Gauge dependence of $c\bar{c}$ potential from Nambu-Bethe-Salpeter wave function in Lattice QCD

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Lattice 2024, Liverpool, 2024/08/02

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Previous works on extracting $q\bar{q}$ potential

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✦ **Phenomenology**:
Cornell potential → $V(r) = -A/r + \sigma r + V_0$

✓ Heavy quarks with finite mass
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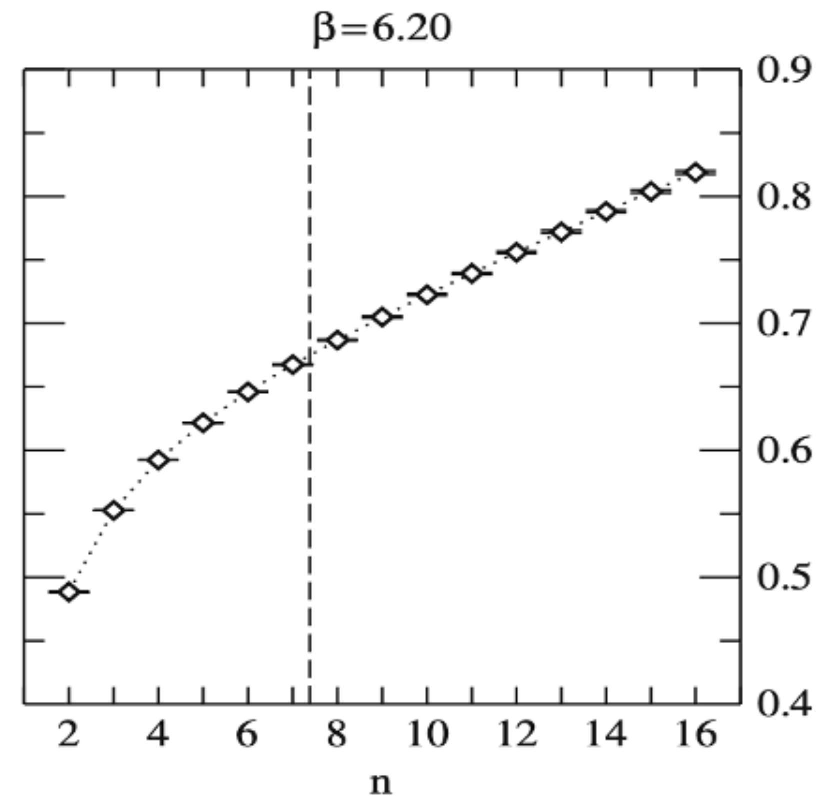

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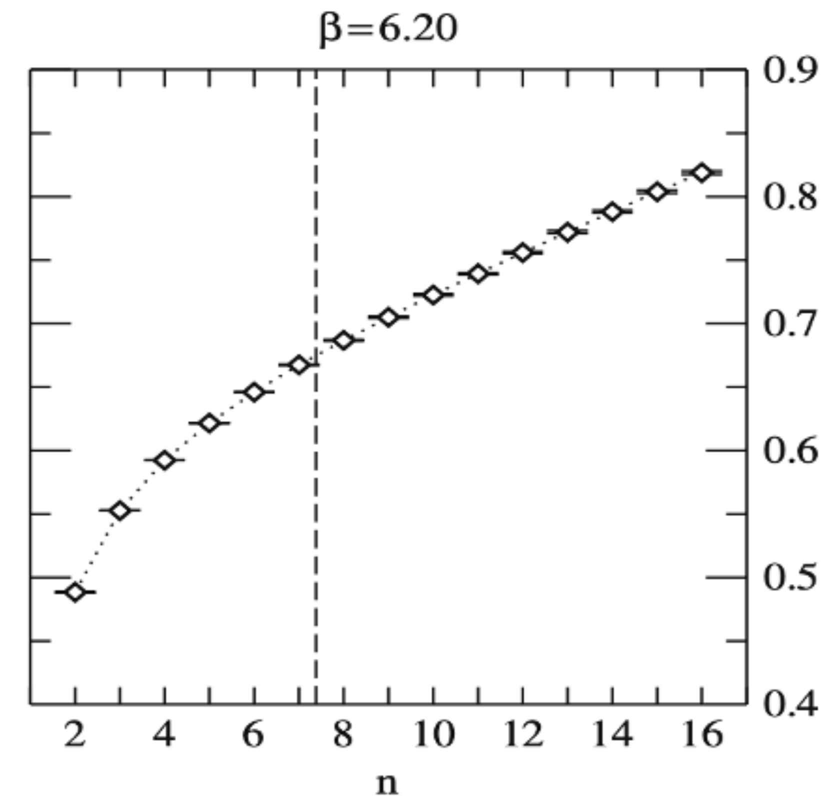

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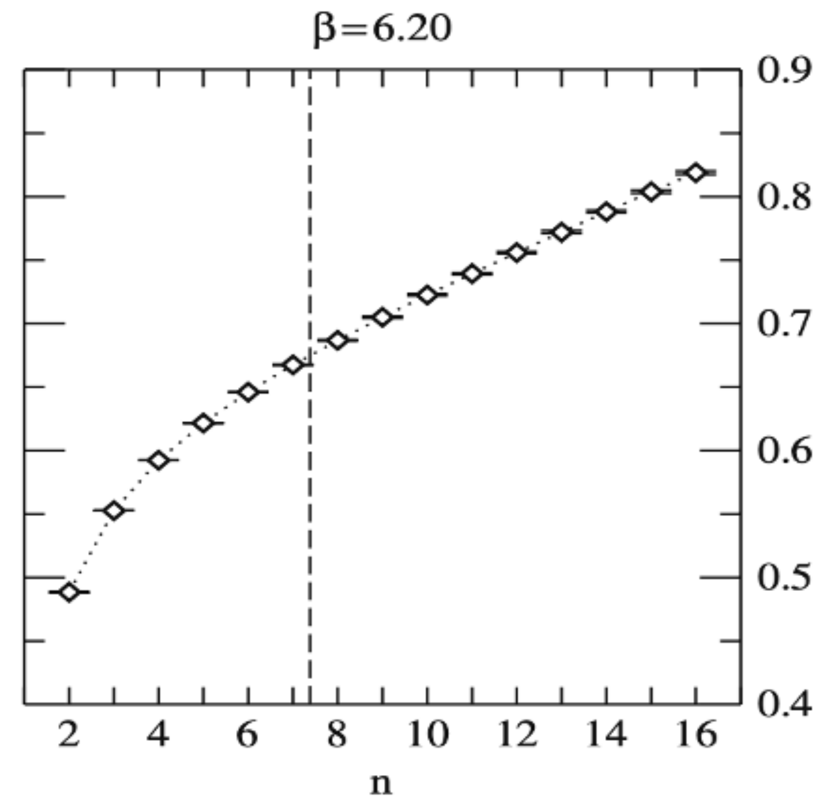

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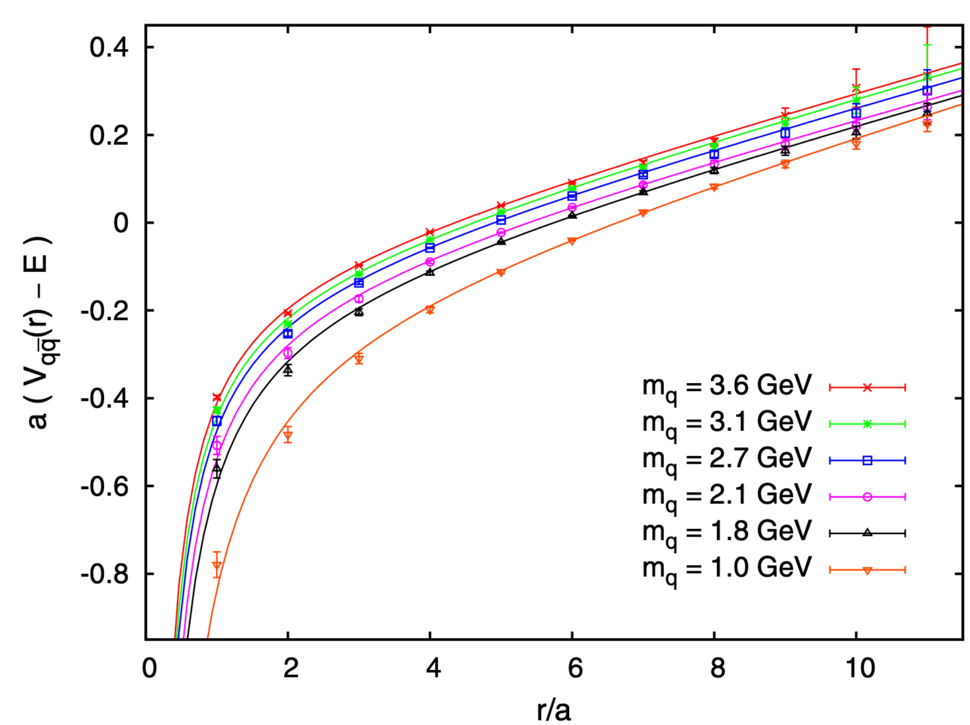
✦ ***NBS amplitude approach***

[Cf.] Kawanai, Sasaki, Phys. Rev. Lett. 107(9):091601, 2011

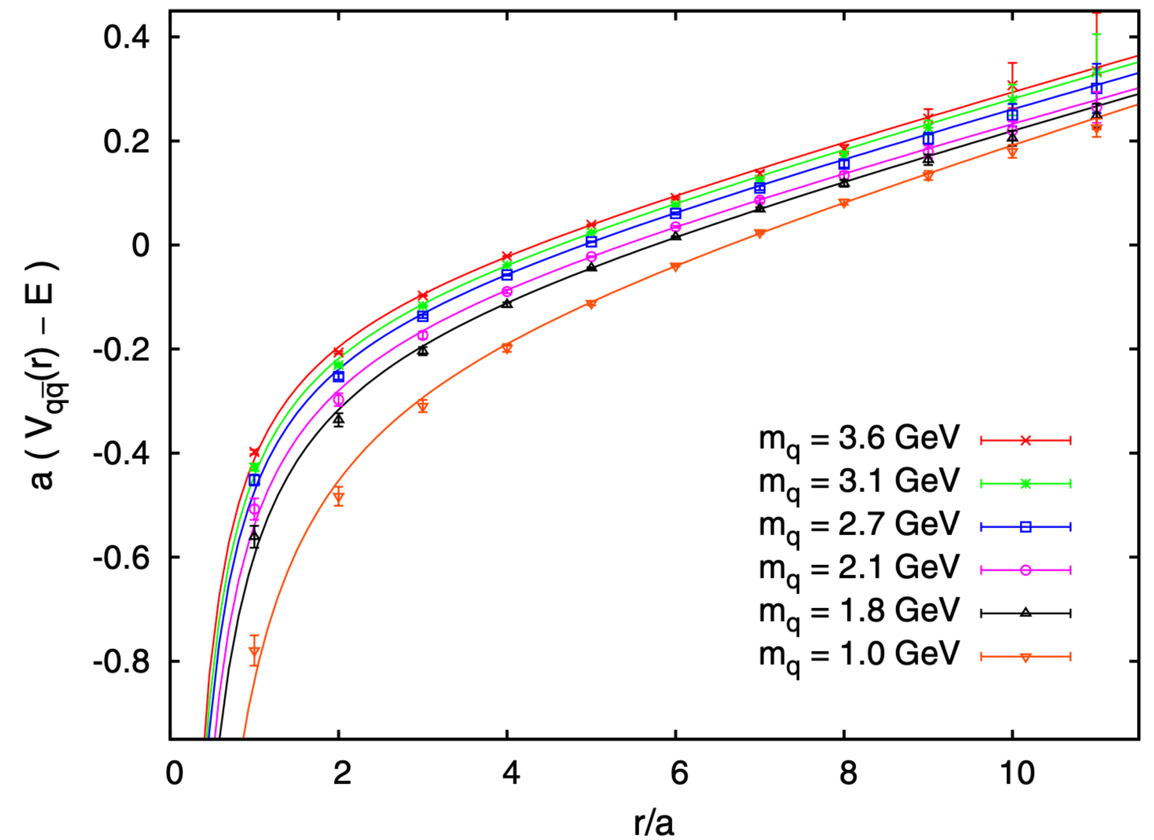
✓ Quarks with finite mass (~ charm quarks)



Motivation

More on NBS amplitude approach

- ✓ Originated from HAL QCD method, modified for the $q\bar{q}$ system
- ✓ Quark mass → ∞: Potential smoothly approaches the Wilson loop result
- ✓ Quark mass ~ charm quark mass m_c: Cornell type potential reproduced



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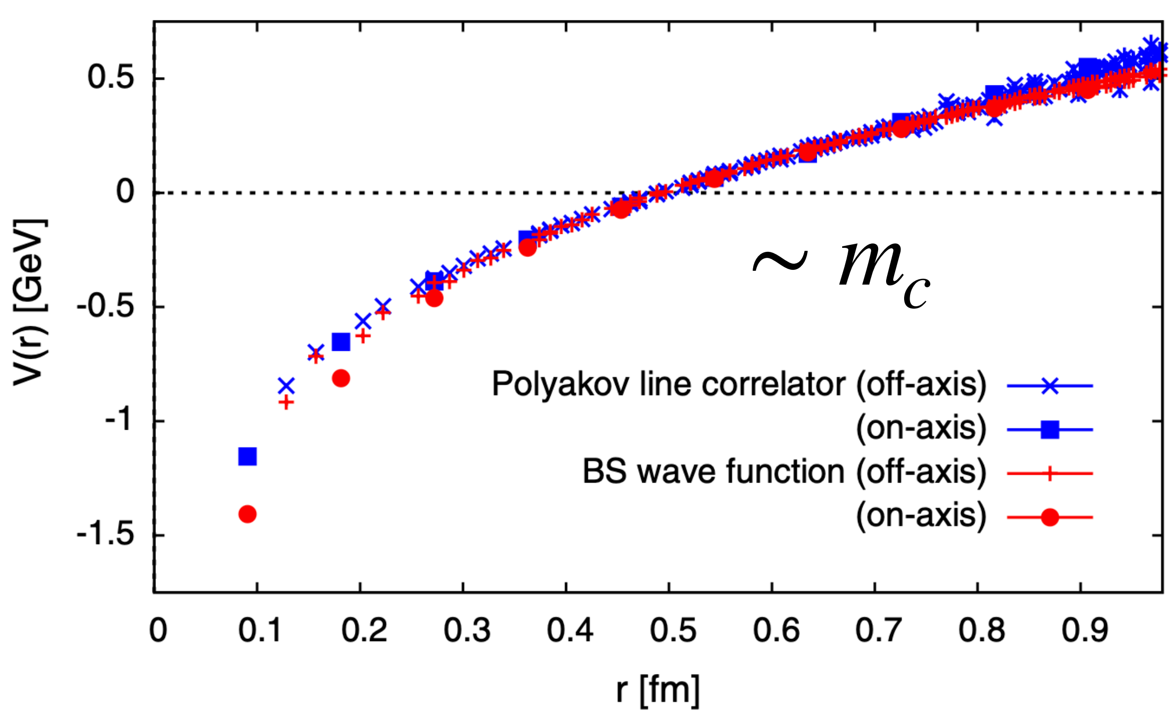

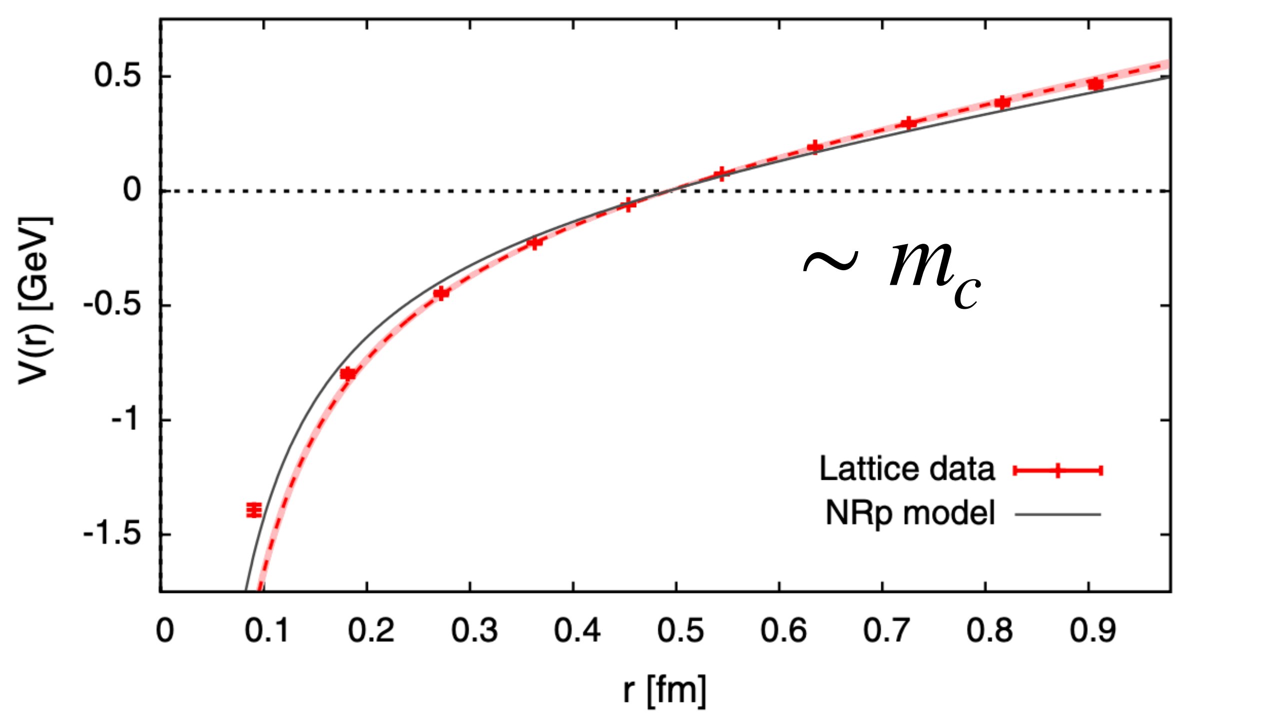

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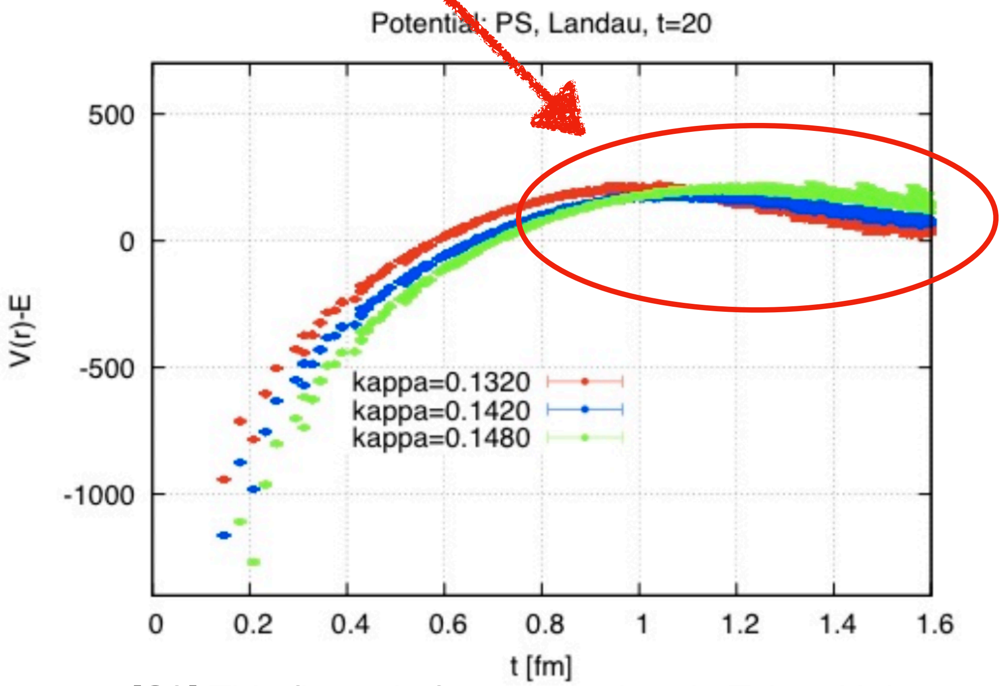
Long-distance behavior of the $q\bar{q}$ potential is **destroyed** in the Landau gauge



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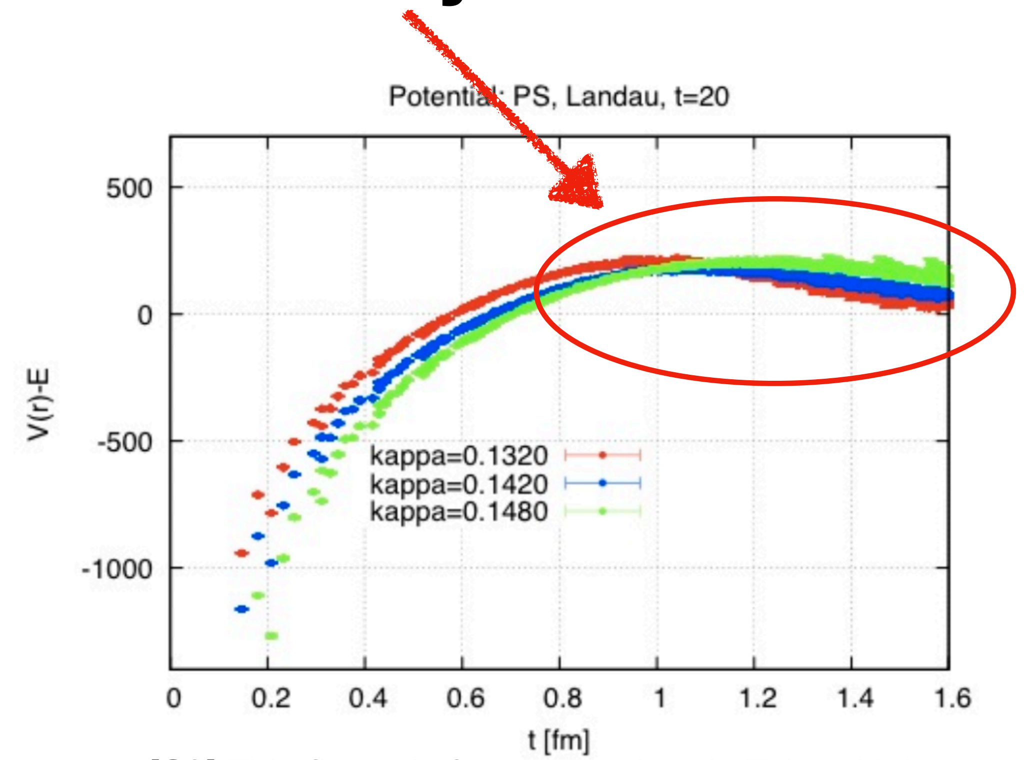

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  - ✓ They extracted them by directly fitting the 2-point correlator in the Landau gauge

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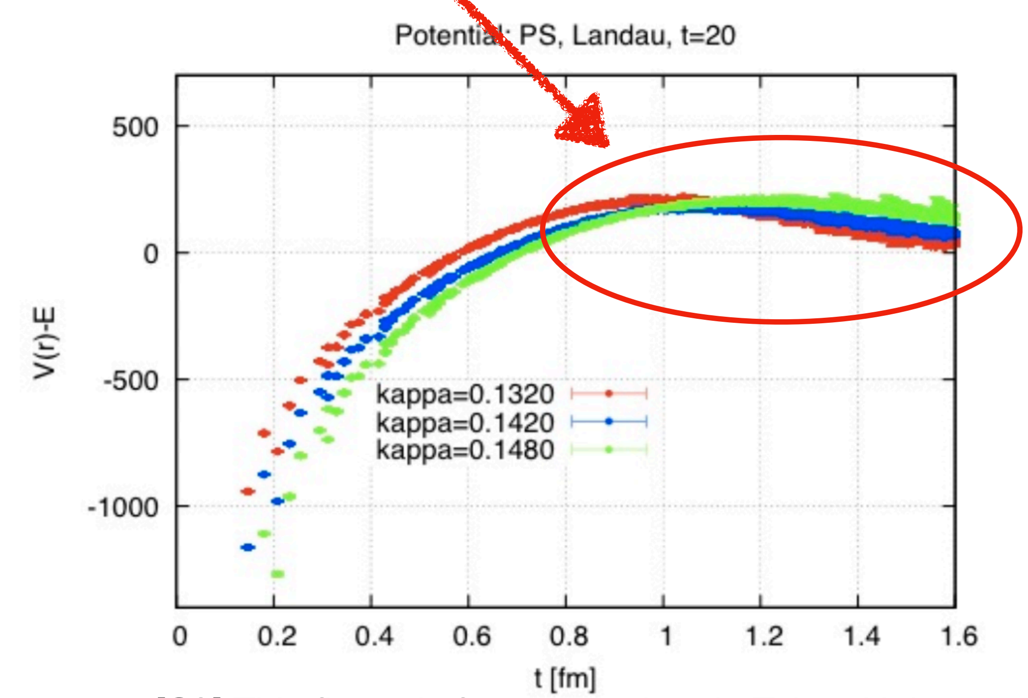

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✦ Renormalizability of the Coulomb gauge is controversial

Coulomb gauge	⟺	**Landau gauge**
Non-renormalizable ⇩ · Existence of the continuum limit? [Cf.] Y.Nakagawa et al., Phys. Rev. D 79, 114504		Renormalizable ⇩ Smooth continuum limit for $q\bar{q}$ potentials

NBS Amplitude Method

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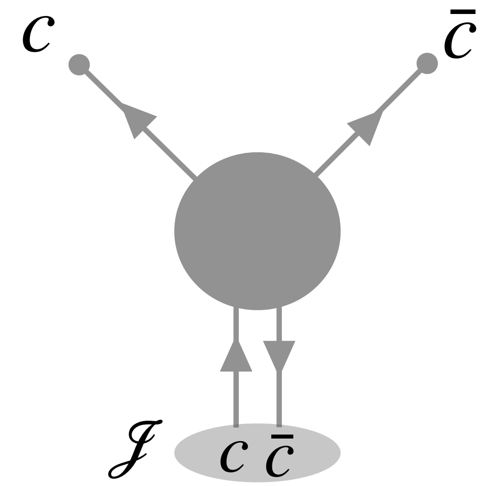
- ✦ **4-point correlator** and **NBS wave function**

NBS Amplitude Method



✦ **4-point correlator** and **NBS wave function**

source

$$C(\mathbf{x}-\mathbf{y},t) = \langle 0 \,|\, T\{\bar{c}(\mathbf{x},t)\Gamma c(\mathbf{y},t)\cdot \mathcal{J}(t=0)\}\,|\,0\rangle$$

$$= \sum_{n=0}^{\infty} a_n \phi_n(\mathbf{x}-\mathbf{y}) e^{-E_n t} \sim a_0 \phi_0(\mathbf{x}-\mathbf{y}) e^{-E_0 t}$$

for large t

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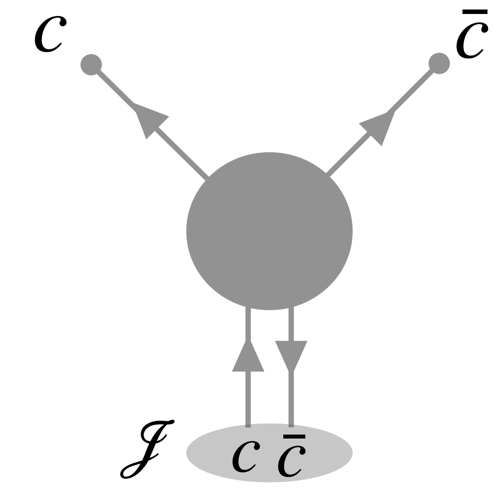

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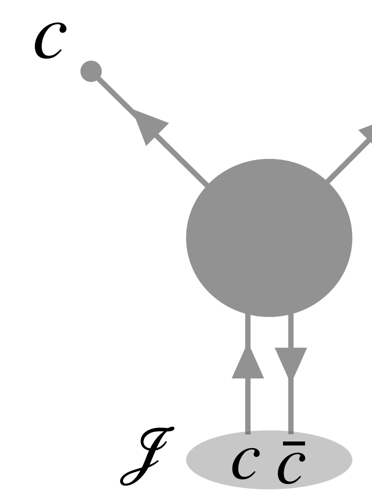

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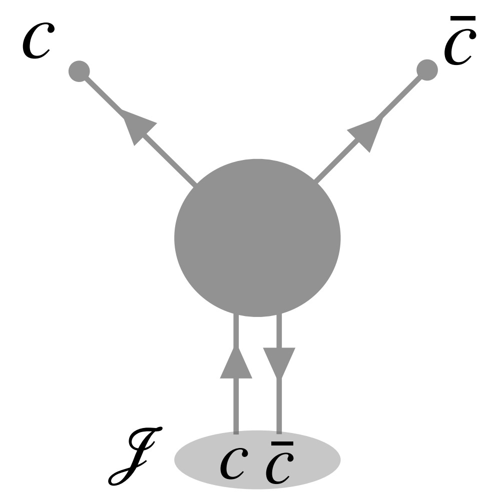
To eliminate the excited state contamination

Ground state saturation is important!

NBS Amplitude Method



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"A_1^+ projection"

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- Schrödinger equation is demanded to be satisfied as

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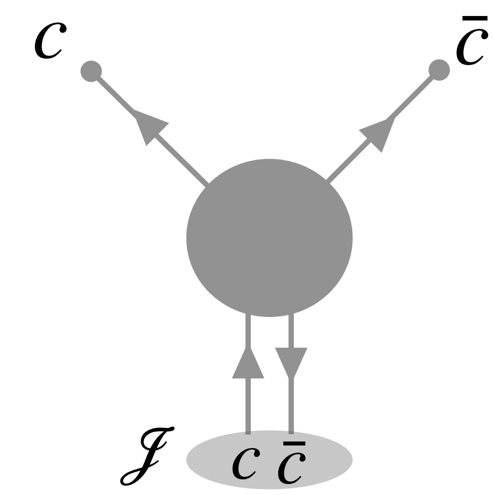

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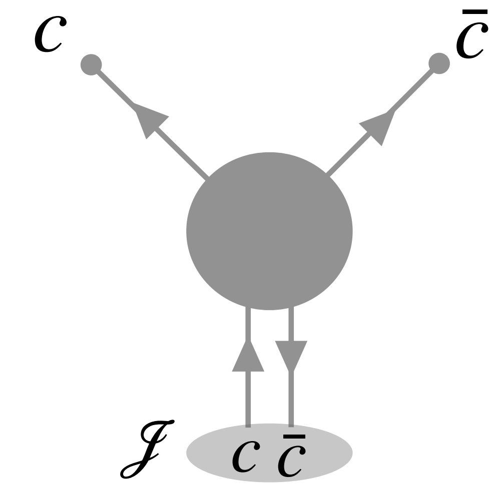

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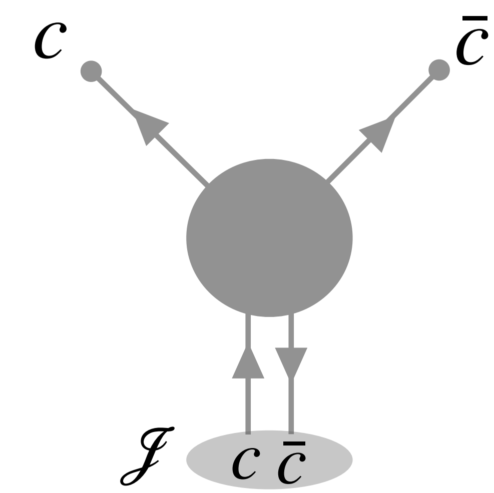

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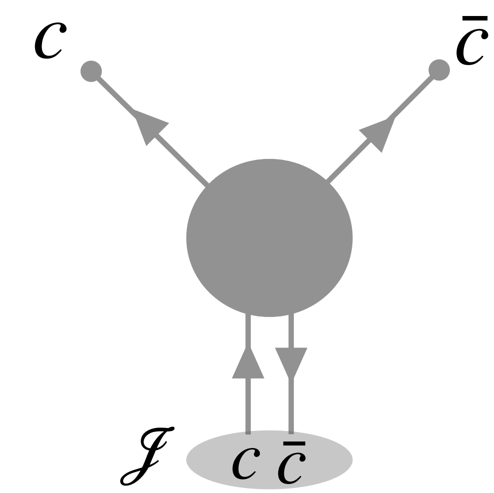

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- ✓ m_c: Charm quark mass (determined later by Kawanai-Sasaki condition)
- ✓ $\hat{V}(r) \equiv V_0(r) + V_s(r)\,\mathbf{s}_1\cdot\mathbf{s}_2 + \cdots$: $c\bar{c}$ potential

$$s_1\cdot s_2 = \begin{cases} -\frac{3}{4} & \text{(PS channel)} \\ \frac{1}{4} & \text{(V channel)} \end{cases}$$

NBS Amplitude Method

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✦ Schrödinger eqs for **PS** and **V:**

$$(M_{PS} - 2m_c)\phi_{PS}(r) = \left[-\frac{\nabla^2}{m_c} + V_0(r) - \frac{3}{4}V_{\rm s}(r)\right]\phi_{PS}(r)$$

$$(M_V - 2m_c)\phi_V(r) = \left[-\frac{\nabla^2}{m_c} + V_0(r) + \frac{1}{4}V_{\rm s}(r)\right]\phi_V(r)$$

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✦ by inversely solving them, the potentials are obtained as

$$V_0(r) = \frac{1}{4m_c}\left[3\frac{\nabla^2\phi_V(r)}{\phi_V(r)} + \frac{\nabla^2\phi_{PS}(r)}{\phi_{PS}(r)}\right] + \frac{1}{4}(3M_V + M_{PS}) - 2m_c$$

$$V_{\rm s}(r) = \frac{1}{m_c}\left[\frac{\nabla^2\phi_V(r)}{\phi_V(r)} - \frac{\nabla^2\phi_{PS}(r)}{\phi_{PS}(r)}\right] + (M_V - M_{PS})$$

✓ **Note:**

For $c\bar{c}$ system, m_c cannot be obtained from the 2-point correlator due to confinement. (Different from the conventional HAL QCD method.)

Kawanai-Sasaki Condition

✦ To determine the charm quark mass

Kawanai and Sasaki proposed

$$V_s(r) = \frac{1}{m_c}\left[\frac{\nabla^2\phi_V(r)}{\phi_V(r)} - \frac{\nabla^2\phi_{PS}(r)}{\phi_{PS}(r)}\right] + (M_V - M_{PS}) \to 0 \text{ *as* } r \to \text{ *large*}$$

which gives Kawanai-Sasaki condition:

$$m_c = -\lim_{r\to\infty} F_{\mathrm{KS}}(r)$$

$$F_{\mathrm{KS}}(r) \equiv \frac{1}{M_V - M_{PS}}\left[\frac{\nabla^2\phi_V(r)}{\phi_V(r)} - \frac{\nabla^2\phi_{PS}(r)}{\phi_{PS}(r)}\right]$$

Kawanai-Sasaki function

Lattice Setup

✦ Gauge configurations:

- $2+1$ flavor dynamical LQCD configurations on $32^3 \times 64$ lattice
  [Cf.] S. Aoki, et al. (PACS-CS Coll.), Phys. Rev. D79, 034503(2009)
  ➡RG improved Iwasaki gauge action ($\beta = 1.90$)
  ➡O(a)-improved Wilson quark action ($\kappa_{ud} = 0.1378100$, $C_{\mathrm{SW}} = 1.715$)

- Pion mass: $155.7\ \mathrm{MeV}$ (Almost the physical pion mass)

- Spatial size: $L = 2.902(42)\ \mathrm{fm}$

- Lattice cutoff: $a^{-1} \sim 2.1753\ \mathrm{GeV}$ ($a = 0.0907(13)\ \mathrm{fm}$)

✦ Charm quark is introduced by the quenched approximation by using the relativistic heavy quark action
[Cf.] Y. Namekawa, et al. (PACS-CS Coll.), Phys. Rev. D84, 074505(2011)

✦ **Coulomb** and **Landau** gauge fixing

✦ Wall source and point sink for quark propagators

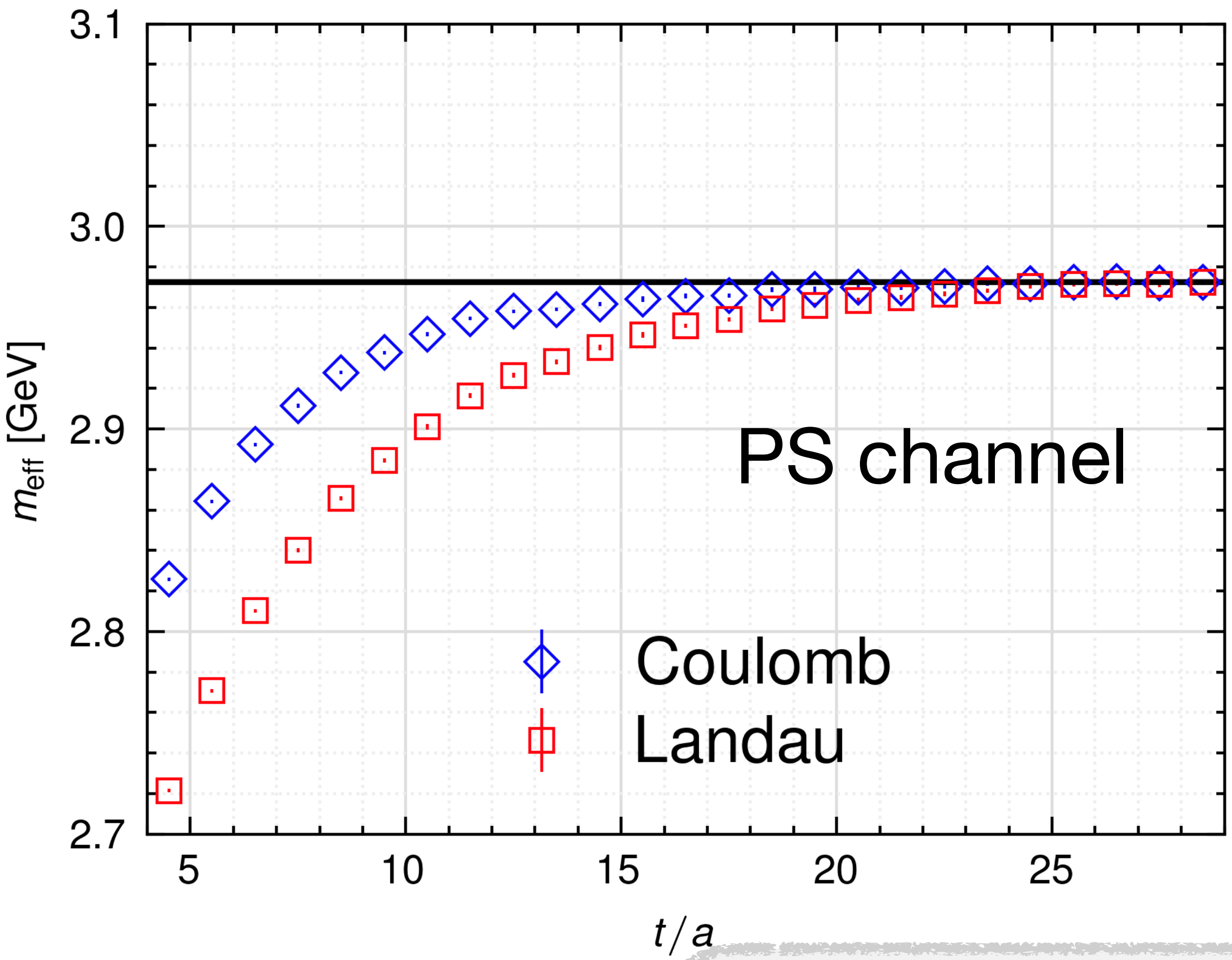
✦ Number of source points: 64

Results

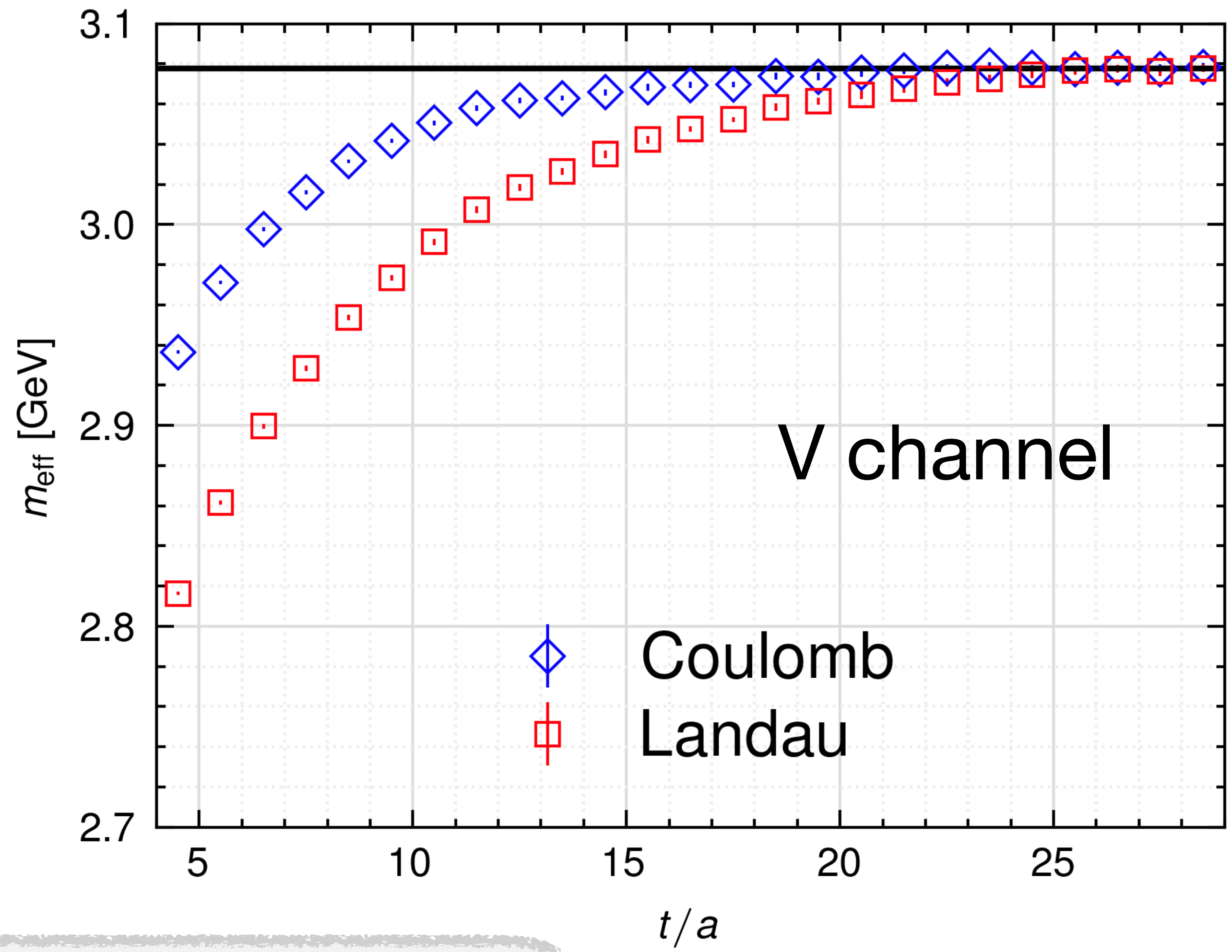
$C(t) = \langle 0 | T\{\underbrace{\bar{c}(t)\Gamma c(t)}_{\text{point sink}} \cdot \underbrace{\mathcal{J}(t=0)}_{\text{wall source}}\} | 0 \rangle \sim A e^{-E_\Gamma t}$ (for large t)

1. $c\bar{c}$ meson masses

$M_{\rm PS} = 2972.5 \pm 0.2$ MeV



$M_{\rm V} = 3077.7 \pm 0.4$ MeV

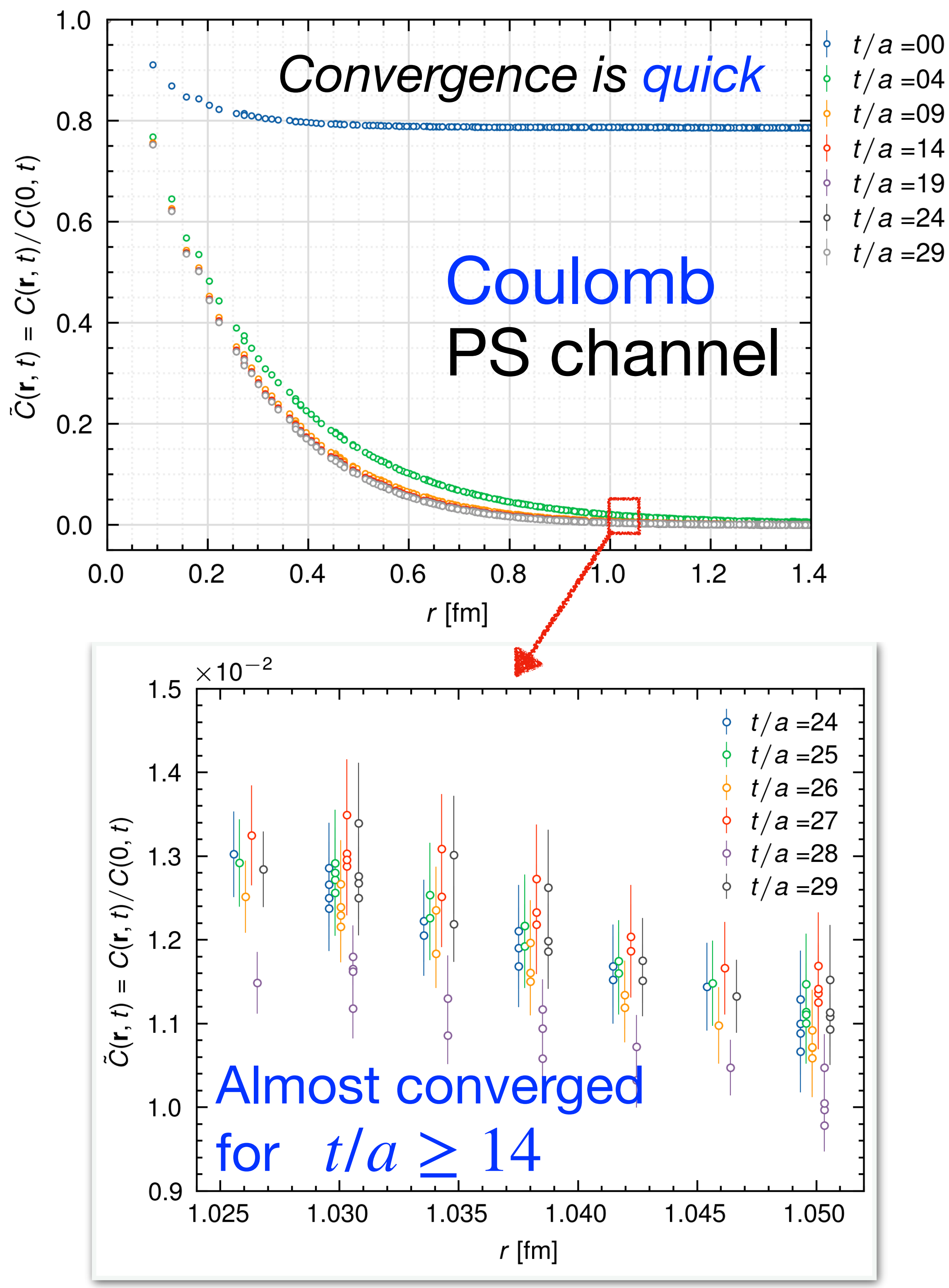


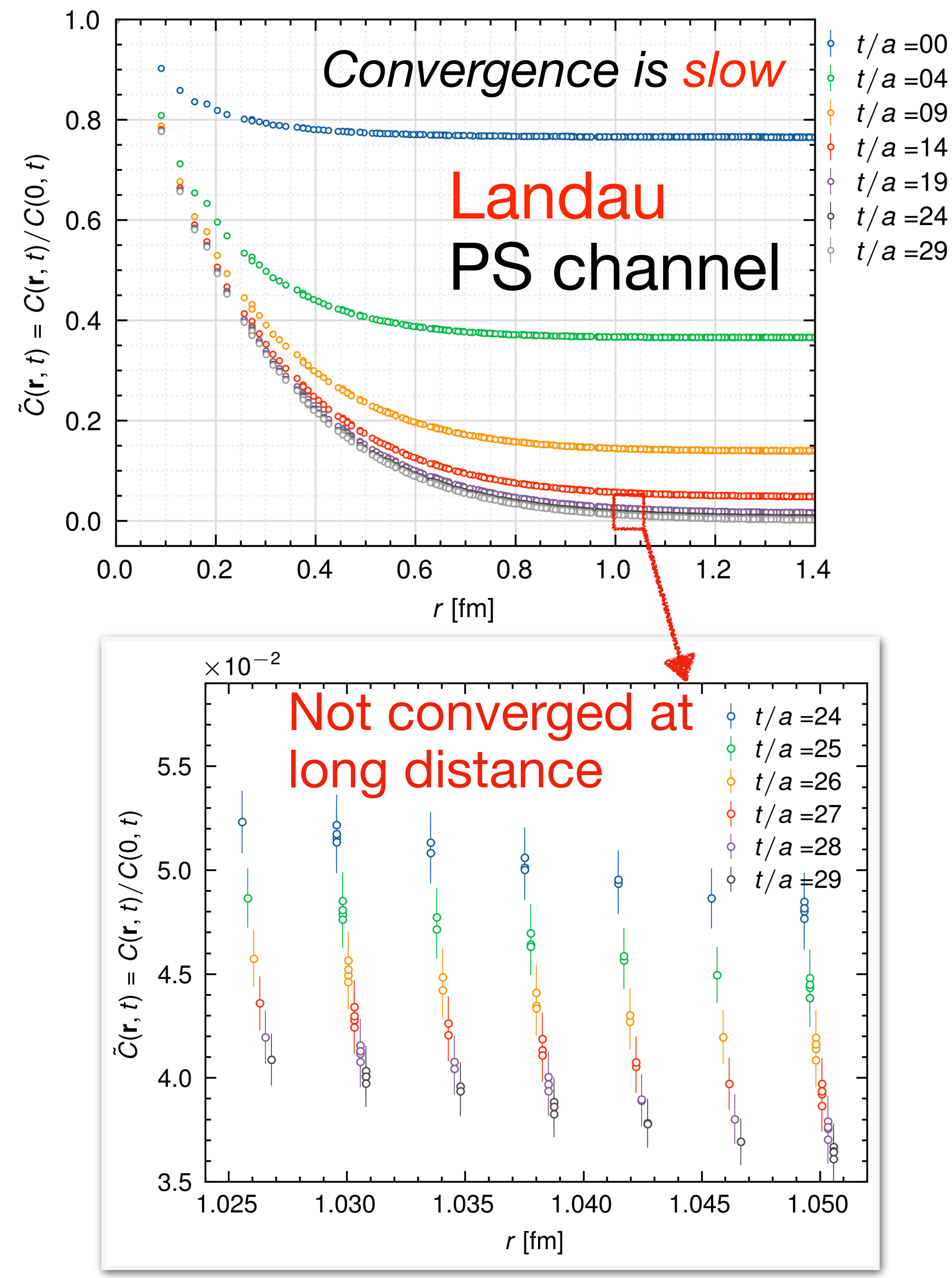
Ground state saturation:
quicker for the Coulomb gauge
slower for the Landau gauge

Results

$$C(\mathbf{x}-\mathbf{y},t) = \sum_n a_n\phi_n(\mathbf{x}-\mathbf{y},t)e^{-E_n t} \sim a_0\phi_0(\mathbf{x}-\mathbf{y})e^{-E_0 t} \text{ (for large } t\text{)}$$

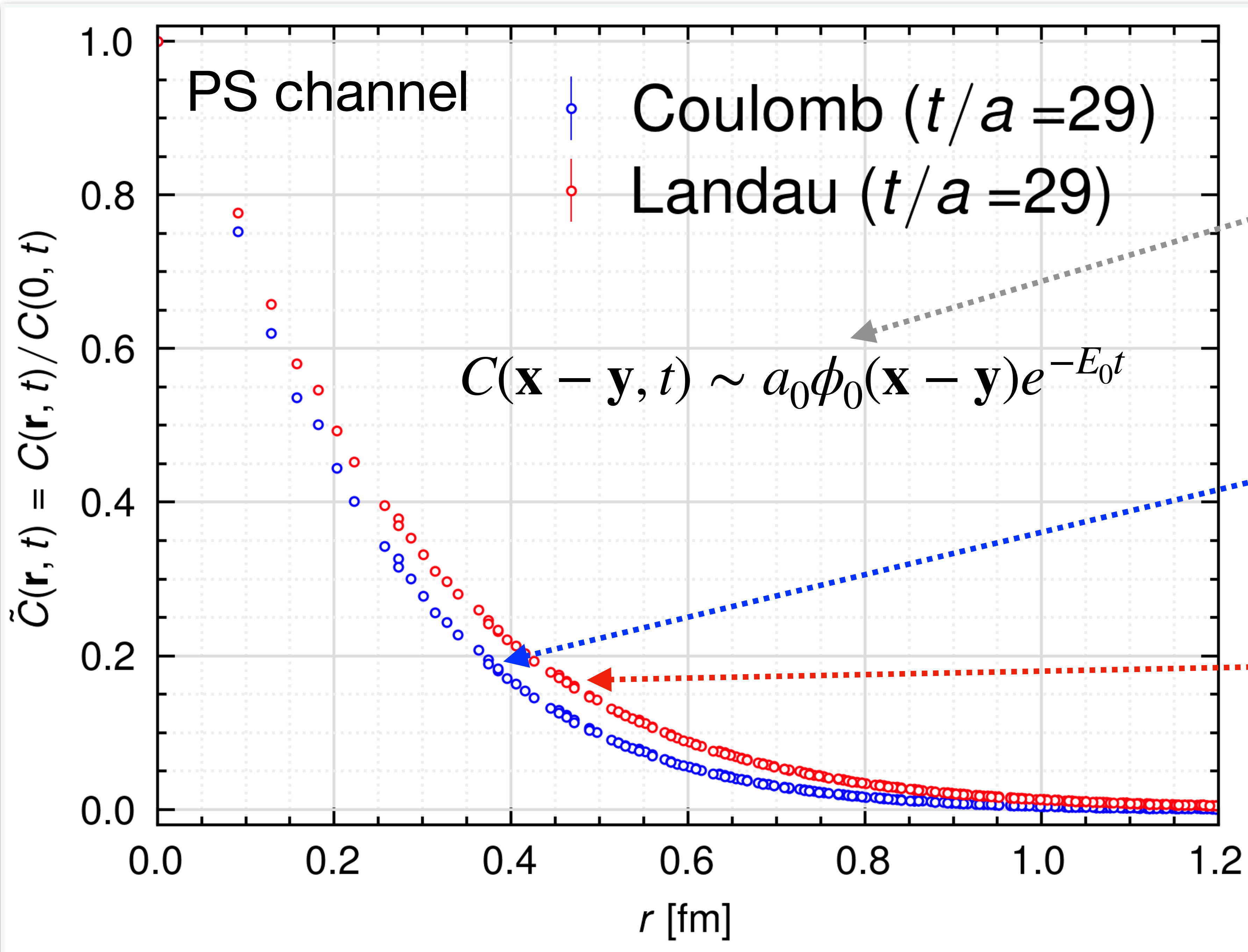
2. Behavior of the 4-point correlators





Results

3. Four-point correlators at large t



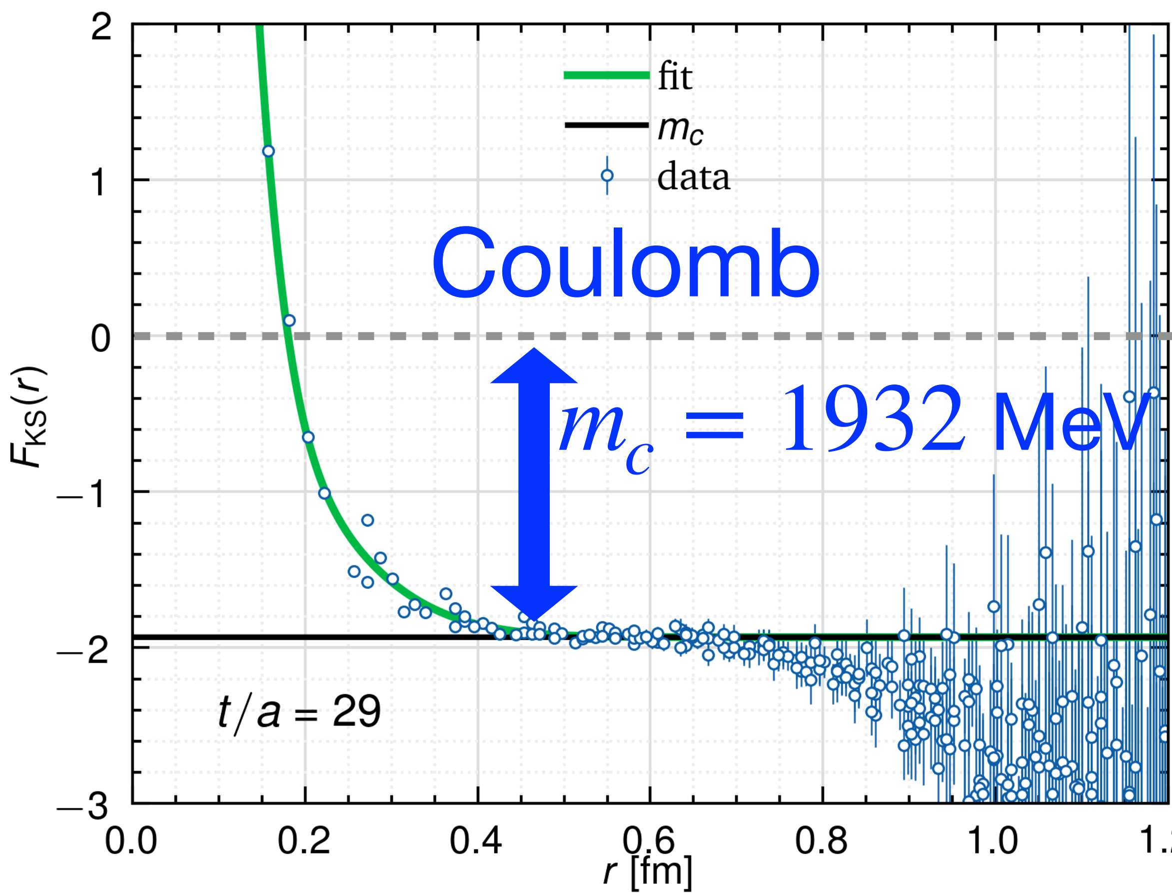
NBS wave function
$\phi_0(\mathbf{x}-\mathbf{y})$

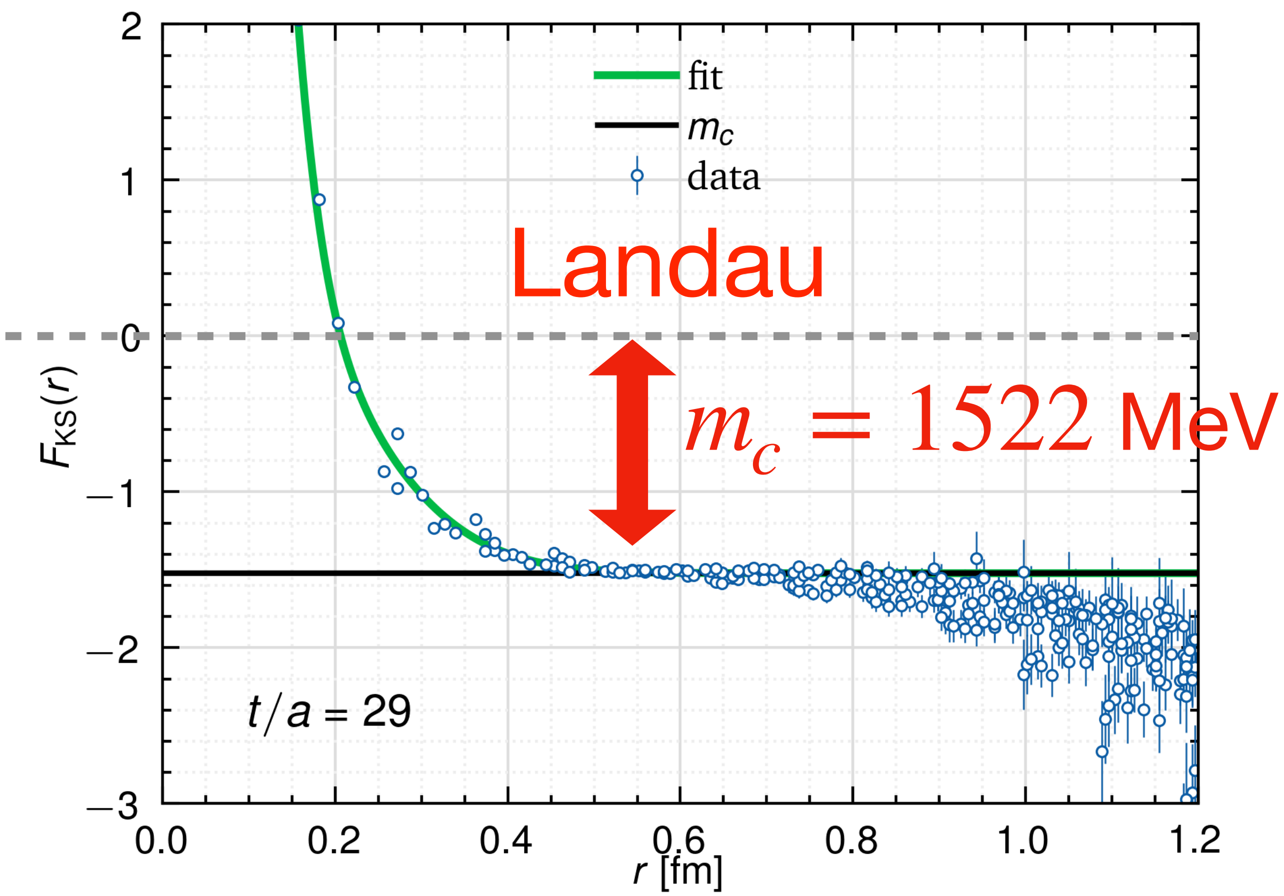
Narrower for the Coulomb gauge

Wider for the Landau gauge

Results

4. Kawanai-Sasaki function





Lighter m_c for the Landau gauge!

Kawanai-Sasaki Condition:

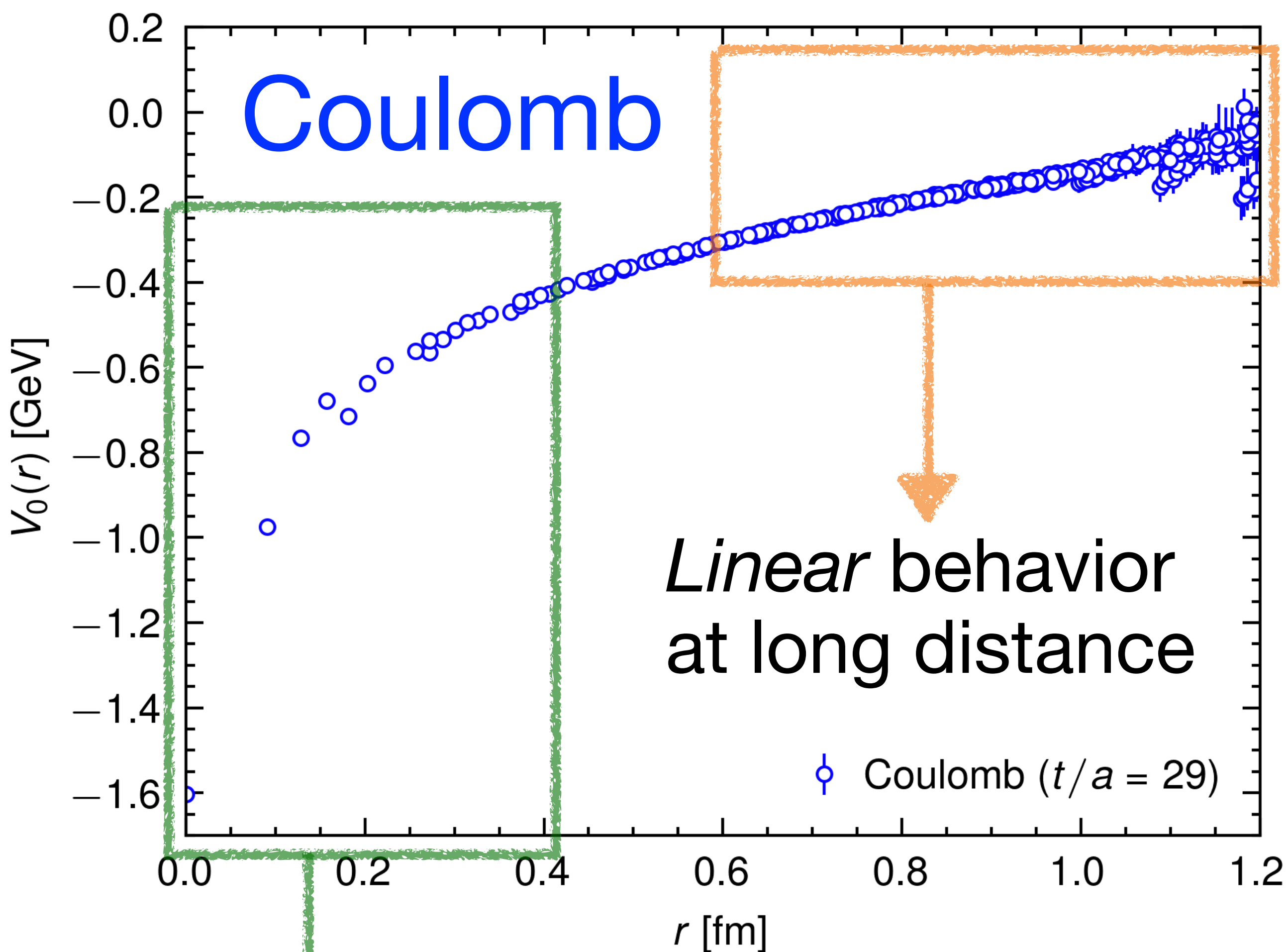
$$m_c = -\lim_{r\to\infty} F_{\mathrm{KS}}(r)$$

Kawanai-Sasaki Function:

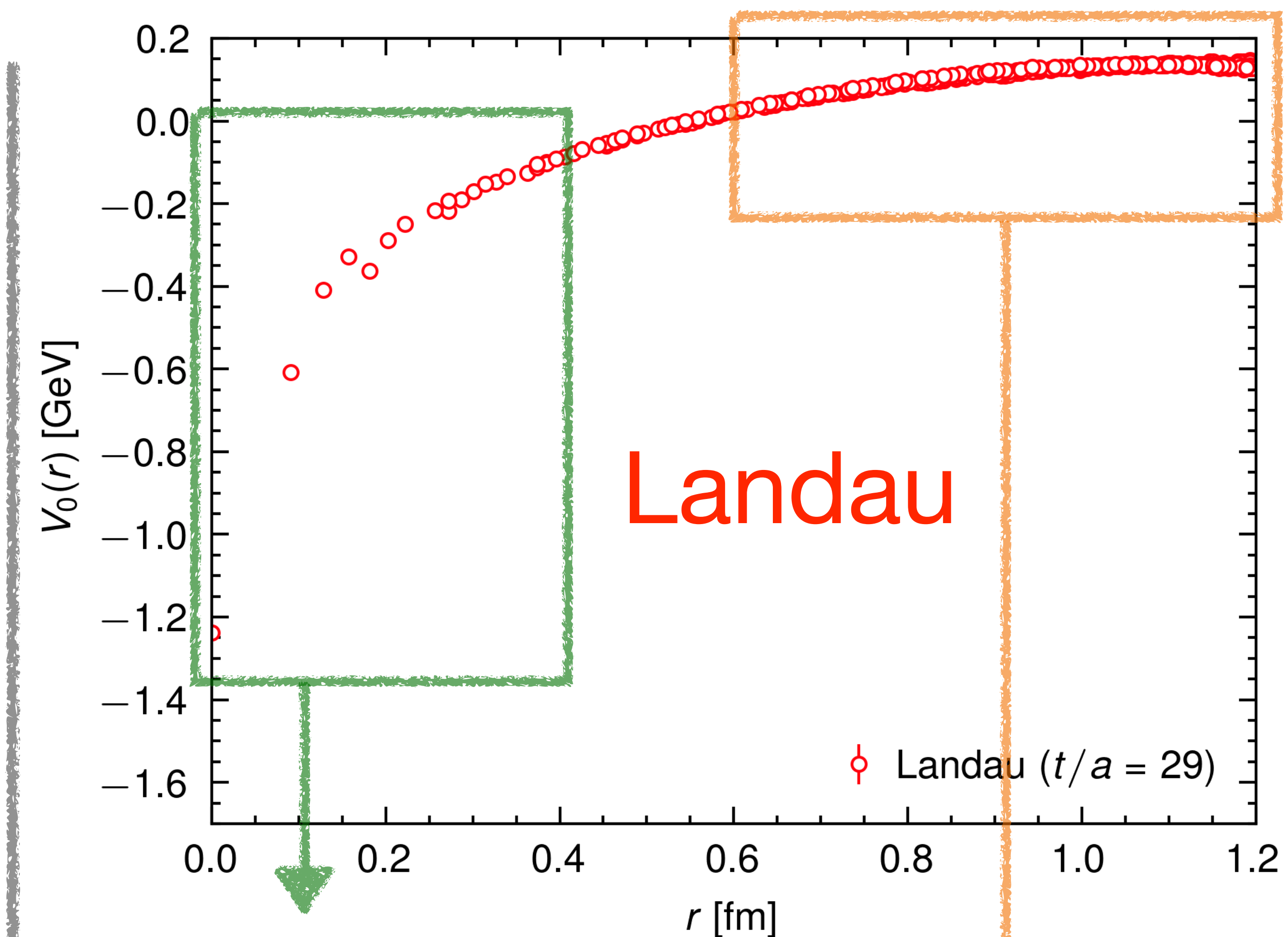
$$F_{\mathrm{KS}}(\mathbf{r}) \equiv \frac{1}{M_V - M_{PS}}\left[\frac{\nabla^2\phi_V(\mathbf{r})}{\phi_V(\mathbf{r})} - \frac{\nabla^2\phi_{PS}(\mathbf{r})}{\phi_{PS}(\mathbf{r})}\right]$$

Results

5. Spin-indep. potential $V_0(r)$



Coulomb-like behavior
at short distance "$-\frac{A}{r}$"



Coulomb-like behavior
at short distance "$-\frac{A}{r}$"

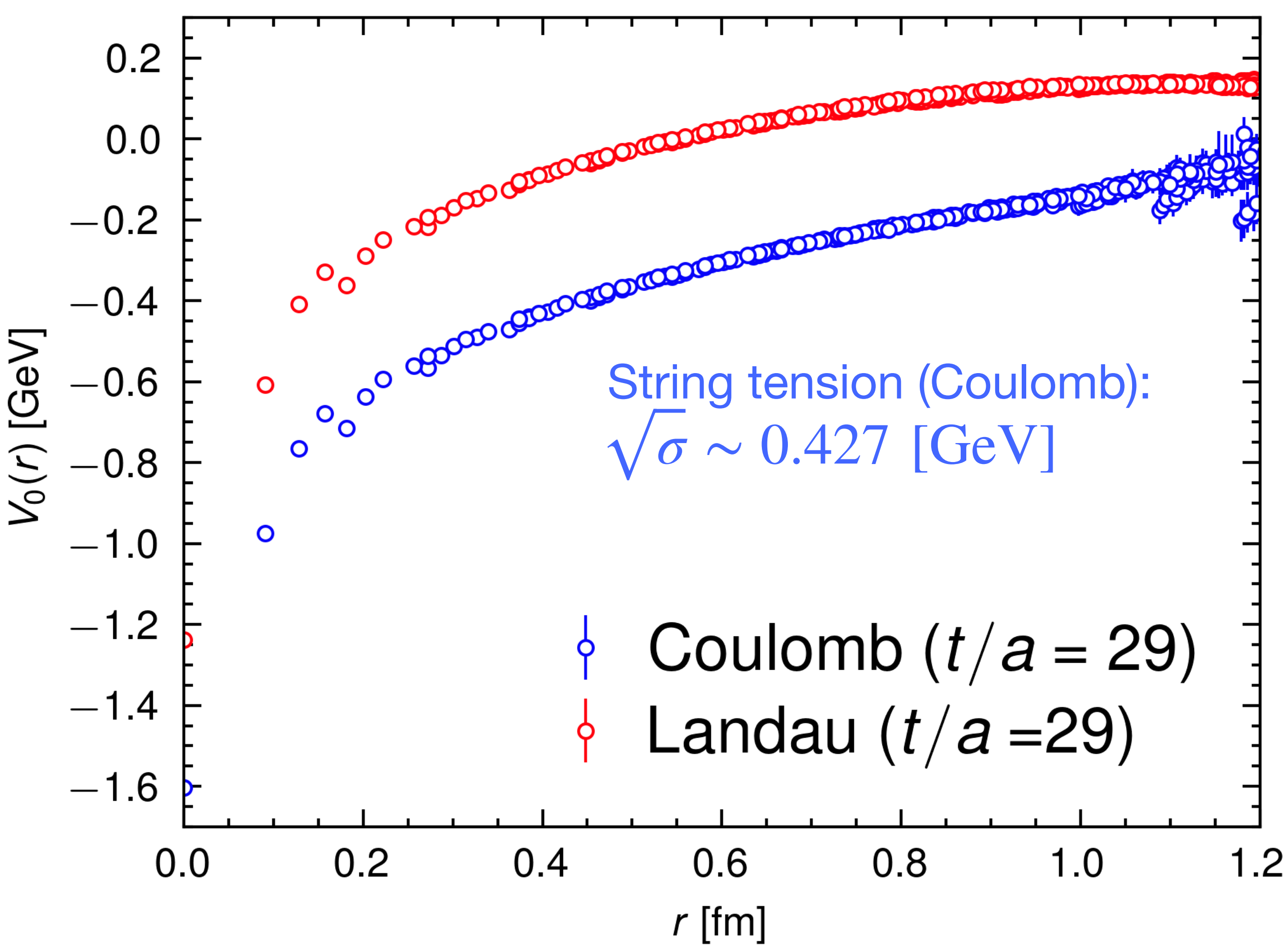
Deviation from linear behavior
is found at long distance

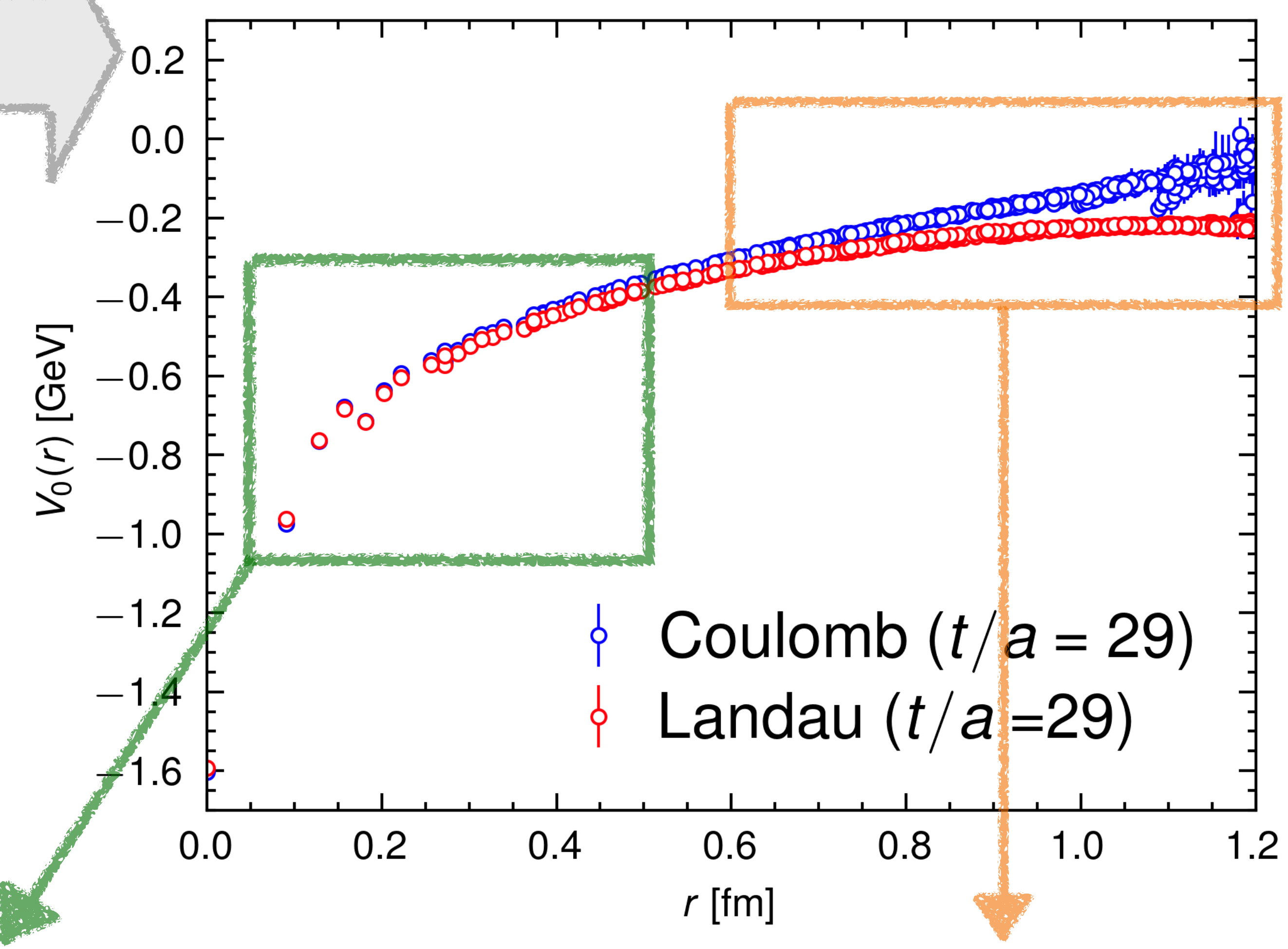
Insufficient ground state saturation!

Results

5. Spin-indep. potential $V_0(r)$

shifted ~ 400 MeV down vertically
for the Landau gauge





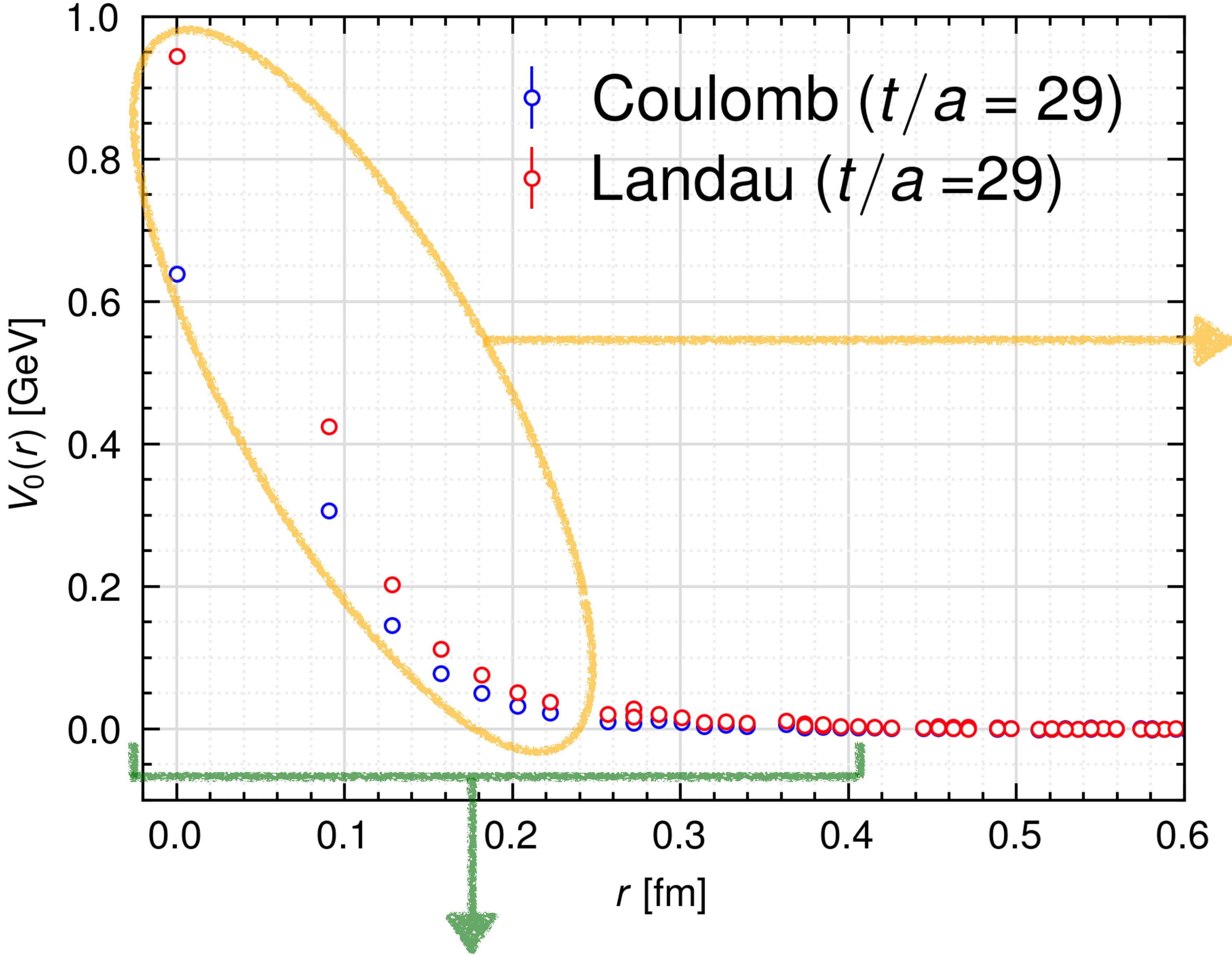
Agreement is very good
at short distance

Deviation from linear behavior
is found at long distance

Insufficient ground state saturation!

Results

5. Spin-dep. potential $V_s(r)$

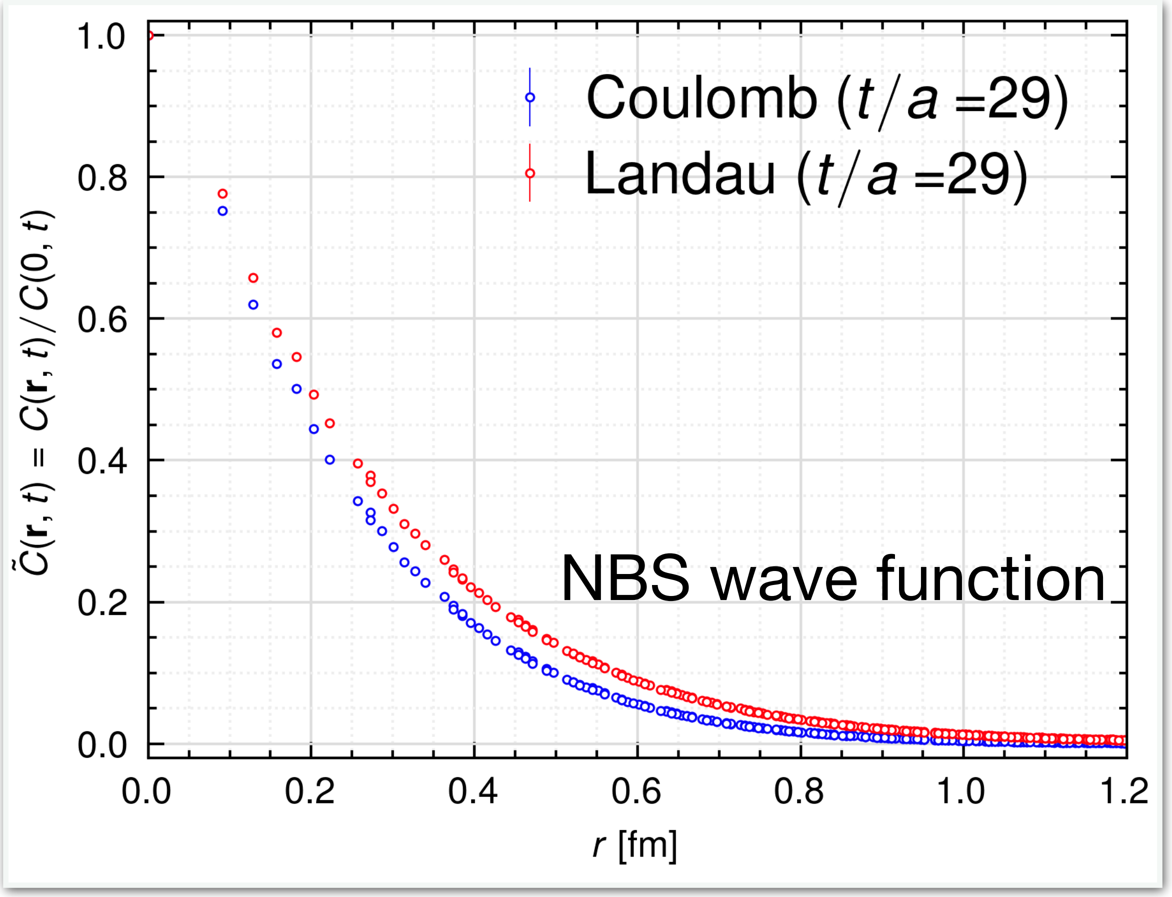


Stronger $V_s(r)$ for the Landau gauge

Short-ranged $V_s(r)$ is obtained (smeared δ-function type)

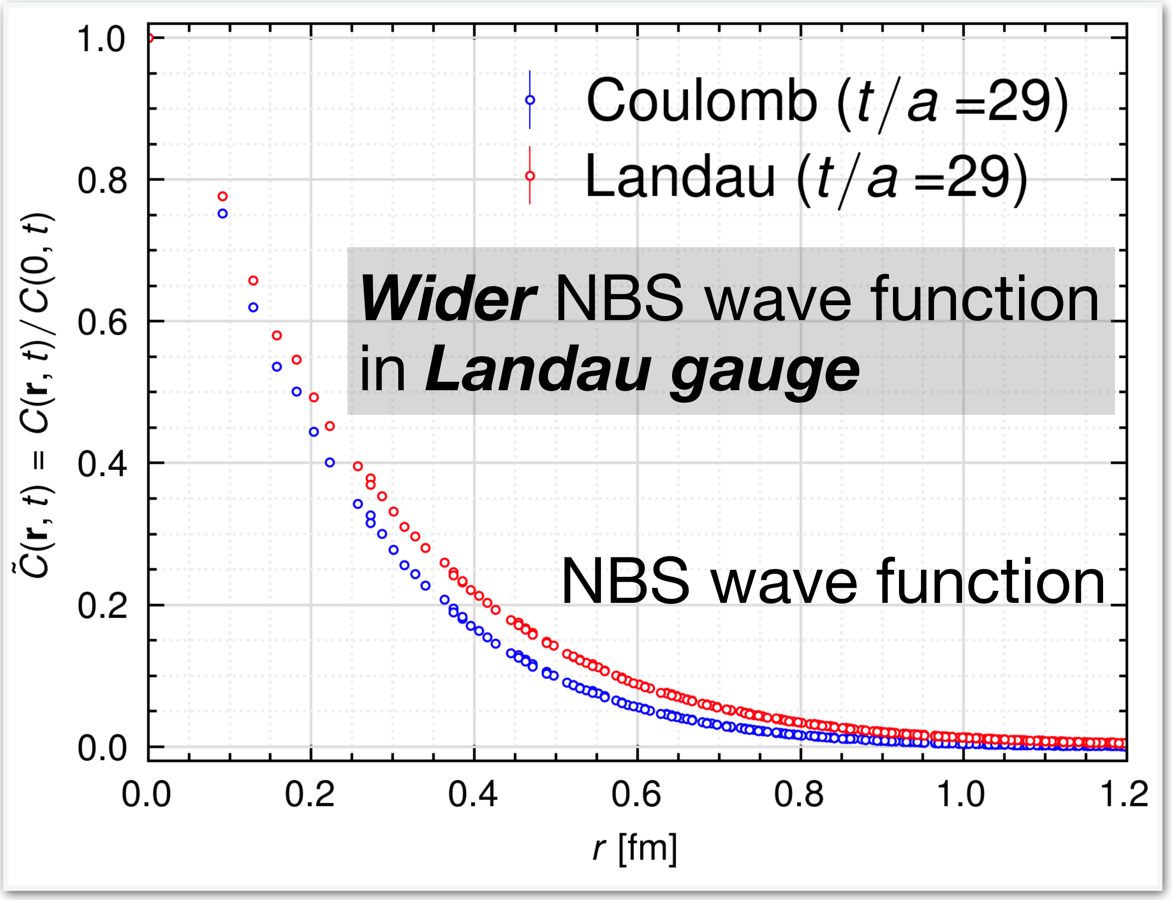
Discussion

Discussion



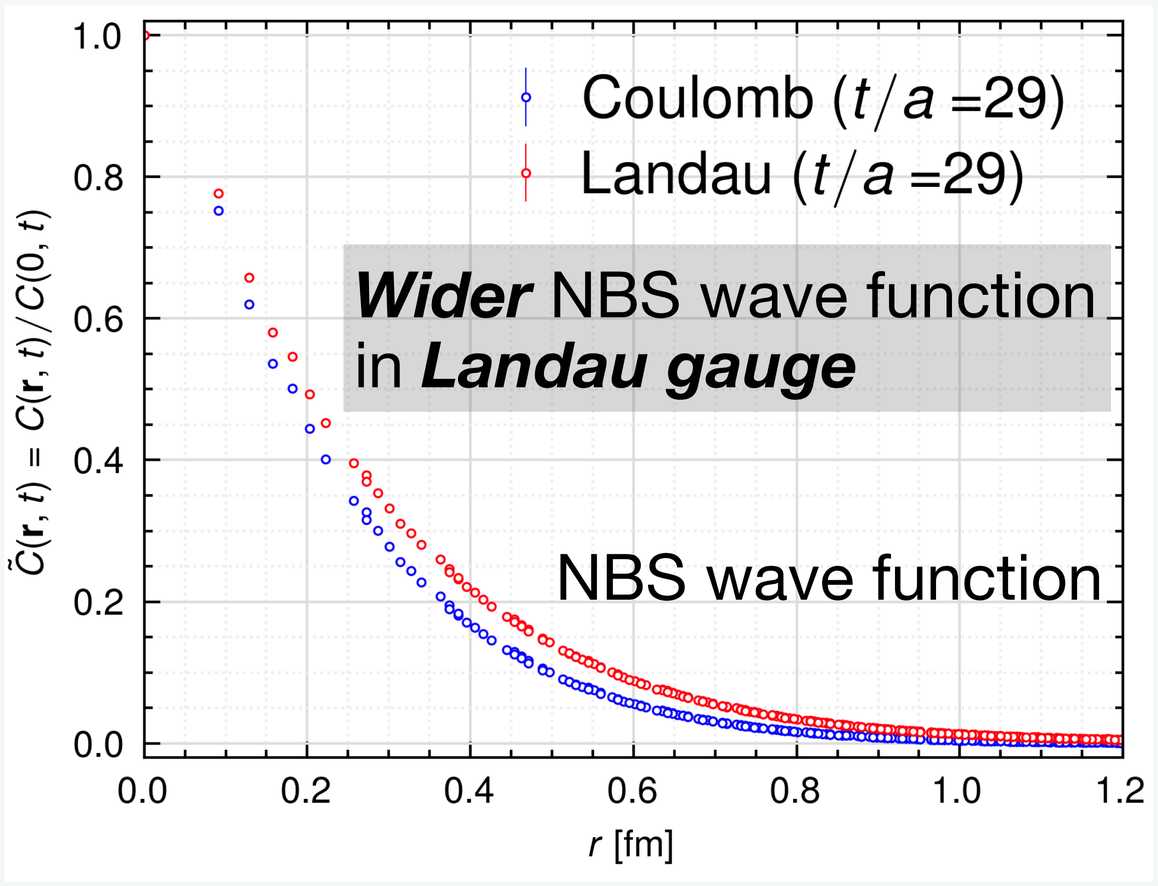
Coulomb (t/a =29)
Landau (t/a =29)
NBS wave function
$\tilde{C}(\mathbf{r}, t) = C(\mathbf{r}, t)/C(0, t)$
r [fm]
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2

Discussion

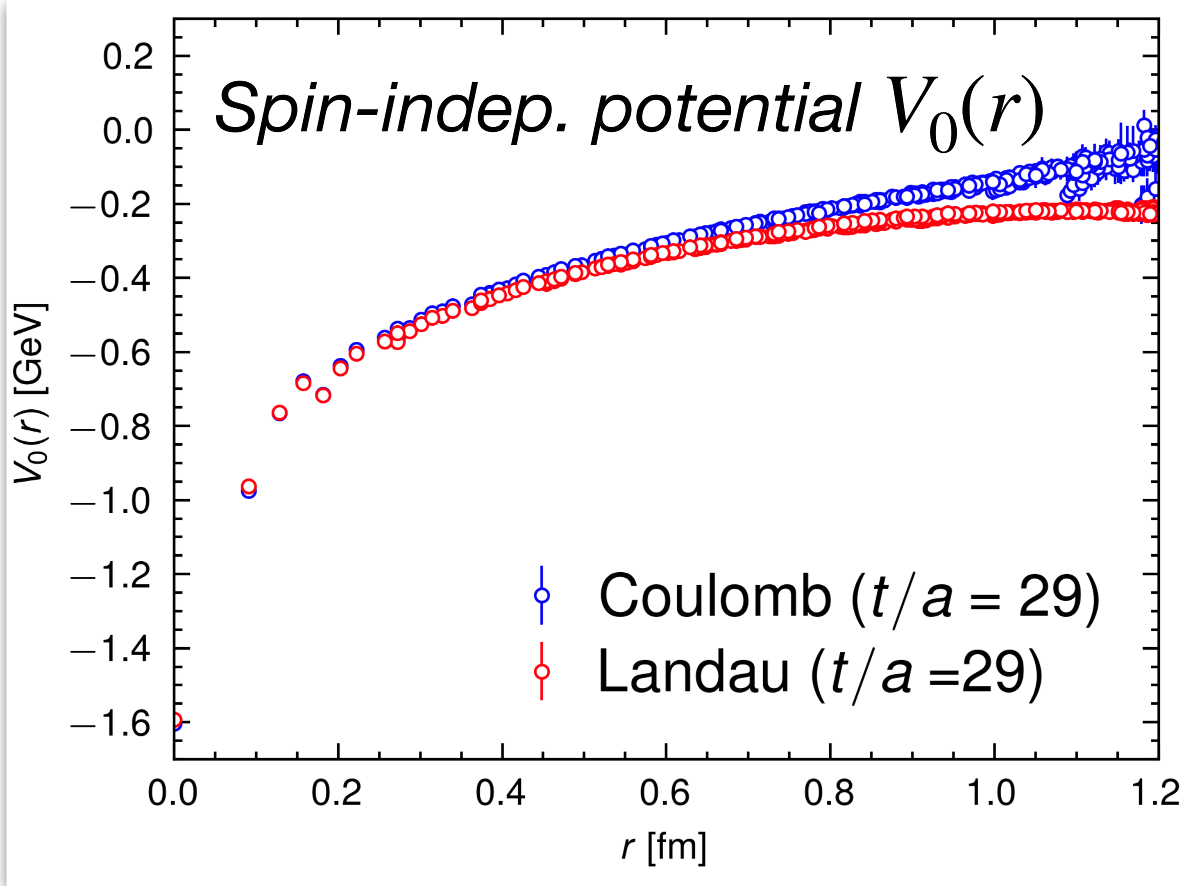


Coulomb (t/a =29)
Landau (t/a =29)
Wider NBS wave function in ***Landau gauge***
NBS wave function
$\tilde{C}(\mathbf{r}, t) = C(\mathbf{r}, t)/C(0, t)$
r [fm]

Discussion

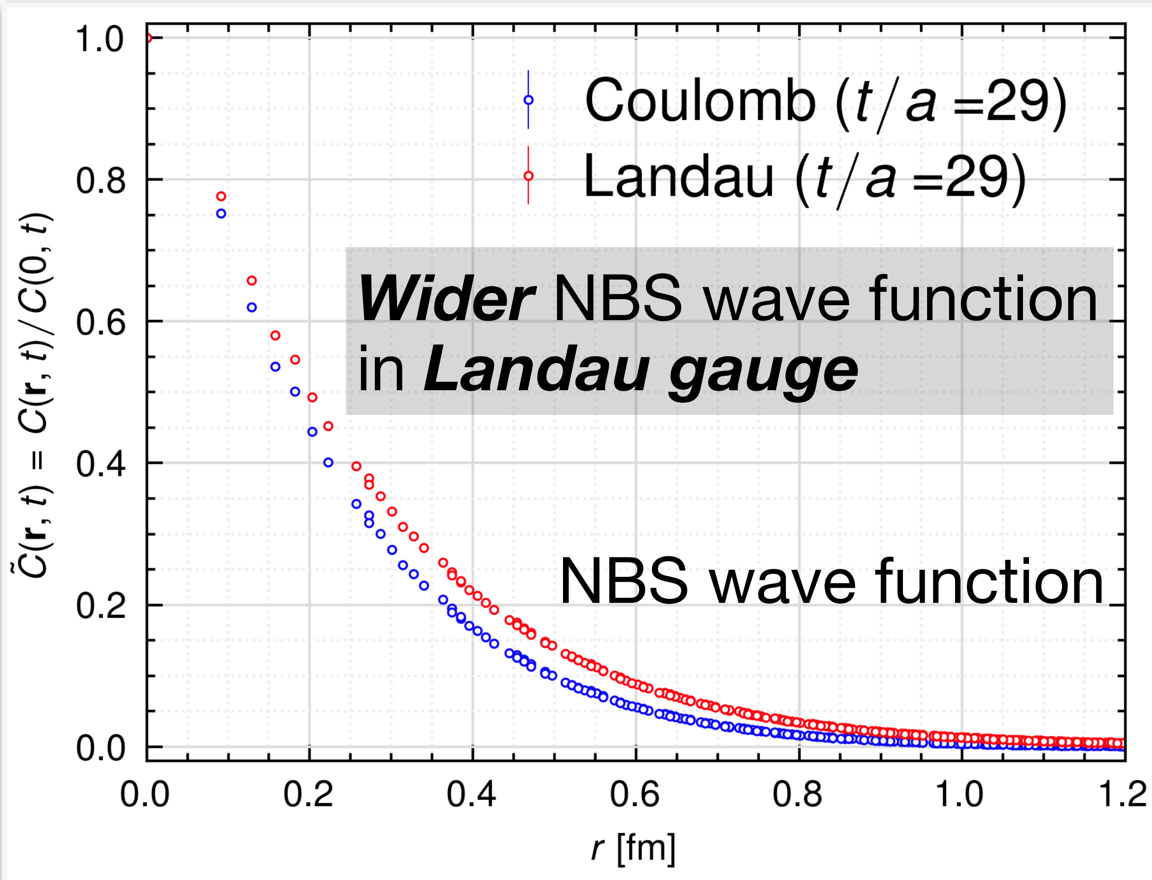


1.0
0.8
0.6
0.4
0.2
0.0
$\tilde{C}(\mathbf{r}, t) = C(\mathbf{r}, t)/C(0, t)$
Coulomb (t/a =29)
Landau (t/a =29)
Wider NBS wave function
in ***Landau gauge***
NBS wave function
0.0
0.2
0.4
0.6
0.8
1.0
1.2
r [fm]

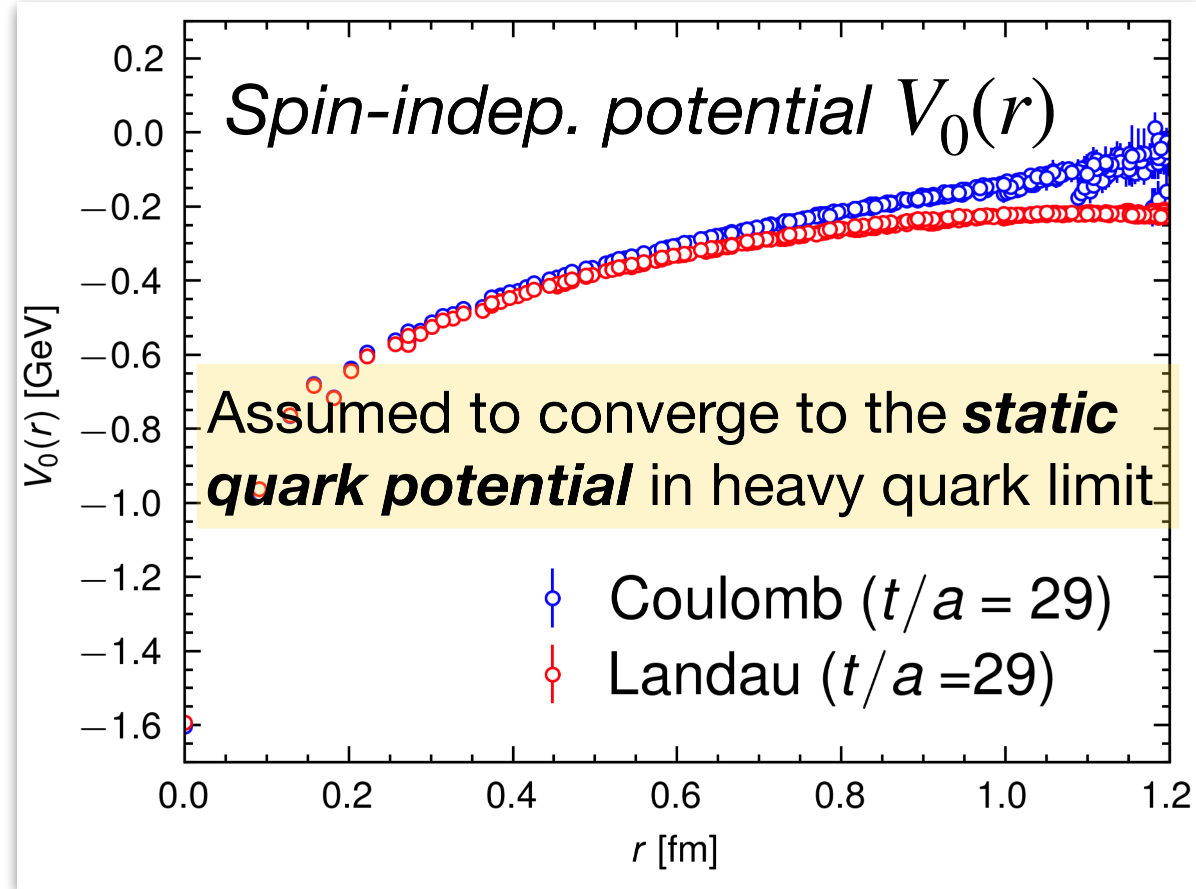


Spin-indep. potential $V_0(r)$
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
−1.4
−1.6
$V_0(r)$ [GeV]
Coulomb (t/a = 29)
Landau (t/a =29)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
r [fm]

Discussion

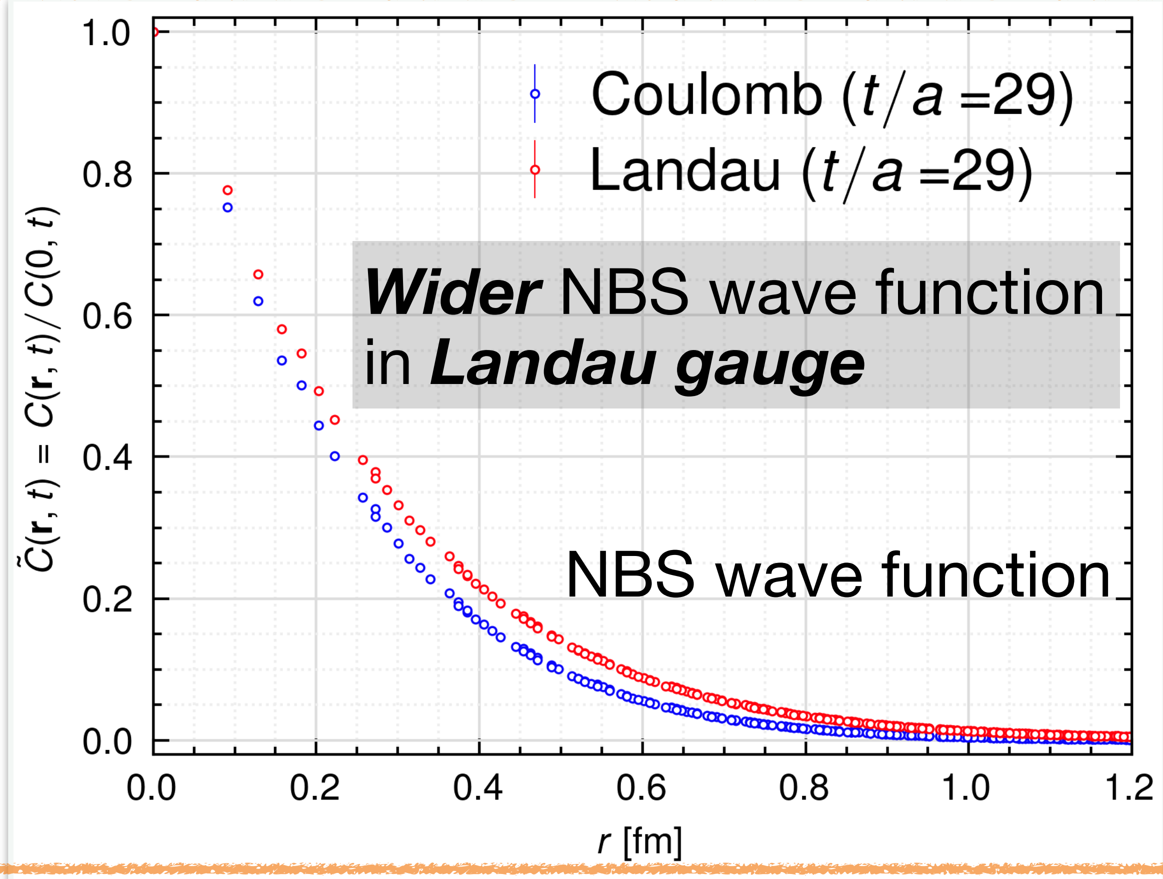


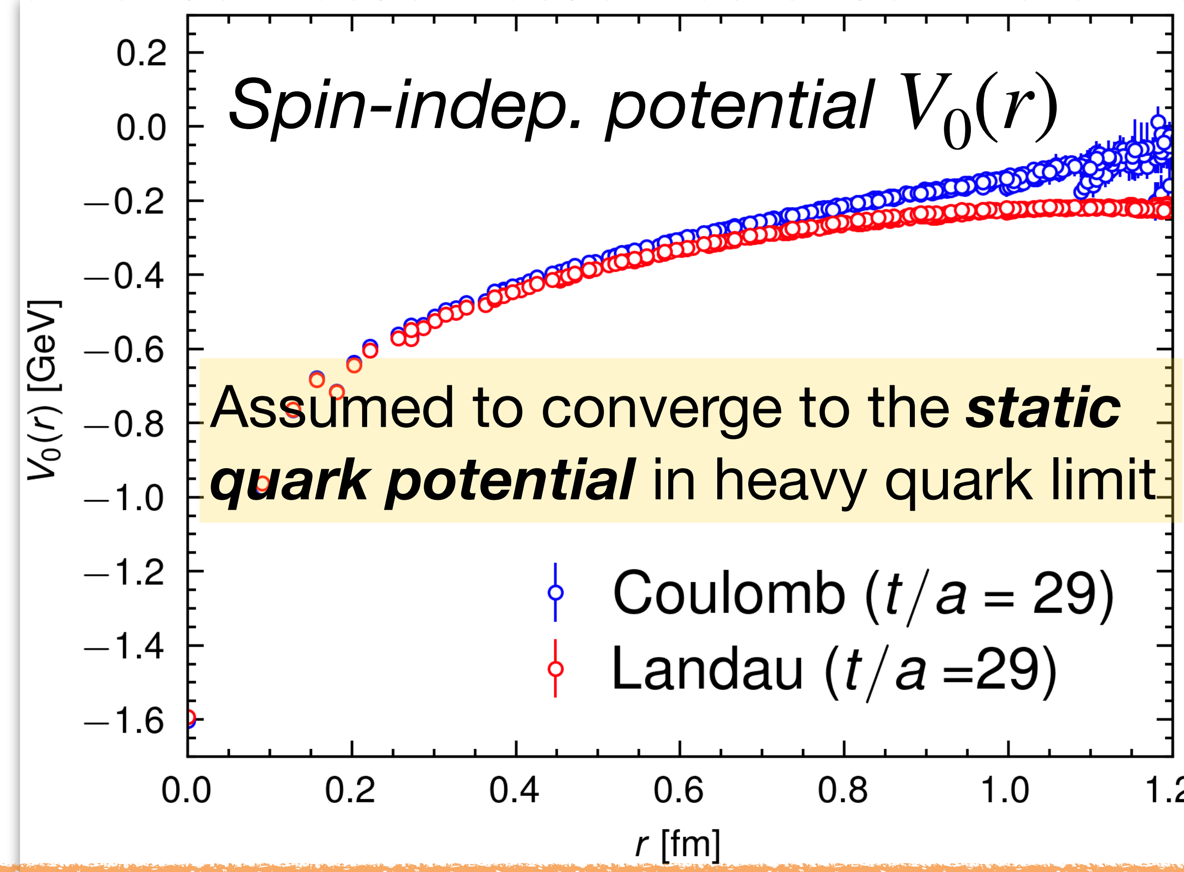
Coulomb (t/a =29)
Landau (t/a =29)
Wider NBS wave function in ***Landau gauge***
NBS wave function
$\tilde{C}(\mathbf{r}, t) = C(\mathbf{r}, t)/C(0, t)$
r [fm]



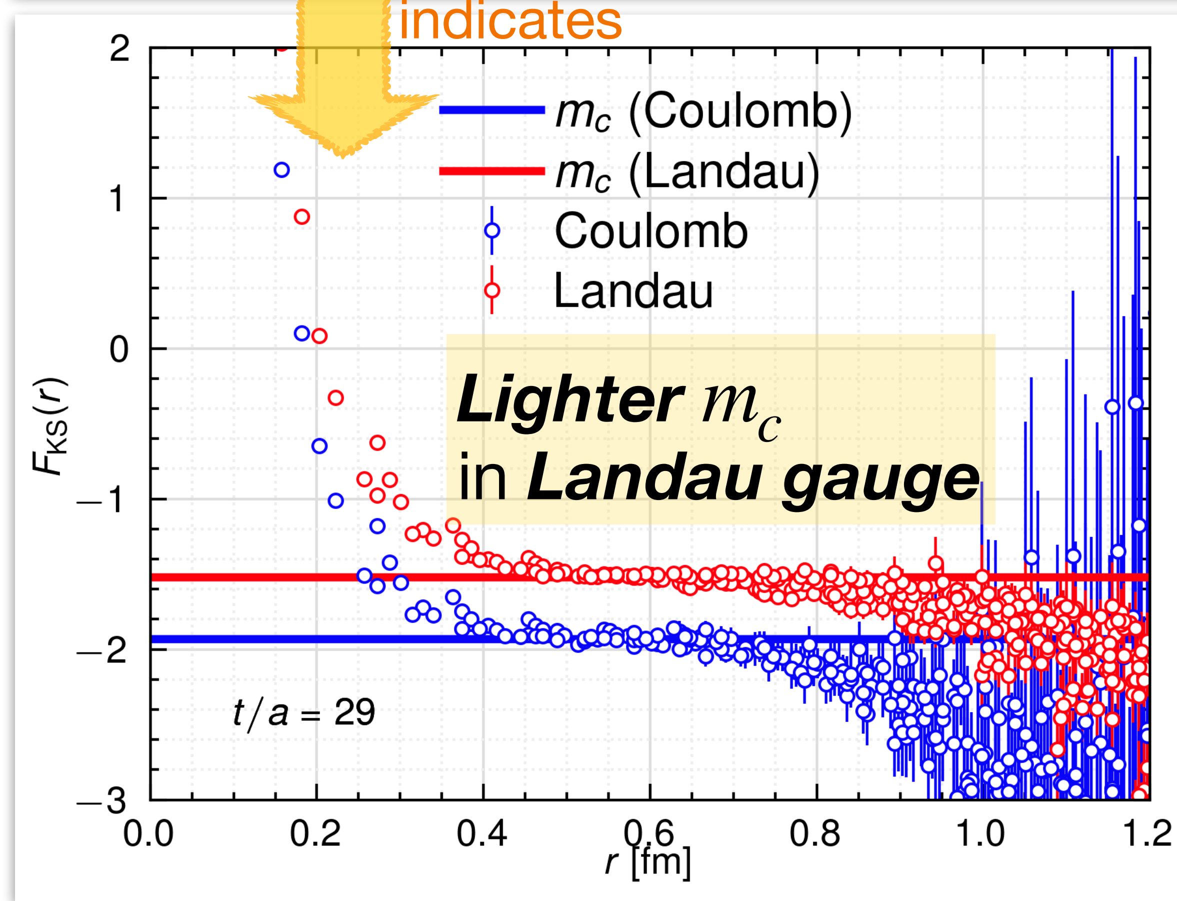
Spin-indep. potential $V_0(r)$
Assumed to converge to the ***static quark potential*** in heavy quark limit
Coulomb (t/a = 29)
Landau (t/a =29)
$V_0(r)$ [GeV]
r [fm]

Discussion

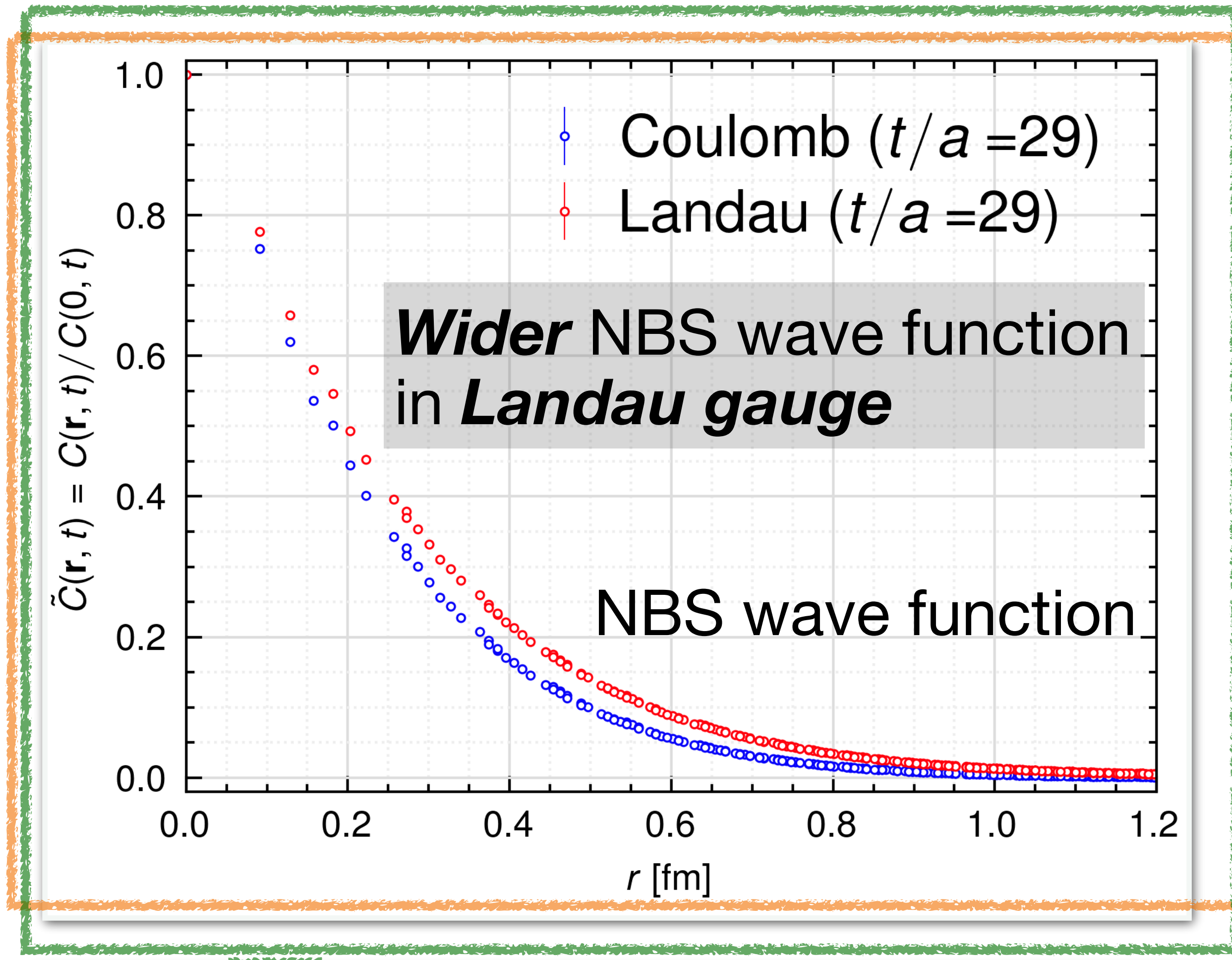




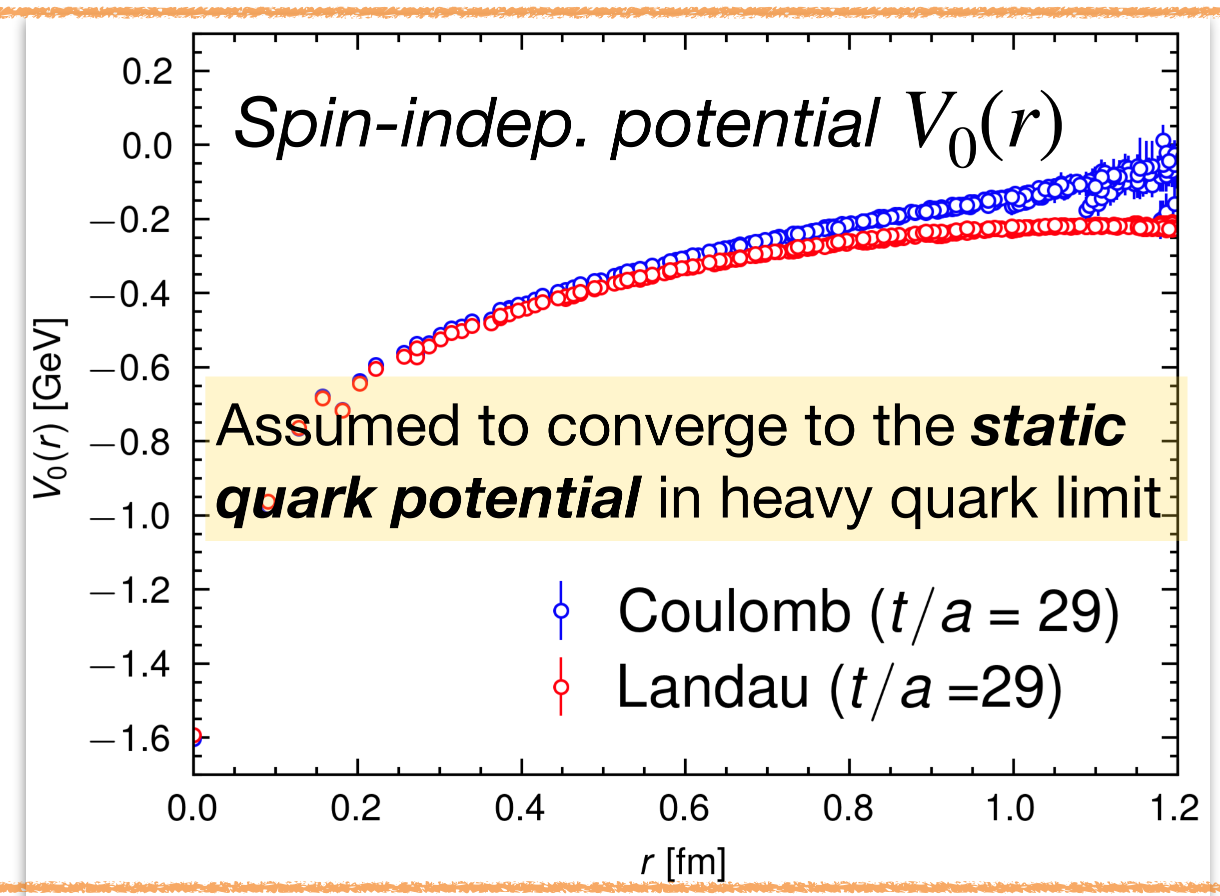
indicates



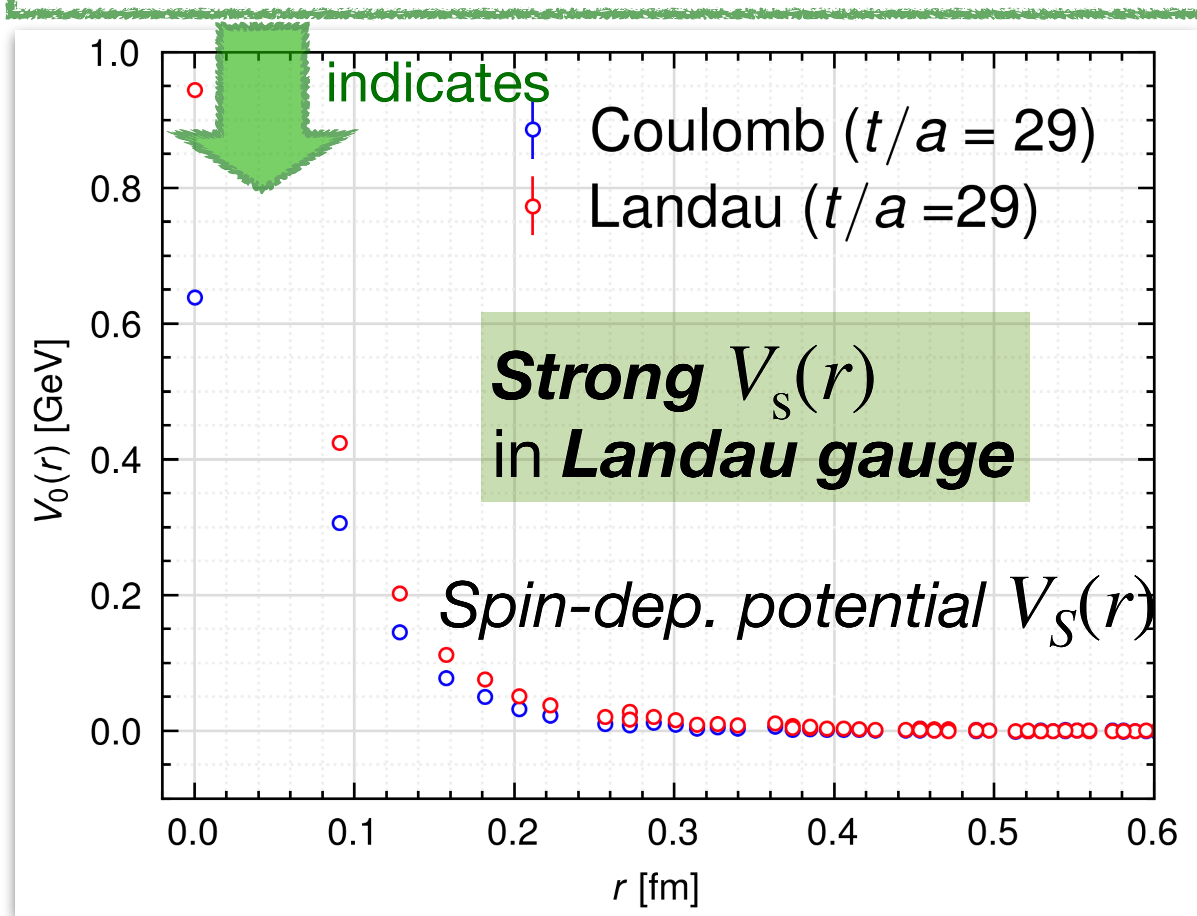
Discussion



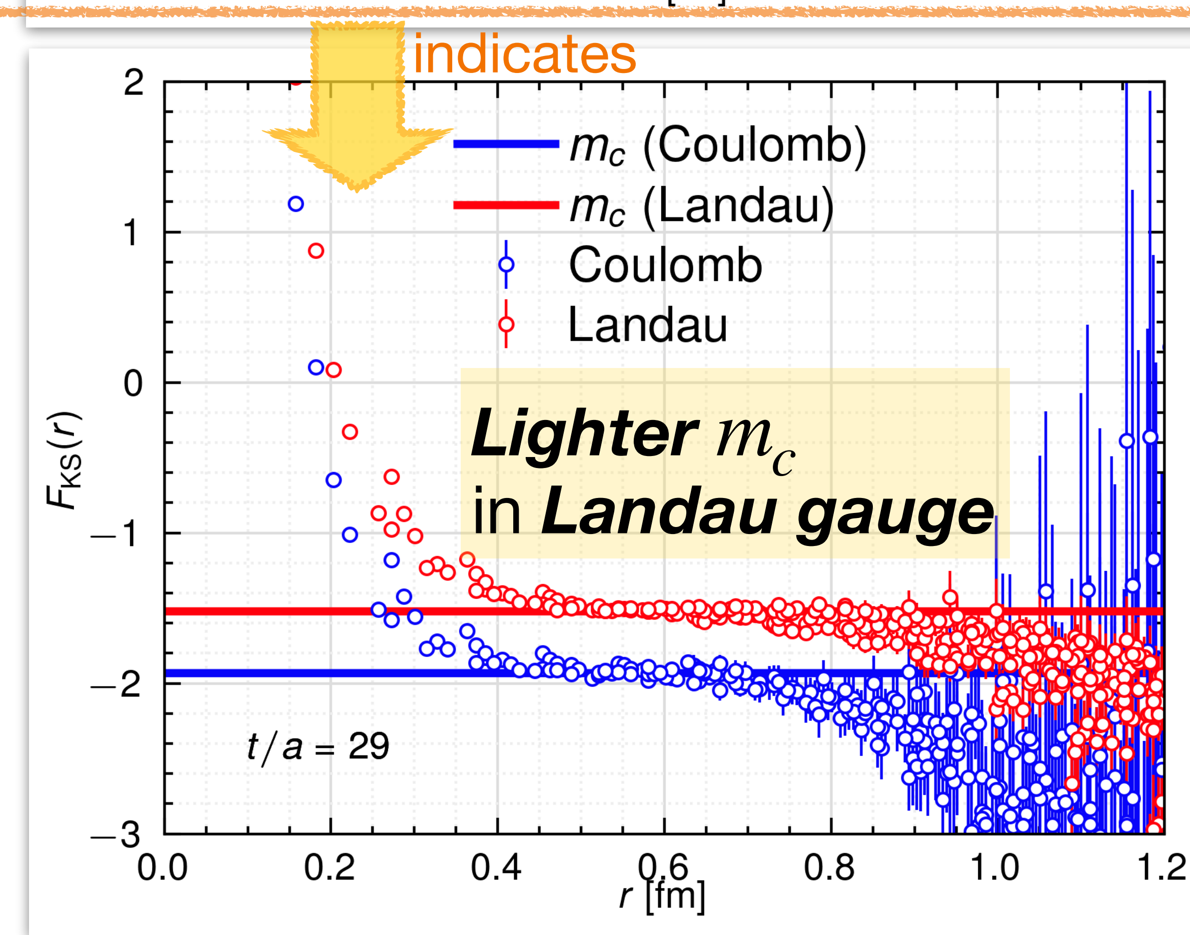
Coulomb (t/a =29)
Landau (t/a =29)
Wider NBS wave function
in ***Landau gauge***
NBS wave function
$\tilde{C}(\mathbf{r}, t) = C(\mathbf{r}, t)/C(0, t)$
r [fm]



Spin-indep. potential $V_0(r)$
Assumed to converge to the ***static quark potential*** in heavy quark limit
Coulomb (t/a = 29)
Landau (t/a =29)
$V_0(r)$ [GeV]
r [fm]



indicates
Coulomb (t/a = 29)
Landau (t/a =29)
Strong $V_s(r)$
in ***Landau gauge***
Spin-dep. potential $V_S(r)$
$V_0(r)$ [GeV]
r [fm]



indicates
m_c (Coulomb)
m_c (Landau)
Coulomb
Landau
Lighter m_c
in ***Landau gauge***
t/a = 29
$F_{KS}(r)$
r [fm]

Comments

✦ We attempted to improve Landau gauge results by **time-dependent NBS amplitude method** *originated from time-dep. HAL QCD method*. But we failed. The reason seems to be the huge contributions from the states above open-charm threshold.

Time-dependent HAL QCD method (N.Ishii et al., PLB712,437(2012).)

1. Consider large t region so that contributions from states above open-charm threshold are suppressed in the 4-point correlator:

$$C(\mathbf{x}-\mathbf{y},t) = \sum_{n=0}^{\infty} a_n \phi_n(\mathbf{x}-\mathbf{y}) e^{-E_n t}$$

2. Schrödinger eq. satisfied by $\phi_n(\mathbf{r})$ below these thresholds

$$(\hat{H}_0 + \hat{V})\phi_n(\mathbf{r}) = (E_n - 2m_c)\phi_n(\mathbf{r})$$

leads us to

$$-\frac{d}{dt}C(\mathbf{r},t) = (2m_c + \hat{H}_0 + \hat{V})C(\mathbf{r},t)$$

which is solved inversely for the potentials.

3. This method allows us to obtain converged potentials from smaller t-region than the original (t-indep.) HAL QCD method.

✦ We plan to use an improved source obtained by the variational method.

Summary

✦ We calculated $c\bar{c}$ potentials and charm quark mass by NBS amplitude method with Kawanai-Sasaki prescription in Landau and Coulomb gauges.

Landau gauge

✓ Slow convergence to ground state

✓ Cornell-type spin-indep. potential whose long distance is destroyed due to slow convergence to groud state

✓ Short-ranged spin-dep. potential

✓ $m_c \simeq 1522$ MeV

Coulomb gauge

✓ Quick convergence to ground state

✓ Cornell-type spin-indep. potential

✓ Short-ranged spin-dep. potential

✓ $m_c \simeq 1932$ MeV

✦ Several features of Landau gauge result compared to Coulomb gauge

- Wider NBS wave function
- Smaller m_c
- Stronger spin-dep. potential
- Good agreement of spin-indep. potential except at long distance

✦ We attempted to improve the convergence of Landau gauge results by the time-dep. NBS amplitude method. It failed due to the contamination above open-charm threshold.

Outlook

✓ We will use an improved source obtained by the variational method in Landau gauge.

✓ We will replace Kawanai-Sasaki's prescription by the one proposed in K.Watanabe, PRD105,074510.