Diquark mass and quark-diquark potential of the 1^+ diquark in Σ_c from Lattice QCD

Soya Nishioka, Noriyoshi Ishii
RCNP, Osaka Univ.

Introduction

Diquark is a composite particle made of two quarks

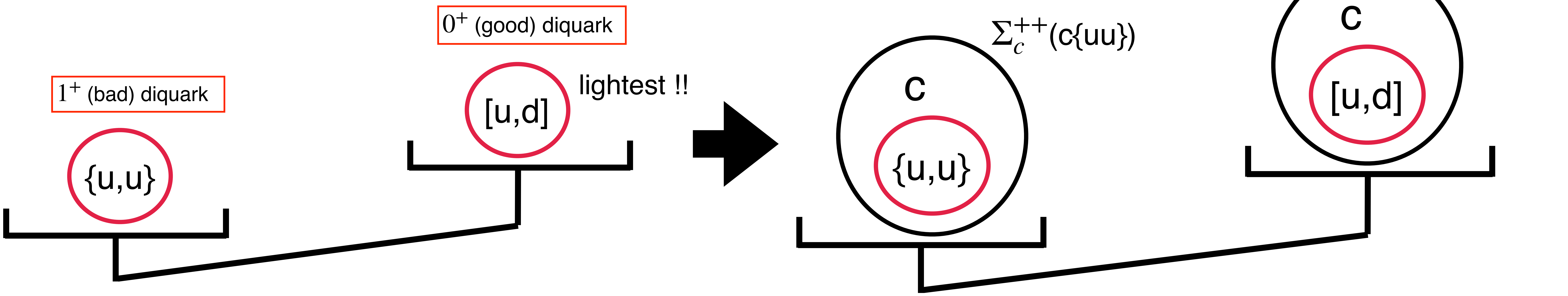
- Diquarks are colored particles

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \cancel{\mathbf{6}}$$

Color $\mathbf{6}$ diquarks are considered to be much heavy.

- **Phenomenologically important diquarks**:
  - good diquark : $J^P = 0^+, I = 0$, color $\bar{\mathbf{3}}$
  - **bad diquark** : $J^P = 1^+, I = 1$, color $\bar{\mathbf{3}}$

favored by (1) one-gluon exchange interaction
(2) instanton-induced interaction



- **Experimental study** : Difficult because of color confinement

Lattice QCD studies of diquarks

Conventional method **may not work** in obtaining diquark mass due to **color confinement**

—- We should not assume that 2-point functions have a pole for colored particles.

$$\boxed{\mathrm{F.T.}\left\langle D(x)D^{\dagger}(0)\right\rangle = \cancel{\frac{i}{q^2 - m^2 + i\epsilon}} + \cdots}$$

cf. Review on quark mass in PDG2020

Diquark masses have been treated as follows.

- M.Hess et al.,PRD.58.11502

  Landau gauge fixing is employed

  Diquark mass is naively obtained from two-point function as a pole-mass.

- C.Alexandrou et al., PRL.97.222002

  A static quark is added to neutralize the system.

  Diquark mass is obtained by neglecting interaction energy between a diquark and a static quark.

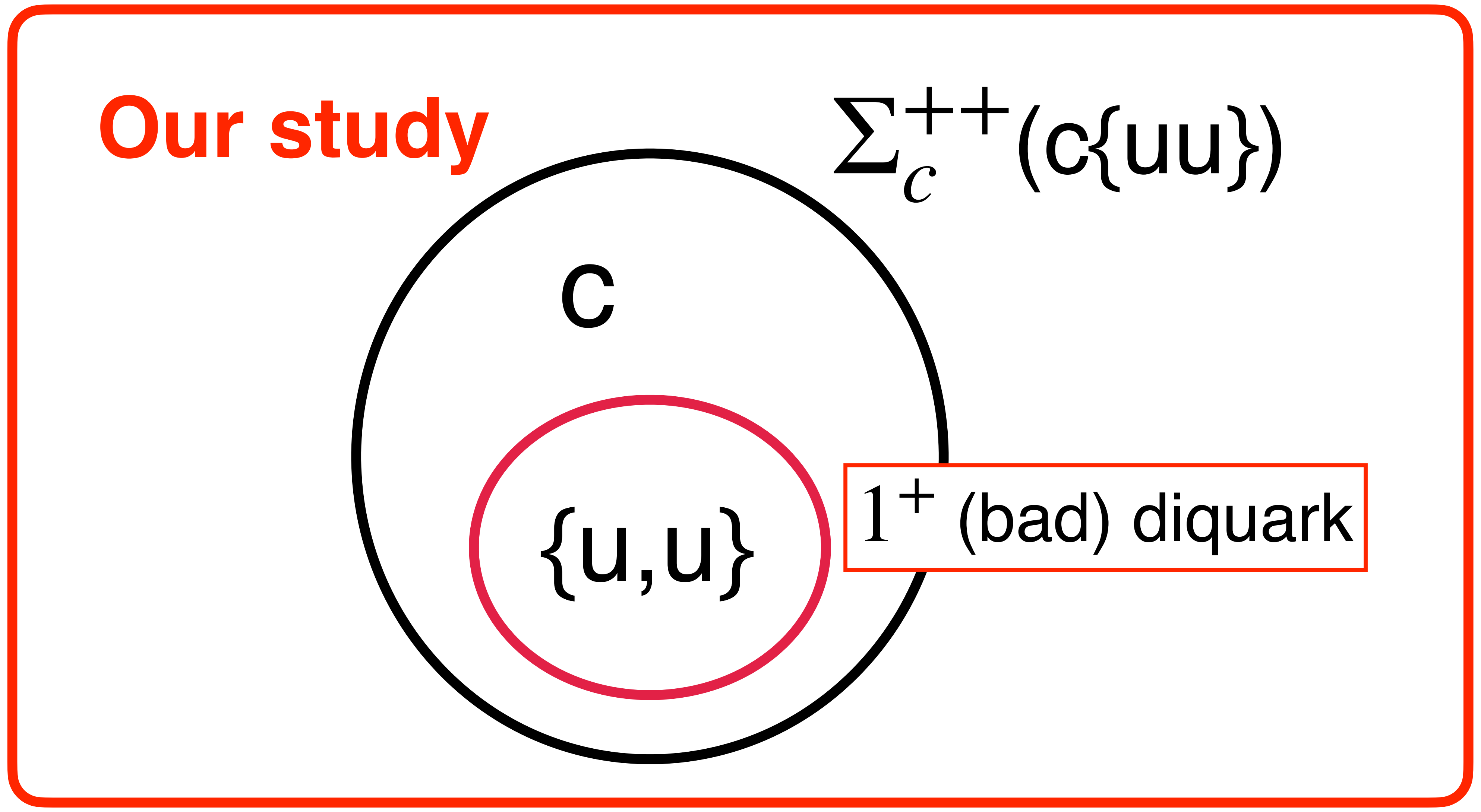
- K.Watanabe, PRD.105.074510

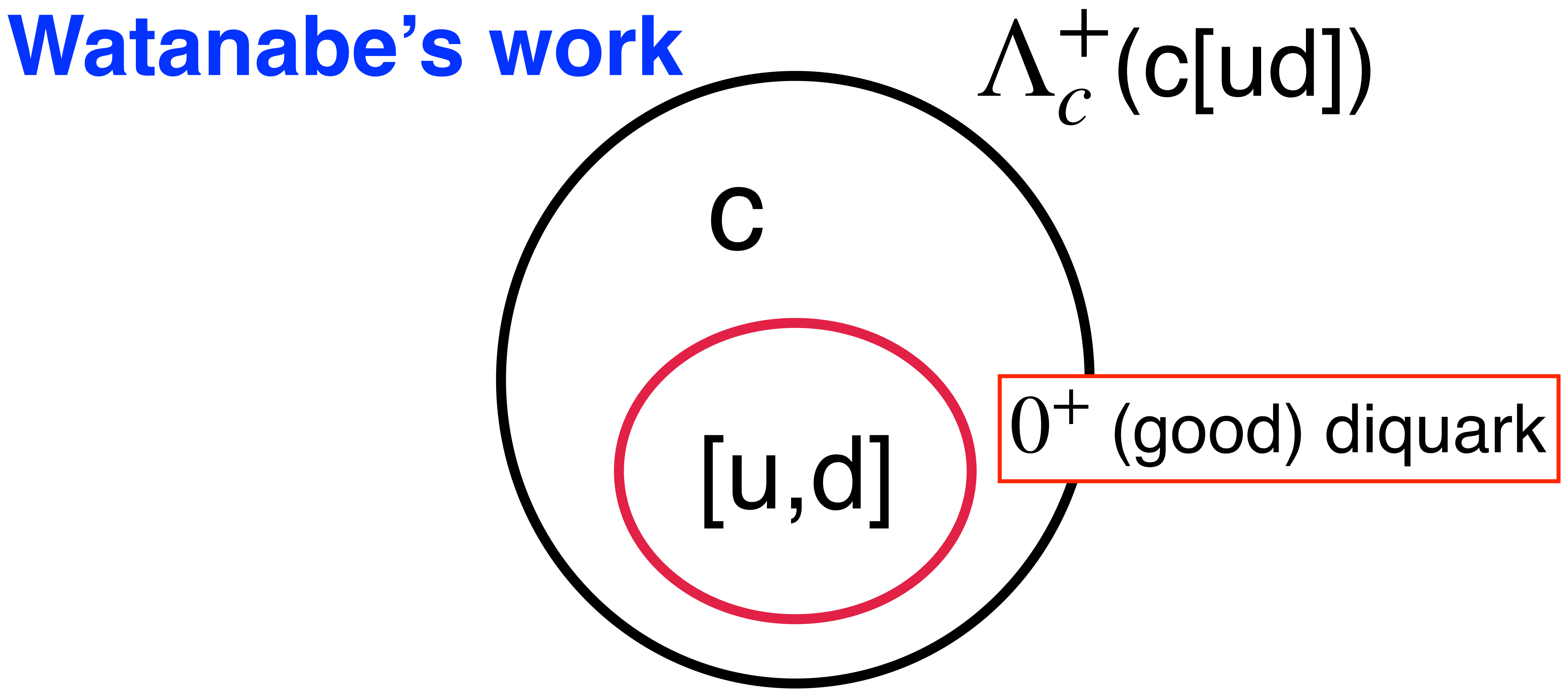
  0^+ diquark mass is treated as a mass parameter of a quark-diquark model

  which is constructed by an extended HAL QCD's potential method.

Our goal

- **We study 1^+ diquark in $\Sigma_c^{++}(c\{uu\})$.**
  We employ a similar strategy as K.Watanabe, PRD105
  where 0^+ diquark in $\Lambda_c(c[ud])$ was studied.





We obtain

- **quark-diquark potential**
  by an extended HAL QCD's potential method from equal-time NBS wave function.
- 1^+ **diquark mass** as a mass parameter of a quark-diquark model.
  Diquark mass is determined by employing a similar prescription
  which was proposed by T.Kawanai and S.Sasaki in ccbar sector.

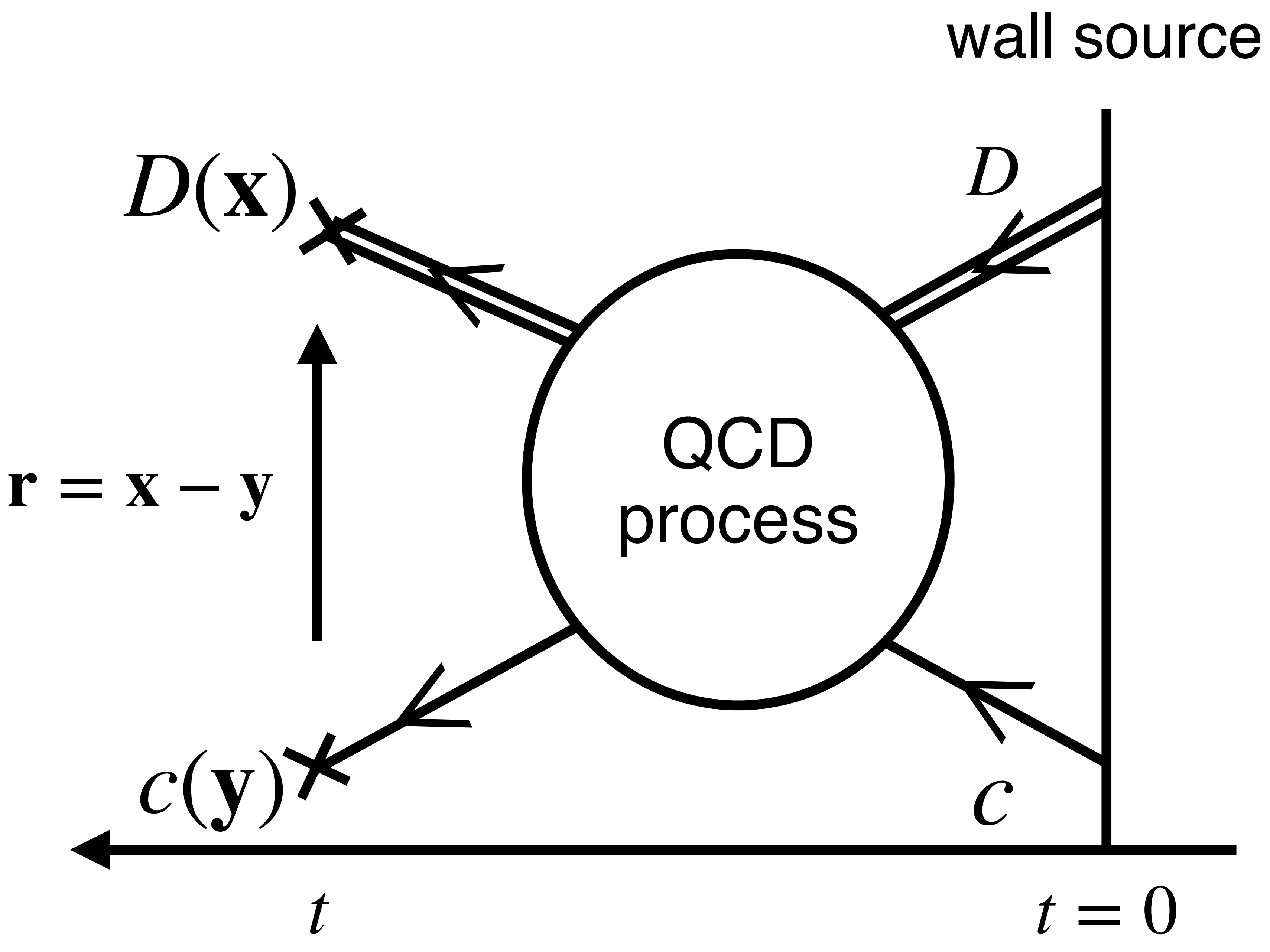
Formalism

Quark-diquark wave function

- **Quark-diquark 4-point function** and its spectral decomposition

$$C(\mathbf{x}-\mathbf{y},t) \equiv \left\langle 0 \left| \mathrm{T}\left[D(\mathbf{x},t)\, c(\mathbf{y},t) \cdot \bar{c}(t=0)\, D^\dagger(t=0) \right] \right| 0 \right\rangle$$

$$= \sum_n \psi_n(\mathbf{x}-\mathbf{y})\, a_n \exp(-M_n t)$$



- 1^+ diquark operator

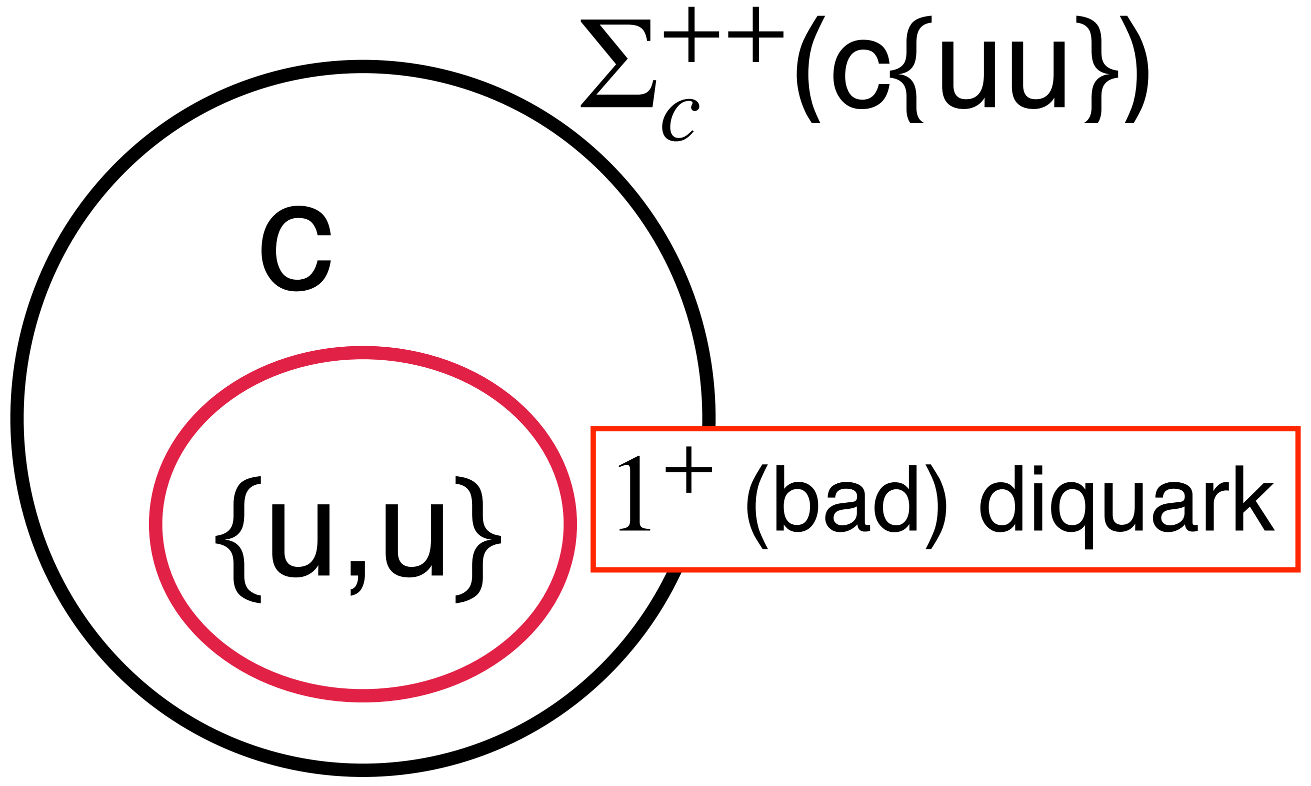
$$D_{ai}(x) \equiv \epsilon_{abc} u_b^T(x) C\gamma_5\gamma_i u_c(x)$$

- **Equal time quark-diquark Nambu-Bethe-Salpeter (NBS) wave function** for Σ_c^{++}

$$\psi_{i\alpha}(\mathbf{x}-\mathbf{y}) \equiv \langle 0 \;\; D_{ai}(\mathbf{x})\, c_{a\alpha}(\mathbf{y}) \;\; \Sigma_c \rangle$$

Rarita-Schwinger form is used for ψ.
Spin of swave Σ_c is 1/2 or 3/2.

$$\boxed{\mathbf{1} \otimes \frac{\mathbf{1}}{\mathbf{2}} = \frac{\mathbf{1}}{\mathbf{2}} \oplus \frac{\mathbf{3}}{\mathbf{2}}}$$



Conventional HAL QCD's potential method

We demand that **NBS wave function** should satisfy Schroedinger eq.

$$\left(-\frac{\nabla^2}{2\mu}+\hat{V}\right)\psi(\mathbf{r}) = E\,\psi(\mathbf{r}) \quad \text{with} \quad \hat{V} \simeq V_0(r) + V_S(r)\mathbf{s}_c\cdot\mathbf{s}_D$$

which split into

Schroedinger eq. in each channel

$$\left(-\frac{\nabla^2}{2\mu}+V_0(\mathbf{r}) - V_s(\mathbf{r})\right)\psi_{1/2}(\mathbf{r}) = \left(M_{1/2} - m_c - m_D\right)\psi_{1/2}(\mathbf{r}) \qquad (J = 1/2)$$

$$\left(-\frac{\nabla^2}{2\mu}+V_0(\mathbf{r}) + \frac{1}{2}V_s(\mathbf{r})\right)\psi_{3/2}(\mathbf{r}) = \left(M_{3/2} - m_c - m_D\right)\psi_{3/2}(\mathbf{r}) \qquad (J = 3/2)$$

We solve them inversely for potentials

- **Central potential**

$$V_0(\mathbf{r}) = \frac{2M_{3/2}+M_{1/2}}{3} - m_c - m_D + \frac{1}{2\mu}\left(\frac{2}{3}\frac{\nabla^2\psi_{3/2}(\mathbf{r})}{\psi_{3/2}(\mathbf{r})} + \frac{1}{3}\frac{\nabla^2\psi_{1/2}(\mathbf{r})}{\psi_{1/2}(\mathbf{r})}\right)$$

- **Spin dependent potential**

$$V_s(\mathbf{r}) = \frac{2}{3}(M_{3/2} - M_{1/2}) - \frac{1}{3\mu}\left(\frac{\nabla^2\psi_{3/2}(\mathbf{r})}{\psi_{3/2}(\mathbf{r})} - \frac{\nabla^2\psi_{1/2}(\mathbf{r})}{\psi_{1/2}(\mathbf{r})}\right)$$

$$\mathbf{s}_c\cdot\mathbf{s}_D = \begin{cases} -1 & (J = 1/2) \\ 1/2 & (J = 3/2) \end{cases}$$

- charm quark mass m_c
- 1^+ diquark mass m_D
- "binding energy" $E \equiv m_{\Sigma_c} - m_c - m_D$
- reduced mass $\mu \equiv \dfrac{1}{1/m_c + 1/m_D}$
- for $J = 1/2, 3/2$
  baryon mass M_J
  wave function ψ_J

We encounter a **PROBLEM**:

Choice of m_c and m_D is not obvious. (2-point function should not be used due to confinement.)

Kawanai-Sasaki prescription to determine diquark mass

Kawanai and Sasaki proposed a self-consistent method to determine quark mass. (PRL107.091601)

By applying their prescription to quark-diquark system, we demand

$$V_s(\mathbf{r}) = \frac{2}{3}(M_{3/2} - M_{1/2}) - \frac{1}{3\mu}\left(\frac{\nabla^2\psi_{3/2}(\mathbf{r})}{\psi_{3/2}(\mathbf{r})} - \frac{\nabla^2\psi_{1/2}(\mathbf{r})}{\psi_{1/2}(\mathbf{r})}\right) \to 0 \quad \text{as} \quad \mathbf{r} \to \infty$$

This leads to

Kawanai-Sasaki condition for quark-diquark system

$$\mu = -\lim_{r\to\infty} \frac{1}{2(M_{3/2} - M_{1/2})}\left(\frac{\nabla^2\psi_{3/2}(\mathbf{r})}{\psi_{3/2}(\mathbf{r})} - \frac{\nabla^2\psi_{1/2}(\mathbf{r})}{\psi_{1/2}(\mathbf{r})}\right)$$

Using m_c from Kawanai-Sasaki prescription in $c\bar{c}$ sector.

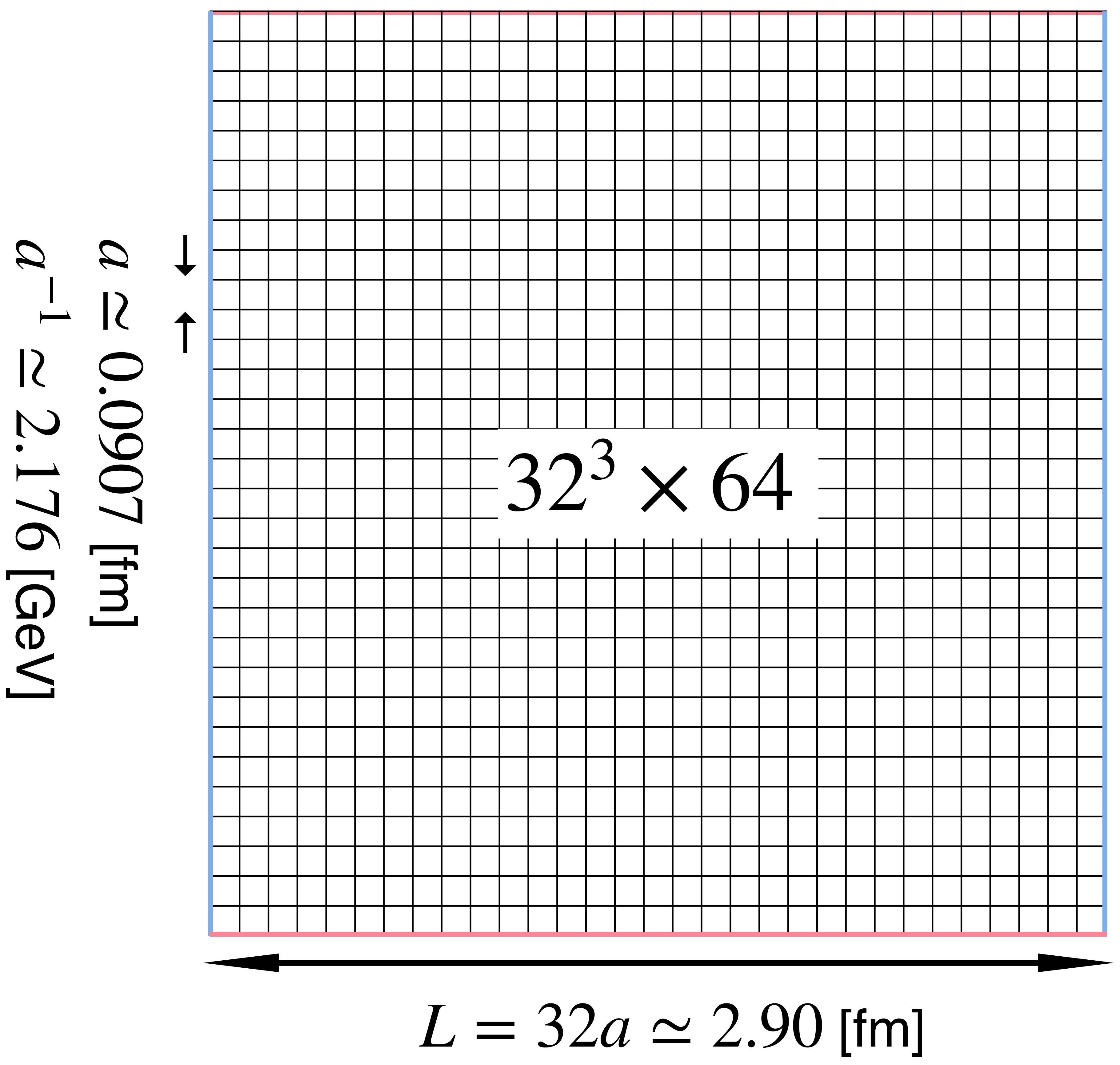
$$m_D = \frac{1}{1/\mu - 1/m_c}$$

- Combining HAL QCD's potential method with Kawanai-Sasaki prescription provides a **self-consistent way** to obtain **diquark mass** and **quark-diquark potential**
- This approach avoids the issue of pole-mass in 2-point function of diquark. (Diquark mass is obtained as a mass parameter of quark-diquark model)

Numerical Results

Lattice QCD setup

- 2+1 flavor QCD gauge config. on $32^3 \times 64$ lattice
  [Ukita et al., PACS-CS Coll., PRD.79.034503]
  - RG improved Iwasaki gauge action $(\beta = 1.90)$
    $O(a)$-improved Wilson quark action $(\kappa_{ud} = 0.137, C_{SW} = 1.715)$
  - Lattice spacing $a = 0.0907$ fm
    $1/a = 2.176$ GeV
    Spacial extension $L = 2.90$ fm
- Charm quark added with quenched approx.
  Relativistic heavy quark action
  [Namekawa et al., PACS-CS Coll., PRD.84.074505]
- Coulomb gauge fixing is employed



These setup reproduce typical hadron mass

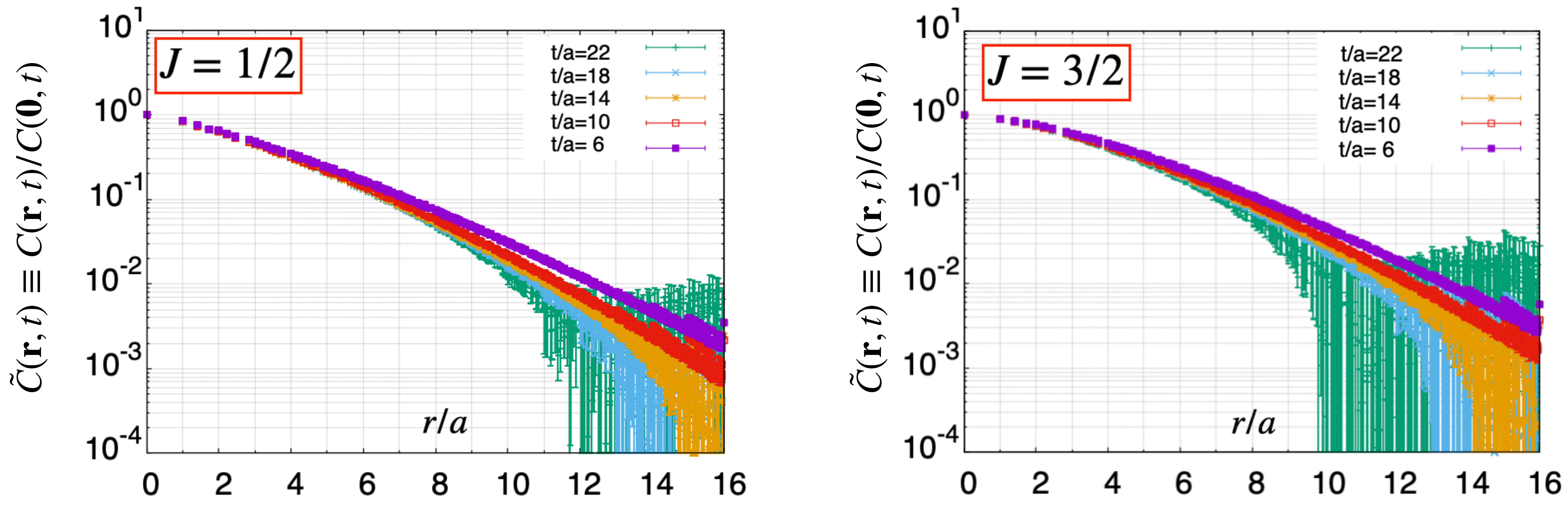
$m_\pi \sim 700\,\mathrm{MeV}$, $m_N \sim 1600\,\mathrm{MeV}$

$m_{\eta_c} \sim 3025\,\mathrm{MeV}$, $m_{J/\psi} \sim 3144\,\mathrm{MeV}$

$m_{\Lambda_c} \sim 2691\,\mathrm{MeV}$

$m_{\Sigma_c(J=1/2)} \sim 2777\,\mathrm{MeV}$, $m_{\Sigma_c(J=3/2)} \sim 2859\,\mathrm{MeV}$

4-point function and NBS wave function



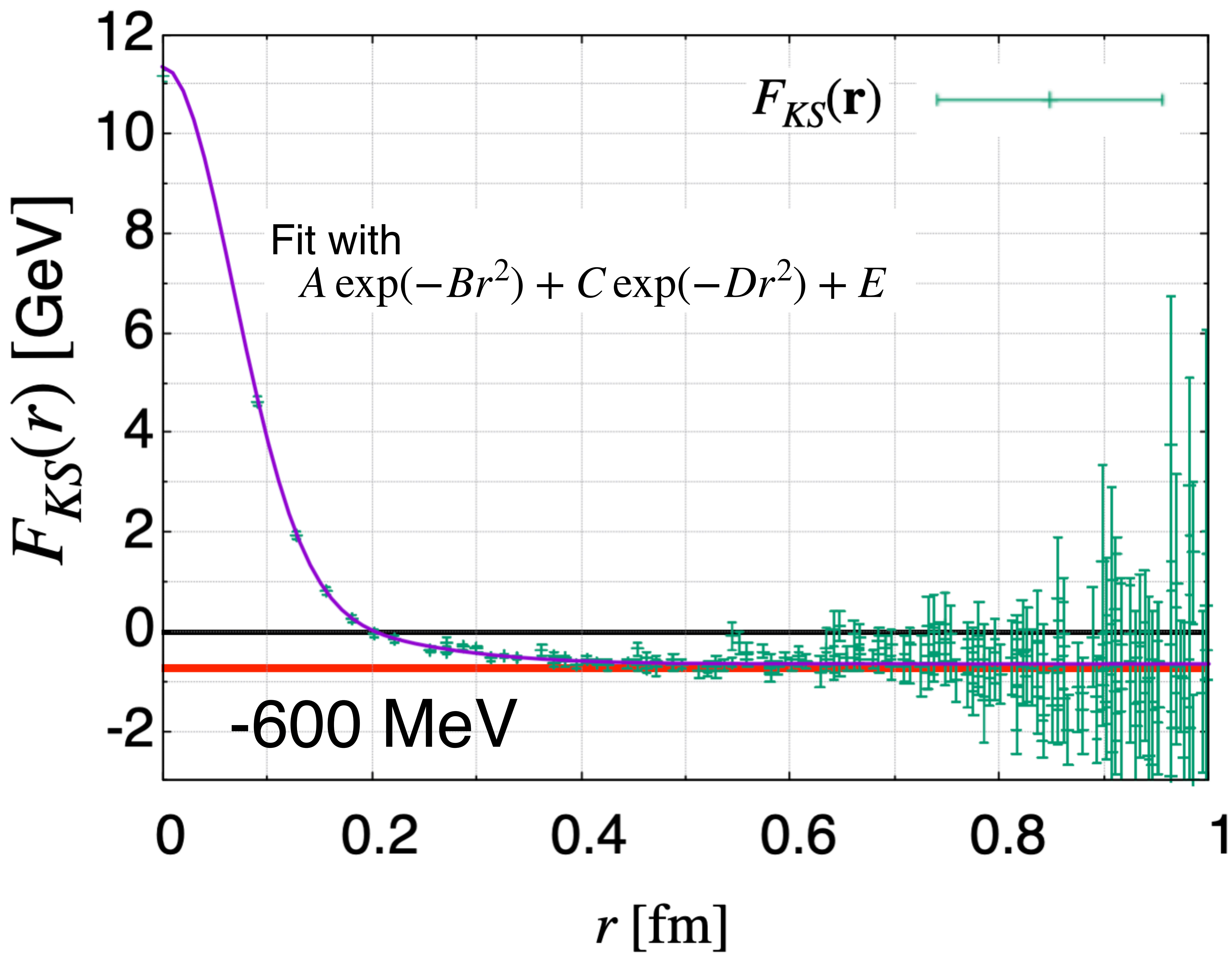
$$C(\mathbf{r},t) = \sum_n \psi_n(\mathbf{r})\, a_n e^{-M_n t}$$

$$\rightarrow \psi_0(\mathbf{r})\, a_0 e^{-M_0 t} \quad \text{for large } t$$

Rough convergence is achieved at t/a = 18 in the region $r < 10a \sim 0.9\,\mathrm{fm}$.

4-point func. at t/a=18 is accepted as a converged **NBS wave func.**
(t/a=22 has too large error bar to be accepted)

Determination of diquark mass



Kawanai-Sasaki function

$$F_{\mathrm{KS}}(\mathbf{r}) \equiv \frac{1}{2(M_{3/2} - M_{1/2})}\left(\frac{\nabla^2\psi_{3/2}(\mathbf{r})}{\psi_{3/2}(\mathbf{r})} - \frac{\nabla^2\psi_{1/2}(\mathbf{r})}{\psi_{1/2}(\mathbf{r})}\right)$$

Reduced mass is determined by

Kawanai-Sasaki condition

$$\mu = -\lim_{r\to\infty} F_{\mathrm{KS}}(\mathbf{r}) \sim 600\,\mathrm{MeV}$$

Diquark mass

$$m_D = \frac{1}{1/\mu - 1/m_c} \simeq 867\,\mathrm{MeV}$$

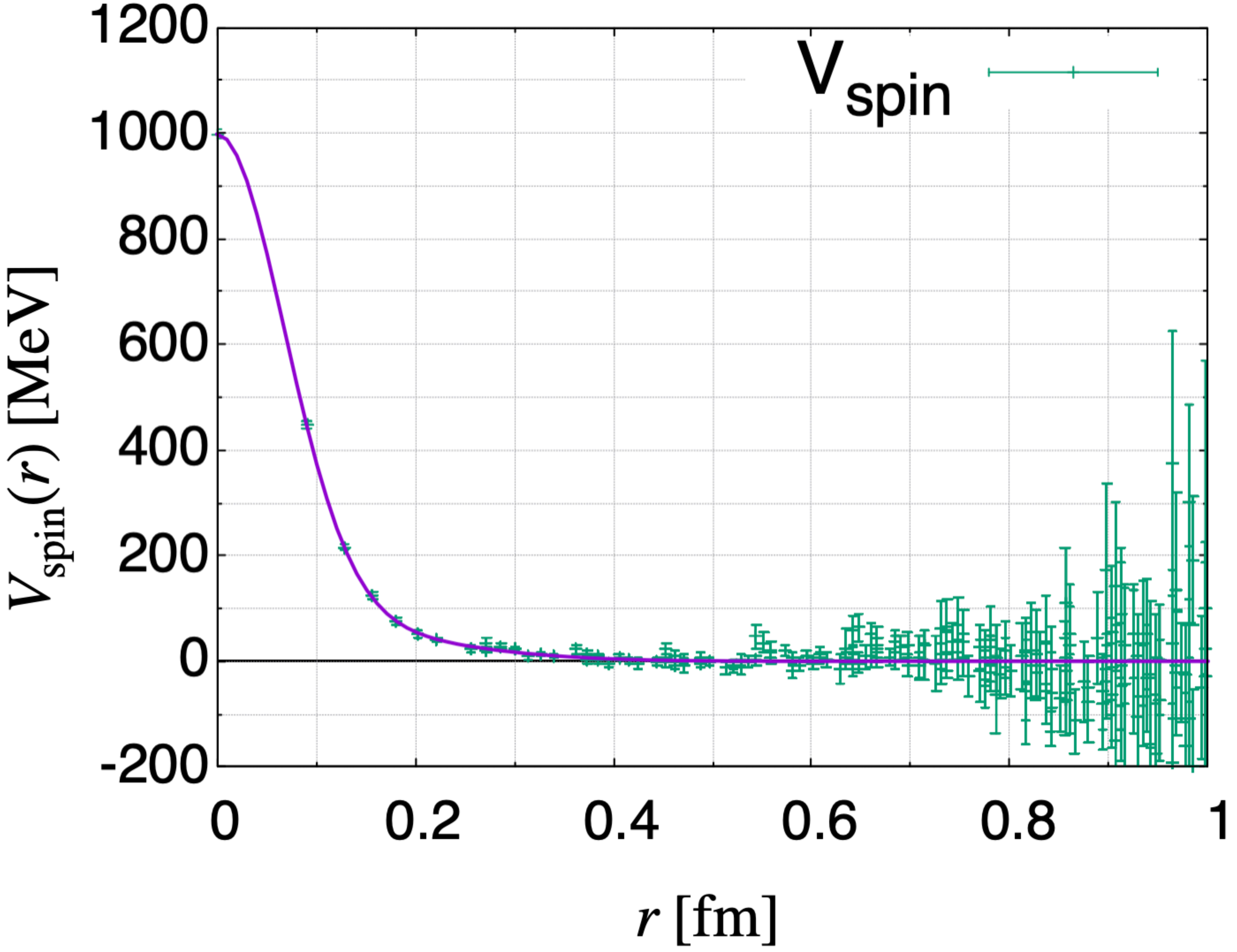
where charm quark mass

$m_c \simeq 1950\,\mathrm{MeV}$ (from ccbar sector)

We feel that m_D might be a bit underestimated because of large noise of F_{KS} at large r

Spin-dependent (quark-diquark) potential

V_{spin} is short-ranged

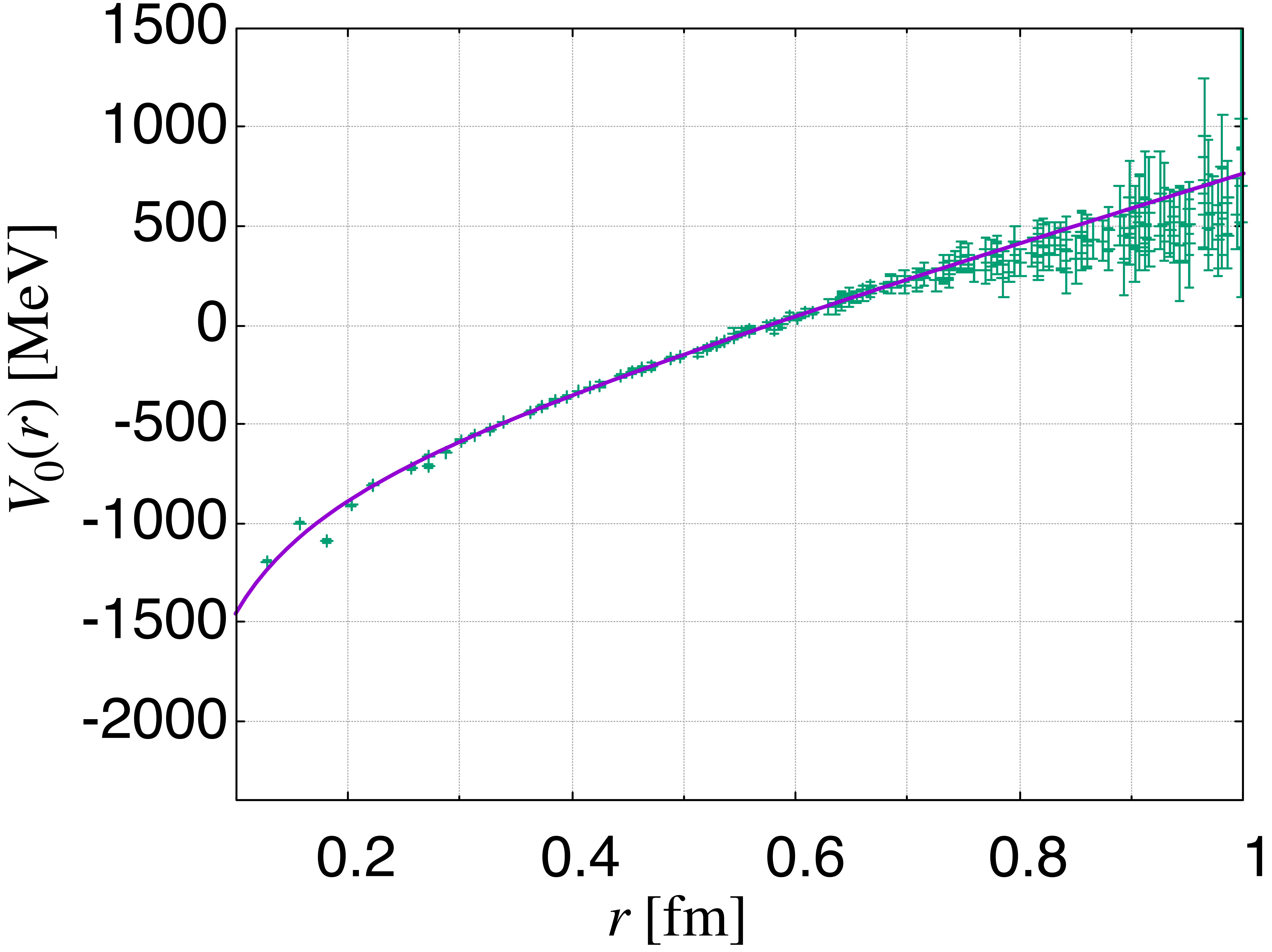


fit with 2-gaussian func. form

$$A \exp(-Br^2) + C \exp(-Dr^2)$$

Spin-indep. (quark-diquark) potential

V_0 is of Cornell-type



Fit with Cornell type func. form

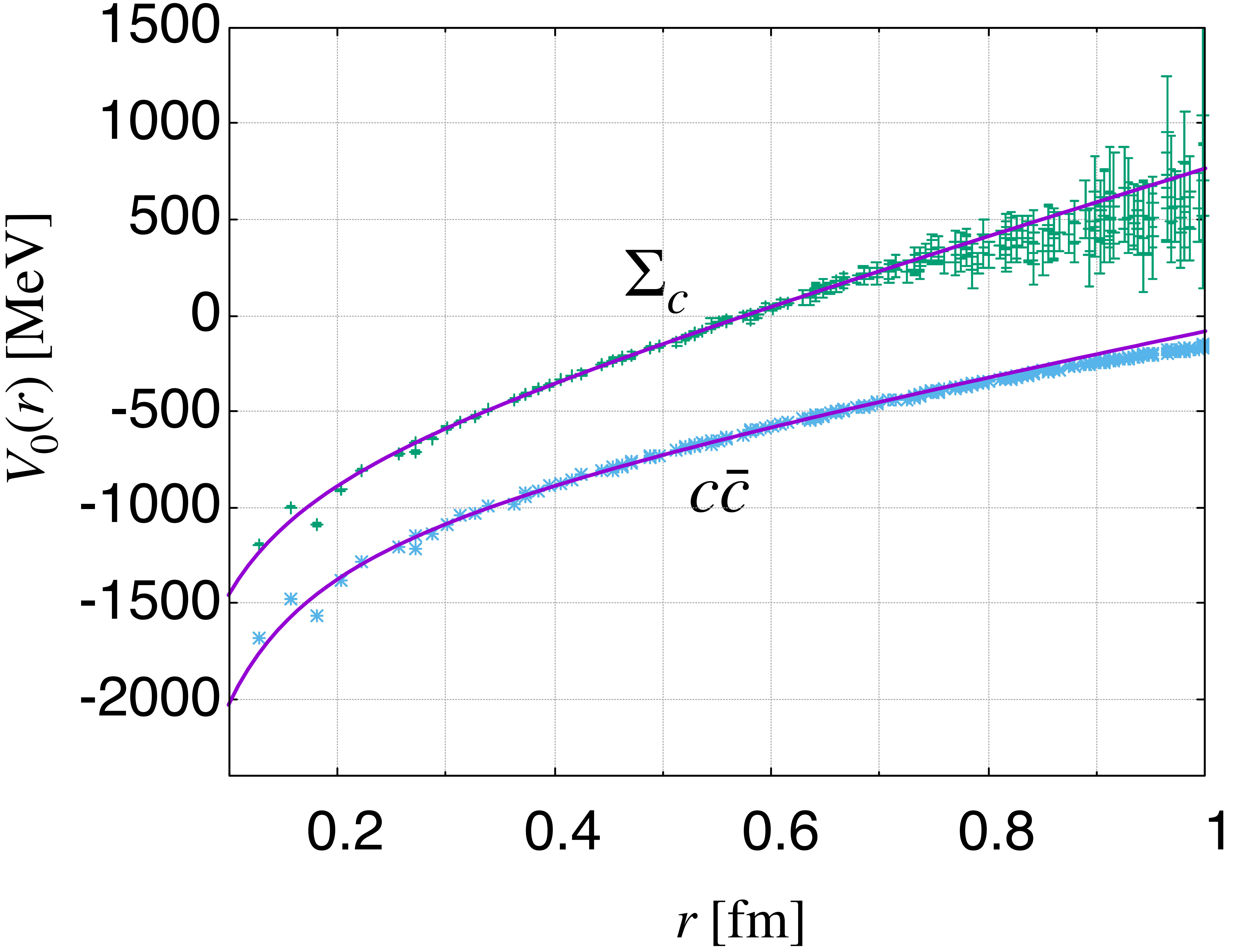
$$-A/r + \sigma r + \text{const}$$

$A = 86$ MeV/fm

$\sqrt{\sigma} = 565$ MeV

Our feeling:
A and σ may be overestimated
due to possible underestimate of m_D.

Quark diquark potential vs $c\bar{c}$ potential



Fitting function

$$-A/r + \sigma r + \text{const}$$

Coulomb coefficient

Σ_c $A = 86$ MeV/fm
$c\bar{c}$ $A = 103$ MeV/fm

String tension

Σ_c $\sqrt{\sigma} = 565$ MeV
$c\bar{c}$ $\sqrt{\sigma} = 459$ MeV

Summary

- **An extended HAL QCD method** was applied to $\mathbf{\Sigma_c}$ (c-{uu}) in the heavy quark mass region to study 1^+ **diquark mass** and **quark-diquark potentials** by 2+1 flavor lattice QCD
  - 1^+**diquark mass** was obtained by using **Kawanai-Sasaki prescription.**
  - **Central potential is of Cornell type**
  - **Spin dependent potential is short-ranged**
  - $m_D \sim 867\,\mathrm{MeV}$ $\sqrt{\sigma} = 565$ MeV $A = 86$ MeV/fm

    Precise evaluation is needed.

    We feel that Diquark mass may be a bit underestimated. σ and A may be a bit overestimated.

    $F_{\mathrm{KS}}(r)$ is noisy at long distance

- **Future plans**
  - Quark mass dependence
  - Comparison of our 1^+ diquark result with 0^+ diquark in K.Watanabe, PRD.105.074510

    (He obtained $m_{0^+} \sim 1.27$ GeV by a similar but different method)

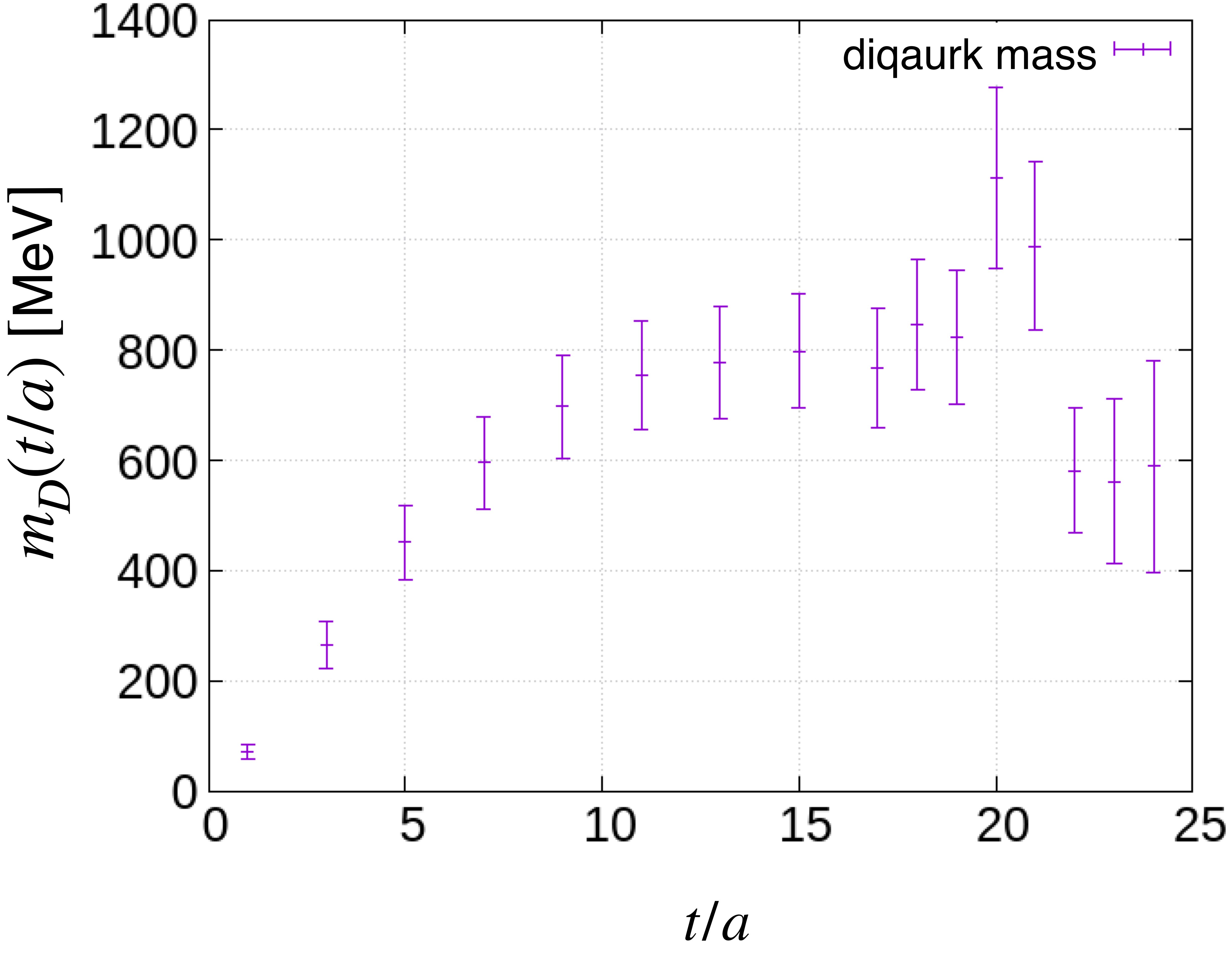
    Several things have to be fixed before the comparison
    - Watanabe employs $m_c \sim 1.840\mathrm{GeV}$ whereas ours is $m_c \sim 1.950\mathrm{GeV}$.

      (This is due to different formalism employed to obtain charm quark mass)
    - Large statistical noise of Kawanai-Sasaki function at long distance has to be improved for precise determination of 1^+ diquark mass.

Backup

Determination of diquark mass

t-dependence of the diquark mass m_D



- m_D increases with t approaching a constant value $m_D \sim 800$ MeV in the region $t \geq 15$.
- In the following slides, we employ $t/a \sim 18$, where sufficient convergence is achieved.

2 point function -> mass split of Σ_c

- 2-point function

$$G_{i\alpha j\beta}(t) \equiv \sum_{\vec{x}} \langle B_{i\alpha}(\vec{x}, t)\bar{B}_{j\beta}(0)\rangle$$

(Baryon operator $B_{i\alpha}(\vec{x}, \vec{y}, t) \equiv D_i^a(\vec{x}, t)c_{a\alpha}(\vec{y}, t)$)

- Let $g_J(t)$ be projection of this operator onto the spin J state

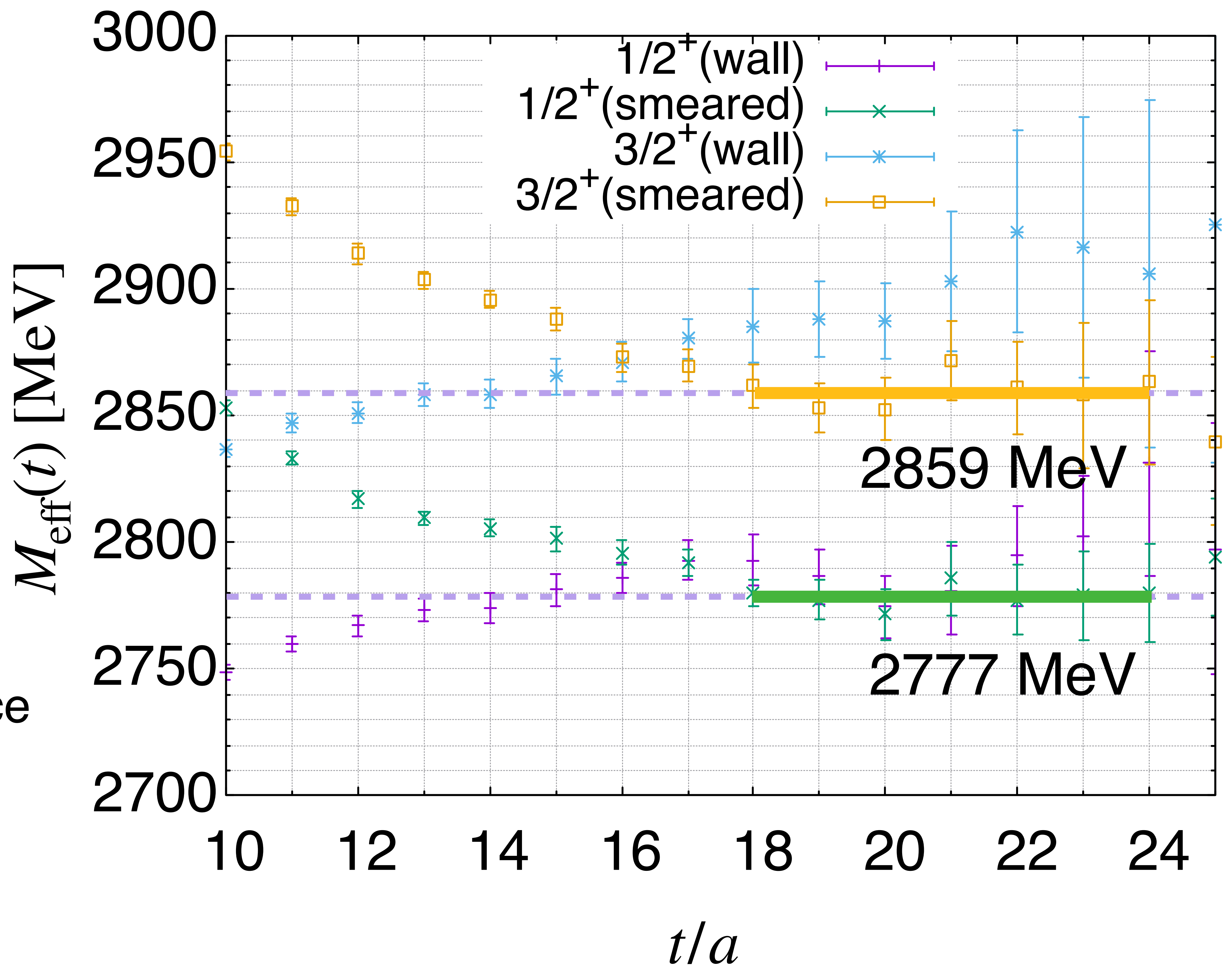
$$g_J(t) \xrightarrow{\text{large t}} Ae^{-M_J t}$$

- Take M_J from the effective mass to obtain the mass difference

$$M_{\mathrm{eff}}^J(t) \equiv \log \frac{g_J(t)}{g_J(t+1)}$$

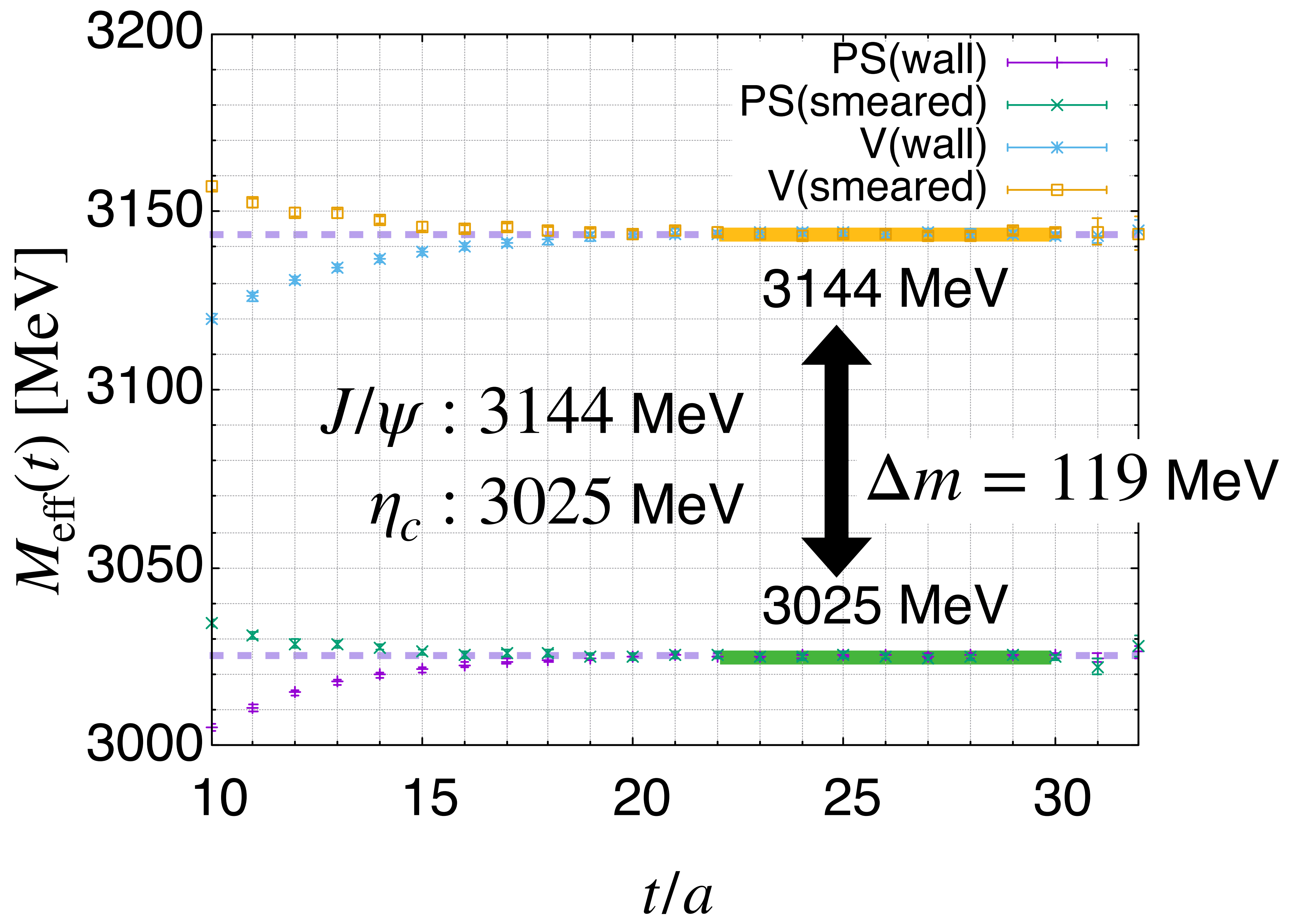
This is called the effective mass,
which asymptotically approaches M_J in the large t region.

effective mass plot



$$\boxed{M_{3/2} - M_{1/2} = 82(8)\text{ MeV}}$$

$c\bar{c}$ sector

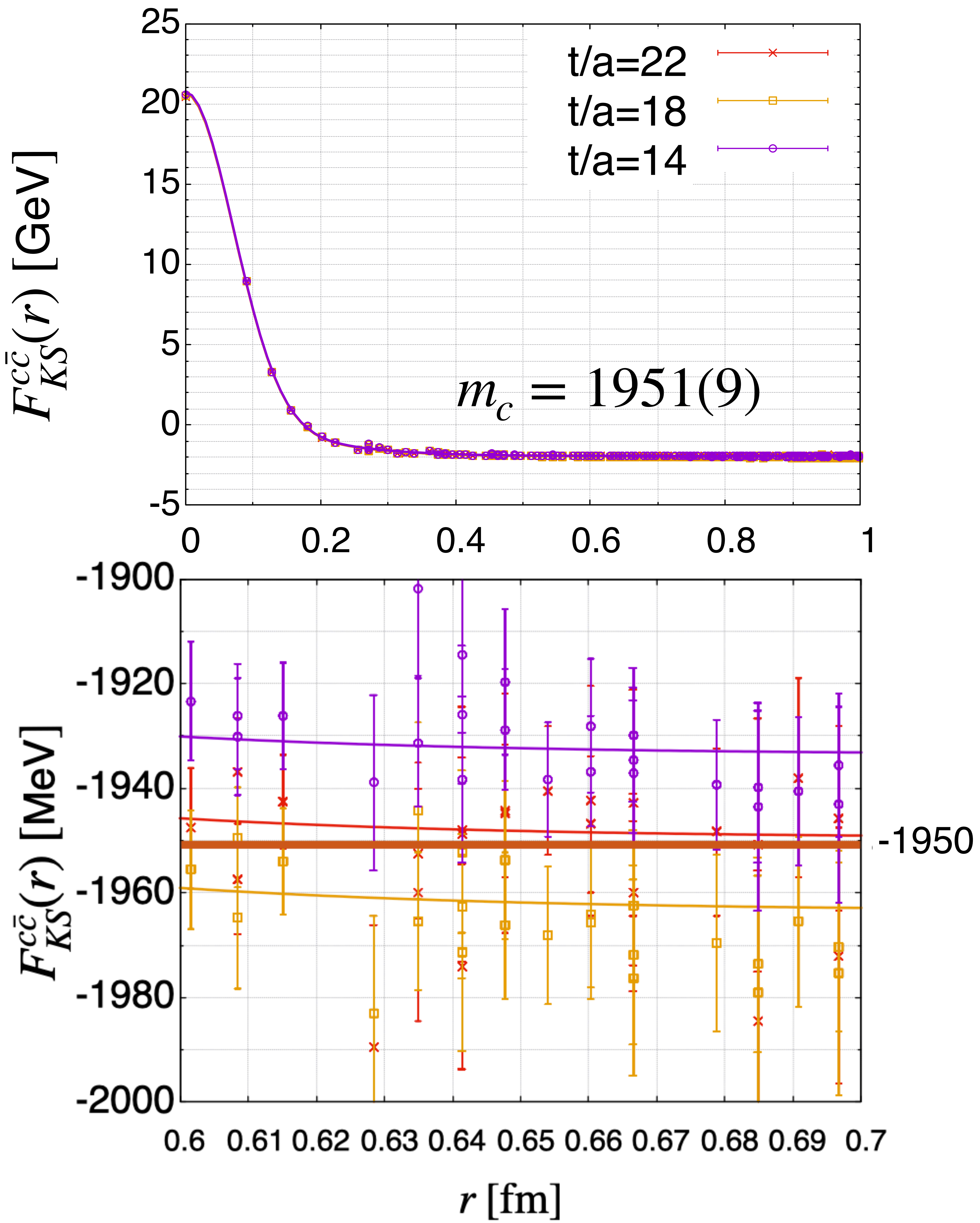


Kawanai-Sasaki condition

$$m_c = -\lim_{r\to\infty} F_{\rm KS}^{c\bar{c}}(\mathbf{r})$$

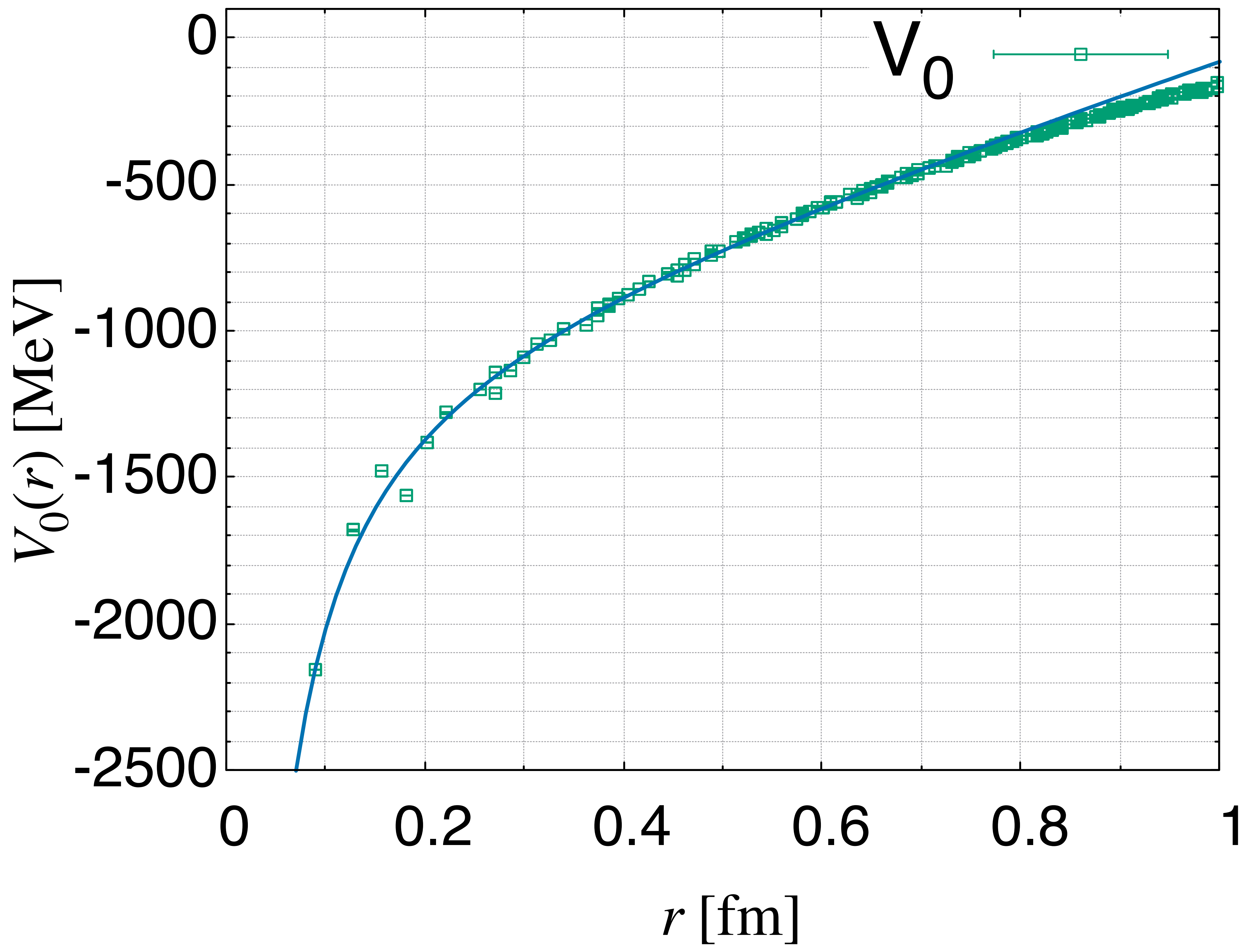
$$F_{\rm KS}^{c\bar{c}}(\mathbf{r}) \equiv \frac{1}{\Delta m}\left(\frac{\nabla^2\psi_{J/\psi}(\mathbf{r})}{\psi_{J/\psi}(\mathbf{r})} - \frac{\nabla^2\psi_{\eta_c}(\mathbf{r})}{\psi_{\eta_c}(\mathbf{r})}\right)$$

➡ **Charm quark mass** : 1950(9) MeV



$c\bar{c}$ sector

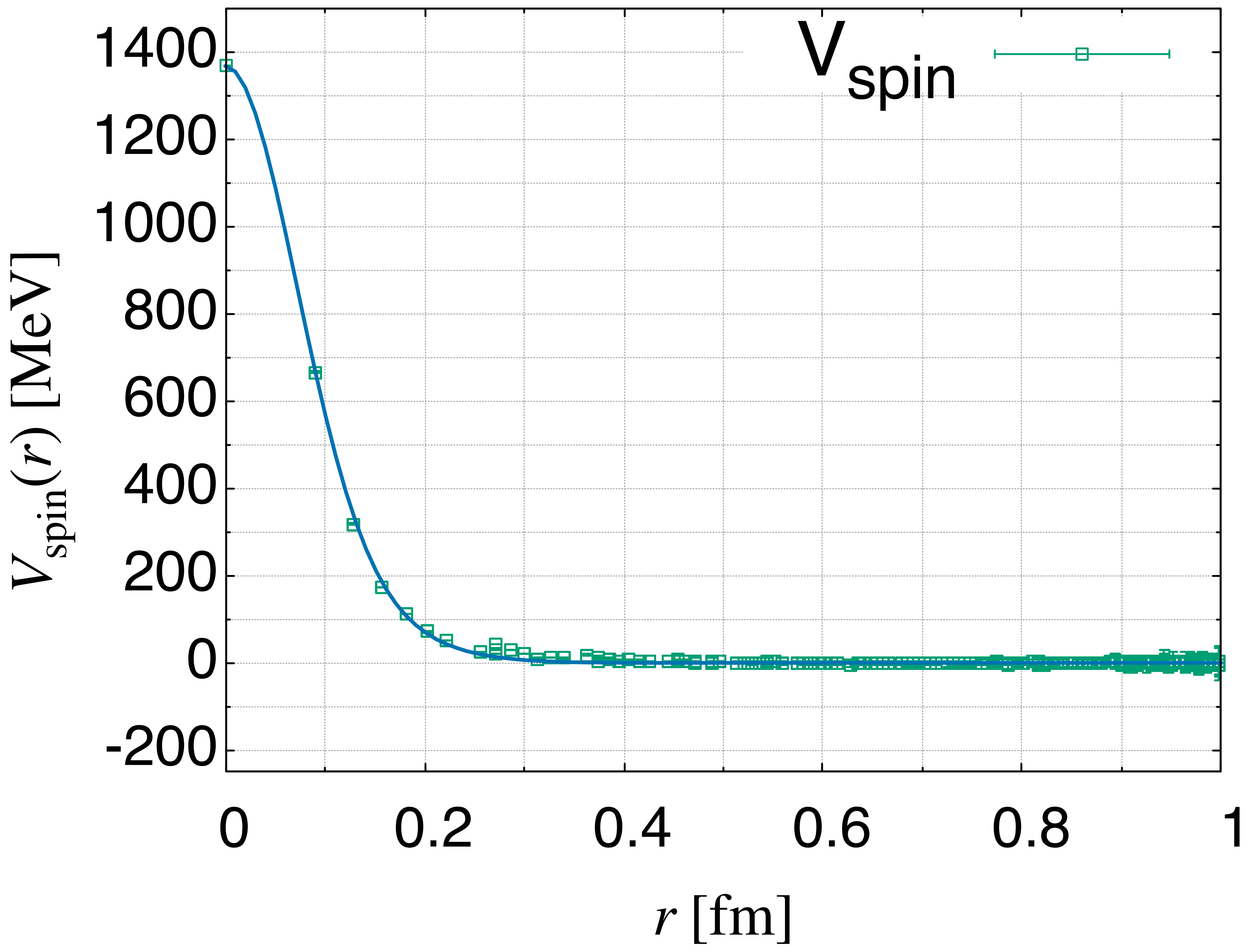
$$-A/r + \sigma r + \text{const}$$



$A \sim 0.526$ (Lattice unit)

$\sqrt{\sigma} = 459$ MeV

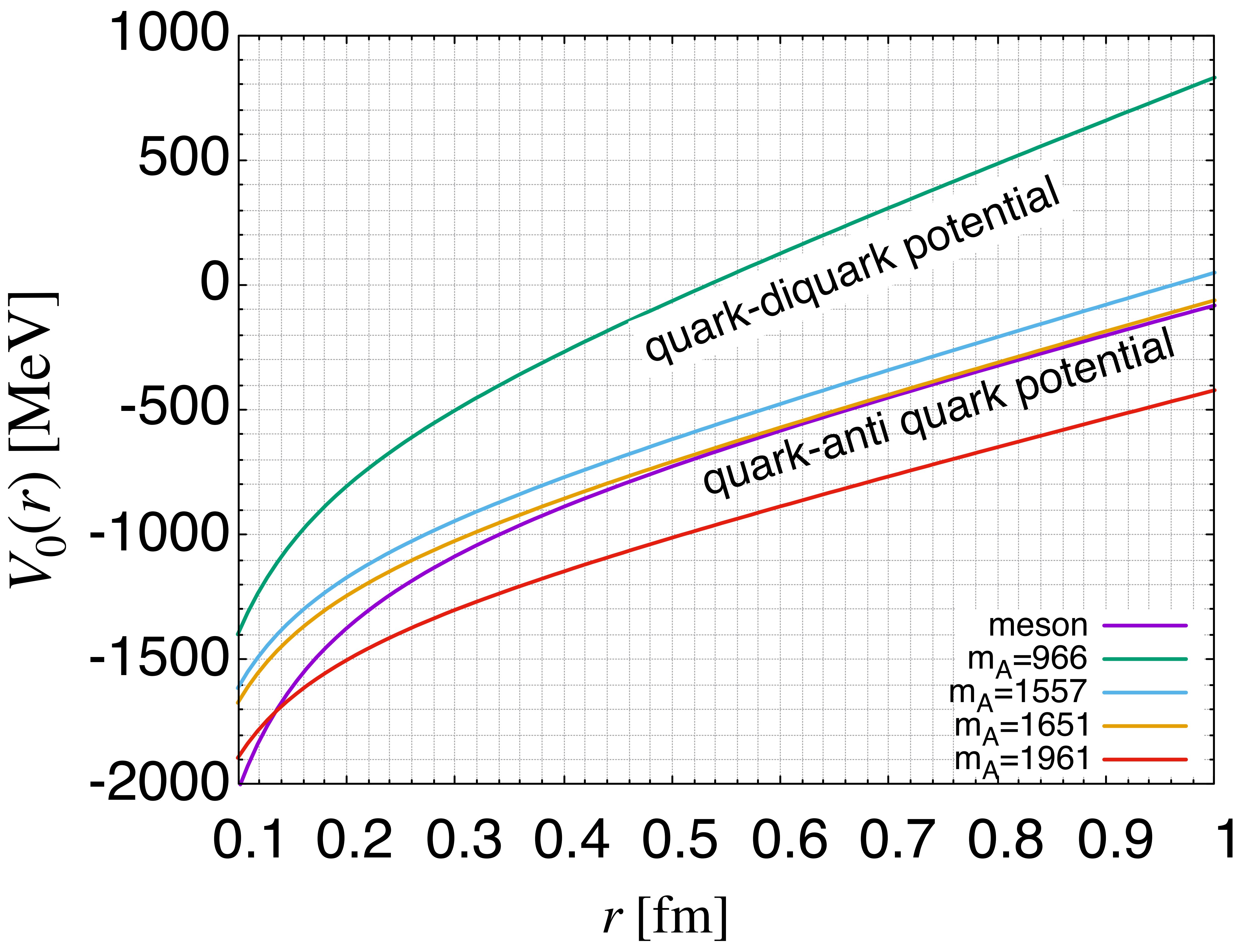
$$A \exp(-Br^2) + C \exp(-Dr^2)$$



$A = 1283$ MeV, $B = 0.80$(Lattice unit)

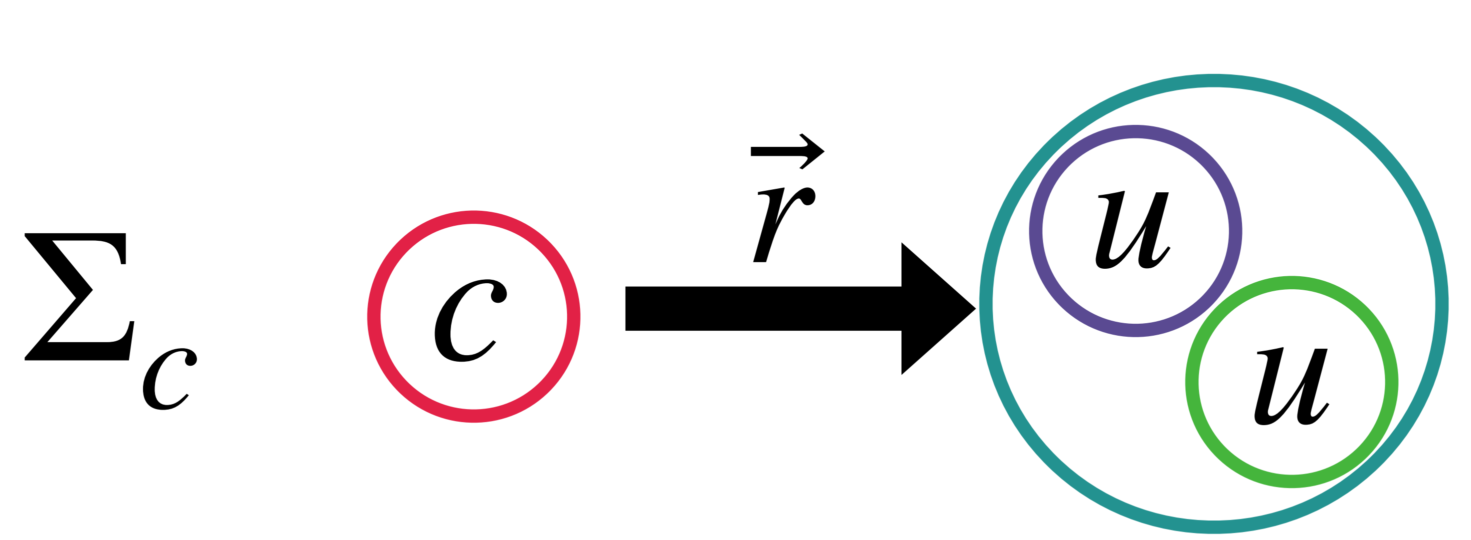
$C = 90$ MeV, $D = 0.14$(Lattice unit)

m_D vs spin independent potential



m_A	966 MeV	1557 MeV	1651 MeV	1961 MeV
$\sqrt{\sigma}$	565 MeV	488 MeV	480 MeV	459 MeV

color $\mathbf{3}$ color $\bar{\mathbf{3}}$



Far enough away, the behavior is the same.

Distance-dependent part of spin-independent potential

$$-A/r + \sigma r = \frac{1}{6\mu}(\tilde{V}(r) - \tilde{V}(\sqrt{\frac{A}{\sigma}}))$$

where、$\tilde{V}(r) = \frac{\nabla^2 \psi_{1/2}}{\psi_{1/2}} + 2\frac{\nabla^2 \psi_{3/2}}{\psi_{3/2}}$

m_D increase ➡ μ increase ➡ σ decrease

Larger m_D may be more natural