Transverse Force Distributions in the Proton from Lattice QCD

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Motivation

- Our understanding of forces in QCD hasn't changed much since the static quark potential.
- High energy scattering off transversely polarised targets yields interesting asymmetries.
- **We present distributions of a "colour-Lorentz" force which are consistent with the observed asymmetries.**
- This formalism offers a new perspective on forces and confinement in QCD.

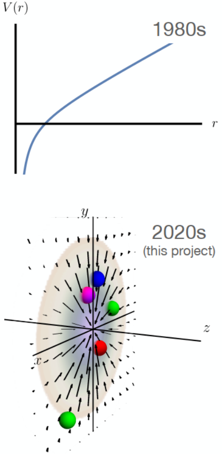

Figure: Changing ideas about QCD forces.

Transversely Polarised Deep Inelastic Scattering

- Scatter longitudinally polarised electrons off transversely polarised proton targets.
- Hadronic tensor is parameterised in terms of structure functions:
  - Unpolarised: $F_1(x, Q^2)$, $F_2(x, Q^2)$.
  - Polarised: $g_1(x, Q^2)$, $\boldsymbol{g_2(x, Q^2)}$.
- g_2 receives contributions from twist-2 and twist-3 operators.
- **Transversely polarised DIS allows for the extraction of the higher twist contributions to g_2.**

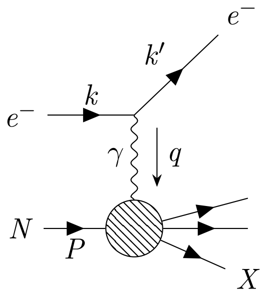

Figure: Feynman diagram for inelastic electron-proton scattering.

Asymmetries in SIDIS Experiments

- Semi-inclusive: measure one final hadronic state X.
- There is an asymmetric distribution of this final state X!
- Sivers asymmetry[a] experimentally verified for many different final states ($\pi^{\pm}$, π^0,...)
- **No consistent understanding of the relationship between higher-twist effects and asymmetries.**

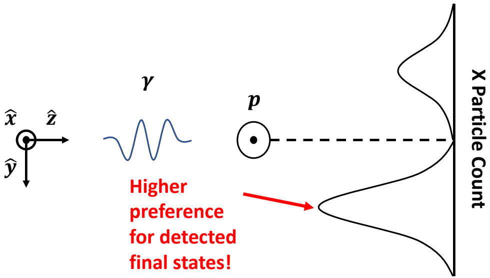

Figure: Cartoon setup of asymmetries in SIDIS.

[a] Sivers, D. *Phys. Rev. D.* 1991.

Heuristic Approach to the Asymmetries

- Final state interactions (FSIs) cause a transverse momentum asymmetry opposite to transverse position asymmetry.
- Net attractive force "pulls" the struck quark in the direction opposite its position asymmetry.
- **Can we image these FSIs? What do they look like? How strong are they?**

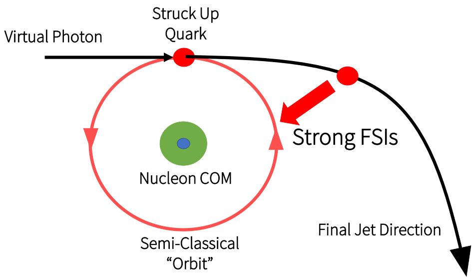

Figure: Semi-classical cartoon of our force picture, with polarisation axis pointing out of the page.

Transverse Forces from DIS

- Transversely polarised DIS allows us to explore higher-twist contributions to observables.
- The twist-3 part of the nucleon structure function $g_2(x, Q^2)$ does not have a single particle interpretation.
- Alternative interpretation: **twist-3 matrix elements represent transverse forces**[1].

$$3\int_{-1}^{1} dx\, x^2 \tilde{g}_2(x) = d_2 = \frac{1}{2mP^+P^+S^x} \langle P, S|\overline{\psi}(0)\gamma^+ gG^{+y}(0)\psi(0)|P, S\rangle \,.$$

- Untangling the gluon field strength tensor component, we find:

$$G^{+y} = \frac{1}{\sqrt{2}}\left(G^{0y} + G^{zy}\right) = -\frac{1}{\sqrt{2}}\left[\vec{E}_c + \vec{v}\times\vec{B}_c\right]^y = -\frac{1}{\sqrt{2}}F^y!$$

[1] Burkardt, M. *Phys. Rev. D.* 2013. arXiv: hep-ph/1510.03112.

Developing Position-Space Densities

- Decompose our matrix element into momentum-dependent form factors, $\Phi_i(-\Delta^2)$, much like electromagnetic form factors.
- Taking the **2D Fourier Transform in the Infinite Momentum Frame yields a position-space density**[2].

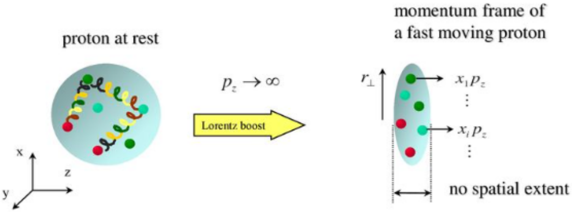

Figure: Infinite Momentum Frame kinematics.

[2] Burkardt, M. *Phys. Rev. D.* 2000. arXiv: hep-ph/0005108.

Recipe for a Density Distribution

Form factor decomposition of our matrix element is[3]

$$\langle p', s' | \overline{\psi}\gamma^{+} i g G^{+i} \psi | p, s \rangle = \overline{u}(p', s') \Big[P^{+}\Delta^{i}\gamma^{+}\Phi_1(t) + M P^{+} i\sigma^{+i}\Phi_2(t) + \frac{1}{M} P^{+}\Delta^{i} i\sigma^{+\Delta}\Phi_3(t) \Big] u(p, s), \quad (1)$$

where $P^{\mu} = (p' + p)^{\mu}/2$, $\Delta^{\mu} = (p' - p)^{\mu}$, $t = -\Delta^2$ and $\sigma^{\mu\Delta} = \sigma^{\mu\nu}\Delta_{\nu}$.

1. **Compute off-forward matrix elements on the lattice.**
2. **Compute form factors for a range of momentum transfers.**
3. **Take 2D Fourier transform to visualise forces in transverse impact parameter space.**

[3] Aslan, F., Burkardt, M, and Schlegel, M. *Phys. Rev. D*. 2019. arXiv: hep-ph/1904.03494.

Full Lattice Details

- Use gauge ensembles generated by CSSM/QCDSF/UKQCD collaborations[4].
- Fermions described by stout-smeared non-perturbatively $\mathcal{O}(a)$ improved Wilson (SLiNC) action[5].
- Use tree-level Symanzik improved gluon action.
- All ensembles at SU(3) symmetric point.

N_f	β	$L^3 \times T$	a (fm)	m_π, m_K (MeV)	t_{sep}/a	N_{meas}
$2+1$	5.50	$32^3 \times 64$	0.074	465	11, 13, 15	3528
$2+1$	5.65	$48^3 \times 96$	0.068	412	11, 14, 17	1074
$2+1$	5.95	$48^3 \times 96$	0.052	418	14, 18, 22	1014

[4] Haar, T. R., Nakamura, Y., and Stüben, H. *EPJ Web Conf.* 2018. arXiv: `hep-lat/1711.03836`.

[5] Cundy, N. et al. *Phys. Rev. D.* 2009. arXiv: `hep-lat/0901.3302`.

Computing Matrix Elements

- Want to compute matrix elements of the twist-3 operator

$$\mathcal{O}^{[5](q)}_{[\sigma\{\mu]\nu\}} = -\frac{g}{6}\overline{\psi}\left(\tilde{G}_{\sigma\mu}\gamma_\nu + \tilde{G}_{\sigma\nu}\gamma_\mu\right)\psi - \text{traces}$$

  where {...} ([...]) denotes (anti-)symmetrisation of indices.

- Compute ratios of three- and two-point functions:

$$\mathcal{R} = \frac{C_{3pt}(\mathbf{p}', t; \mathbf{q}, \tau; \mathcal{O})}{C_{2pt}(\mathbf{p}', t)}\left[\frac{C_{2pt}(\mathbf{p}', t)\, C_{2pt}(\mathbf{p}', \tau)\, C_{2pt}(\mathbf{p}, t-\tau)}{C_{2pt}(\mathbf{p}, t)\, C_{2pt}(\mathbf{p}, \tau)\, C_{2pt}(\mathbf{p}', t-\tau)}\right]^{\frac{1}{2}}$$

$$\mathcal{R} \overset{t\gg\tau\gg 0}{\propto} \langle p', s' | \mathcal{O} | p, s \rangle$$

- $\mathcal{O}^{[5]}$ mixes with lower dimensional operators → need to compute those matrix elements as well.

Two State Ratio Fits

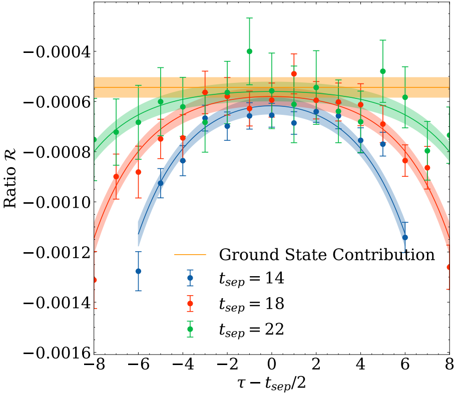

Figure: Ratio fit proportional to the forward matrix element d_2.

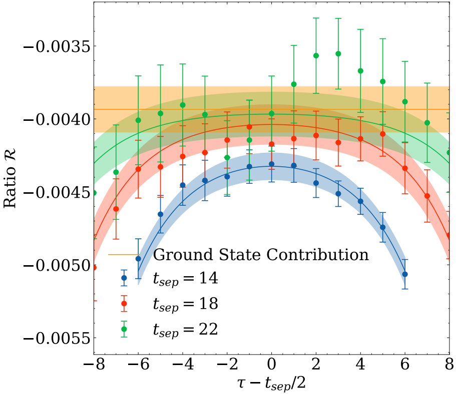

Figure: Ratio fit proportional to the corresponding mixing operator.

Preliminary d_2 Extrapolation

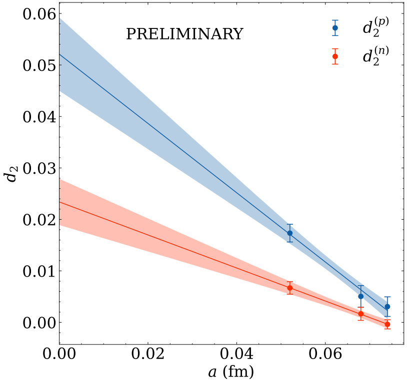

Figure: Continuum extrapolation of $d_2^{(p)}$ and $d_2^{(n)}$.

- Use our three lattice spacings to extrapolate to the continuum.
- **No quark mass effects included.**
- Sensitive to renormalisation procedure and mixing coefficient calculation.
- Renormalise in the RI′-MOM scheme following RQCD procedure[a].
- Running additional lattice spacings to refine this extrapolation.

[a] Bürger, S. et al. *Phys. Rev. D.* 2022. arXiv: hep-lat/2111.08306.

Computing Form Factors

- Form factor decomposition of our matrix element is[6]

$$\langle p', s' | \overline{\psi} \gamma^+ igG^{+i} \psi | p, s \rangle = \overline{u}(p', s') \Big[P^+ \Delta^i \gamma^+ \Phi_1(t) + MP^+ i\sigma^{+i} \Phi_2(t) + \frac{1}{M} P^+ \Delta^i i\sigma^{+\Delta} \Phi_3(t) \Big] u(p, s), \quad (2)$$

where $P^\mu = (p' + p)^\mu / 2$, $\Delta^\mu = (p' - p)^\mu$, $t = -\Delta^2$ and $\sigma^{\mu\Delta} = \sigma^{\mu\nu}\Delta_\nu$.

- Model t dependence with an a^2-corrected dipole function:

$$\Phi_i(t) = \frac{\Phi_i(0) + b_i a^2}{\left(1 + t\left(\frac{1}{\Lambda_i^2} + c_i a^2\right)\right)^2}$$

[6] Aslan, F., Burkardt, M, and Schlegel, M. *Phys. Rev. D*. 2019. arXiv: `hep-ph/1904.03494`.

Form Factor Results - Φ_1 and Φ_3

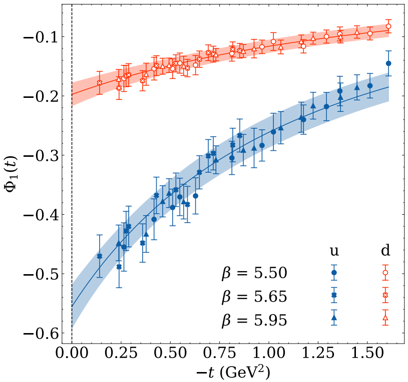

Figure: Results for the Φ_1 form factor.

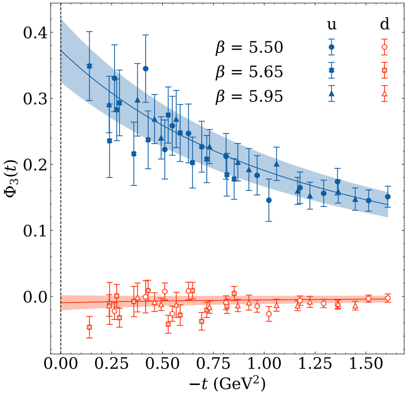

Figure: Results for the Φ_3 Form Factor.

Impact Parameter Space Distributions

- Take 2D Fourier transform to visualise in transverse impact parameter space.

$$\mathcal{F}^{i}_{s's}(\mathbf{b}_\perp) = -2\sqrt{2}P^{+}b^{i}\frac{d}{db_\perp^2}\tilde{\Phi}_1(\mathbf{b}_\perp^2)$$
$$+\sqrt{2}m_N\epsilon^{ij}S^{j}\tilde{\Phi}_2(\mathbf{b}_\perp^2) - \frac{\sqrt{2}\epsilon^{jk}S^{k}}{m_N}\left(2\delta^{ij}\frac{d}{db_\perp^2}\tilde{\Phi}_3(\mathbf{b}_\perp^2) + 4b^{i}b^{j}\frac{d^2}{d(b_\perp^2)^2}\tilde{\Phi}_3(\mathbf{b}_\perp^2)\right)$$

- Overlay resulting vector field on quark density distributions[7],

$$\rho(\mathbf{b}_\perp) = \frac{1}{2}\left[\tilde{F}_1(\mathbf{b}_\perp^2) + \frac{b^{j}\epsilon^{ji}S^{i}}{M_N}\frac{d}{db_\perp^2}\tilde{F}_2(\mathbf{b}_\perp^2)\right], \quad \tilde{F}(\mathbf{b}_\perp^2) = \int\frac{d^2\Delta_\perp}{(2\pi)^2}e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}F(t)$$

[7] Diehl, M. and Hägler, Ph. *Eur. Phys. J. C.* 2005. arXiv: `hep-ph/0504175`.

Visualising Quark Densities and Force Densities

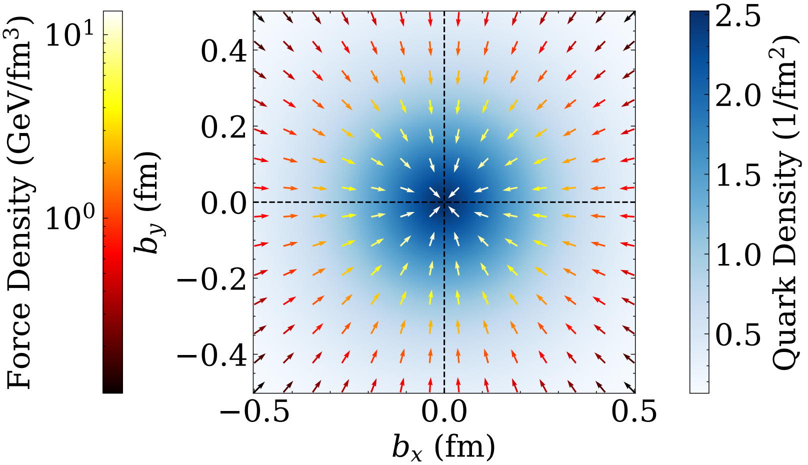

Figure: Force density in an unpolarised proton.

Impact Parameter Space Distributions

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Visualising Quark Densities and Force Densities

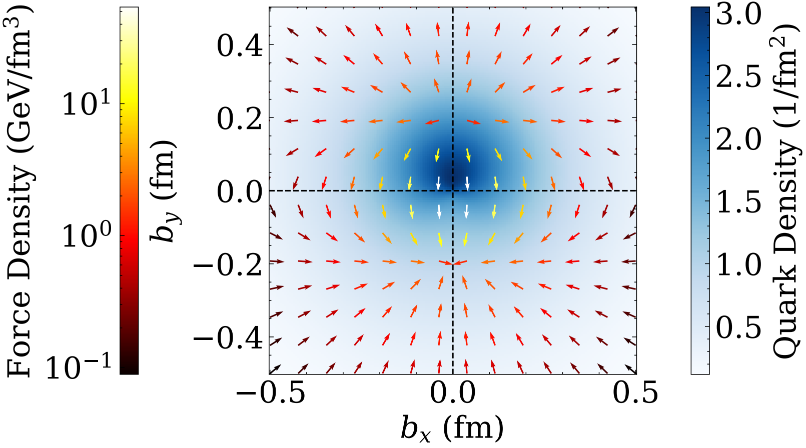

Figure: Force density in a proton polarised in the $\hat{x}$ direction.

Summary and Conclusions

- Transverse force tomography is a novel perspective on forces in QCD.
- We have produced novel images of the distribution of "colour-Lorentz" forces that act in polarised DIS.
- Force distributions indicate large local forces, on the order of ~ 3 GeV/fm - **3x the QCD string tension**.
- Expand momentum range to better assess model dependence of forces.
- **These images are simple, intuitive representations of how asymmetries can be generated in semi-inclusive DIS.**

References I

- Aslan, F., M Burkardt, and M Schlegel. “Transverse force tomography”. In: *Phys. Rev. D* 100 (9 2019). arXiv: hep-ph/1904.03494.
- Burkardt, M. “Impact parameter dependent parton distributions and off-forward parton distributions for $\zeta \to 0$”. In: *Phys. Rev. D* 62 (7 2000), p. 071503. arXiv: hep-ph/0005108.
- — .“Transverse force on quarks in deep-inelastic scattering”. In: *Phys. Rev. D* 88.11 (2013). arXiv: hep-ph/1510.03112.
- Bürger, S. et al. “Lattice results for the longitudinal spin structure and color forces on quarks in a nucleon”. In: *Phys. Rev. D* 105.5 (2022). arXiv: hep-lat/2111.08306.
- Cundy, N. et al. “Non-perturbative improvement of stout-smeared three flavour clover fermions”. In: *Phys. Rev. D* 79 (2009), p. 094507. arXiv: hep-lat/0901.3302.

References II

- Diehl, M. and Ph Hägler. "Spin densities in the transverse plane and generalized transversity distributions". In: *Eur. Phys. J. C* 44.1 (2005), pp. 87–101. arXiv: hep-ph/0504175.
- Edwards, Robert G. and Bálint Joó. "The Chroma Software System for Lattice QCD". In: *Nuclear Physics B - Proceedings Supplements* 140 (2005), pp. 832–834.
- Haar, T. R., Y. Nakamura, and H. Stüben. "An update on the BQCD Hybrid Monte Carlo program". In: *EPJ Web Conf.* 175 (2018), p. 14011. arXiv: hep-lat/1711.03836.
- Sivers, D. "Hard-scattering scaling laws for single-spin production asymmetries". In: *Phys. Rev. D* 43 (1 1991), pp. 261–263. DOI: 10.1103/PhysRevD.43.261. URL: https://link.aps.org/doi/10.1103/PhysRevD.43.261.

Acknowledgements

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Operator Mixing and Renormalisation

- Our operator mixes with lower dimensional operators, contaminating the signal.
- We incorporate this mixing when renormalising in the RI′-MOM scheme:

$$\mathcal{O}_R^{[5]}(\mu) = Z^{[5]}(a\mu)\left(\mathcal{O}^{[5]}(a) + \frac{1}{a}\frac{Z^{\sigma}(a\mu)}{Z^{[5]}(a\mu)}\mathcal{O}^{\sigma}(a)\right)$$

- Mixing coefficient determined both through LPT and non-perturbatively.
- Multiplicative renormalisation constant $Z^{[5]}(a\mu)$ computed using the procedure outlined by RQCD[9].
- Cannot match to $\overline{\text{MS}}$ at this time as perturbative calculations not available.

[9] Bürger, S. et al. *Phys. Rev. D*. 2022. arXiv: `hep-lat/2111.08306`.

Mixing Coefficient Calculation

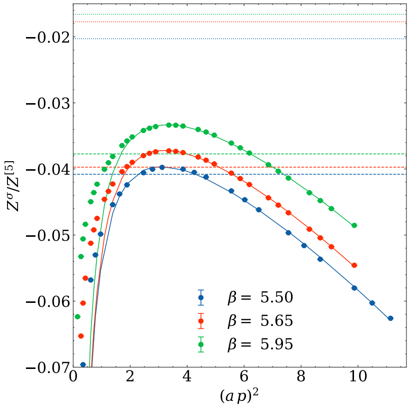

Figure: Non-perturbative calculation of the mixing coefficient $Z^\sigma/Z^{[5]}$.

- We compute the amputated 3-pt Greens function on the lattice and match it to the continuum tree-level result:

$$\mathrm{Tr}\left[\Gamma_R^{[5]}(p)\Gamma_{tree}^{\sigma}(p)^{-1}\right]_{p^2=\mu^2} = 0$$

- We fit the data using the form:

$$\frac{Z^\sigma}{Z^{[5]}} = \frac{A}{(ap)^2} + B + C(ap)^2 + D(ap)^4$$

- Extract the constant piece B.

RI′-MOM Procedure[10]

1. Compute $Z^{[5]}$ on each lattice by matching to tree-level results,

$$\frac{1}{12}\,\mathrm{Tr}\left[\Gamma_R^{[5]}(p)\Gamma_{tree}^{[5]}(p)^{-1}\right]_{p^2=\mu^2} = 1.$$

2. Choose a reference scale $\mu_0 = 2$ GeV and compute the ratio $Z^{[5]}(\mu)/Z^{[5]}(\mu_0)$ on all lattices.
3. Extrapolate this ratio to the continuum and define it as $R(\mu, \mu_0)$.
4. $Z^{[5]}(\mu')$ for each lattice, at some intermediate scale μ', is then calculated as

$$Z^{[5]}(\mu') = R(\mu', \mu_0) Z^{[5]}(\mu_0).$$

5. Evolve to some common scale μ through the one-loop formula,

$$Z^{[5]}(\mu) = \left(\frac{\alpha_s(\mu')}{\alpha_s(\mu)}\right)^{-B} Z^{[5]}(\mu'), \qquad B = \frac{1}{\frac{11}{3}N_c - \frac{2}{3}N_f}\left(3N_c - \frac{1}{6}\left(N_c - \frac{1}{N_c}\right)\right)$$

[10] Bürger, S. et al. *Phys. Rev. D.* 2022. arXiv: `hep-lat/2111.08306`.

a^2 Extrapolation for d_2

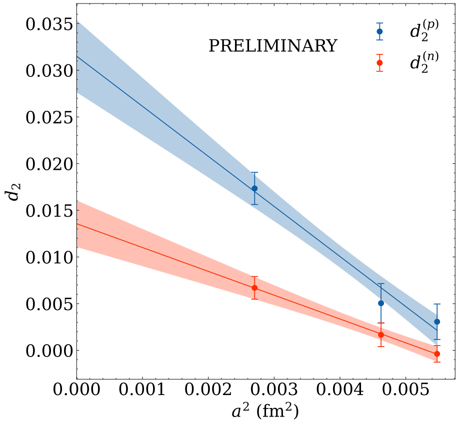

Figure: Linear extrapolation for d_2 in a^2.

Φ_2 Form Factor and Resulting Force Distribution

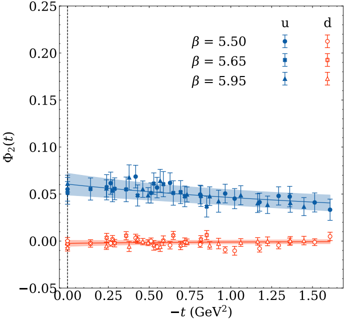

Figure: Results for the Φ_2 Form Factor.

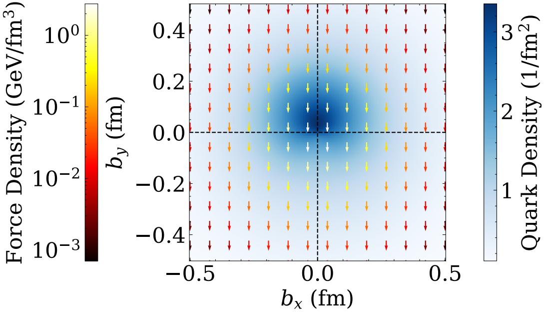

Figure: Force distribution due to Φ_2 in a $\hat{x}$-polarised proton.

Model Dependence and Force Magnitude Estimates

- Operator is $\overline{\psi}\gamma^+ gG^{+i}\psi$, but the force comes from G^{+i} $\rightarrow$ need to remove quark density dependence.
- Assume the weighted force factorises:

$$\mathcal{F}^i_{s'\,s}(\mathbf{b}_\perp) = \rho_{s'\,s}(\mathbf{b}_\perp) F^i_{s'\,s}(\mathbf{b}_\perp)$$

- Assess model dependence of force magnitudes using n-order pole fits

$$\Phi_i(t) = \frac{\Phi_i(0)}{\left(1 + \frac{t}{\Lambda_i^2}\right)^n}, \qquad n = 2, 3, 4.$$

- For scale, continuum QCD string tension ≈ 1 GeV/fm

Model Dependence and Force Magnitude Estimates

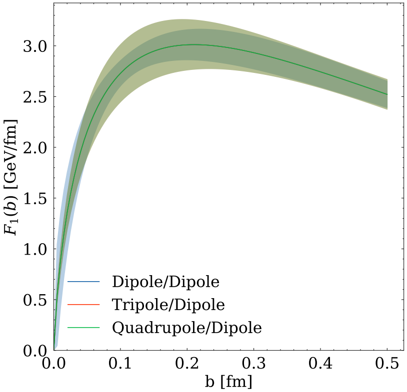

Figure: Model-dependent estimates for force magnitude due to Φ_1 form factor.

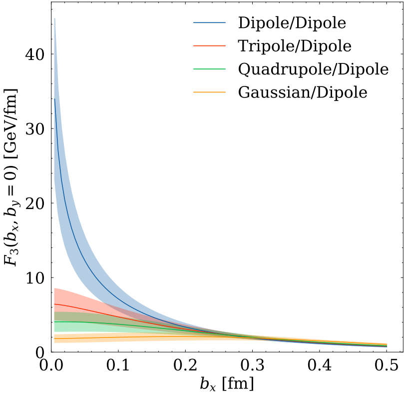

Figure: Model-dependent estimates for force magnitude due to Φ_3 form factor.