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中国工程物理研究院研究生院
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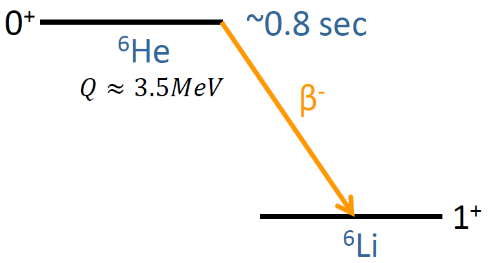
-1984-

Exploring Nuclear Beta Decay Through Lattice Effective Field Theory

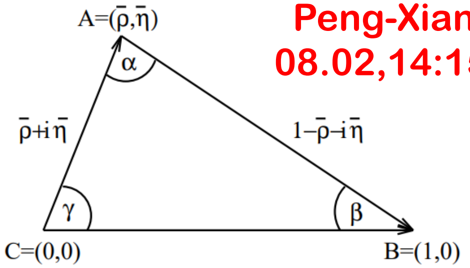
Teng Wang 2024.8.2
Collaborate with **Xu Feng** and **Bing-Nan Lu**

Nuclear β-Decay on the Lattice: Opportunities and Challenges

★ **Nuclear β-decay is related to various fundamental problems in physics.**

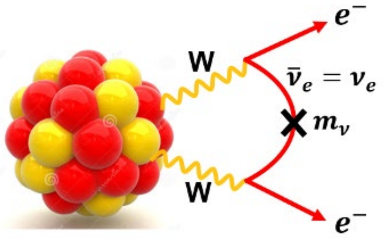


^{6}He β-decay spectrum
and exotic currents



super allowed β-decay
and CKM unitarity

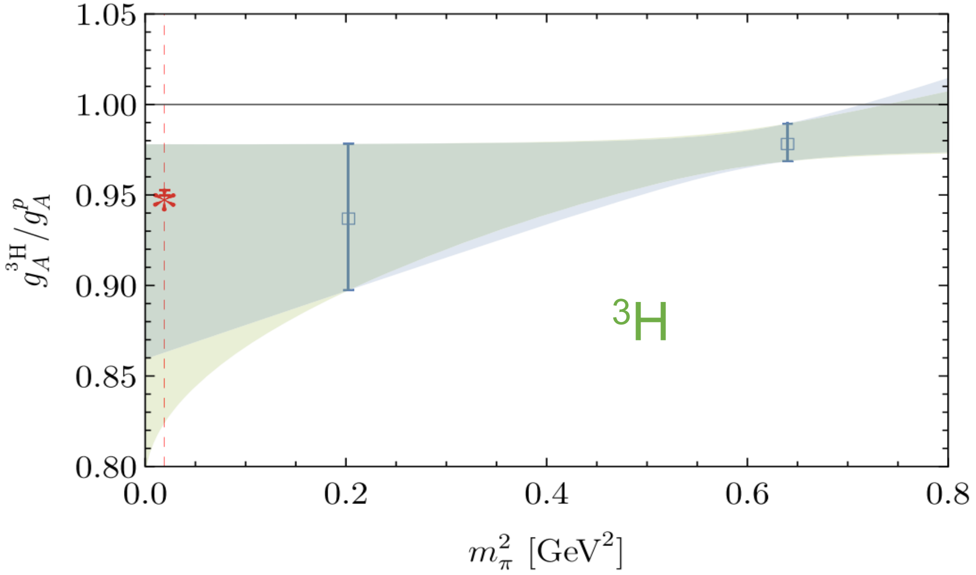
Peng-Xiang Ma's talk
08.02,14:15-14:35 ,LT3



neutrinoless double β-decay
and neutrino physics

Zi -Yu Wang's talk
07.31,LT1

★ **Lattice QCD has made impressive progresses on the β-decay of few-body nuclei.**



M. J. Savage et al, Phys. Rev. Lett. 119, 062002 (2017)

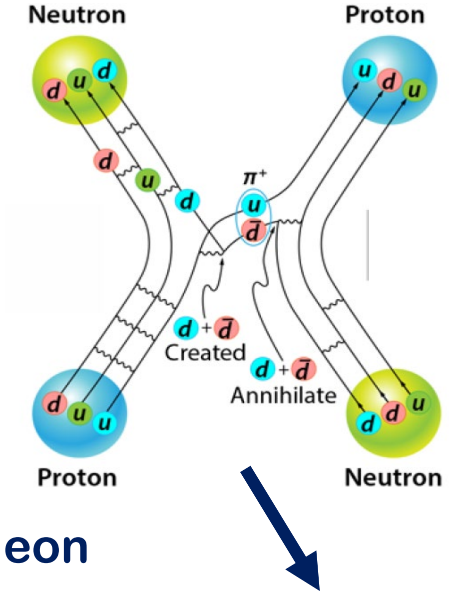
A. Parreño et al, Phys. Rev. D 103, 074511 (2021)

What about heavier nuclei?

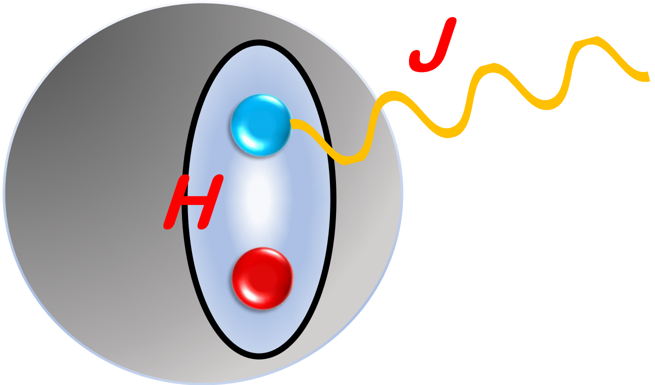
From QCD to Chiral Effective Field Theory

★ **Chiral Effective field theory(EFT) provides a shortcut to the problem.**

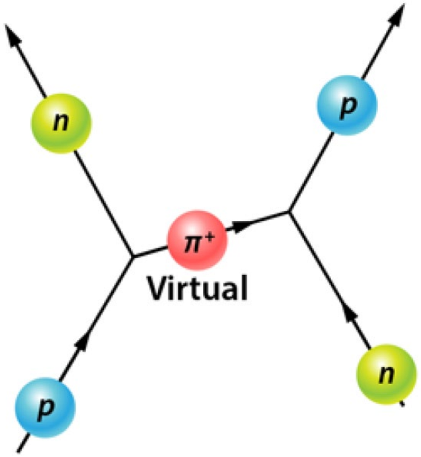
- Low energy effective field theory of QCD, works for $Q << \Lambda_{\chi} \approx 1 \mathrm{GeV}$
- Nucleons and pions as DOFs.
- Spontaneously broken chiral symmetry $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$
- Order by order calculation through power counting.



★ **Chiral EFT can provide the interaction of a nucleon with another nucleon inside the nucleus and with external fields.**



- Chiral EFT can well describe nuclear β-decay.
- How to solve the nuclear many-body problem?



Lattice Effective Field Theory: an Introduction

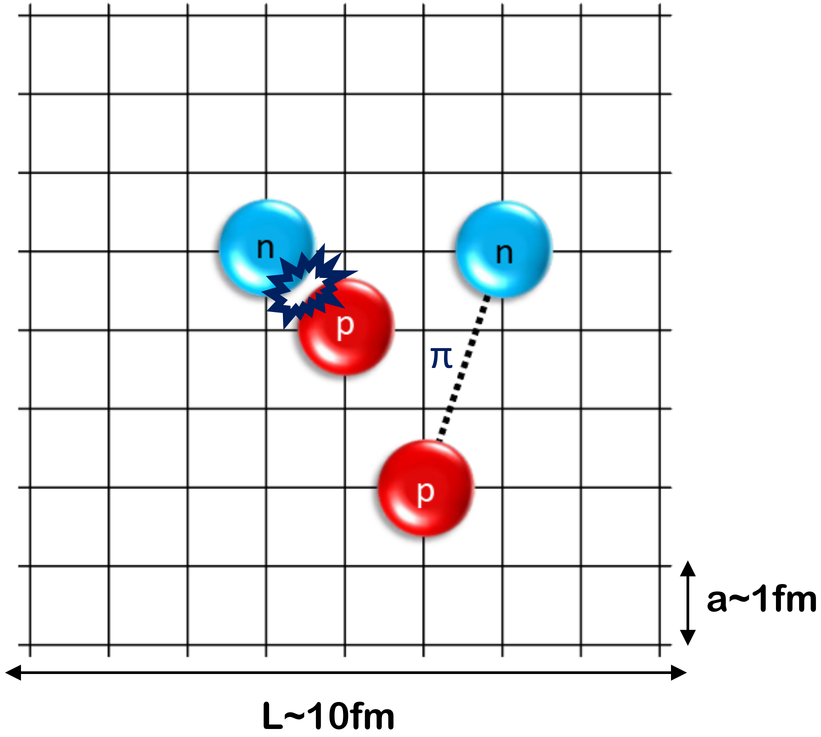
★ Lattice effective field theory(Lattice EFT) is a many body method to study nucleus on the lattice.

Lattice EFT = Chiral EFT + Lattice Method

Review: Dean Lee, Prog. Part. Nucl. Phys. 63, 117 (2009)

Lähde, Meißner, “Nuclear Lattice Effective Field Theory”, Springer (2019)

★ Main features of Lattice EFT.



- Discretize space-time into an Lt×L×L×L lattice.
- Nucleons are point-like particles on the site.
- Discretize the nuclear force, including the strong and Coulomb force on the lattice.
- lattice spacing a~1fm and lattice size L~10fm.

An Analogy to Lattice QCD

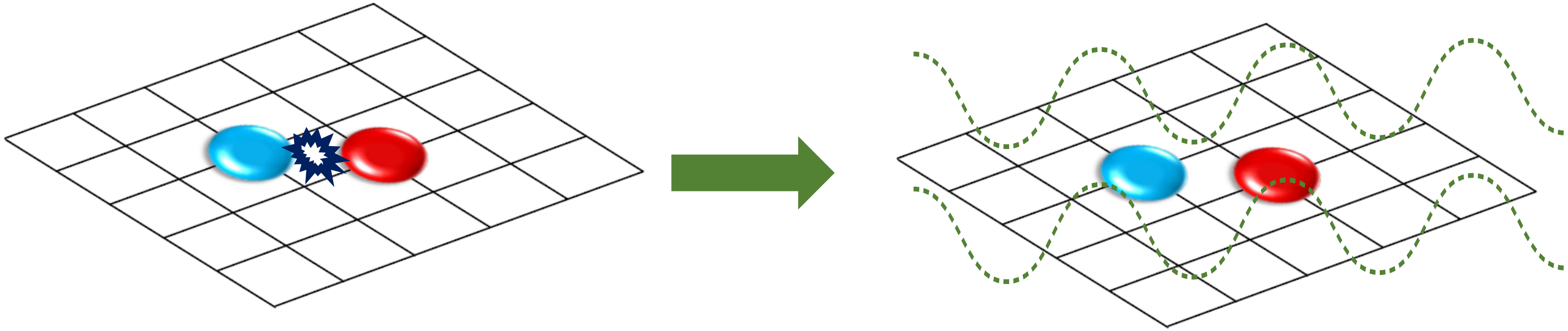
★ **Calculations in Lattice EFT are quite similar to Lattice QCD.**

- States are evolved via Euclidean time projection.

$$|\Phi_g\rangle = \lim_{T\to\infty} e^{-HT}|\Phi_i\rangle$$

- The NN interaction is decoupled into fermion-boson interaction via the auxiliary field transformation.

$$\boxed{\exp\left[-\frac{C}{2}\left(N^\dagger N\right)^2\right] = \sqrt{\frac{1}{2\pi}}\int ds \exp\left[-\frac{s^2}{2} + \sqrt{C}\, s\left(N^\dagger N\right)\right]}$$

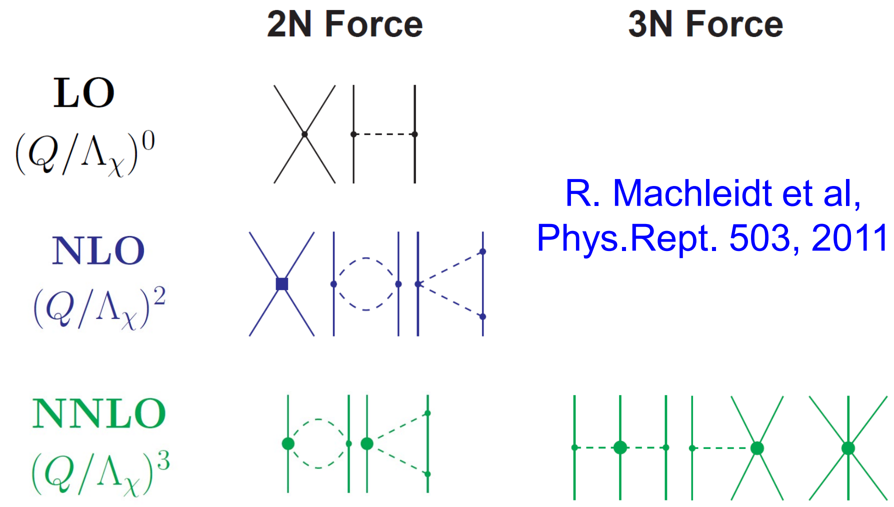


- The multi-dimensional integral over the auxiliary field is evaluated through Monte Carlo.

$$\langle\Phi|e^{-HT}|\Phi\rangle = \int \mathcal{D}s\; e^{-\frac{1}{2}s^2} Z[s]$$

Chiral Nuclear Force

★ We use NNLO chiral nuclear force.

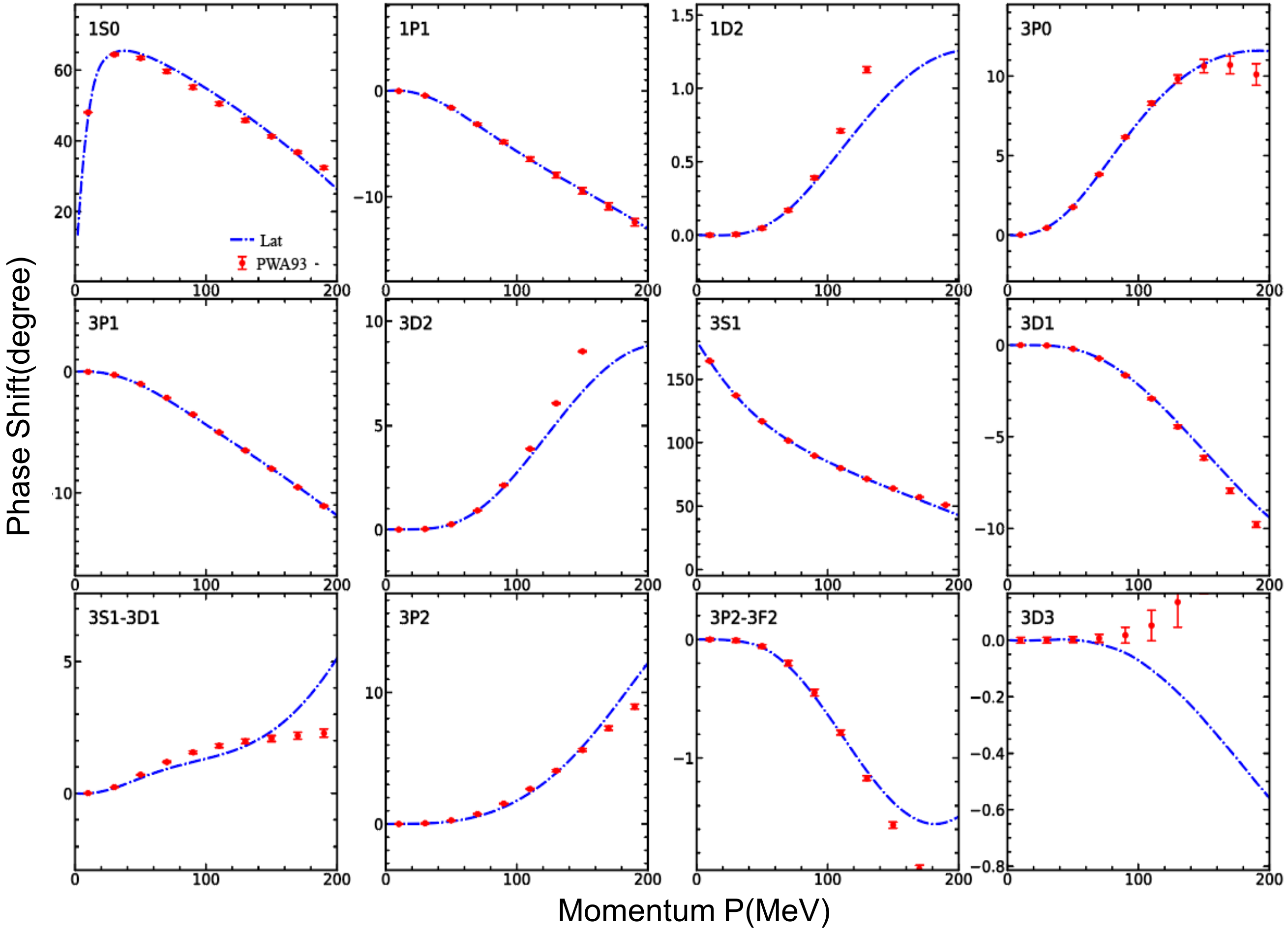


R. Machleidt et al, Phys.Rept. 503, 2011

$$
\begin{aligned}
V_{2N} = \Big[& B_1 + B_2(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + C_1 q^2 + C_2 q^2 (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \\
& + C_3 q^2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + C_4 q^2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \\
& + C_5 \frac{i}{2} (\boldsymbol{q} \times \boldsymbol{k}) \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) + C_6 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}) \\
& + C_7 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q})(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \Big] \\
& - \frac{g_A^2 f_\pi(q^2)}{4F_\pi^2} \left[\frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q})}{q^2 + M_\pi^2} + C'_\pi \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \right] (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)
\end{aligned}
$$

9 LECs to fit, 10 data points for each channel

★ **The 2N LECs are fitted to PWA93 n-p scattering phase shifts**

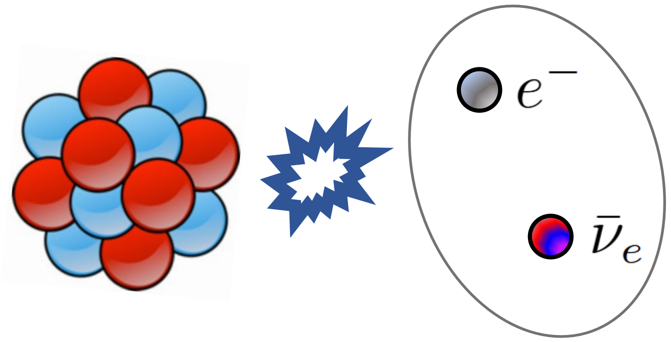


experiment data: https://nn-online.org

Fitting Method: B. N. Lu et al, Phys. Lett. B 760(2016)

Gamow Teller Matrix Element and Axial Current

★ **We focus on allowed nuclear β-decay.**

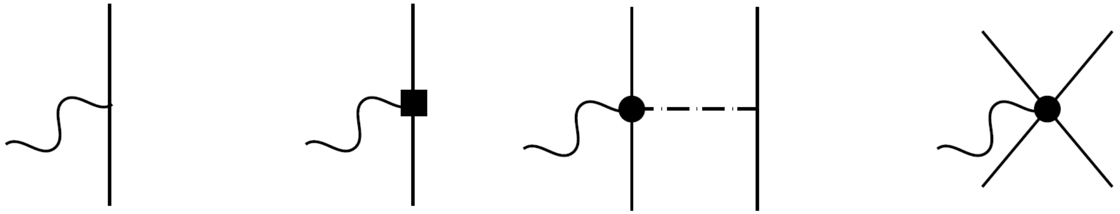


$$L_{e^-+\bar{\nu}_e} = 0$$

★ **There are 2 types of matrix elements, Fermi and Gamow-Teller type, with the later more interesting.**

Fermi	$\langle \Psi_f \| \| J_V^0 \| \| \Psi_i \rangle$	constranied by CVC
GT	$\langle \Psi_f \| \| J_A^i \| \| \Psi_i \rangle$	sensitive to NS

★ **We use nuclear axial currents up to N3LO consistently derived from chiral EFT**



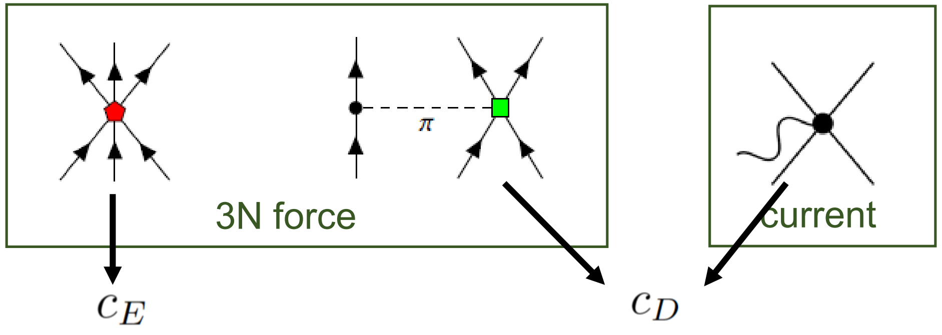
LO **N2LO** **N3LO**

A. Baroni et al, Phys. Rev. C, 93 015501, 2016

H. Krebs et al, Annals of Physics 378, 2017

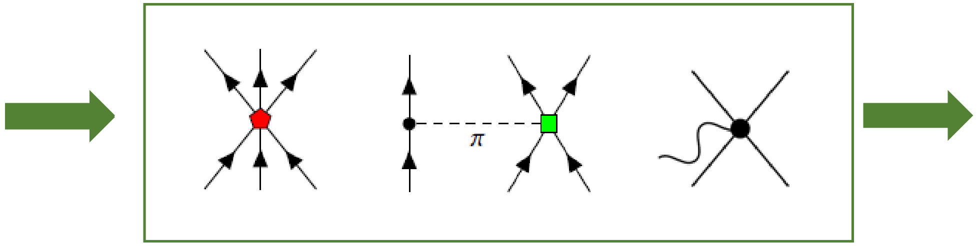
Determine 3N LECs: ^{3}H binding energy and ^{3}H β-decay

★ There are two unknown LECs, c_D and c_E, in the 3N force, the former also appears in the N3LO contact axial current .



★ ^{3}H is an ideal system to help determine the LECs.

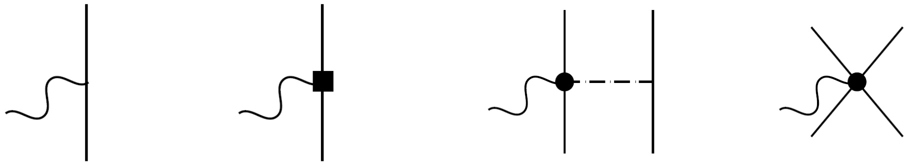
$$E(^3\mathrm{H}) = -8.482\mathrm{MeV}$$
$$\mathrm{GT}(^3\mathrm{H} \to ^3\mathrm{He}) = 0.9511(13)$$



$$c_D = 1.01(12)$$
$$c_E = 0.81(1)$$

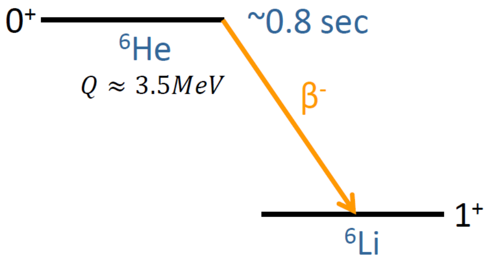
★ Contributions to the Gamow-Teller matrix element (GTME) from currents at different orders.

LO	N2LO	N3LO(OPE)	N3LO(CT)
0.9614(86)	-0.0041(11)	-0.0311(34)	0.0146(28)



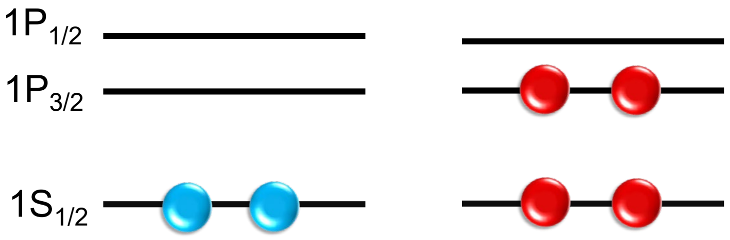
Predicting ^{6}He β-decay: Trial States' Preparation

★ We validate our method through calculating the GTME of ^{6}He→^{6}Li β-decay process.

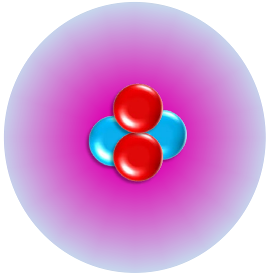


$$\mathrm{GT}(t)=\frac{\sqrt{2J_f+1}}{g_A}\frac{\langle\Phi_f|e^{-\frac{1}{2}Ht}\mathcal{J}e^{-\frac{1}{2}Ht}|\Phi_i\rangle}{\sqrt{\langle\Phi_i|e^{-Ht}|\Phi_i\rangle\langle\Phi_f|e^{-Ht}|\Phi_f\rangle}}$$

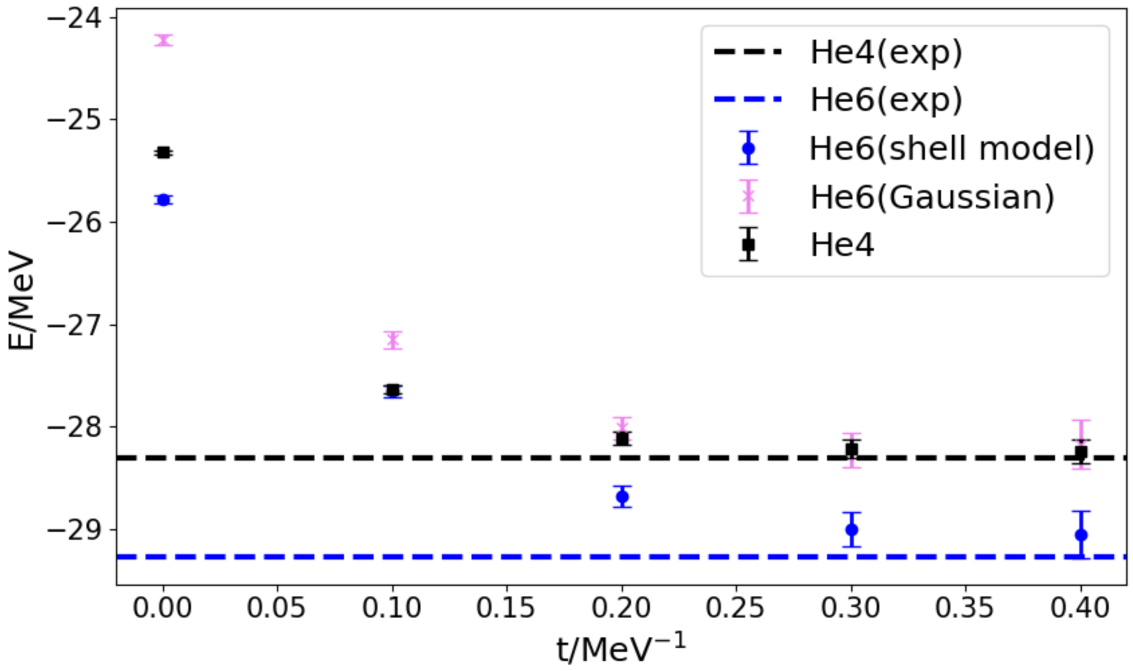
★ We find the convergence rate to ^{6}He ground depends strongly on the choice of the trial state.



shell model trial state



Gaussian trial state



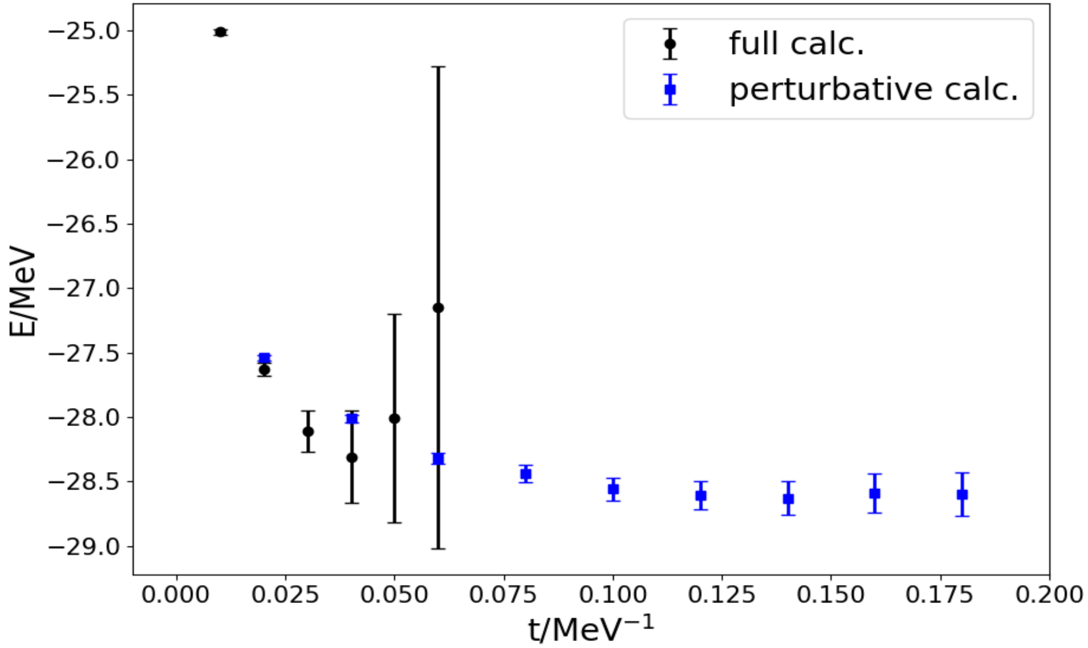
Predicting ^{6}He β-decay: Overcoming Sign Problem

★ **Among various competing uncertainties, statistical errors induced by the sign problem is the dominant.**

- Monte Carlo Statistical error
- Model dependence
- Finite volume effect···

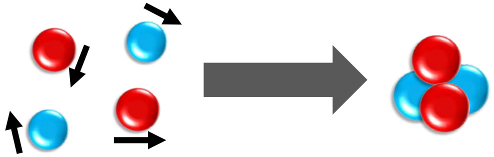
$$\int \mathcal{D}s\; e^{-\frac{1}{2}s^2} Z[s]$$

complex



★ **Wigner SU4 symmetry can help reduce sign problem**

$$N = \begin{bmatrix} a_{p\uparrow} \\ a_{p\downarrow} \\ a_{n\uparrow} \\ a_{n\downarrow} \end{bmatrix}$$



E. Wigner, Phys. Rev. 51, 106 (1937)

J.-W. Chen et al, Phys. Rev. Lett. **93**, 242302 (2004)

★ **Making use of the SU4 symmetry, we ease the sign problem using perturbative method**

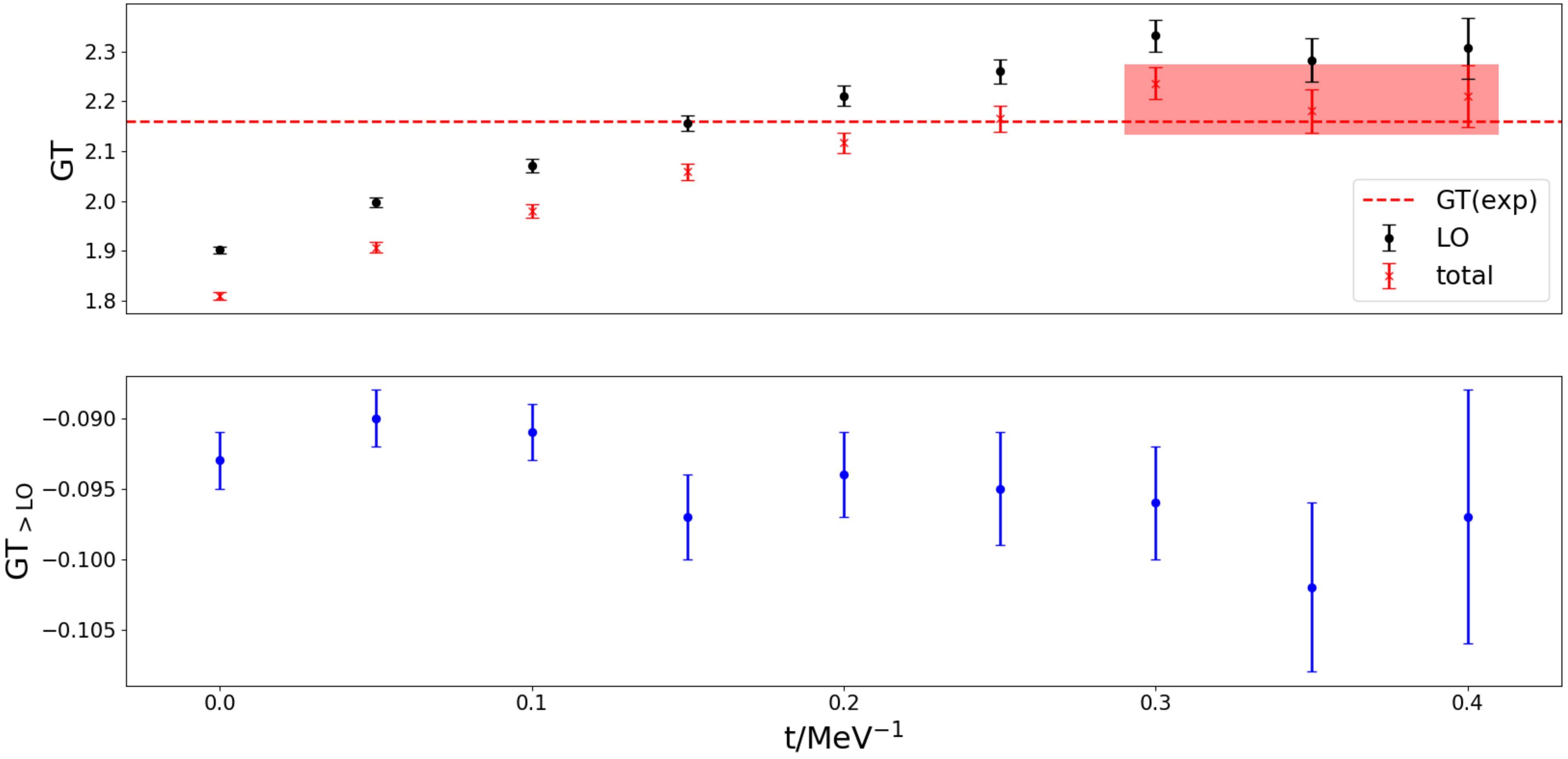
$$H = H_{\mathrm{LO}} + \Delta V$$

$$\mathrm{GT} = \mathrm{GT}^{(0)} + \mathrm{GT}^{(1)} + \mathcal{O}(\Delta V^2)$$

B. N. Lu et al, Phys. Rev. Lett. 128, 242501 (2022)

$$H_{\mathrm{LO}} = K + \underbrace{\frac{C_2}{2!}\sum_{\boldsymbol{n}} :\tilde{\rho}^2(\boldsymbol{n}): + \frac{C_3}{3!}\sum_{\boldsymbol{n}} :\tilde{\rho}^3(\boldsymbol{n}):}_{\text{SU4 symmetric}} + \underbrace{\frac{C_I}{2!}\sum_{I,\boldsymbol{n}} :\tilde{\rho}_I^2(\boldsymbol{n}): + V_{\mathrm{OPE}}^{\Lambda'_\pi}}_{\text{SU4 breaking}}$$

Predicting ^{6}He β-decay: Result



	LO	N2LO	N3LO(OPE)	N3LO(CT)	Total-LO	Total	Exp
GT	2.304(64)	-0.024(9)	-0.105(1)	0.030(2)	-0.099(10)	2.205(65)	2.161(4)

Summary and Outlook

★ We calculated nuclear β-decay through lattice EFT for the first time.

★ 3N LECs are determined from ^{3}H binding energy and β-decay.

★ ^{6}He β-decay is predicted in good consistency with the experiment.

★ Tests for heavier nuclei are required.

★ A direct application on various EW processes in the nuclei is straightforward.

thank You!