Long-range interactions in double heavy tetraquarks: $\bar{Q}\bar{Q}qq$

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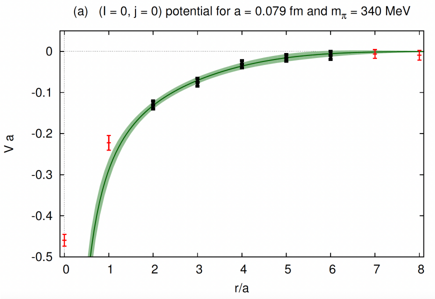
Outline

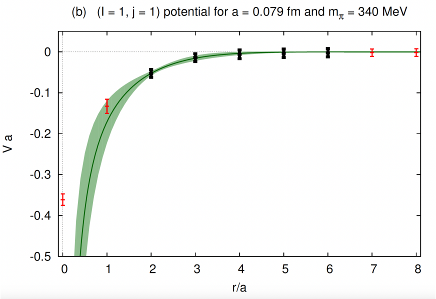
Lattice $\bar{b}\bar{b}ud$ static potentials

Bicudo et al, PRD 96 (2017) 054510

$J^P = 1^+$ $\bar{b}\bar{b}ud$ potentials in $I = j_l$ (isospin=spin of light quarks) channels

$m_\pi = 340$ MeV





Functional form fits fed into Schrödinger Eq. $\Rightarrow$ deeply bound-state ~ -90 MeV B.E

- New more interesting potentials have been obtained, see Bicudo, Tuesday 14:05
- Focus here is $J^P = 1^+$, for other quantum numbers, see Hoffmann, Tuesday 14:25

Theoretical Overview

Aim: To understand long- and intermediate-range interactions in $\bar{b}\bar{b}ud$ using Chrial Effective Field Theory

$\hookrightarrow$ Static limit translated to Heavy Quark Limit where $B \approx B^*$ degenerated (no spin effects included)

$\hookrightarrow$ Main long & intermediate ranged contributions to $B^{(*)}B^{(*)}$ static potential are:

1. One pion exchange, having range $\mathcal{O}\left(\frac{1}{m_\pi}\right)$
2. 2 pion exchange, driven by S-wave πB interaction - range $\mathcal{O}\left(\frac{1}{2m_\pi}\right)$
3. 2 pion exchange, driven by P-wave πB interaction - range $\mathcal{O}\left(\frac{1}{2m_\pi}\right)$

Theoretical Caveats

↪ Subsystem interactions are very crucial here

↪ Treatment of the 2 pion exchange potential at $m_\pi = 340$ MeV is highly non-trivial;

1. at this pion mass the $f_0(500)/\sigma$ pole is very close to the $2m_\pi$ threshold $\pi\pi$ rescattering is very essential
   Hanhart, Pelaez, Rios, PRL 100 (2008)152001; HadSpec, PRL 118, (2017) 022002
2. around $m_\pi = 340$ MeV, the πB S-wave interaction also develops a bound state, e.g for D-sector Liu et al. PRD 87 (2013) 014508
3. additional B^* left-hand-cut (LHC), same as discussed in "T_{cc}^+ Saga"
   Monday, Sasa Prelovsek, Sebastian Dawid: Thursday, Sinya Aoki, $\cdots$

Goal: Proper inclusion of the subsystem interactions and their interplay
LHC + RHC

One Pion Exchange Potential

- The LO contribution is one pion exchange - OPE
  Mark B. Wise, PRD 45(1992)2188; G. Burdman, J. F. Donoghue, PLB 280(1992)287

$$\mathcal{L}_{\mathrm{LO}} = i\mathrm{Tr}[\bar{H}_a v_\mu D^\mu_{ba} H_b] + g_\pi \mathrm{Tr}[\bar{H}_a H_b \gamma_\nu \gamma_5] u^\nu_{ba} \quad (1)$$

heavy field $H = \frac{1+\not{v}}{2}\left[\not{V} + iP\gamma_5\right], U = \exp\left(\frac{\sqrt{2}i\phi}{F}\right)$

- OPE potential in the momentum space is

$$V_\pi(q) = (I_1 \cdot I_2)\ \frac{g_\pi^2}{F_\pi^2}\ \frac{(q\cdot\epsilon_2)(q\cdot\epsilon_4^*)}{q^2 - m_\pi^2} \qquad I_1 \cdot I_2 = \begin{cases} -\frac{3}{4} & \text{for } I = 0 \\ \frac{1}{4} & \text{for } I = 1 \end{cases}$$

- The S-wave position space potential is

$$V_\pi(r) = (I_1 \cdot I_2)\ \frac{g_\pi^2}{3F_\pi^2}\left(m_\pi^2\ \frac{e^{-m_\pi r}}{4\pi r} - \delta(r)\right) \quad (2)$$

OPE Comparison

- $g_\pi = 0.5$, almost m_π independent
  UKQCD Collaboration, JHEP10(1998)010; Detmold et al. PRD 85,114508(2012)
- $F_\pi = 114$ MeV at $m_\pi = 340$ MeV Bicudo et al. PRD 96, 054510 (2017)

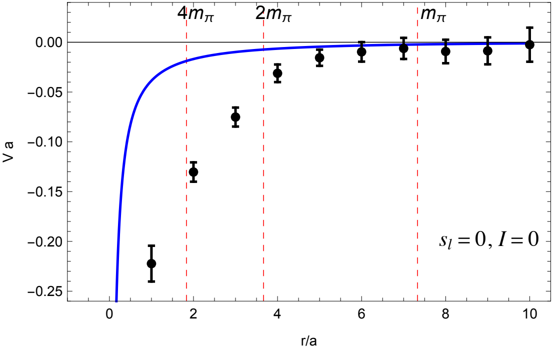


Figure: OPE potential in lattice units ($a = 0.079$ fm)

Intermediate Range Potential

- Possible 2 pion exchange diagrams - without $\pi\pi$ correlation

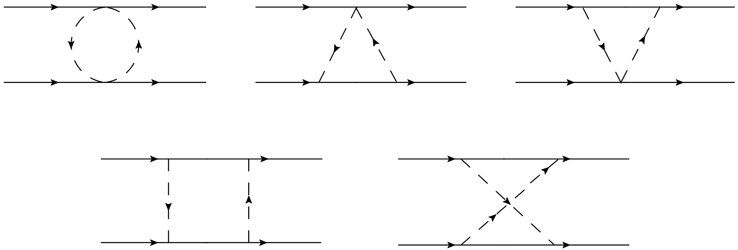


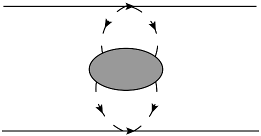
Figure: Two Pion exchange between two B mesons.

- Contributions from the correlated pions are discussed in the following

Prescription to deal with 2PE

- 2PE potential can be calculated by appropriately multiplying together the relevant $B\pi$ scattering diagrams, as NN Donoghue, PLB 643(2006)165

- For the $\pi\pi$ subsystem, unitarity requires the inclusion of the $\pi\pi$ rescattering Hanhart, Pelaez, Rios, PRL 100(2008)152001



- $\pi\pi$ amplitudes are described by a polynomial times Omnès function - Omnès solution Omnès, Nuovo Cim. 8 (1958) 316; Hanhart, PLB 715(2012)170

2 pion exchange, driven by S-wave πB interaction

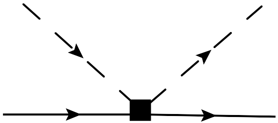


Figure: πB scattering

πB Scattering Amplitude

- The $B\pi$ scattering amplitude using $\mathcal{L}_{\text{NLO}}$ from Liu PRD 87, 014508(2013)

$$V(s,t,u) = \frac{1}{F_\pi^2}\left[\frac{C_{\text{LO}}}{4}(s-u) - 4C_0 h_0 + 2C_1 h_1 - 2C_{24}H_{24}(s,t,u) + 2C_{35}H_{35}(s,t,u)\right]$$

LECs $h_{0,1,24,35} \sim m_Q = M_H$. For arbitrary heavy mass $h_i^Q = h_i^c \, \frac{M_H}{M_D}$

- At threshold, the S-wave amplitude is

$$V_{\text{thr}}^{H\pi,I} = \frac{m_\pi^2 M_H}{F_\pi^2}\left[\frac{C_{\text{LO}}^I}{m_\pi} - \frac{2}{M_D}\left(2h_0^c + h_1^c + 2h_{24}^c - h_{35}^c\right)\right] \equiv M_H \tilde{V}^I \tag{3}$$

- The S-wave $H\pi$ scattering length Guo, Hanhart, Meißner, EPJ A 40,(2009)171

$$a_0^I = -\frac{M_H T_{\text{thr,NR}}^I}{4\pi(M_H + m_\pi)}, \qquad \text{with} \qquad T_{\text{thr}}^I = \frac{V_{\text{thr}}^{H\pi,I}}{\left[1 - V_{\text{thr}}^{H\pi,I} G_{\text{thr}}^\Lambda\right]} \tag{4}$$

Scattering Length Approximation

- Using non-rel. unitarized amplitude

$$a_0^I = \frac{\tilde{V}^I}{4\pi}\left\{1 - \frac{\tilde{V}^I}{16\pi^2}\left[-2\Lambda + M_\pi \log\frac{M_\pi^2}{(\sqrt{M_\pi^2+\Lambda^2}+\Lambda)^2}\right]\right\}^{-1} \tag{5}$$

Hard cutoff $\Lambda \sim 700$ MeV is matched (at $B\pi$ threshold) to the sub. constant of the dim. reg. scalar loop function Guo et al. NPA 773(2006)78

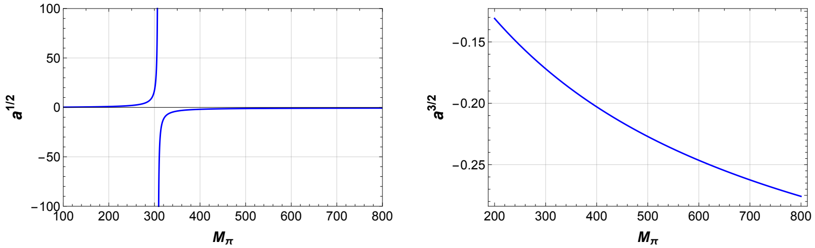


Figure: $B\pi$ scattering length in the HQL: $a^{1/2}$ (Left), $a^{3/2}$ (Right). Bound state in 1/2 channel around $m_\pi = 340$ MeV. Also for the $D\pi$ case Liu PRD 87, 014508(2013)

2PE via Dispersion Relation

$\hookrightarrow$ The scalar-isoscalar potential can be written as Donoghue, PLB 643(2006)165

$$V_\sigma(q^2) = \frac{2}{\pi} \int_{2m_\pi}^{\infty} d\mu \; \mu \frac{\text{Im}\mathcal{M}(s,\mu^2)}{\mu^2 + q^2} \tag{6}$$

$\text{Im}\mathcal{M}(s,\mu^2)$ is the discontinuity in the $B\bar{B} \to \pi\pi$ transition amplitude, properly matched to $\pi B \to \pi B$ scattering length, following Hanhart, PLB 715(2012)170

$$\text{Im}\mathcal{M}^{I=0} = -\frac{3}{2} i\pi \left(a^{(+)}\right)^2 \sqrt{1 - \frac{4m_\pi^2}{s}} \left|\Omega^0(s)\right|^2 \Theta(s - 4m_\pi^2) \tag{7}$$

$a^{(+)} = \frac{1}{3}(a^{1/2} + 2a^{3/2}) = -0.54^{+0.19}_{-0.59}$ fm by fitting h_is to the lattice data

$\hookrightarrow$ The 2PE potential in the scalar-isoscalar channel is

$$V_\sigma(r) = -3 \left(a^{(+)}\right)^2 \int_{2m_\pi}^{\infty} d\mu \sqrt{\mu^2 - 4m_\pi^2} \; \frac{e^{-\mu r}}{4\pi r} \left|\Omega^0(\mu^2)\right|^2 \tag{8}$$

Comparison with lattice data

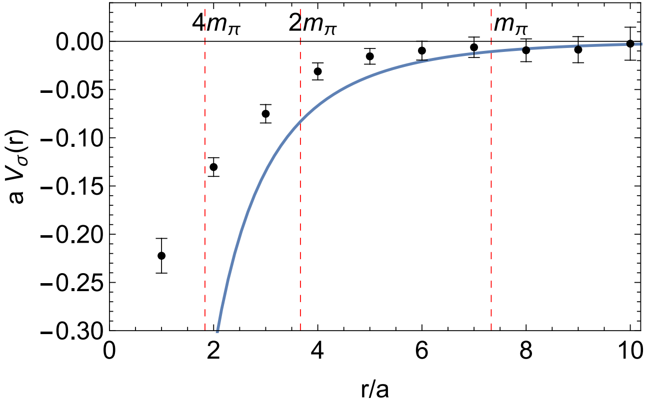
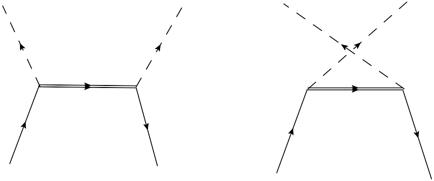


Figure: Scalar-isoscalar 2PE potential driven by S-wave πB interaction. Data is from Bicudo et al. PRD 96,054510(2017)

2 pion exchange, driven by P-wave πB interaction

Inclusion of the πB P-wave Interaction

- $\pi B \to B^* \to \pi B$ process is equivalent to $B\bar{B} \to \pi\pi$ in the crossed channel



- The B^* exchange amplitude

$$\hat{A}(s) = -\frac{\sqrt{3}g_\pi^2}{\sqrt{2}F_\pi^2}(\vec{q_1}\cdot\vec{q_2})m_{B^*}\left(\frac{1}{t-m_{B^*}^2}+\frac{1}{u-m_{B^*}^2}\right) \tag{9}$$

projected on $\pi\pi$ S-wave is

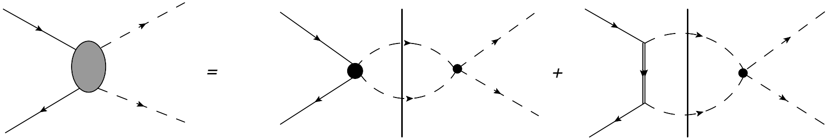
$$\hat{A}_0(s) = \frac{\sqrt{3}\pi g_\pi^2}{4\sqrt{2}F_\pi^2}\sqrt{s-4M_\pi^2} \tag{10}$$

- $\hat{A}_0(s)$ with $\pi\pi$ rescattering and πB scattering is included-framework?

The Khuri-Treiman Formalism

Khuri and Treiman PR119(1960)1115; Niecknig, Kubis, Schneider EPJC72(2012)2014

Amplitude $\Gamma(s) = A(s) + \hat{A}(s)$, where $A(\hat{A})$ has only right (left) hand cut



- $\Gamma(s)$ can be reconstructed dispersively, for $\pi\pi$ S-wave

$$\Gamma_0(s) = \hat{A}_0(s) + \Omega_0(s)\left[P_0 + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty} dz \frac{\hat{A}_0(z)\sin\delta_0(z)}{z(z-s-i\epsilon)|\Omega_0(z)|}\right], \quad P_0 = 2\sqrt{6}\pi a^{(+)}$$

- Ladder-type 2PE, identified as $(\hat{A}_{0,t-\text{channel}})^2$, must be subtracted from the $|\Gamma_0|^2$

$$\mathrm{Im}\mathcal{M}(s,t) = -\left[\Gamma_0(s)\sigma\Gamma_0^*(s) - |\hat{A}_0|^2\sigma\right]. \quad (11)$$

- The modified potentials will be (using Donoghue, PLB 643 (2006) 165)

$$V_\sigma(r) = -\frac{1}{32\pi^3}\int_{2m_\pi}^{\infty} d\mu \, \frac{e^{-\mu r}}{r}\sqrt{\mu^2 - 4m_\pi^2}\left[\left|\Gamma_0(\mu^2)\right|^2 - \left|\hat{A}_0\right|^2\right]. \quad (12)$$

Results comparison with data

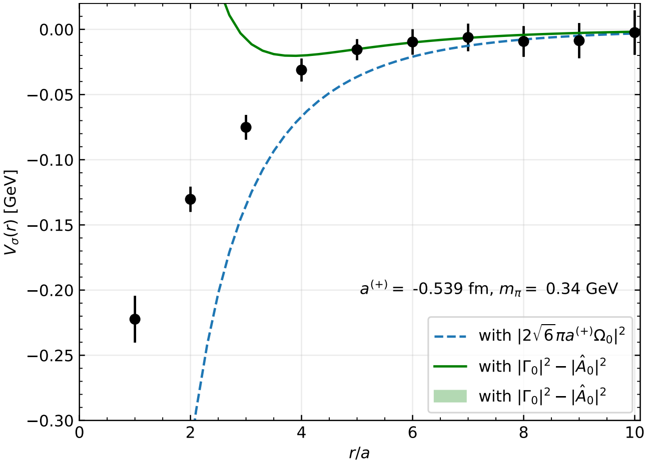


Figure: Scalar-isoscalar 2PE potential with LHCs and subtracted ladder-type 2PE.
Uncertainty is from fitting h_is to πD lattice data, $a^{(+)} = -0.54^{+0.19}_{-0.59}$ fm

Conclusions

Summary:

- The long range part of the lattice potentials is well-described by the OPE and scalar-isoscalar 2PE potentials
- The uncertainty band arising from $a^{(+)}$ is large for the intermediate range
- Finite volume corrections (not discussed) are analyzed and found small

Outlook:

- Due to the $B\pi$ bound state, $a^{(+)}$ gets large at $m_\pi = 340$ MeV. Better to use energy dependent $B\pi$ scattering amplitude instead of s.l. approx.
- Other intermediate range contributions are needed to be considered, e.g. the vector-isovector channel, namely the ρ exchange

Thanks for your attention!

Back Up Slides

Finite volume corrections

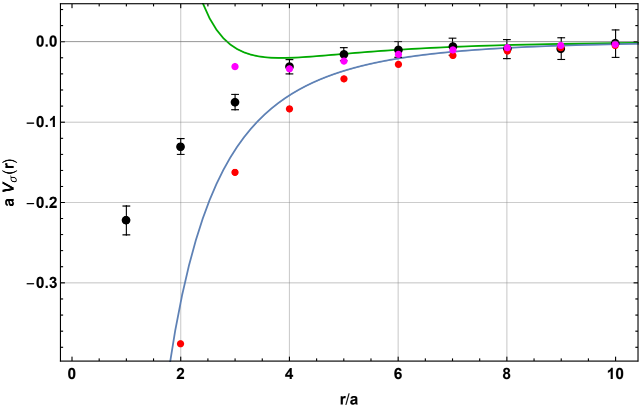
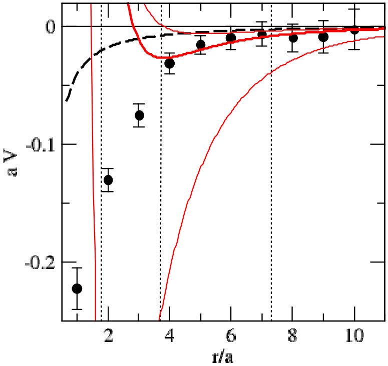


Figure: Full sigma exchange potential with finite volume corrections.

Final Results



Lattice Convention (New)

The lattice potentials which we aim to compare are in basis of the spin of the light d-o-f, j_l. The corresponding basis in terms of $B^{(*)}B^{(*)}$ mesons are

$$|s_b = 1, j_l = 0, J = 1\rangle = \frac{1}{2}\big(|B^*B\rangle - |BB^*\rangle\big) + \frac{1}{\sqrt{2}}|B^*B^*\rangle \tag{13}$$

s_b spin of the bottom quarks pair, J is the total momentum of the state. For the Isoscalar OPE interaction, utilizing the relation

$$|B^*B\rangle_{I=0} = \frac{1}{\sqrt{2}}\Big(|B^{*+}B^-\rangle - |B^{*-}B^+\rangle\Big) \tag{14}$$

$$|s_b = 1, j_l = 0, J = 1\rangle = \frac{1}{\sqrt{2}}\Big(|B^*B\rangle_{I=0} + |B^*B^*\rangle_{I=0}\Big) \tag{15}$$

The OPE in this channel is

$$V_\pi = \frac{1}{2}V_\pi(B^*B)_{I=0} + \frac{1}{2}V_\pi(B^*B^*)_{I=0} \tag{16}$$