Study on the P-wave form factors of the B_s to D_s semi-leptonic decays from inclusive lattice simulations

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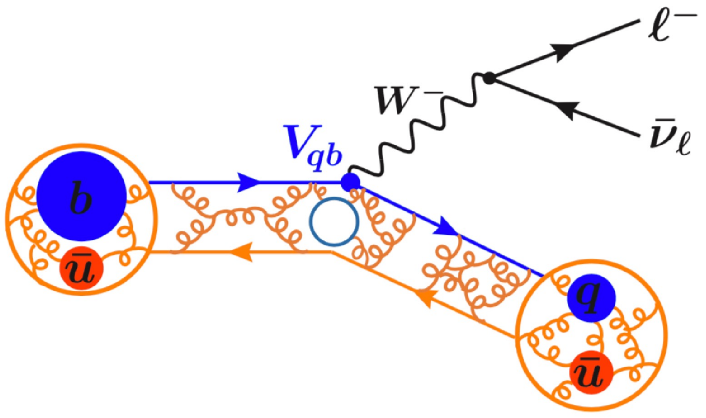
Introductions

Semi-leptonic decays

- we investigate $\boldsymbol{B_s \to X_{cs} l \nu_l}$
- in the quark level it corresponds to $\boldsymbol{b \to c l \nu_l}$
- it is a CKM-favoured decay corresponding to $\boldsymbol{V_{cb}}$, a fundamental parameter in SM
- in principle X_{cs} can be any single/multiple particle state with valence content cs, but here we only consider the single particle state D_s meson: $\boldsymbol{D_s, D_s^*, D_{s0}^*, D_{s1}, D_{s1}', D_{s2}^*}$
- for l, we only consider **light leptons**
- two problems
  - tension between inclusive and exclusive $\boldsymbol{V_{cb}}$
  - 1/2 vs 3/2 puzzle



$$B_s \to X_{cs}\ l\ \nu_l$$

$$v = \frac{p_{B_s}}{M_{B_s}} \qquad v' = \frac{p_X}{M_X} \qquad q$$

$$\omega = v \cdot v' \qquad p_{B_s} = p_X + q$$

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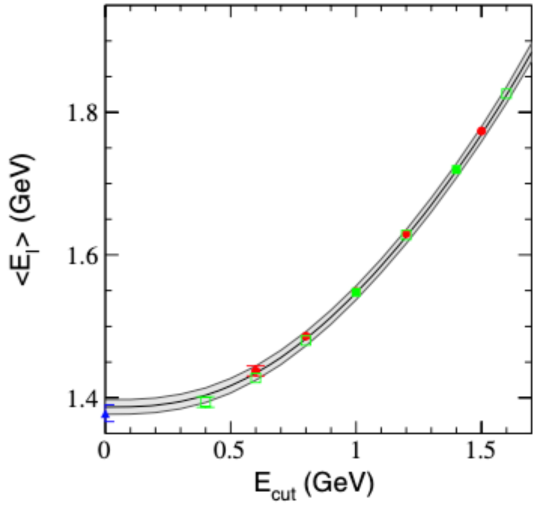
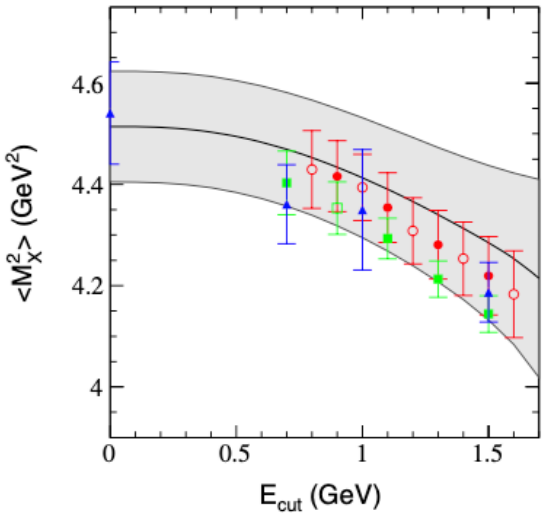
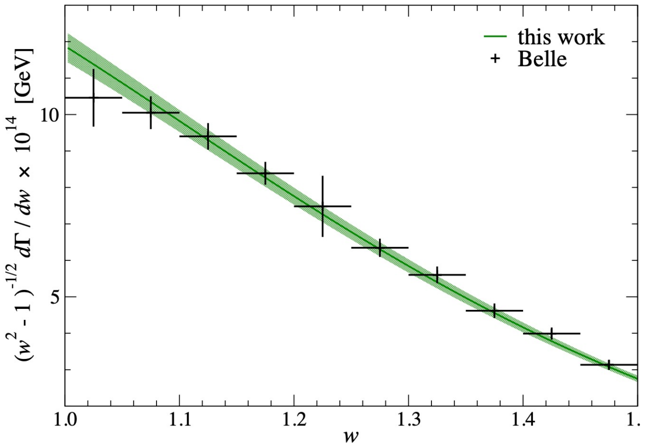
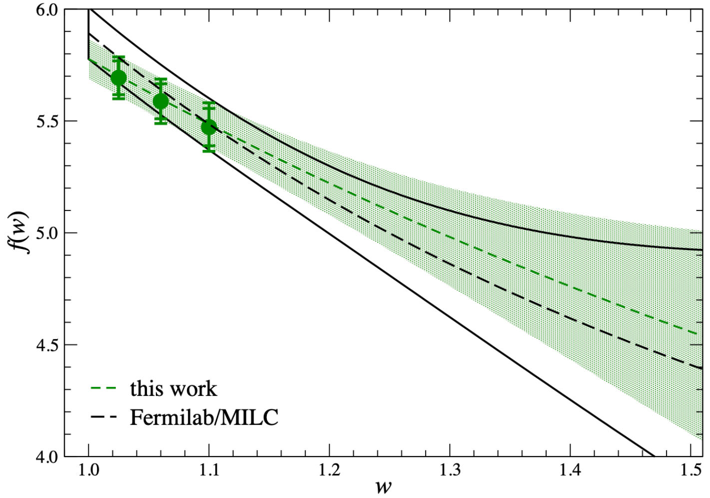
Inclusive and exclusive determinations of $|V_{cb}|^2$

OPE expansion of moments for inclusive decays ← inclusive ($\approx 42 \pm 1\times10^{-3}$) — **experimental data** — exclusive ($\approx 38 \pm 1\times10^{-3}$) → **lattice data + CLN/BGL parameterizations of exclusive form factors**









$$X = X^{(0)} + \frac{\alpha_s}{\pi} X^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 X^{(2)} + \left(\frac{\mu_\pi}{m_b}\right)^2 X^{(\pi)} + \left(\frac{\mu_G}{m_b}\right)^2 X^{(G)} + \left(\frac{\mu_D}{m_b}\right)^3 X^{(D)} + \left(\frac{\mu_{LS}}{m_b}\right)^2 X^{(LS)} + \ldots$$

[Gambino et al., *PRD*89.1(2014):1–13]

$$F(z) = \frac{1}{P_F(z)\phi_F(z)} \sum_{k=0}^{N_F} a_{F,k} z^k, \quad z(\omega) = \frac{\sqrt{(\omega+1)} - \sqrt{2}}{\sqrt{(\omega+1)} + \sqrt{2}}$$

[Aoki et al.:2306.05657]

Inclusive and exclusive determinations of $|V_{cb}|^2$

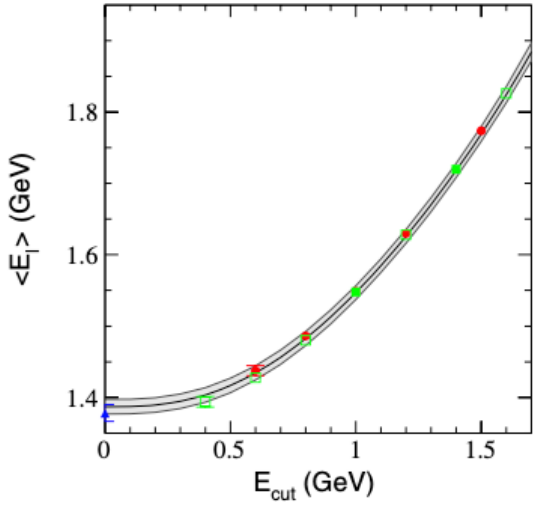
OPE expansion of moments for inclusive decays

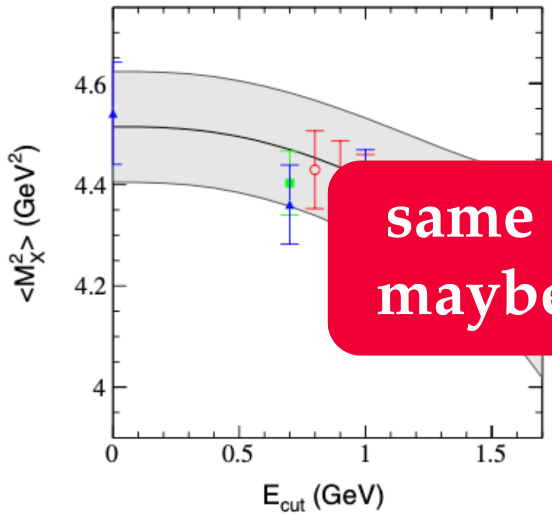
← inclusive — $\approx 42 \pm 1\times 10^{-3}$

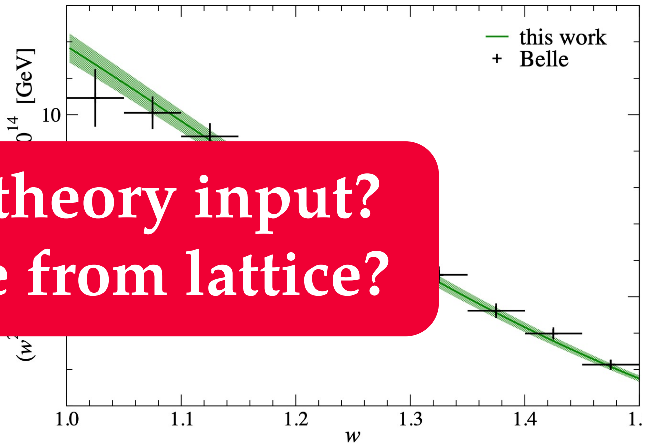
experimental data

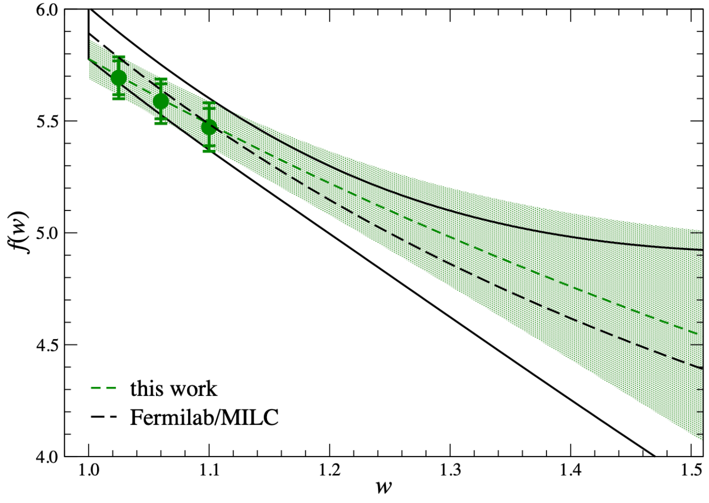
exclusive → — $\approx 38 \pm 1\times 10^{-3}$

lattice data + CLN/BGL parameterizations of exclusive form factors









same theory input?
maybe from lattice?

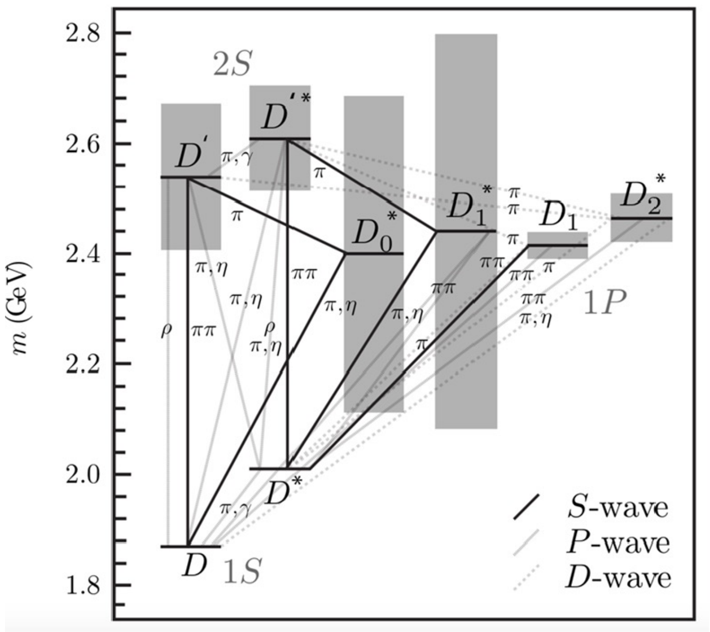
$$X = X^{(0)} + \frac{\alpha_s}{\pi} X^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 X^{(2)} + \left(\frac{\mu_\pi}{m_b}\right)^2 X^{(\pi)} + \left(\frac{\mu_G}{m_b}\right)^2 X^{(G)} + \left(\frac{\mu_D}{m_b}\right)^3 X^{(D)} + \left(\frac{\mu_{LS}}{m_b}\right)^2 X^{(LS)} + \ldots$$

[Gambino et al.,*PRD*89.1(2014):1–13]

$$F(z) = \frac{1}{P_F(z)\phi_F(z)} \sum_{k=0}^{N_F} a_{F,k} z^k \, , \; z(\omega) = \frac{\sqrt{(\omega+1)} - \sqrt{2}}{\sqrt{(\omega+1)} + \sqrt{2}}$$

[Aoki et al.:2306.05657]

1/2 and 3/2 puzzle



j^P	J^P	
$(1/2)^- \equiv S$	0^-	D
	1^-	D^*
$(1/2)^+ \equiv P_{1/2}$	0^+	D_0^*
	1^+	D_1'
$(3/2)^+ \equiv P_{3/2}$	1^+	D_1
	2^+	D_2^*

$$j = L + s_{\text{light quark}}$$

$$J = j + s_{\text{heavy quark}}$$

$D \approx 20\%$

$D^* \approx 50\%$

$D_{3/2}^{**} \approx 15\%$

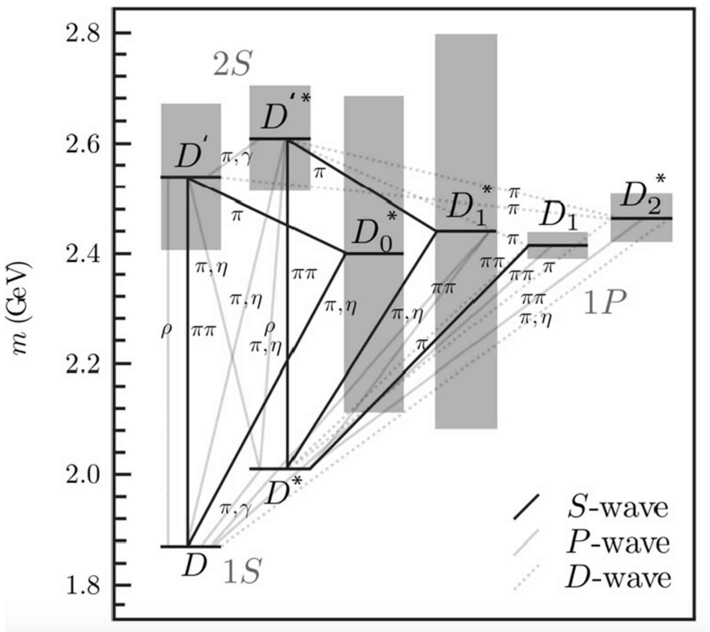
$? \approx 15\%$

- heavy quark limit $m_Q \to \infty$
- OPE expansions

- $\tau_{1/2}(\omega = 1) \ll \tau_{3/2}(\omega = 1)$
- transition form factors from B to $D_{3/2}^{**}$ and $D_{1/2}^{**}$, and thus their decay ratios, can be expressed by **Isgur-Wise form factor $\tau_{1/2}$ and $\tau_{3/2}$**

$$\Gamma_{1/2} \ll \Gamma_{3/2}$$

1/2 and 3/2 puzzle



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	1^+	D_1'
$(3/2)^+ \equiv P_{3/2}$	1^+	D_1
	2^+	D_2^*

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⬇

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- transition form factors from B to $D^{**}_{3/2}$ and $D^{**}_{1/2}$, and thus their decay ratios, can be expressed by **Isgur-Wise form factor $\tau_{1/2}$ and $\tau_{3/2}$**

⬇

$\Gamma_{1/2} \ll \Gamma_{3/2}$

understanding the excited state contribution to $B \to X_c l \nu_l$ is important, in particular the P-wave contribution

what is the nature of the remaining 15%?

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Methods

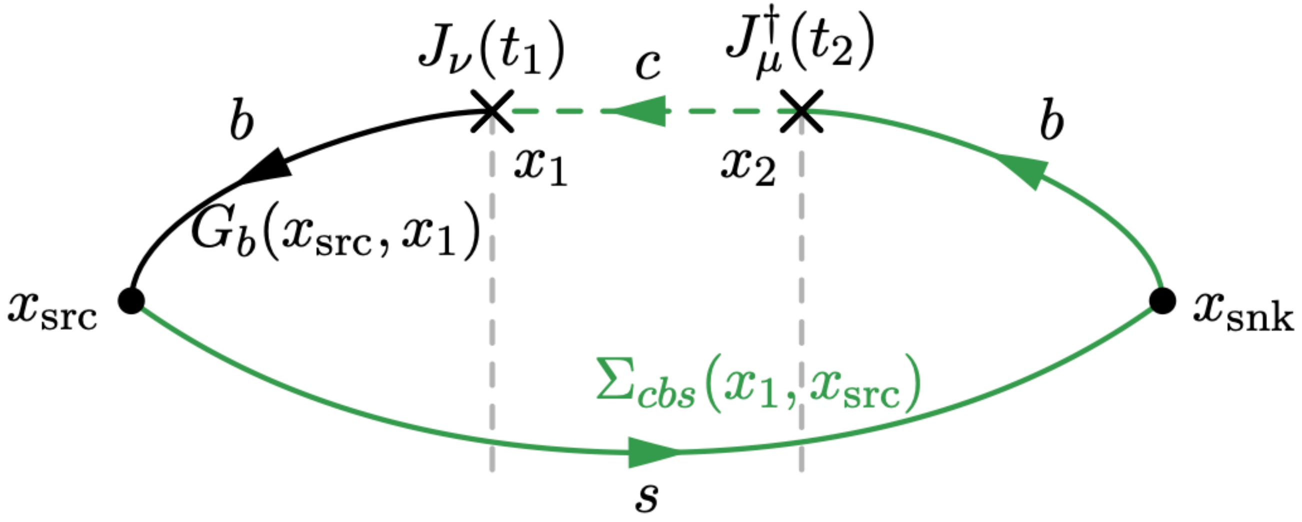
Four-point correlators

same theory input for exclusive and inclusive investigations of V_{cb}

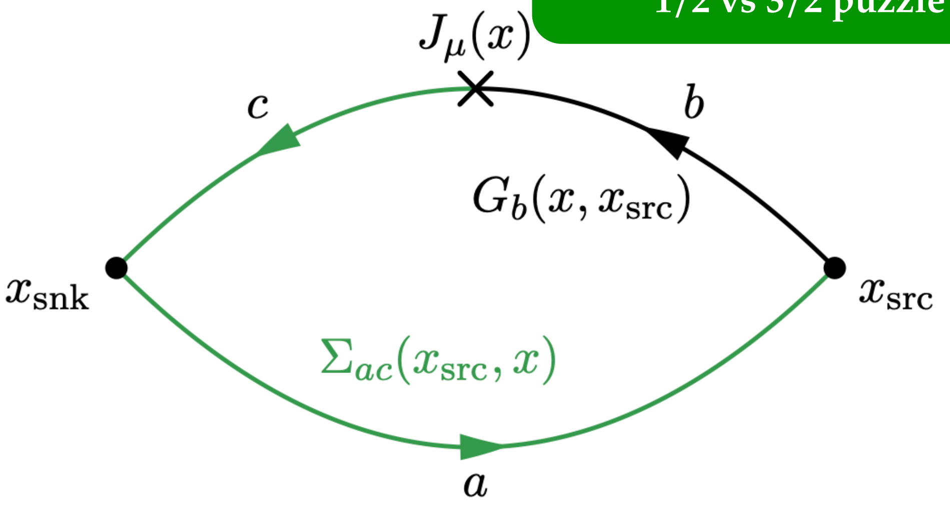
$$C_{J_\mu J_\nu}(\boldsymbol{q},t)$$
$$\equiv \int d^3x \frac{e^{i\boldsymbol{q}\cdot\boldsymbol{x}}}{2M_{B_s}} \langle B_s | J_\mu^\dagger(\boldsymbol{x},0) e^{-\hat{H}t} J_\nu(0) | B_s \rangle$$
$$= \sum_{X_c} \frac{1}{2M_{B_s} 2E_{X_c}} \langle B_s | J_\mu^\dagger(0) | X_c, -\boldsymbol{q} \rangle \langle X_c, -\boldsymbol{q} | J_\nu(0) | B_s \rangle e^{-E_{X_c} t}$$

$$J_\mu \equiv V_\mu - A_\mu, V_\mu = \bar{b}\gamma^\mu c, A_\mu = \bar{b}\gamma^\mu\gamma^5 c$$

- **inclusive information** can be extracted from $C_{J_\mu J_\nu}$, **see Ryan Kellermann's talk later**
- $C_{J_\mu J_\nu}$ cotains contributions from **all possible final states**, we can investigate the excited state information without constructing the **complicated operators for excited D_s state**



extract P-wave contributions to investigate 1/2 vs 3/2 puzzle

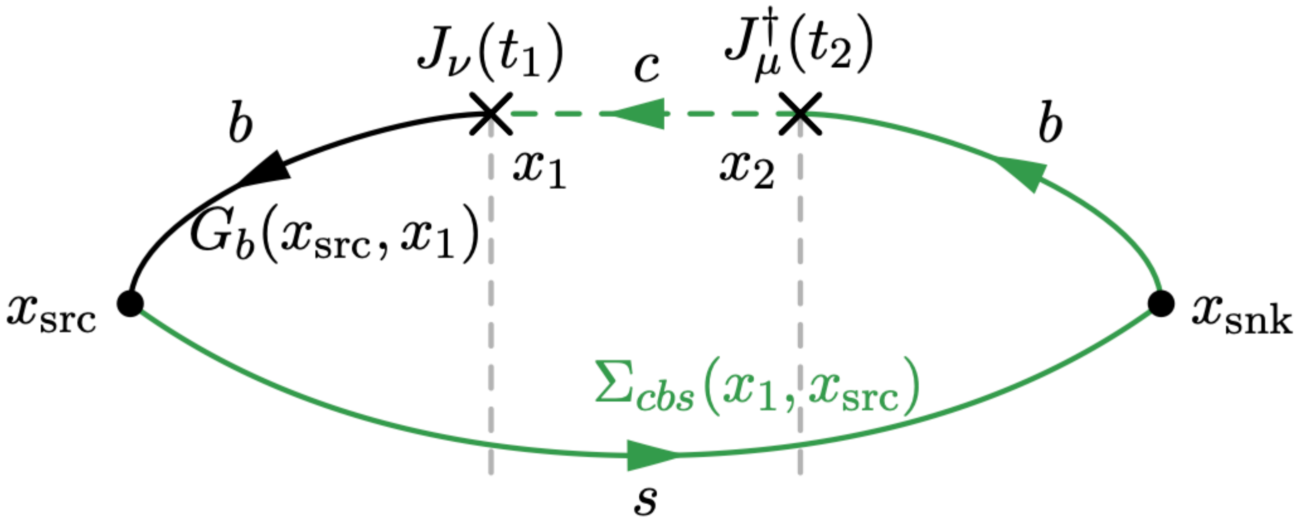


Lattice setup

$$C_{J_\mu J_\nu}(\boldsymbol{q},t)$$
$$\equiv \int d^3x \frac{e^{iq\cdot x}}{2M_{B_s}} \langle B_s | J_\mu^\dagger(\boldsymbol{x},0) e^{-\hat{H}t} J_\nu(0) | B_s \rangle$$
$$= \sum_{X_c} \frac{1}{2M_{B_s} 2E_{X_c}} \langle B_s | J_\mu^\dagger(0) | X_c, -\boldsymbol{q} \rangle \langle X_c, -\boldsymbol{q} | J_\nu(0) | B_s \rangle e^{-E_{X_c} t}$$

$$J_\mu \equiv V_\mu - A_\mu, V_\mu = \bar{b}\gamma^\mu c, A_\mu = \bar{b}\gamma^\mu\gamma^5 c$$

- lattice size: $24^3 \times 64$
- lattice spacing: $a \approx 0.11$ fm
- $M_\pi \approx 330$ MeV
- $M_{B_s} \approx 5370$ MeV
- $M_{D_s} \approx 1680$ MeV
- 2+1-flavour DWF actions with approximately physical masses are utilized for light quarks
- relativistic-heavy quark action for b and c quarks

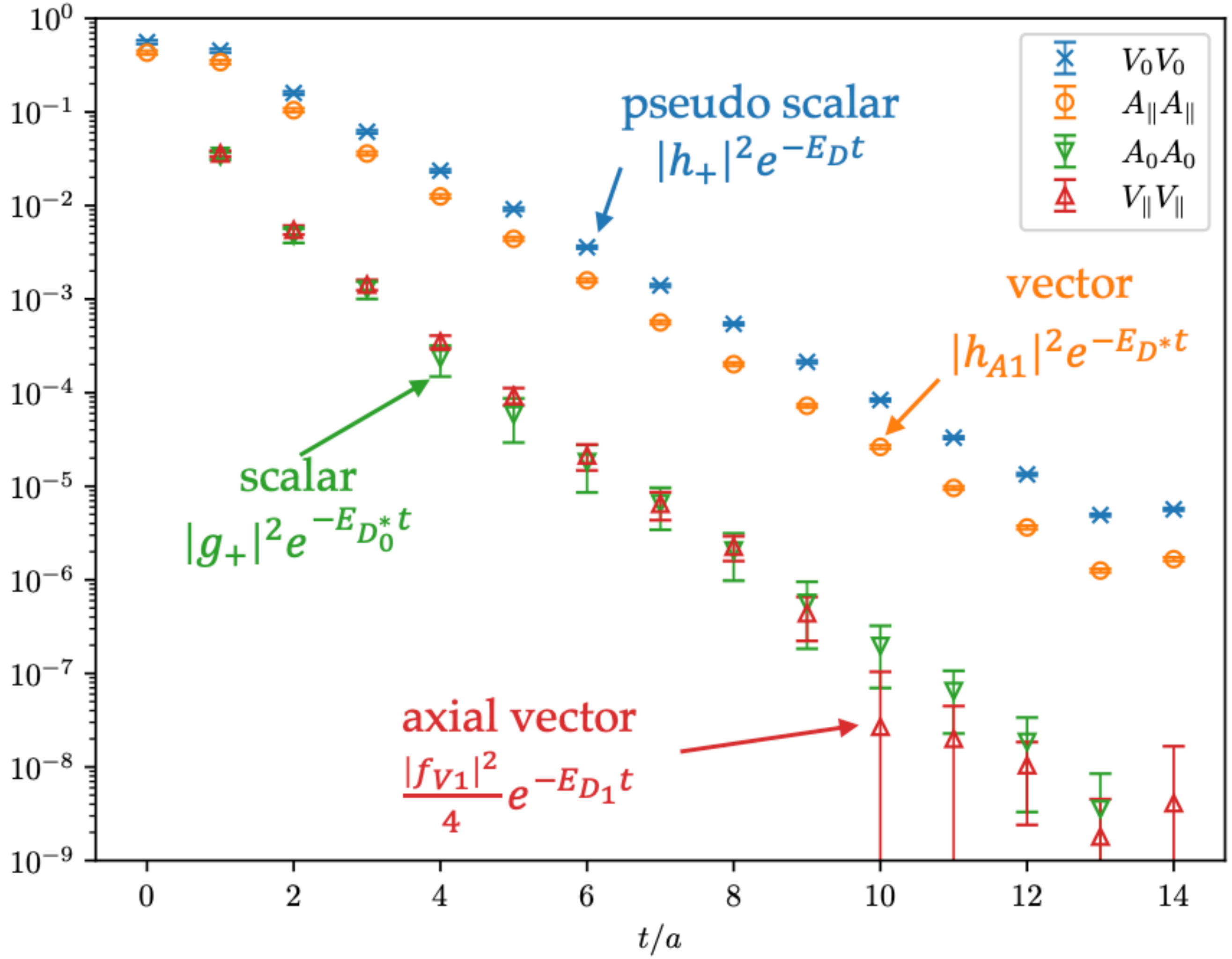


- $t_{snk} - t_{src} = 20$, $t_2 - t_{src} = 14$, fixed
- t_1 range from 0 to 14 $\Longrightarrow t = t_2 - t_1$ range from 14 to 0
- we work in the rest frame of B_s, $v = (1, 0, 0, 0)$
- $\boldsymbol{p}_X = -\boldsymbol{q} = (q_k, q_k, q_k)$
- we adopt an almost non-perturbative renormalization for our current, from [El-Khadra et. al.,PRD.64(2001):014502]

Zero-recoil results

four-point correlators at zero recoil

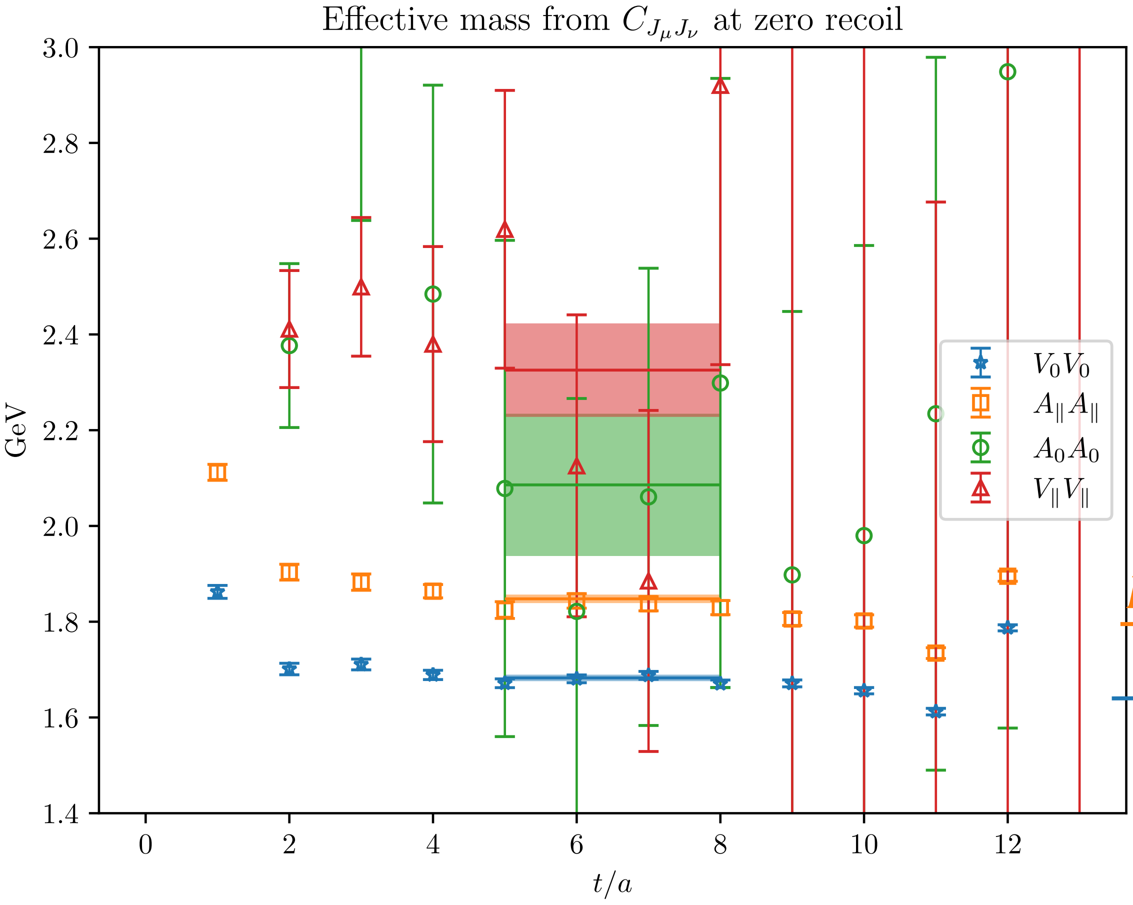
at zero recoil limit, parity is well defined, thus parity symmetry dedicates the isolation of final states



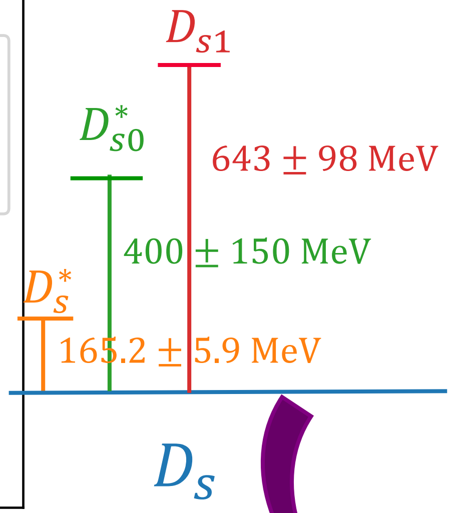
- S – wave : $C_{V_0V_0} \approx |h_+|^2 e^{-E_D t}$, $C_{A_{\parallel}A_{\parallel}} \approx |h_{A1}|^2 e^{-E_{D^*} t}$
- $P_{1/2}$ – wave : $C_{A_0A_0} \approx |g_+|^2 e^{-E_{D_0^*} t}$
- $P_{3/2}$ – wave : $C_{V_{\parallel}V_{\parallel}} \approx \frac{|f_{V1}|^2}{4} e^{-E_{D_1} t}$

the magnitudes of *P*-wave contributions are around 1/10 smaller than those of the *S*-wave contributions, but it is still visible in the lattice simulation

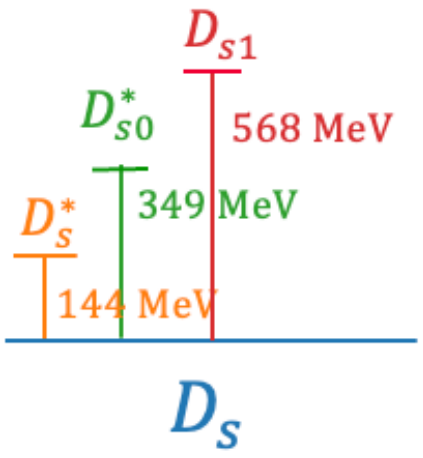
comparison of the effective mass and the fitted mass



- $S-$wave : $C_{V_0V_0} \approx |h_+|^2 e^{-E_D t}$, $C_{A_\parallel A_\parallel} \approx |h_{A1}|^2 e^{-E_{D^*} t}$
- $P_{1/2}-$wave : $C_{A_0A_0} \approx |g_+|^2 e^{-E_{D_0^*} t}$
- $P_{3/2}-$wave : $C_{V_\parallel V_\parallel} \approx \frac{|f_{V1}|^2}{4} e^{-E_{D_1} t}$



PDG values of D_s mass splitting

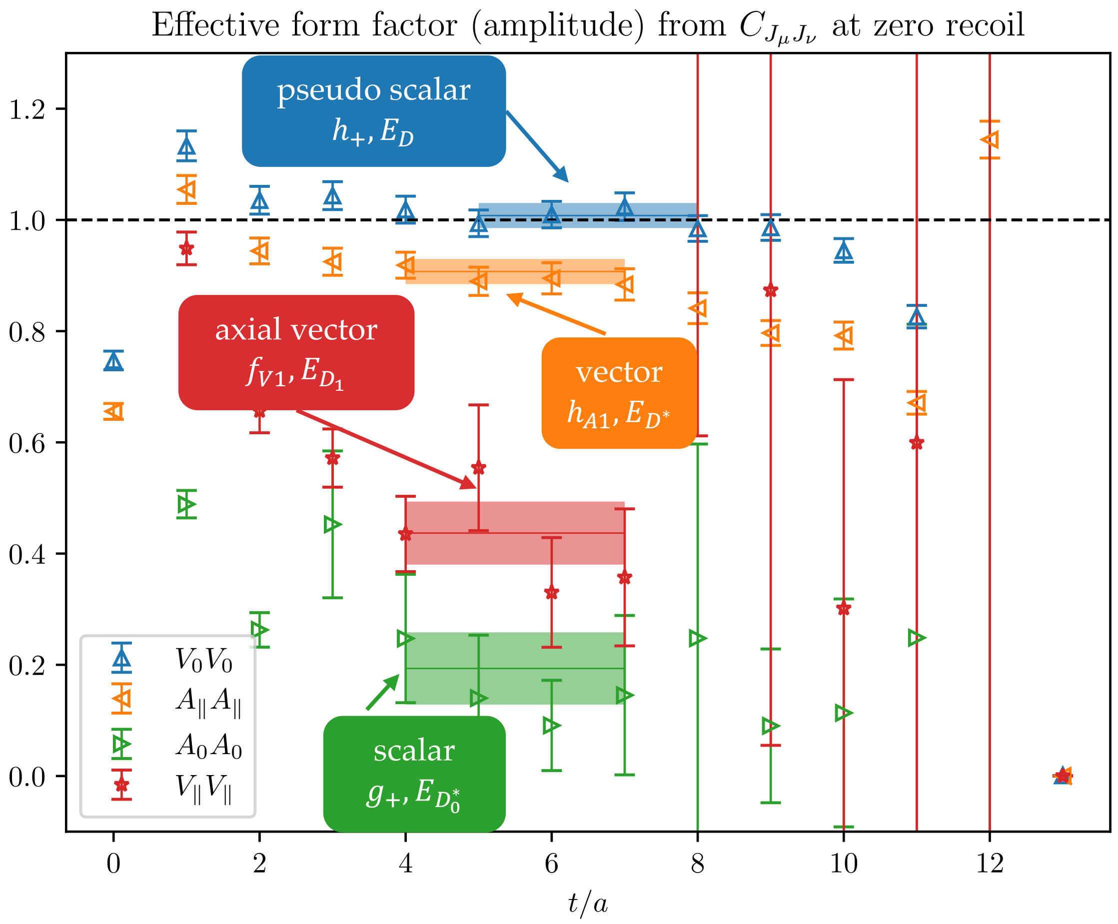


consistent with in mind the coarse nature of this study

comparision of the fitted and effective form factor

effective form factor

$$\sqrt{\left(C_{J_\nu J_\mu}(t)e^{m_{\text{eff}}t} + C_{J_\nu J_\mu}(t+1)e^{m_{\text{eff}}(t+1)}\right)/2}$$



- S-wave form factors at zero recoil

$$\langle B_s | V_\mu | D_s \rangle \propto h_+(\omega)(v_\mu + v'_\mu)$$
$$\langle B_s | A_\mu | D_s^*, \epsilon_\mu \rangle \propto (\omega + 1) h_{A1}(\omega)\epsilon_\mu$$

- $h_+(\omega = 1) \approx 1, h_{A1}(\omega = 1) \approx 0.88$

$$\underbrace{B_s}_{} \rightarrow \underbrace{X_{cs}}_{} \; \underbrace{l\,\nu_l}_{}$$

$$v = \frac{p_{B_s}}{M_{B_s}} \quad v' = \frac{p_X}{M_X} \quad q$$

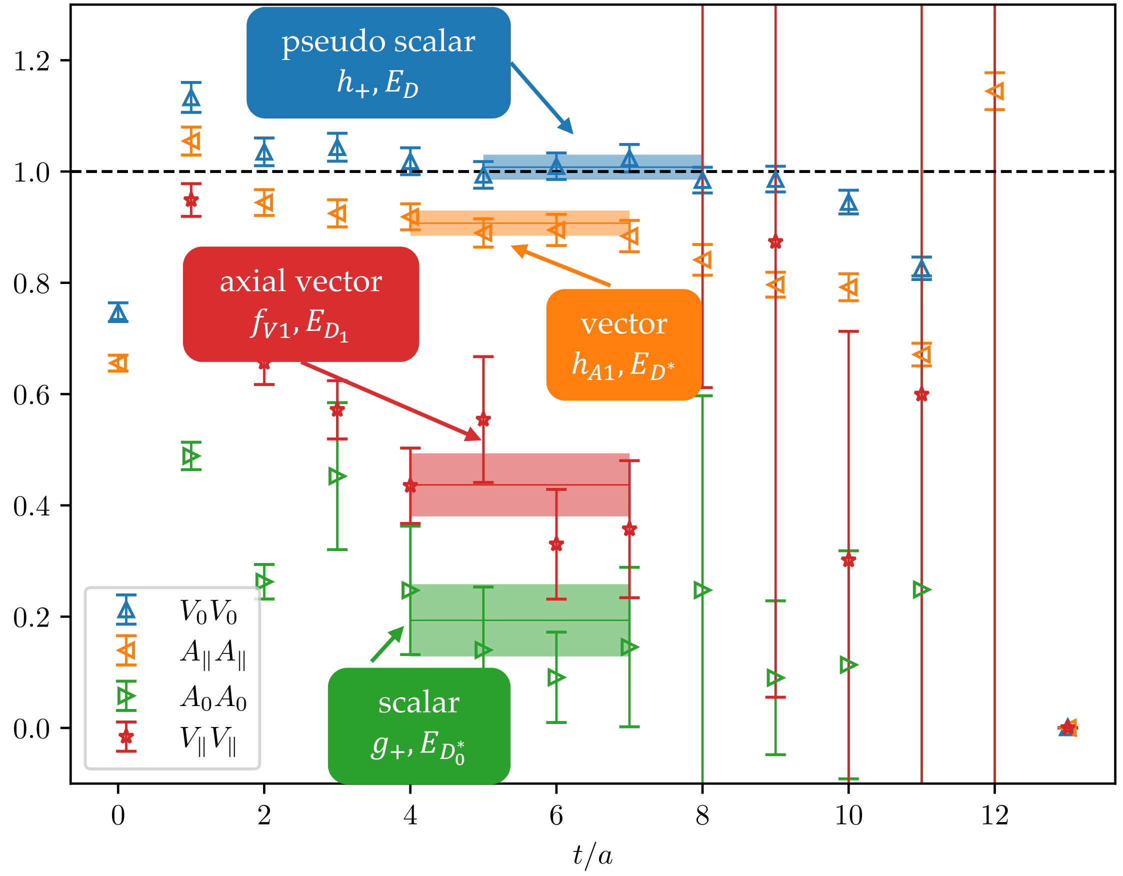
$$\omega = v \cdot v' \qquad p_{B_s} = p_X + q$$

comparision of the fitted and effective form factor

effective form factor

$$\sqrt{\left(C_{J_\nu J_\mu}(t)e^{m_{\text{eff}}t} + C_{J_\nu J_\mu}(t+1)e^{m_{\text{eff}}(t+1)}\right)/2}$$

Effective form factor (amplitude) from $C_{J_\mu J_\nu}$ at zero recoil



- P-wave form factors at zero recoil

$$\langle B_s|A_\mu|D_{s0}^*\rangle \propto g_+(\omega)(v_\mu + v'_\mu)$$
$$\langle B_s|V_\mu|D'_{s1}, \epsilon_\mu\rangle \propto g_{V1}(\omega)\epsilon_\mu$$
$$\langle B_s|V_\mu|D_{s1}, \epsilon_\mu\rangle \propto f_{V1}(\omega)\epsilon_\mu$$

- according to heavy quark effective theory [*Leibovich et. al.*,PRD.57.1:308–330]
  - $g_+(1) = -\frac{3}{2}(\epsilon_c + \epsilon_b)(\overline{\Lambda^*} - \overline{\Lambda})\zeta(1)$
  - ~~$g_{V1}(1) = (\epsilon_c - 3\epsilon_b)(\overline{\Lambda^*} - \overline{\Lambda})\zeta(1)$~~
  - $f_{V1}(1) = -\frac{8}{\sqrt{6}}(\epsilon_c)(\overline{\Lambda'} - \overline{\Lambda})\tau(1)$

$\epsilon_c = \frac{1}{2m_c} \approx 0.801$

$\epsilon_b = \frac{1}{2m_c} \approx 0.12$

difference of spin-averaged mass

$P_{1/2} - S : \bar{\Lambda}^* - \bar{\Lambda} = 0.36\text{GeV}$

$P_{3/2} - S : \bar{\Lambda}' - \bar{\Lambda} = 0.4\text{GeV}$

$\tau_{1/2}(1) = \frac{1}{2}\zeta(1), \tau_{3/2}(1) = \frac{1}{\sqrt{3}}\tau(1)$

$\tau_{1/2}, \tau_{1/2}$: Isgur-Wise form factors

$\tau_{1/2} = 0.156(48)$

$\tau_{3/2} = 0.277(42)$

$\tau_{1/2} \approx \tau_{3/2}$

More words about $\tau_{1/2} \ll \tau_{3/2}$

- it comes from the following sum rule and the asumption of the radial ground state dominace

$$\frac{1}{4} = \sum_m \left|\tau_{3/2}^{(m)}\right|^2 - \sum_n \left|\tau_{1/2}^{(n)}\right|^2$$

$$\tau_{1/2} = \tau_{1/2}^{(0)}, \tau_{3/2} = \tau_{3/2}^{(0)}$$

- it is a sum rule in the zero-recoil limit

Non-zero recoil (preliminary)

Fitting formula

$$C_{V_0V_0} = \frac{e^{-E_D t}}{4E_D M_D}\left[h_+ (E_D + M_D) - h_- (E_D - M_D)\right]^2 + \frac{e^{-E_{D_1'} t}}{4E_{D_1'} M_{D_1'}} 3q_k^2 \left[g_{V1} + g_{V2} + \frac{E_{D_1'}}{M_{D_1'}} g_{V3}\right]^2 + \frac{e^{-E_{D_1} t}}{4E_{D_1} M_{D_1}} 3q_k^2 \left[f_{V1} + f_{V2} + \frac{E_{D_1}}{M_{D_1}} f_{V3}\right]^2$$

$$C_{A_0A_0} = \frac{e^{-E_{D^*} t}}{4E_{D^*} M_{D^*}} 3q_k^2 \left[\left(\frac{E_{D^*} + M_{D^*}}{M_{D^*}}\right) h_{A1} - h_{A2} - \frac{E_{D^*}}{M_{D^*}} h_{A3}\right]^2 + \frac{e^{-E_{D_0^*} t}}{4E_{D_0^*} M_{D_0^*}} \left[g_+ \left(E_{D_0^*} + M_{D_0^*}\right) - g_- \left(E_{D_0^*} - M_{D_0^*}\right)\right]^2 + \frac{e^{-E_{D_2^*} t}}{4E_{D_2^*} M_{D_2^*}} \cdots$$

- **S wave**
  - ***D*** **(0⁻)**

$$\langle B, v | V_\mu | D, v' \rangle = \left[h_+(v_\mu + v'_\mu) + \boxed{h_-(v_\mu - v'_\mu)}\right] \sqrt{M_B M_D} \quad (3.87)$$

$$\langle B, v | A_\mu | D, v' \rangle = 0 \quad (3.88)$$

  - ***D**** **(1⁻)**

$$\langle B, v | V_\mu | D^*, v', \sigma \rangle = \left[\boxed{h_V \varepsilon_{\mu\alpha\beta\gamma} \epsilon^\alpha v'^{,\beta} v^\gamma}\right] \sqrt{M_B M_{D^*}} \quad (3.89)$$

$$\langle B, v | A_\mu | D^*, v', \sigma \rangle = i\left[(\omega + 1) h_{A1} \epsilon_\mu - \boxed{(\epsilon \cdot v)\left(h_{A2} v_\mu + h_{A3} v'_\mu\right)}\right] \sqrt{M_B M_{D^*}} \quad (3.90)$$

- **P wave, $j = \frac{1}{2}$**
  - **D_0^* (0⁺)**

$$\langle B, v | V_\mu | D_0^*, v' \rangle = 0 \quad (3.91)$$

$$\langle B, v | A_\mu | D_0^*, v' \rangle = \left[g_+(v_\mu + v'_\mu) + \boxed{g_-(v_\mu - v'_\mu)}\right] \sqrt{M_B M_{D_0^*}} \quad (3.92)$$

  - **D_1' (1⁺)**

$$\langle B, v | V_\mu | D_1', v', \sigma \rangle = \left[g_{V1} \epsilon_\mu + \boxed{(\epsilon \cdot v)\left(g_{V2} v_\mu + g_{V3} v'_\mu\right)}\right] \sqrt{M_B M_{D_1'}}, \quad (3.93)$$

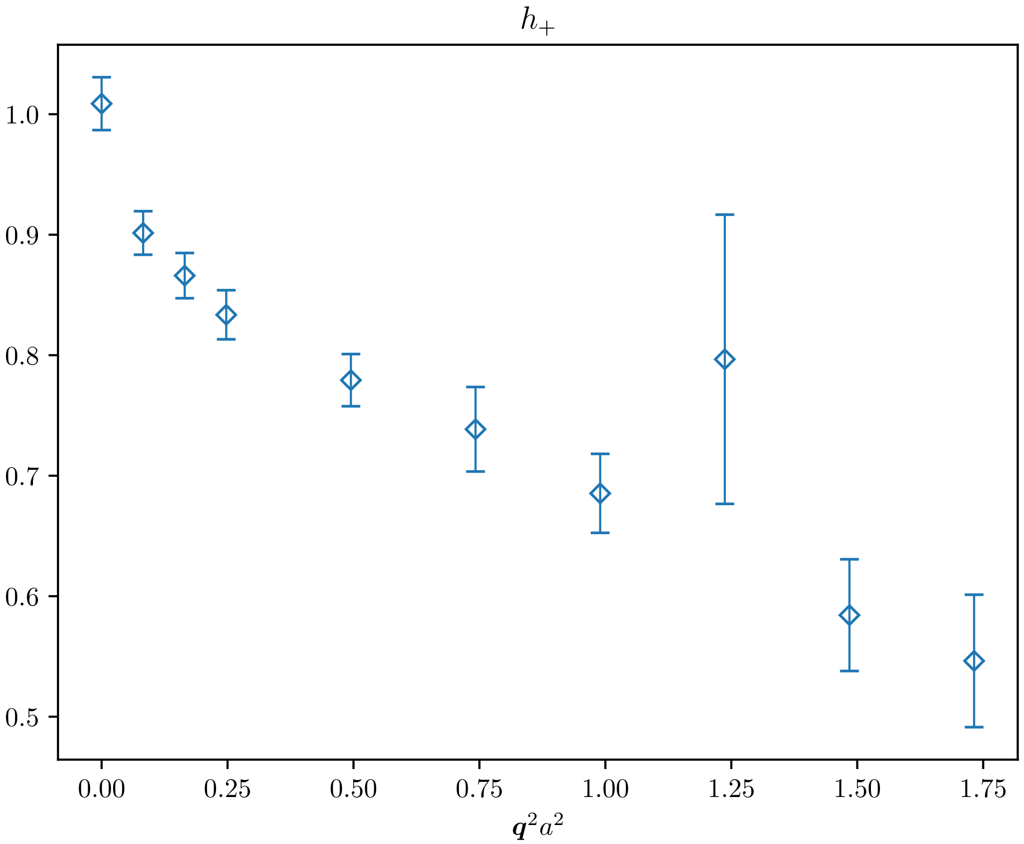
$$\langle B, v | A_\mu | D_1', v', \sigma \rangle = -i\left[\boxed{g_A \varepsilon_{\mu\alpha\beta\gamma} \epsilon^\alpha v'^{,\beta} v^\gamma}\right] \sqrt{M_B M_{D_1'}}. \quad (3.94)$$

- **P wave, $j = \frac{3}{2}$**
  - **D_1 (1⁺)**

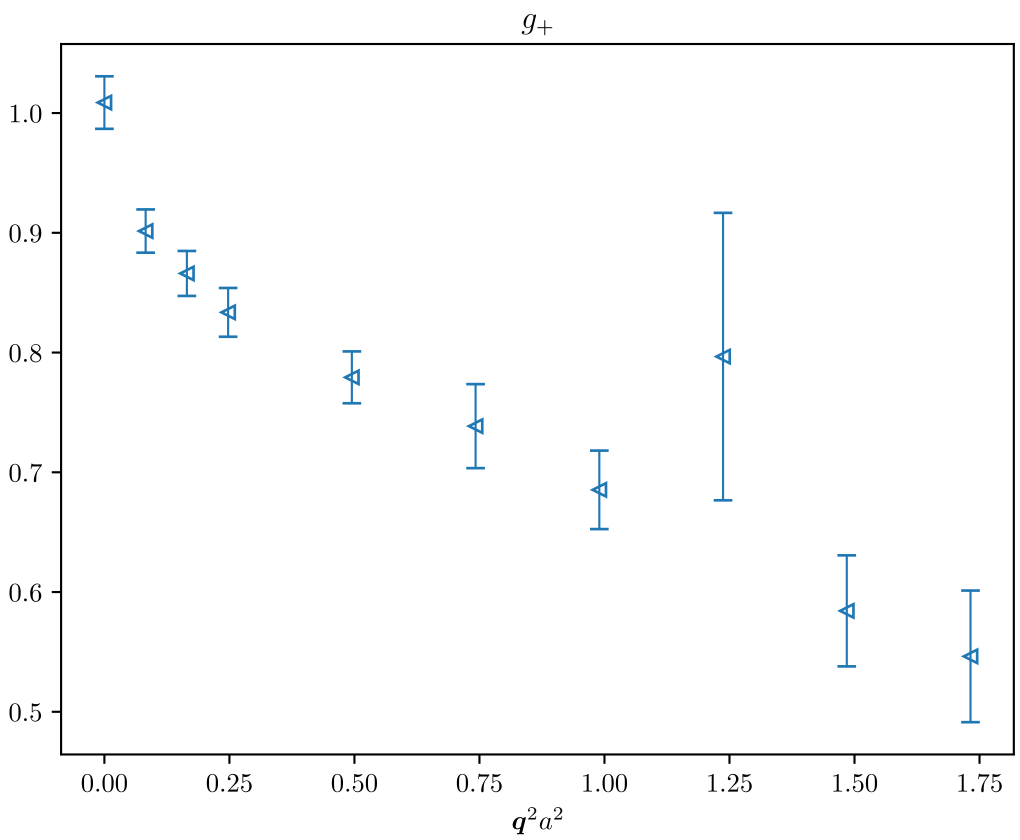
$$\langle B, v | V_\mu | D_1', v', \sigma \rangle = \left[f_{V1} \epsilon_\mu + \boxed{(\epsilon \cdot v)\left(f_{V2} v_\mu + f_{V3} v'_\mu\right)}\right] \sqrt{M_B M_{D_1}}, \quad (3.95)$$

$$\langle B, v | A_\mu | D_1', v', \sigma \rangle = -i\left[\boxed{f_A \varepsilon_{\mu\alpha\beta\gamma} \epsilon^\alpha v'^{,\beta} v^\gamma}\right] \sqrt{M_B M_{D_1}}. \quad (3.96)$$

Form factors

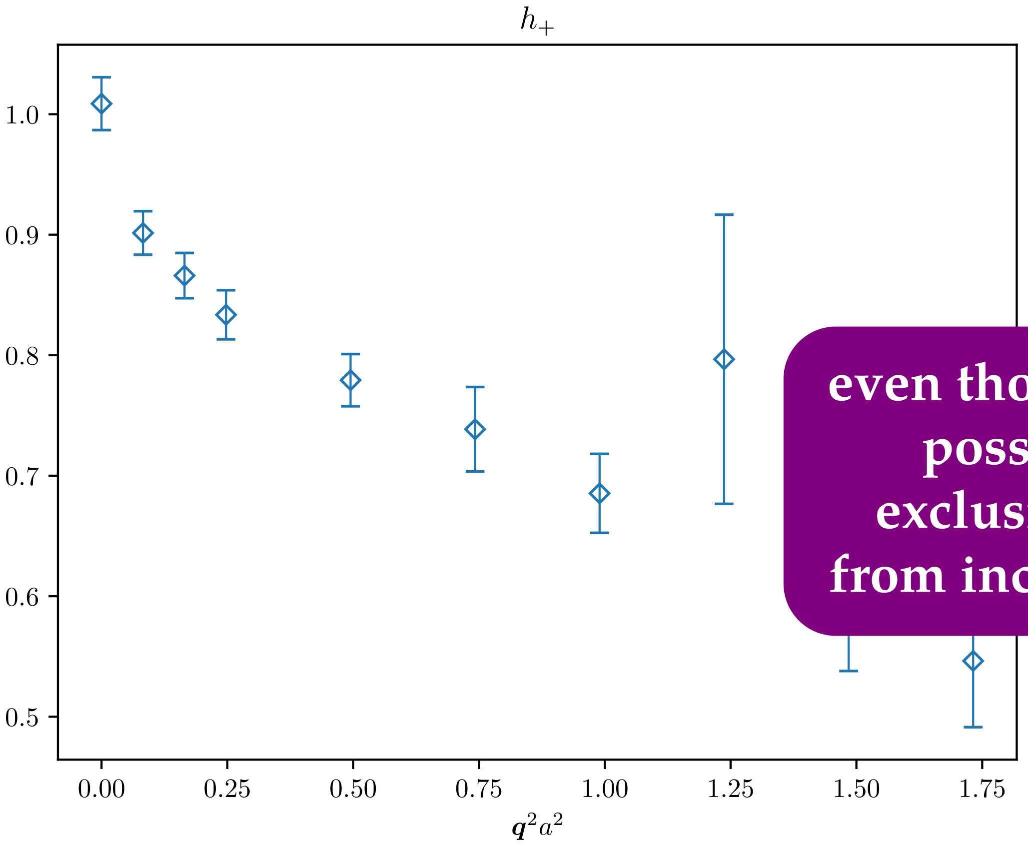


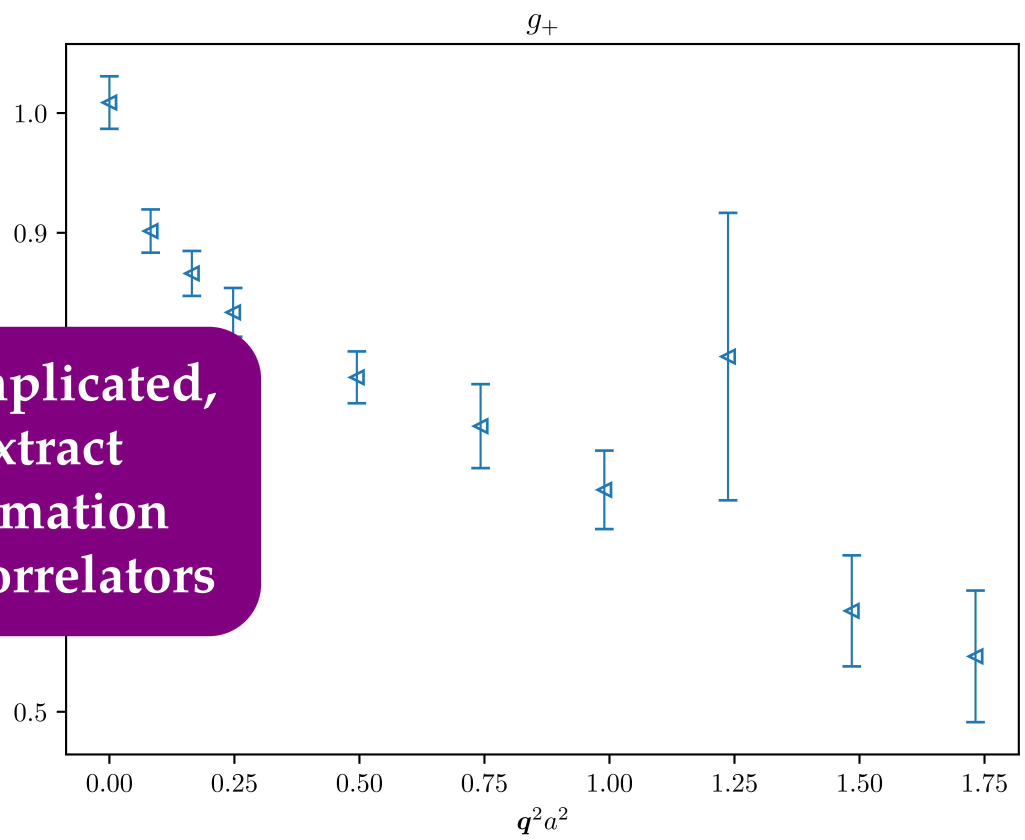
D_s, S-wave



D_{s0}^*, P-wave

Form factors





even though complicated, possible to extract exclusive information from inclusive correlators

D_s, S-wave

D_{s0}^*, P-wave

Summary and prospect

Summary

- as a preliminary study, we show the feasibility to extract exclusive informations from inclusive correlators
- we investigate the implication of the information extracted from the zero-recoil limit

Prospest

- we should investigate the non-zero recoil correlators in more depth to get the dependency of those form factors in the full kinematical range
- we should fit them with BGL parameterization
- we should investigate the continuum and infinite volumn limit