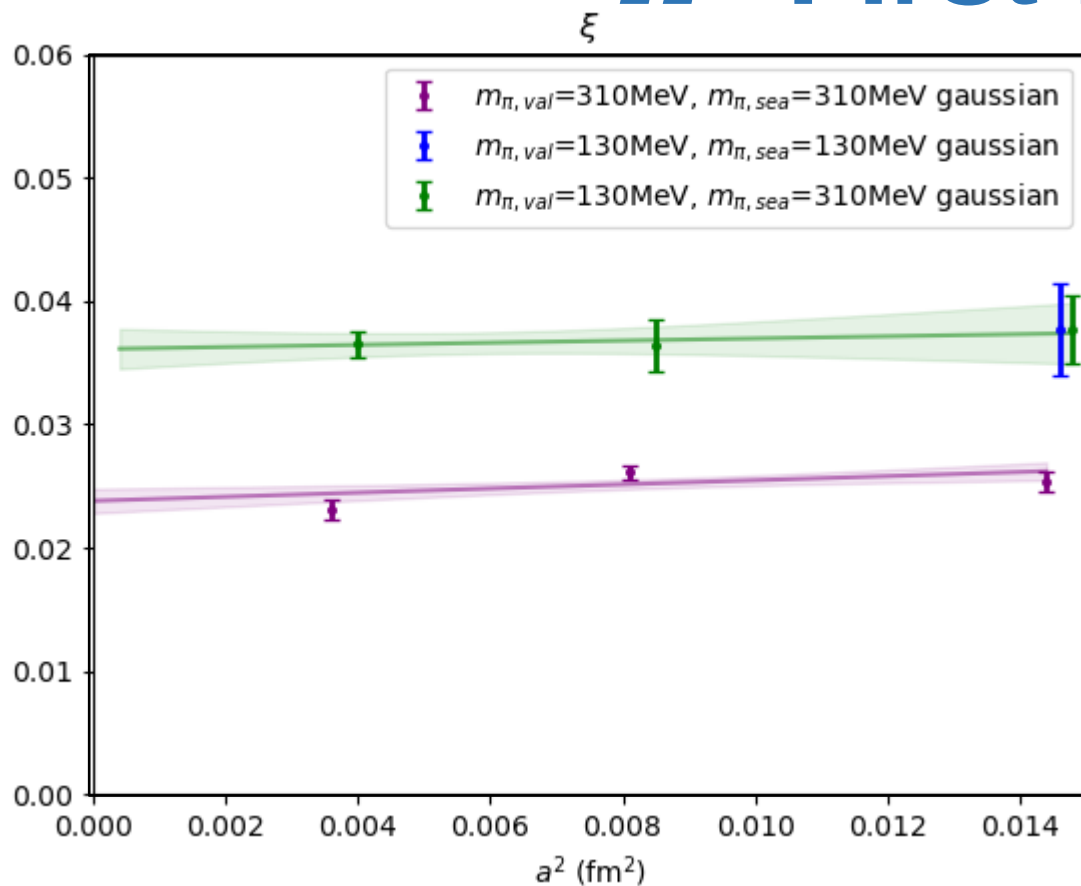


our preliminary results: $\langle \xi \rangle = 0.0341(27)$

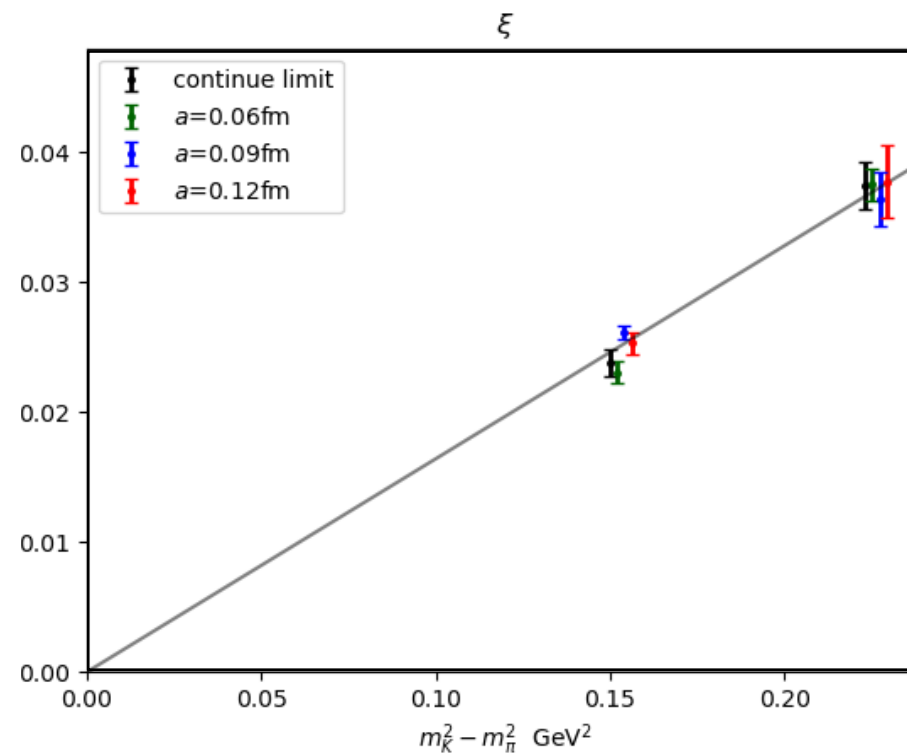
M	RI'	order	$\langle \xi^1 \rangle_M$	a_1^M
K	SMOM	NNLO	$0.0320_{-12}^{+11}(3)_r(13)_a(11)_m$	$0.0533_{-19}^{+18}(6)_r(22)_a(18)_m$
K	SMOM	NLO	$0.0327_{-12}^{+11}(6)_r(14)_a(11)_m$	$0.0545_{-20}^{+18}(9)_r(23)_a(18)_m$

Result from Bali, G.S., Braun, V.M., Bürger, S. *et al.* *J. High Energ. Phys.* **2019**, 65 (2019).

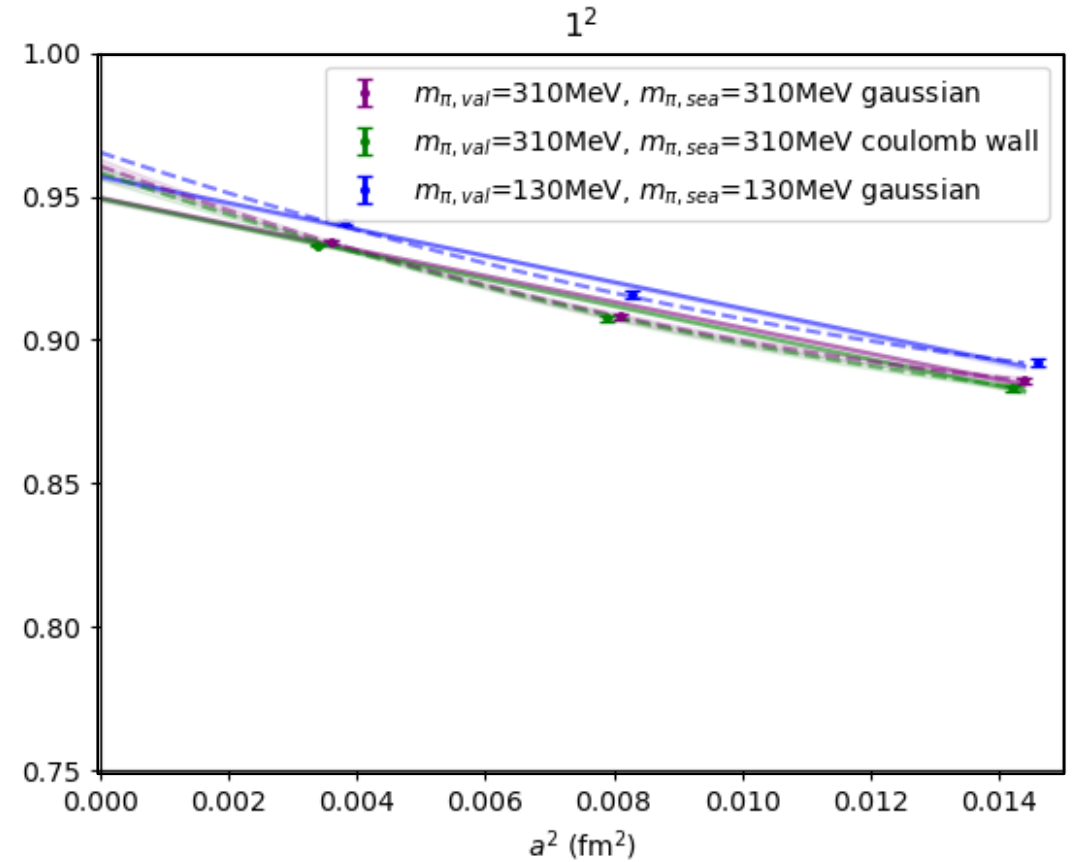
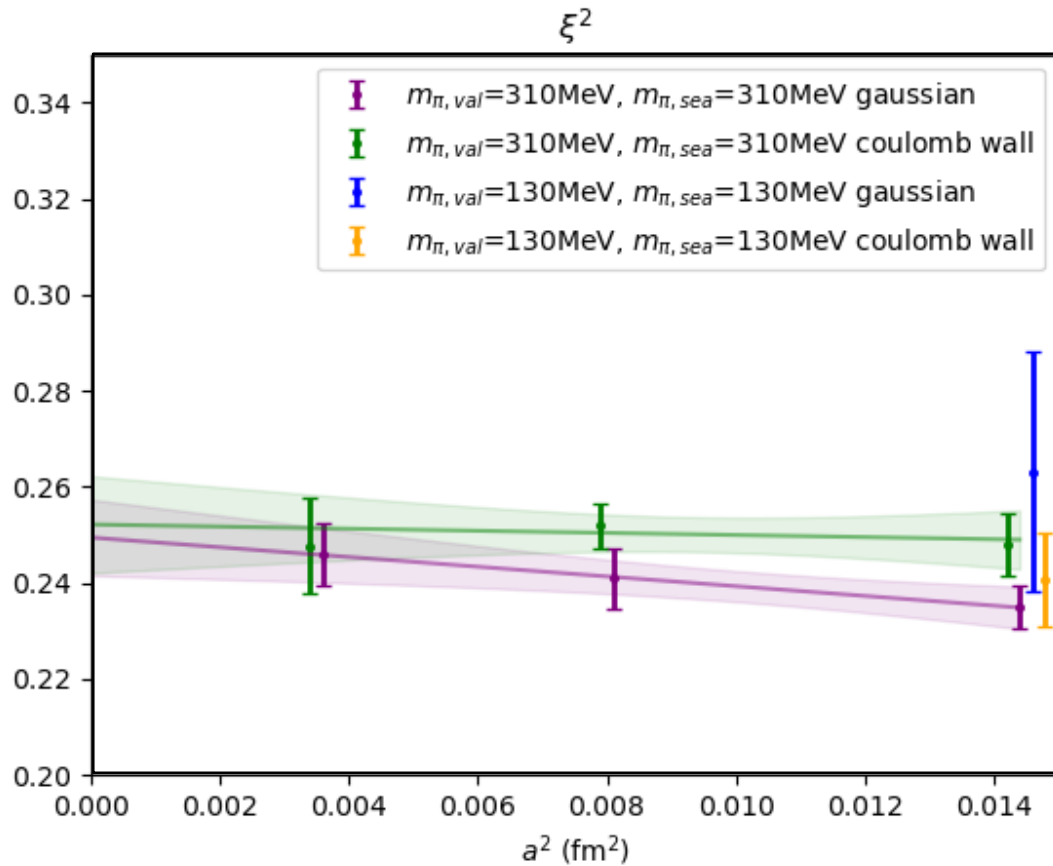
K^* First Moments



our preliminary results: $\langle \xi \rangle = 0.0374(19)$



π Second Moments

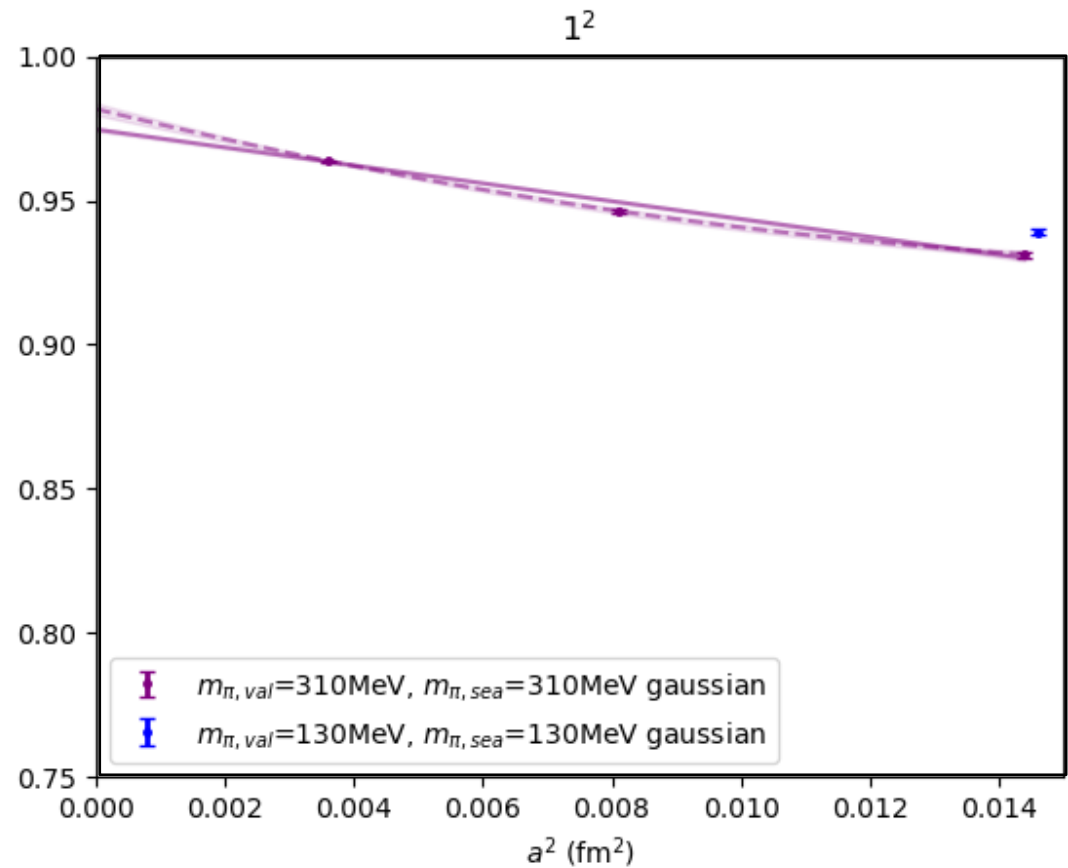
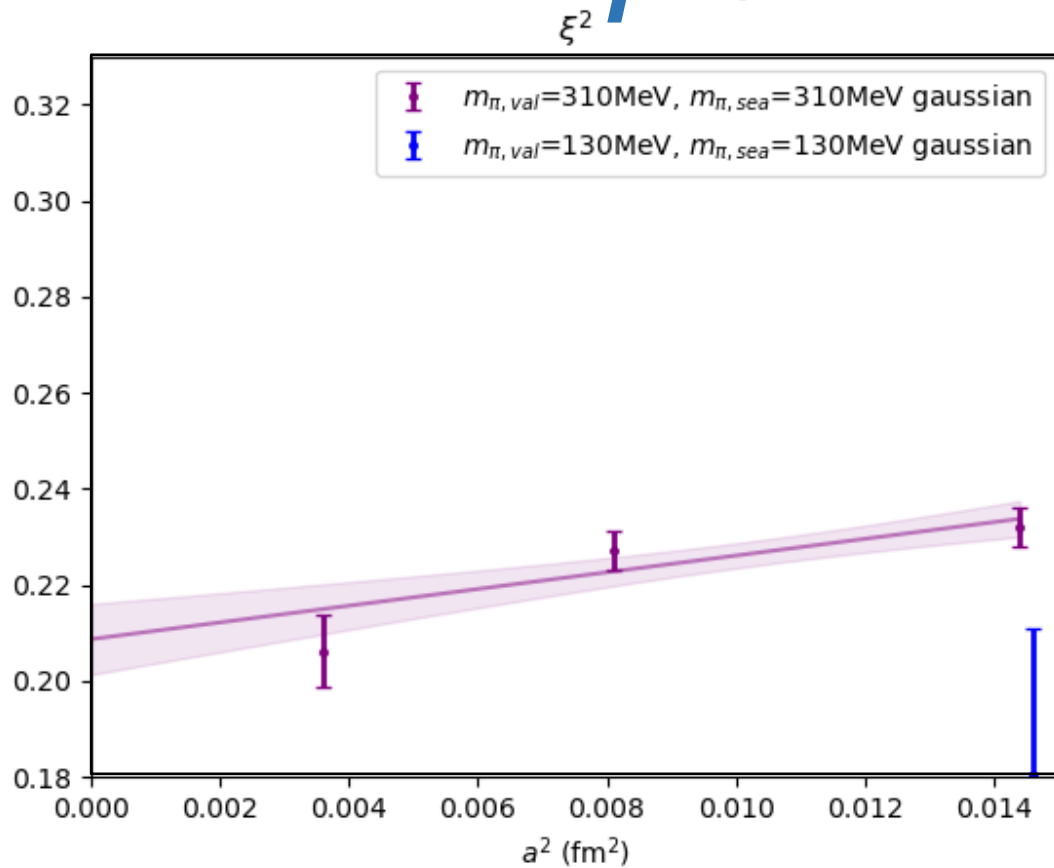


our preliminary results: $\langle \xi^2 \rangle = 0.2494(79)$

M	RI'	order	$\langle \xi^2 \rangle_M$
π	SMOM	N ³ LO	$0.240_{-6}^{+6}(2)_r(3)_a(2)_m$
π	SMOM	NNLO	$0.234_{-6}^{+6}(4)_r(4)_a(2)_m$
π	SMOM	NLO	$0.227_{-6}^{+6}(5)_r(5)_a(2)_m$

Result from Bali, G.S., Braun, V.M., Bürger,
S. et al. *J. High Energ. Phys.* **2019**, 65 (2019).

ϕ Second Moments



our preliminary results: $\langle \xi^2 \rangle = 0.2087(73)$

Preliminary Results

	a12m310	a09m310	a06m310	continue
π	0.2349(45)	0.2409(61)	0.2460(66)	0.2494(79)
K	0.2289(31)	0.2342(42)	0.2283(61)	0.2330(66)
	a12m130			
π	0.263(25)			
K	0.249(14)			

	a12m310	a09m310	a06m310	continue
ρ	0.2373(55)	0.2295(53)	0.2078(98)	0.208(10)
K^*	0.2322(46)	0.2292(46)	0.2050(90)	0.2106(88)
ϕ	0.2320(39)	0.2271(39)	0.2062(73)	0.2087(73)
	a12m130			
ρ	0.193(70)			
K^*	0.192(51)			
ϕ	0.181(30)			

$$\xi_2^\pi = 0.300(41), \quad \xi_2^K = 0.258(32)$$

Results from J. Hua et al. (Lattice Parton Collaboration),
Phys. Rev. Lett. 129, 132001 (2022).

Except the physical point, the mass dependence of second moments are very small

Summary

- Compute the first moments for the K and the second moments for the K and π . it is consistent with the previous calculation of RQCD collaboration.
- Compute the first moments for the K^* and the second moments for the ρ , K^* and ϕ .