Studies of nucleon isovector structure with the PACS10 superfine lattice

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for PACS Collaboration

Introduction & Lattice QCD

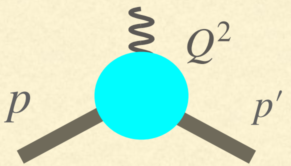
Computational cost roughly increases $\propto a^{-\gamma_0} m^{-\gamma_1} L^{\gamma_2}$ with power laws $\gamma_0, \gamma_1, \gamma_2 > 1$

Internal structure of the nucleon

Form factor describes *the internal structure* : $F(q^2) = \int \rho(\boldsymbol{r})\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}}\, d^3r$

low-q^2 quantities

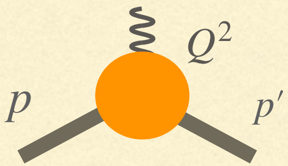
Target : coupling $g_l = G_l(0)$, Radius $\langle r_l^2 \rangle = -\frac{6}{G_l(0)} \frac{dG_l(q^2)}{dq^2}\Big|_{q^2\to 0}$



$$= \bar{u}(p')\left[\frac{(p'+p)^\mu}{2M}\frac{G_E(q^2) - \frac{q^2}{4M^2}G_M(q^2)}{1-\frac{q^2}{4M^2}} + i\sigma^{\mu\nu}\frac{q_\nu}{2M}G_M(q^2)\right]u(p)$$

$\langle N(p')\,|\,\bar{q}\gamma_\mu q\,|\,N(p)\rangle$

$$\to \langle r_E^2 \rangle,\ \mu,\ \langle r_M^2 \rangle$$



$$= \bar{u}(p')\left[\gamma_\mu\gamma_5 F_A(q^2) + iq^\mu\gamma_5 F_P(q^2)\right]u(p)$$

$\langle N(p')\,|\,\bar{q}\gamma_\mu\gamma_5 q\,|\,N(p)\rangle$ $\quad \to \langle r_A^2 \rangle$, $g_A = F_A(0)$, $g_{\pi NN}$, g_P*

Local current

Evaluation of $\langle r_l^2 \rangle$ or g: Model independent analysis, z-expansion

$$G_l(z) = \sum_{k=0}^{\infty} c_k z^k,\ z = (\sqrt{t_{cut}+q^2} - \sqrt{t_{cut}})/(\sqrt{t_{cut}+q^2} + \sqrt{t_{cut}}) \text{ with } t_{cut} = \begin{cases} 4m_\pi^2\ (l = E, M) \\ 9m_\pi^2\ (l = A) \end{cases}$$

[1] E. Shintani *et al.*, Phys. Rev. D **99**, 014510(2019), (Erratum; Phys. Rev. D **102**, 019902 (2020).)
[2] R. Tsuji *et al.*, Phy. Rev. D **109**, 094505 (2019).

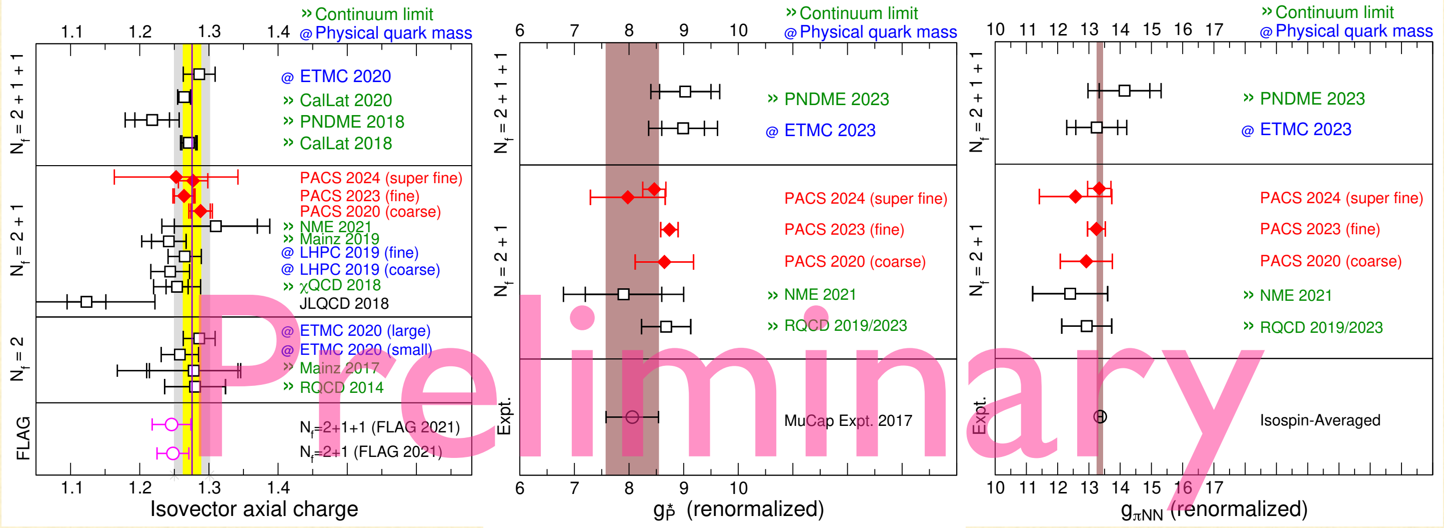
Nucleon structures from lattice QCD

Uncertainties in the calculation

- Statistical error
- Excited-state contamination
- Model-dependence in the analysis
- Chiral-Continuum-Finite-size extrapolation

PACS2020[1] + PACS2023[2]:

✔ Improvement by AMA
✔ Tuning the smearing
✔ Model-independent method
- Large-volume at physical point



This talk: Preliminary results towards the continuum limit

Numerical results I

- Preliminary results for superfine 256^4 lattice
  : G_E, G_M, F_A, g_A and Lattice spacing effect

Program for Promoting Researches on the Supercomputer Fugaku
Large-scale lattice QCD simulation and development of AI technology

Simulation details -PACS10

	128^4	160^4	256^4
L [fm]	$\sim (10.9\ \mathrm{fm})^3$	$\sim (10.1\ \mathrm{fm})^3$	$\sim (10.1\ \mathrm{fm})^3$
m_π [GeV]	0.135	0.138	~ 0.142
m_K [GeV]	0.497	0.505	~ 0.514
M_N [GeV]	~ 0.935	~ 0.947	~ 0.959
Cutoff [GeV]	2.3	3.1	4.7
	coarse	fine	Super fine
Lattice spacing	$\sim 0.085\ \mathrm{fm}$	$\sim 0.063\ \mathrm{fm}$	$\sim 0.04\ \mathrm{fm}$

Fugaku co-design outcome: [Ishikara *et al.*, CPC(2023)]
QCD Wide SIMD (QWS) Library for Fugaku
Resources: Fugaku in HPCI System Research Project (hp200062, hp200167, hp210112, hp220079, hp230199)

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	coarse	fine	Super fine
L [fm]	$\sim (10.9\ \mathrm{fm})^3$	$\sim (10.1\ \mathrm{fm})^3$	$\sim (10.1\ \mathrm{fm})^3$

Eliminate uncertainties

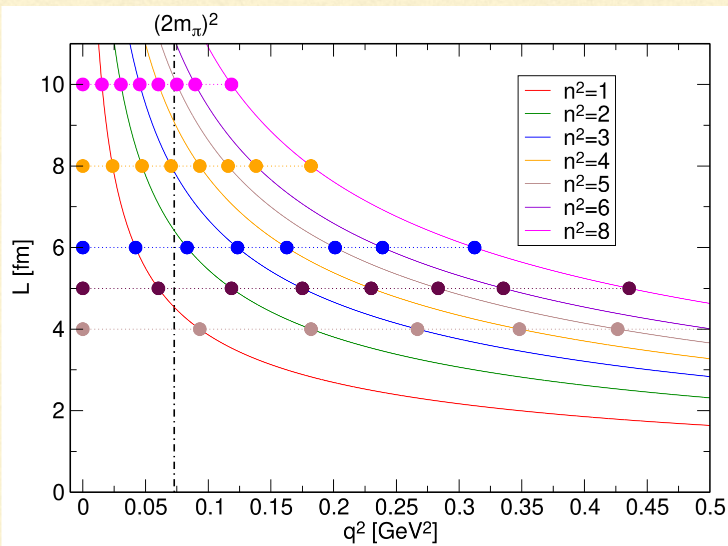
- Finite Volume effect
- Chiral extrapolation

$\otimes$

Access low-q^2 region

$$q^2 = (2\pi/L)^2 \times |\boldsymbol{n}|^2$$

= PACS10



Good resolution in Low q^2 can be achieved by a large-volume lattice QCD

i.e. our simulation is the BEST to seek the low q^2 region!

Preliminary

Program for Promoting Researches on the Supercomputer Fugaku
Large-scale lattice QCD simulation and development of AI technology

Simulation details -PACS10

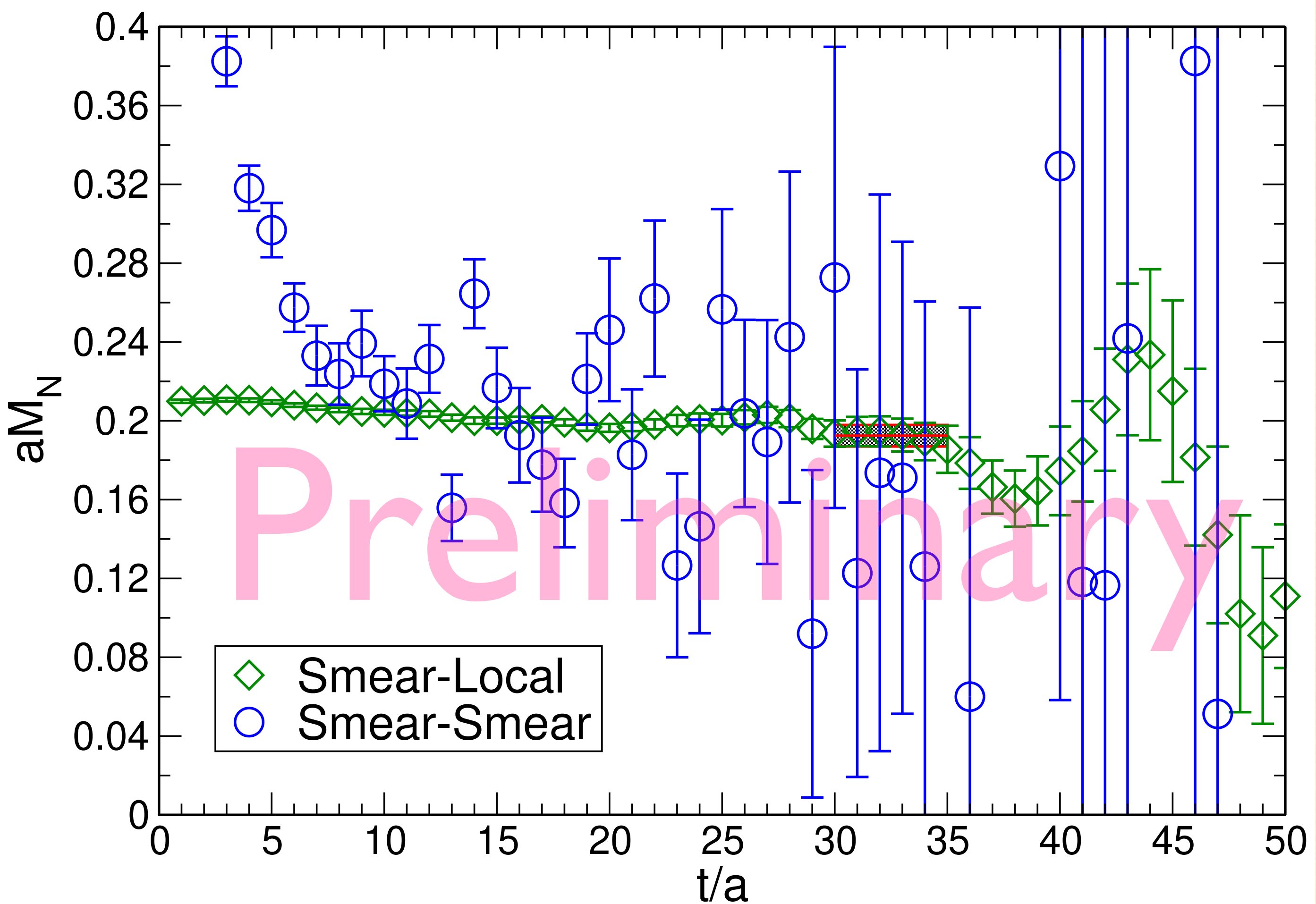
	L	$t_{\rm sep}$	$\epsilon_{\rm high}$	$\epsilon_{\rm low}$	$N_{\rm org}$	$N_{\rm G}$	$N_{\rm conf}$	$N_{\rm meas}$	Fit range
coarse	128	10	10^{-8}	—	1	128	20	2,560	[3:7]
		12	10^{-8}	—	1	256	20	5,120	[4:8]
		14	10^{-8}	—	2	320	20	6,400	[5:9]
		16	10^{-8}	—	4	512	20	10,240	[6:10]
fine	160	13	10^{-8}	—	1	64	76	4,864	[4:8]
		16	10^{-8}	—	3	192	76	14,592	[6:10]
		19	10^{-8}	—	4	768	76	58,368	[7:10]
Super fine	256	20	10^{-8}	—	1	16	24	384	[8:12]
		29	10^{-8}	—	1	16	50	800	[13:17]

* $N_{\rm org}$ and N_G: the number of high- and low-precision calculations; $N_{\rm meas}$: the number of measurements ($N_{\rm meas} = N_{\rm org} \times N_G$)

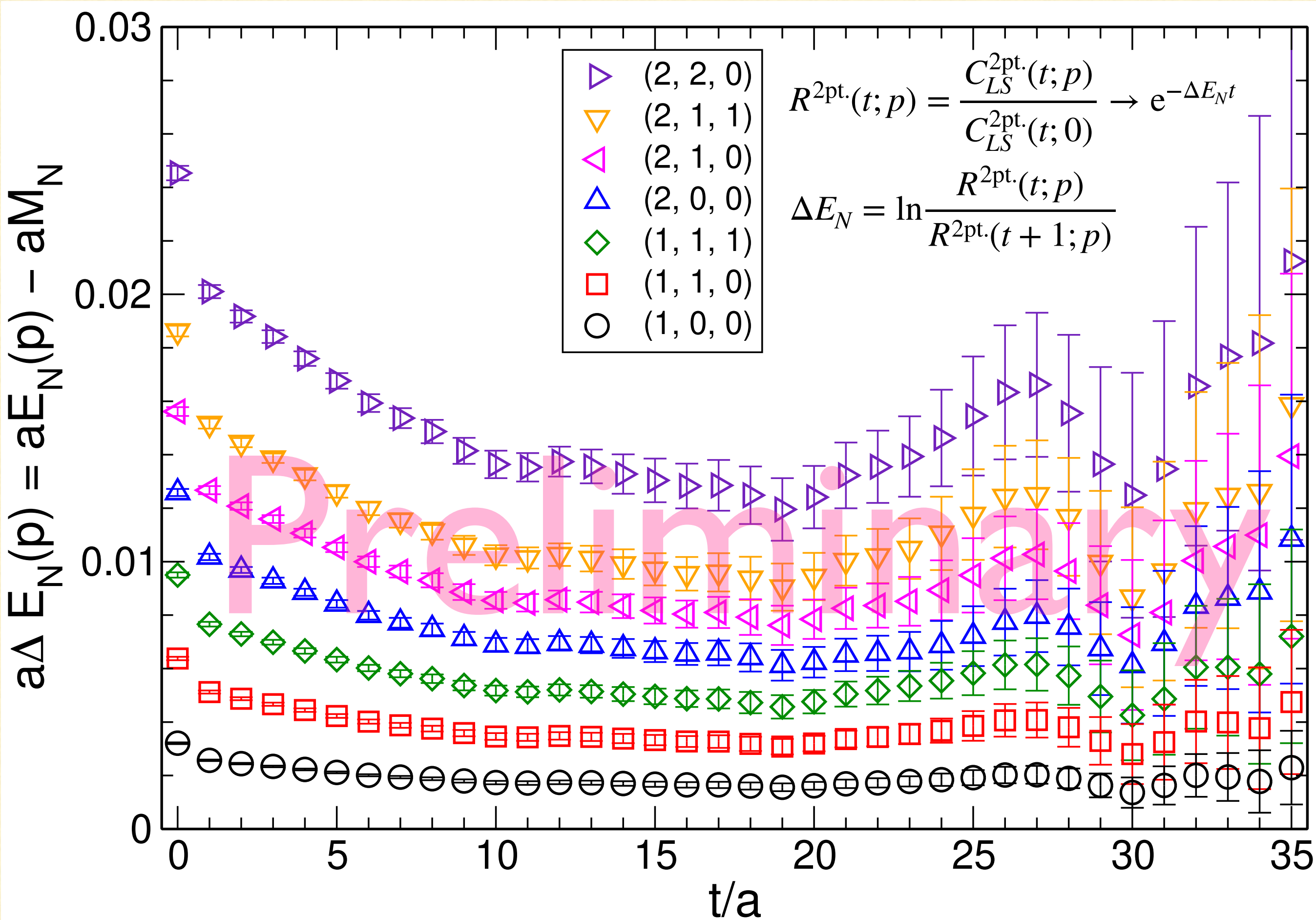
* The low precision calculation use a fixed number of iteration for the stopping condition as several GCR iterations using tiny SAP domain size with $O(10)$ deflation fields.

Effective mass

$aM_N = 0.192(6),\ M_N = 0.922(26)$ GeV



Energy difference



Preliminary

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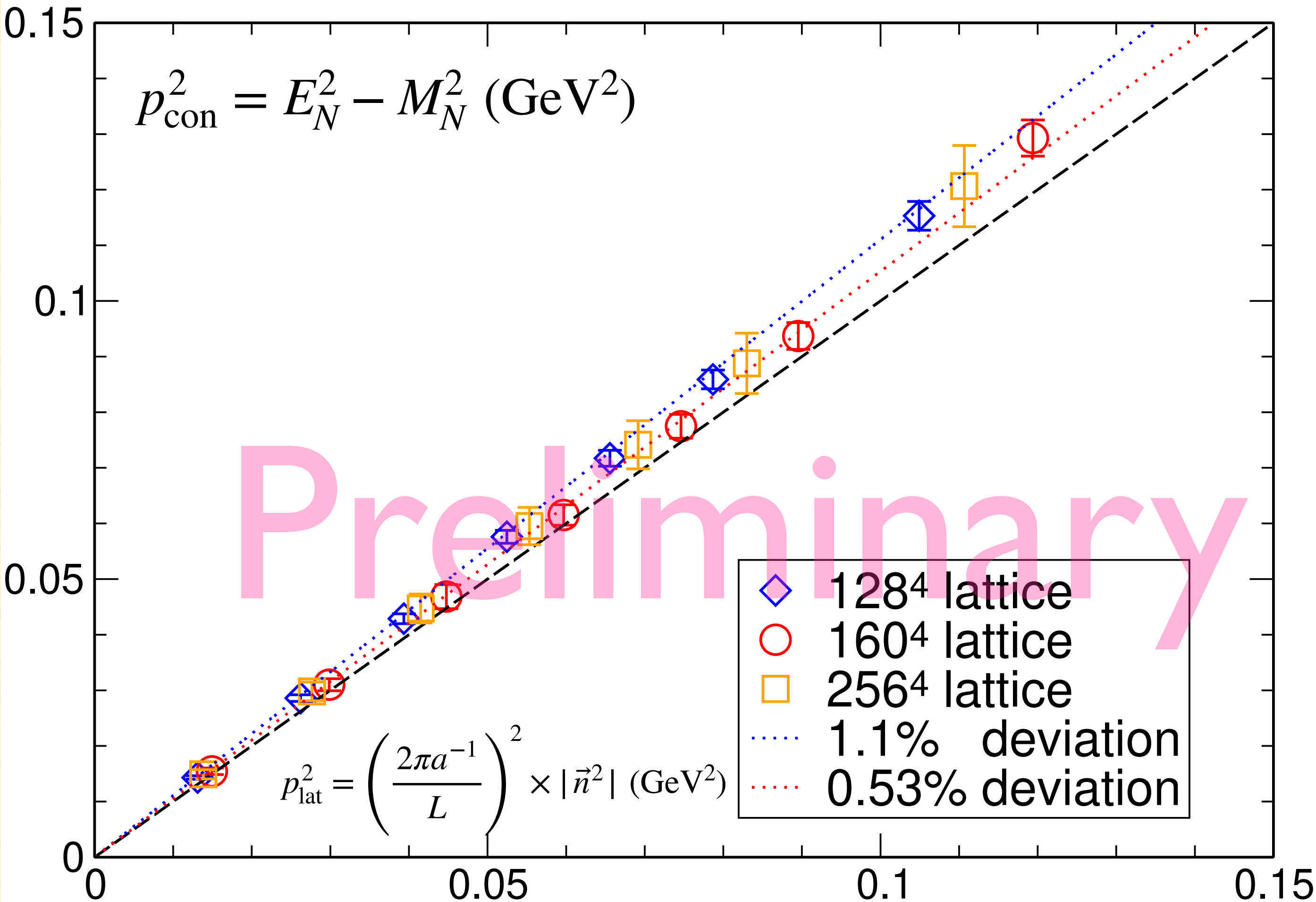
Dispersion relation

Lattice spacing effect is small



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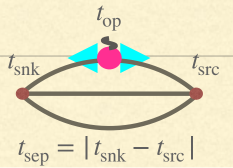
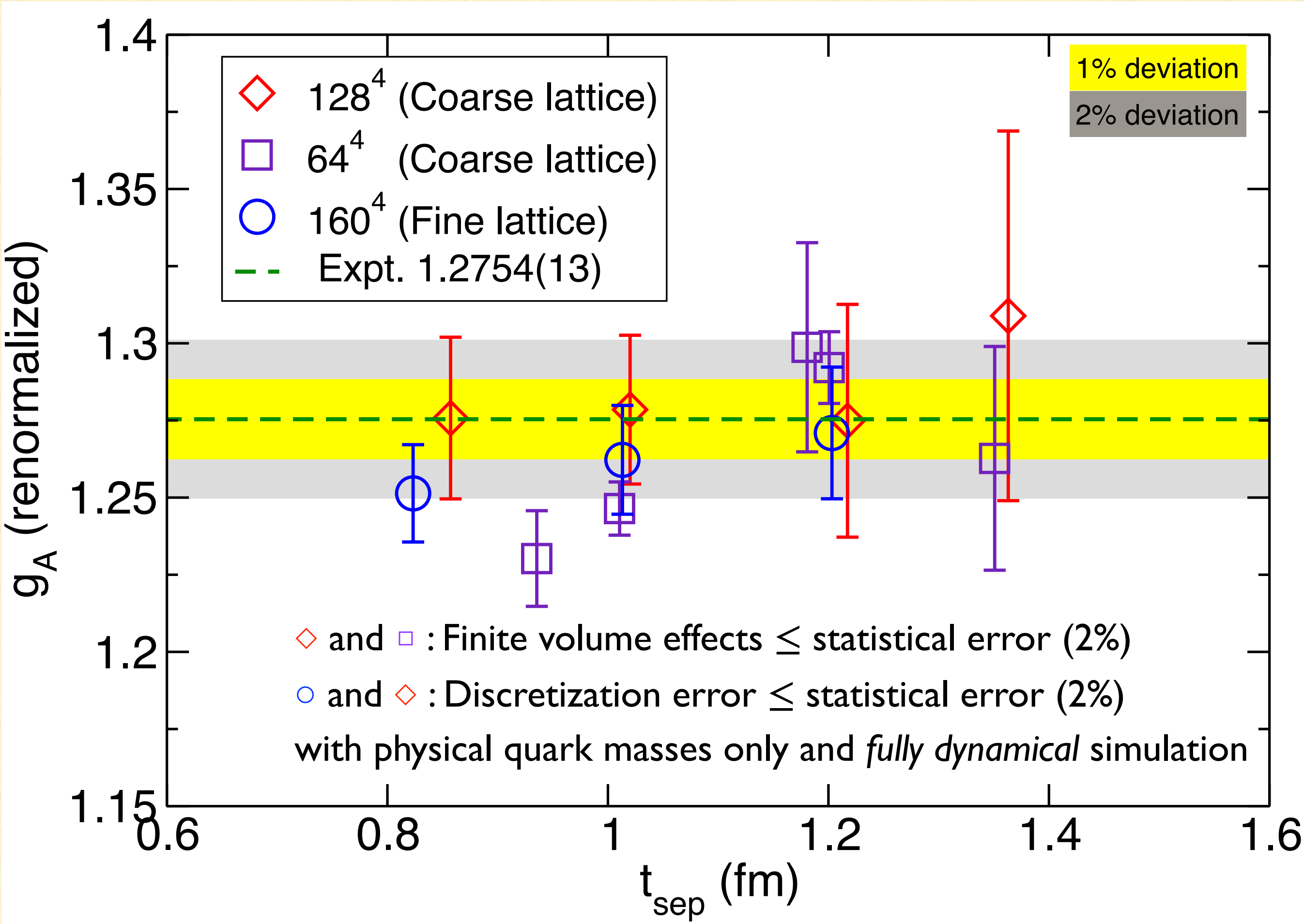
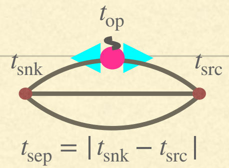


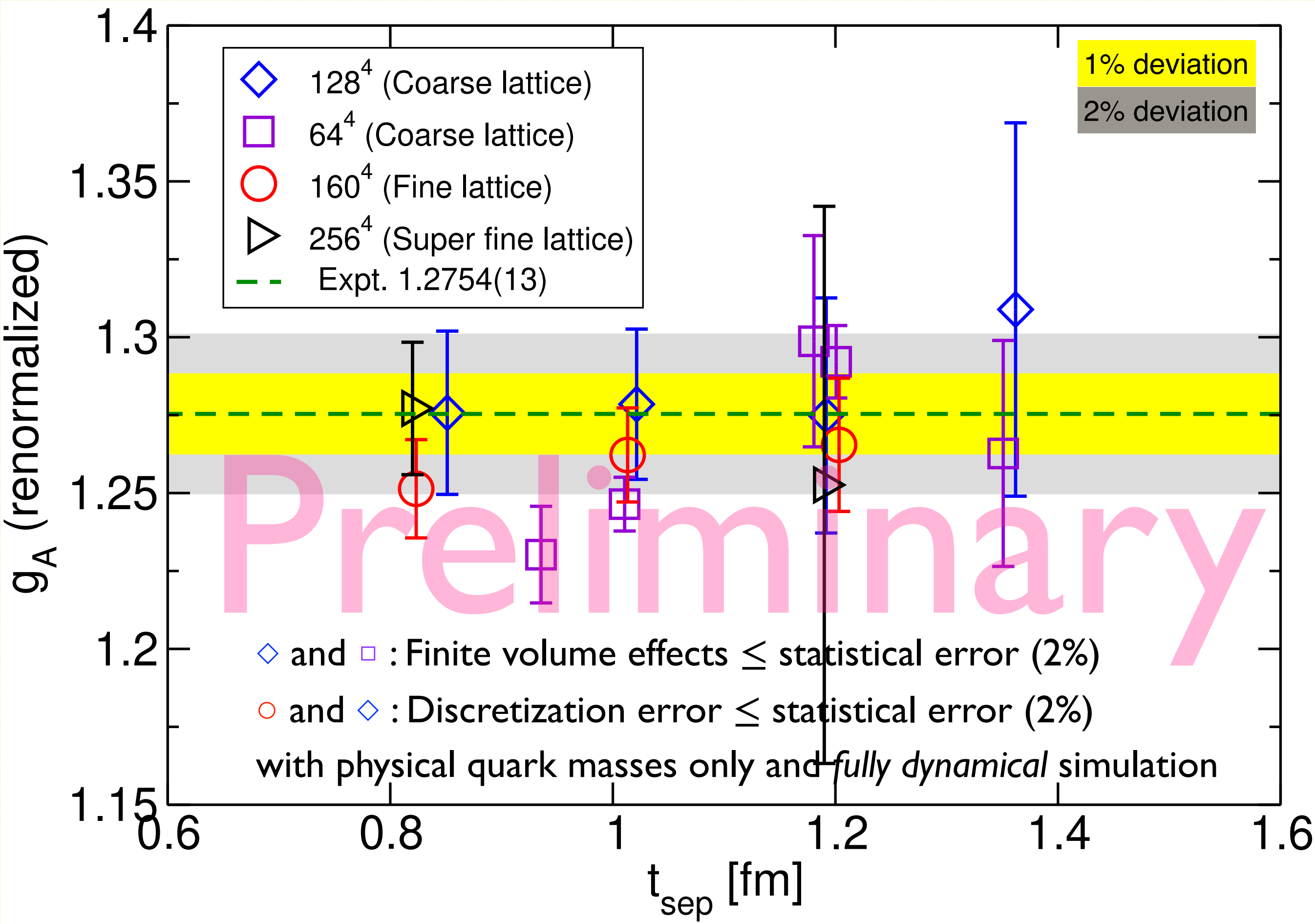
Axial-vector coupling



Preliminary

Axial-vector coupling





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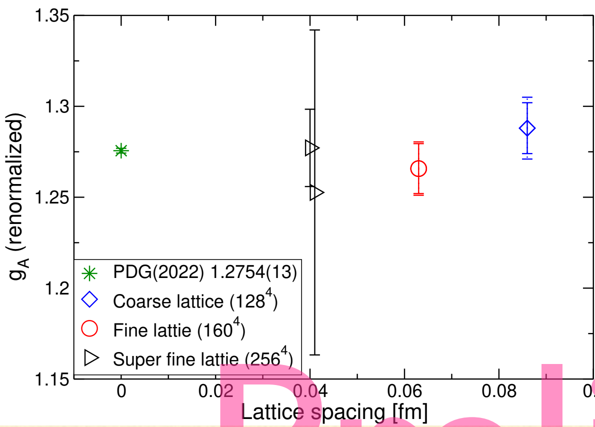
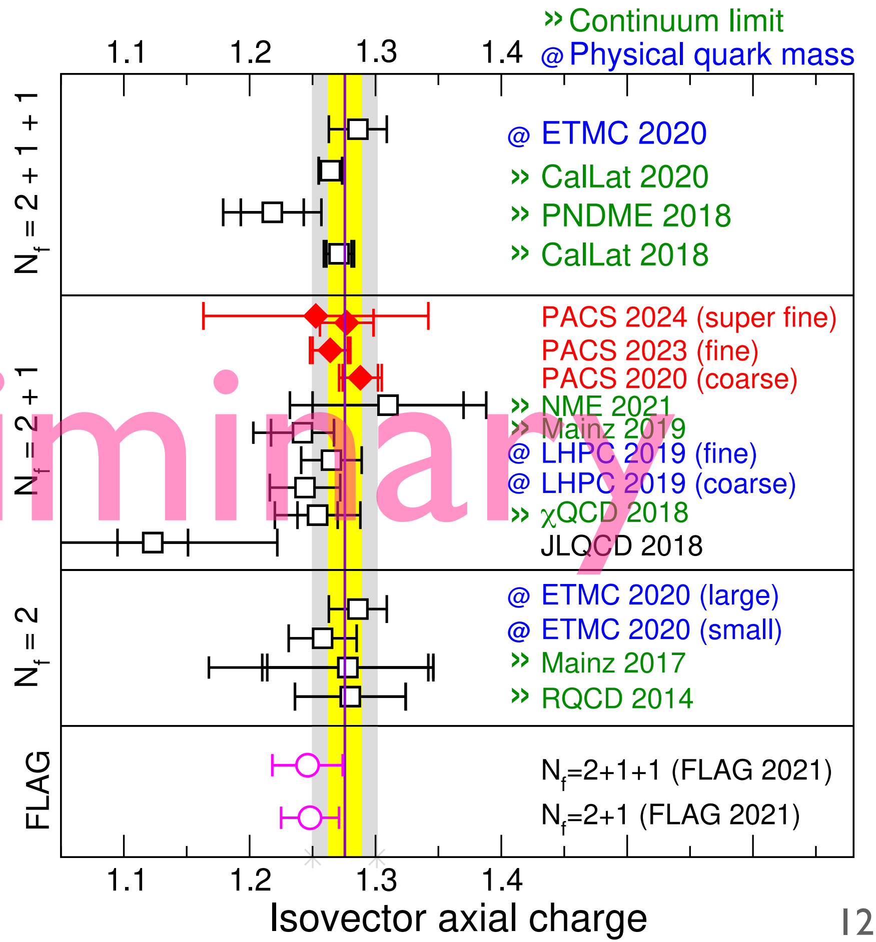


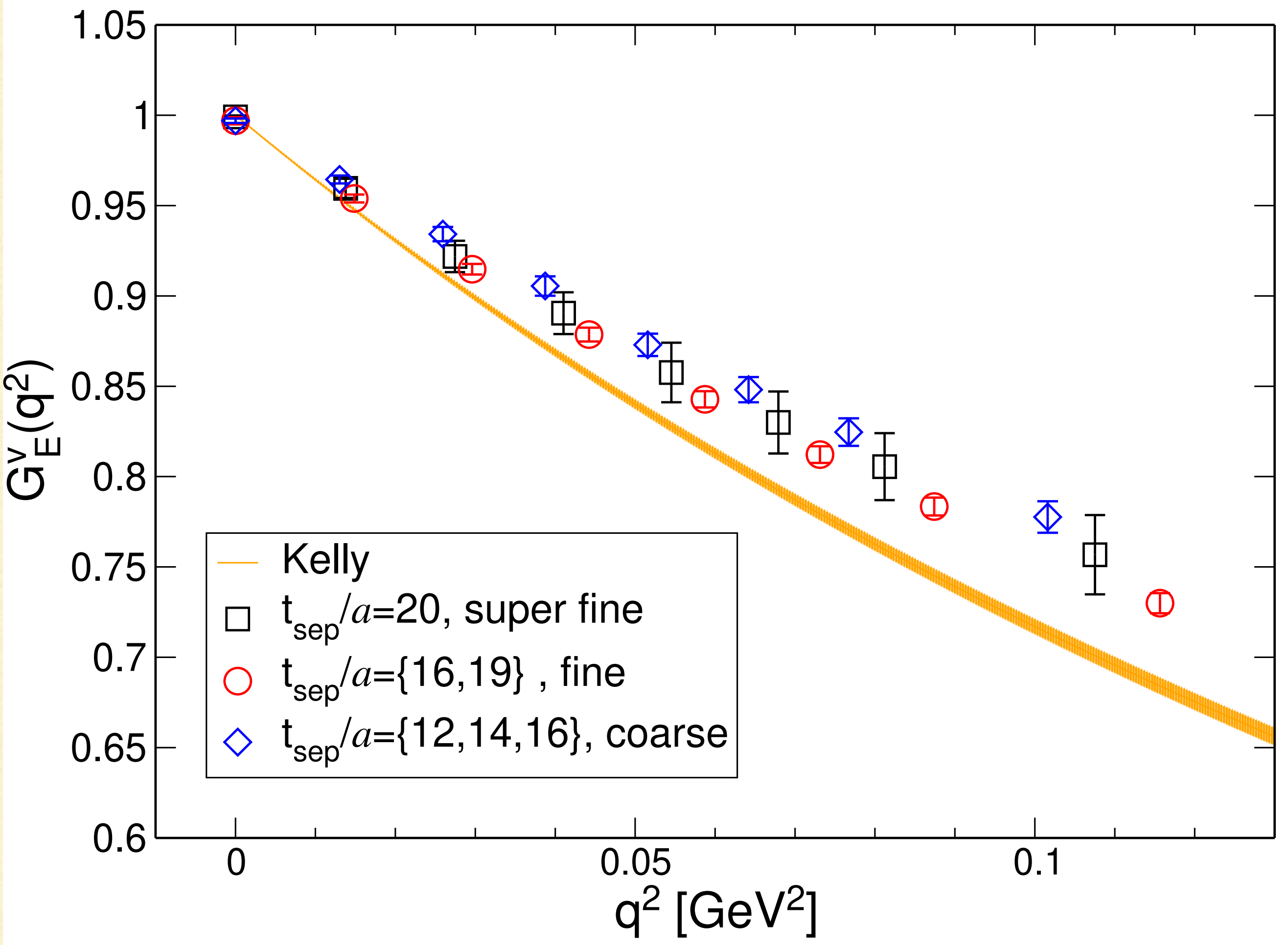


Preliminary

Reproduce the PDG value within the statistical precision for every PACS10 ensembles

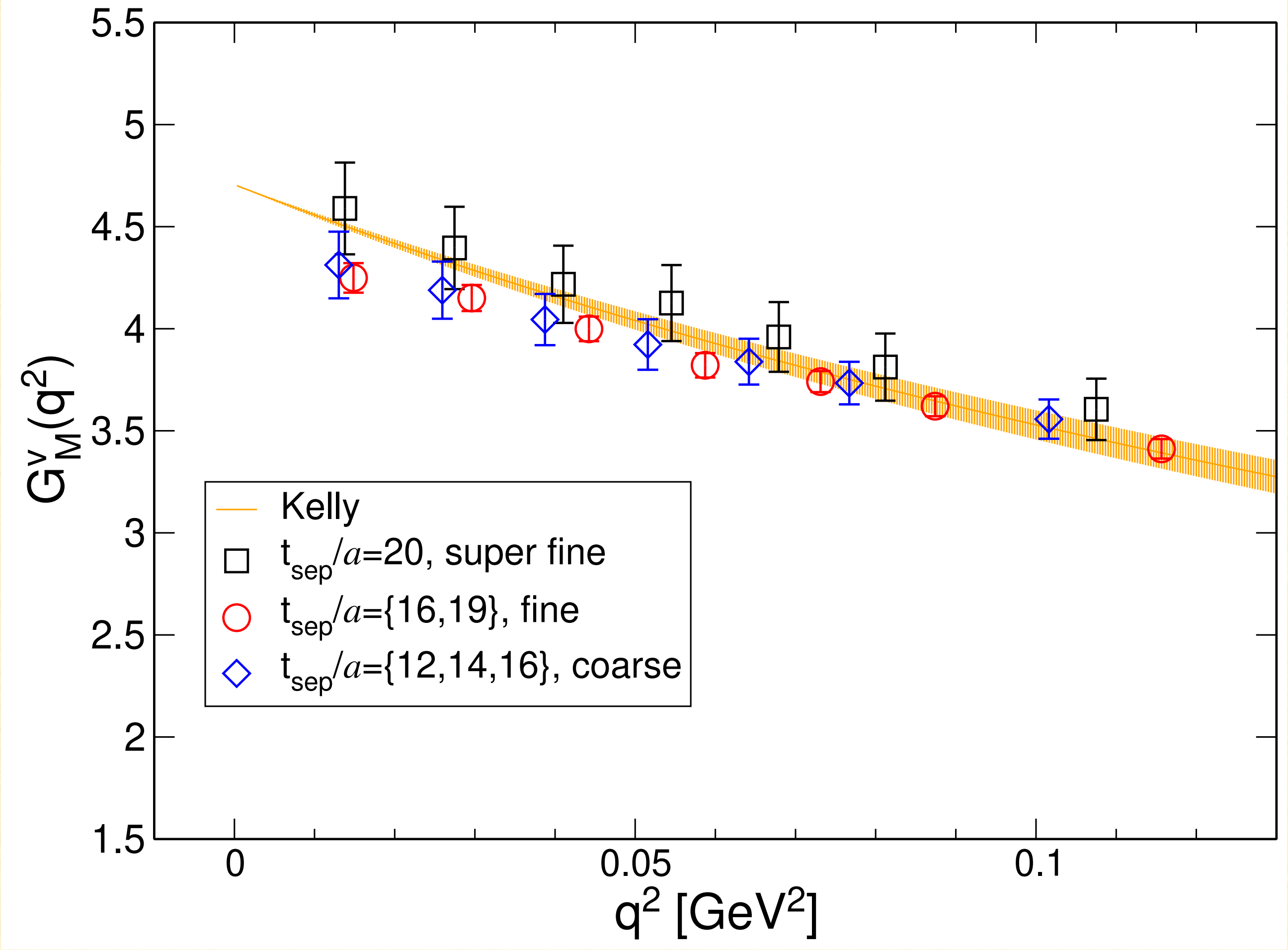
Further calculation is necessary

Electric form factor



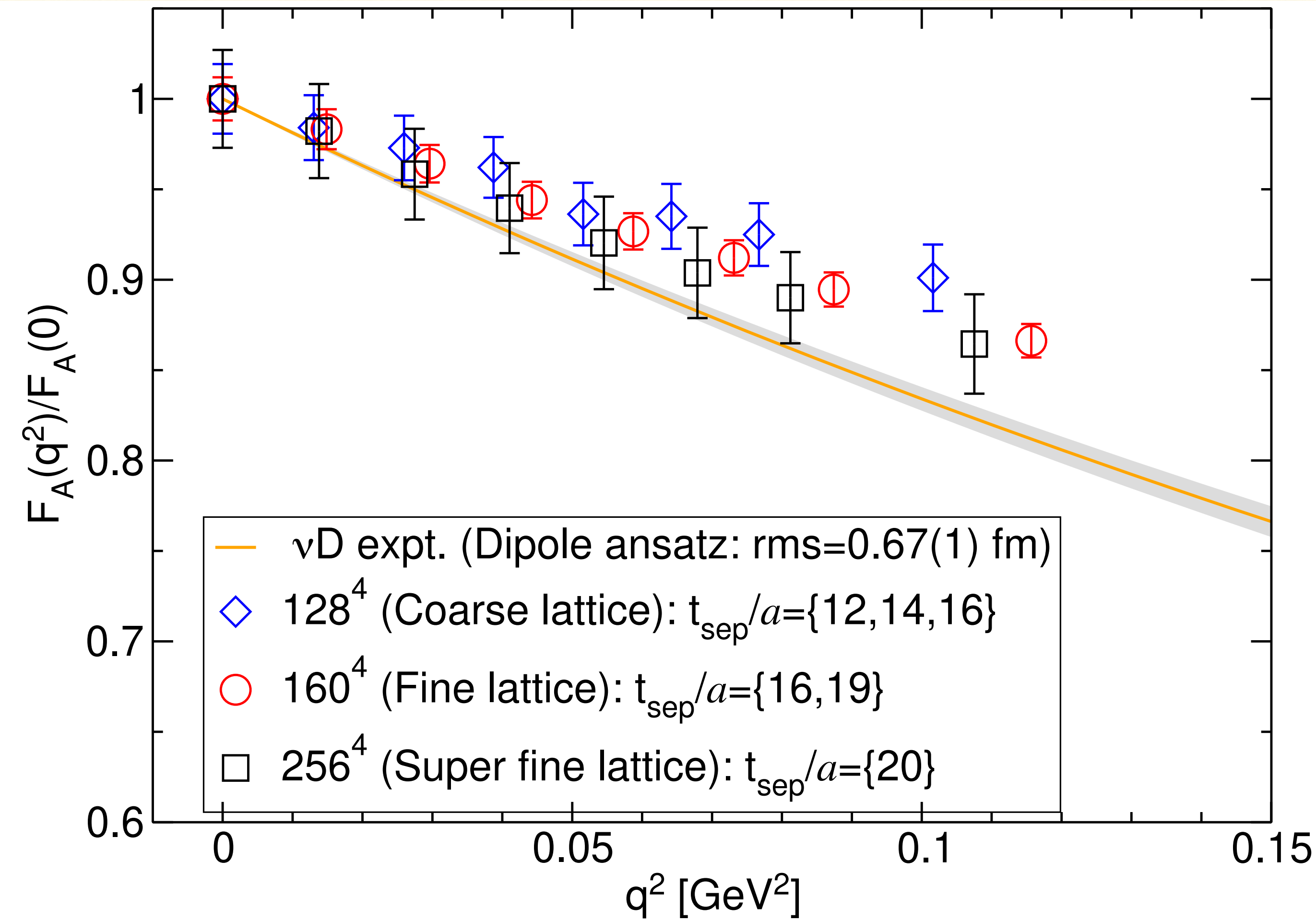
Preliminary

Magnetic form factor



Preliminary

Axial form factor



Lattice spacing effect

Do we need the $O(a)$ improvement of the current?

$O(a)$ improved current $A_\alpha^{\rm imp} = A_\alpha + c_A a \partial_\alpha P$ → PCAC relation

$$m_{\rm PCAC}^{\rm pion} \equiv \frac{m_\pi^2 f_\pi}{2\langle 0 | P^+(0) | \pi \rangle} \quad \longleftrightarrow \quad m_{\rm PCAC}^{\rm nucl} \equiv \frac{\langle N_{\rm snk} \partial_\mu A_\mu(x) \overline{N}_{\rm src} \rangle}{2\langle N_{\rm snk} P(x) \overline{N}_{\rm src} \rangle}$$

- Pion 2-pt function
- Zero momentum
- Improvement is small

$$(m_{\rm PCAC}^{\rm pion})^{\rm imp} = m_{\rm PCAC}^{\rm pion} + \frac{a c_A m_\pi^2}{2}$$

$m_{\rm PCAC}^{\rm pion} = m_{\rm PCAC}^{\rm nucleon}$ in $a \to 0$

$\to (m_{\rm PCAC}^{\rm pion})^{\rm imp} = (m_{\rm PCAC}^{\rm nucleon})^{\rm imp}$

$\to \Delta = m_{\rm PCAC}^{\rm pion} - m_{\rm PCAC}^{\rm nucleon}$

$\sim$ discretization error

- Nucleon 3-pt function
- Nonzero momentum
- Improvement works

$$(m_{\rm PCAC}^{\rm nucl})^{\rm imp} = m_{\rm PCAC}^{\rm nucl} - \frac{a c_A q^2}{2}$$

In the continuum limit, $m_{\rm PCAC}^{\rm pion}$ and $m_{\rm PCAC}^{\rm nucl}$ should be identical.

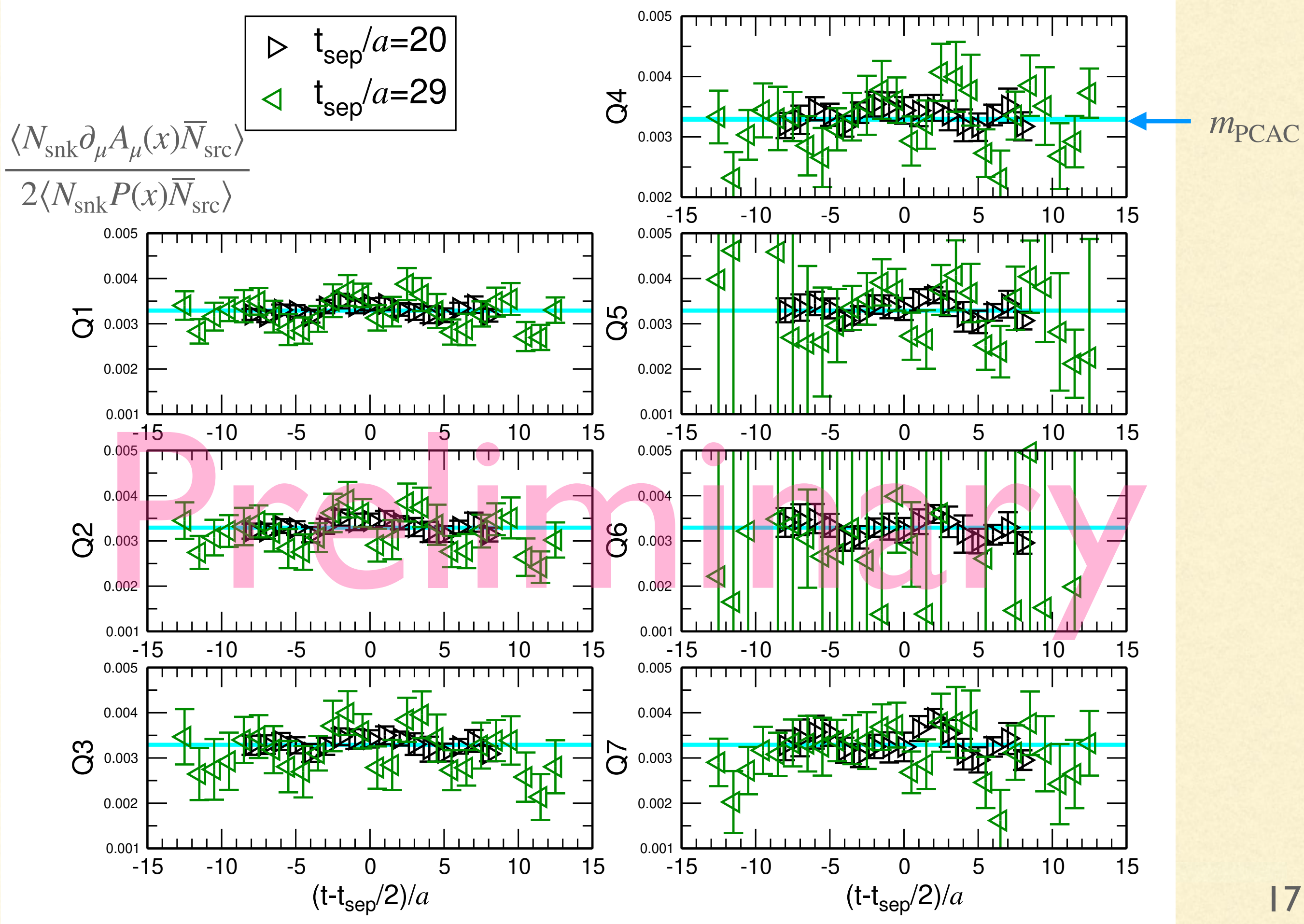
→ a difference can be attributed to lattice spacing effect

$$\partial_t C_{A_4}^{53}(t;\mathbf{q}) = \frac{1}{2a}\left\{ C_{A_4}^{53}(t+a;\mathbf{q}) - C_{A_4}^{53}(t-a;\mathbf{q}) \right\}, \text{ and } \partial_k C_{A_k}^{53}(t;\mathbf{q}) = \frac{i}{a}\sin(q_k a) C_{A_k}^{53}(t;\mathbf{q})$$

$$\to m_{\rm nucl}^{\rm AWTI} = \frac{\frac{1}{2a}\left\{ C_{A_4}^{53}(t+a;\mathbf{q}) - C_{A_4}^{53}(t-a;\mathbf{q}) \right\} - \frac{i}{a}\sin(q_k a) C_{A_k}^{53}(t;\mathbf{q})}{2C_P^3(t;\mathbf{q})}$$

PCAC quark masses

$A^{\rm imp}_\alpha = A_\alpha + c_A a \partial_\alpha P$ effect is small

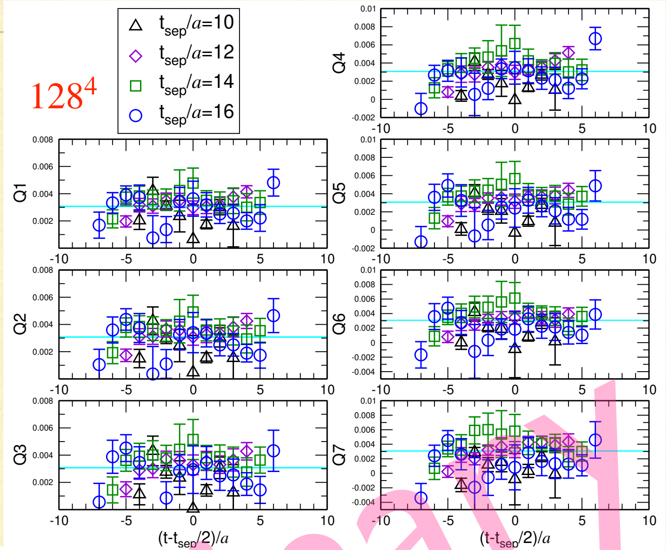


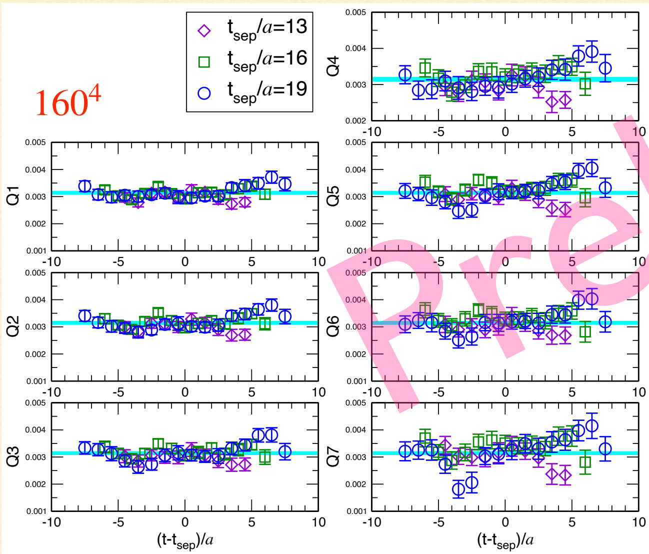
R. Tsuji et al., Phys. Rev. D 109, 094505 (2024) + PhD thesis

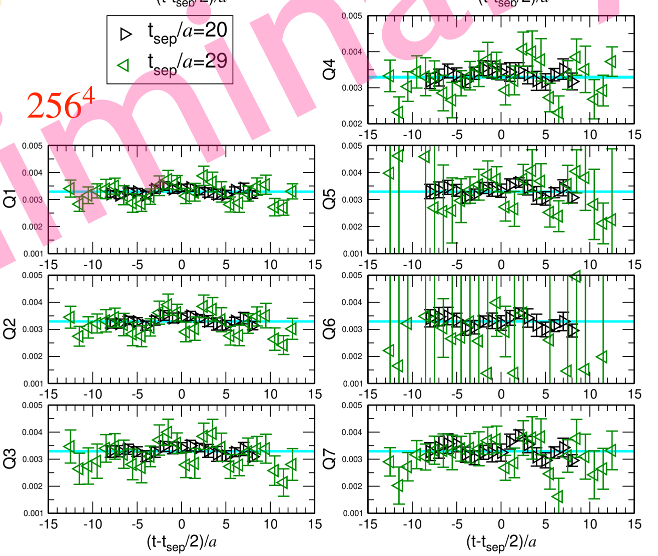
PCAC quark masses

$A_\alpha^{\mathrm{imp}} = A_\alpha + c_A a \partial_\alpha P$ effect is small

for every PACS10 ensembles







Preliminary

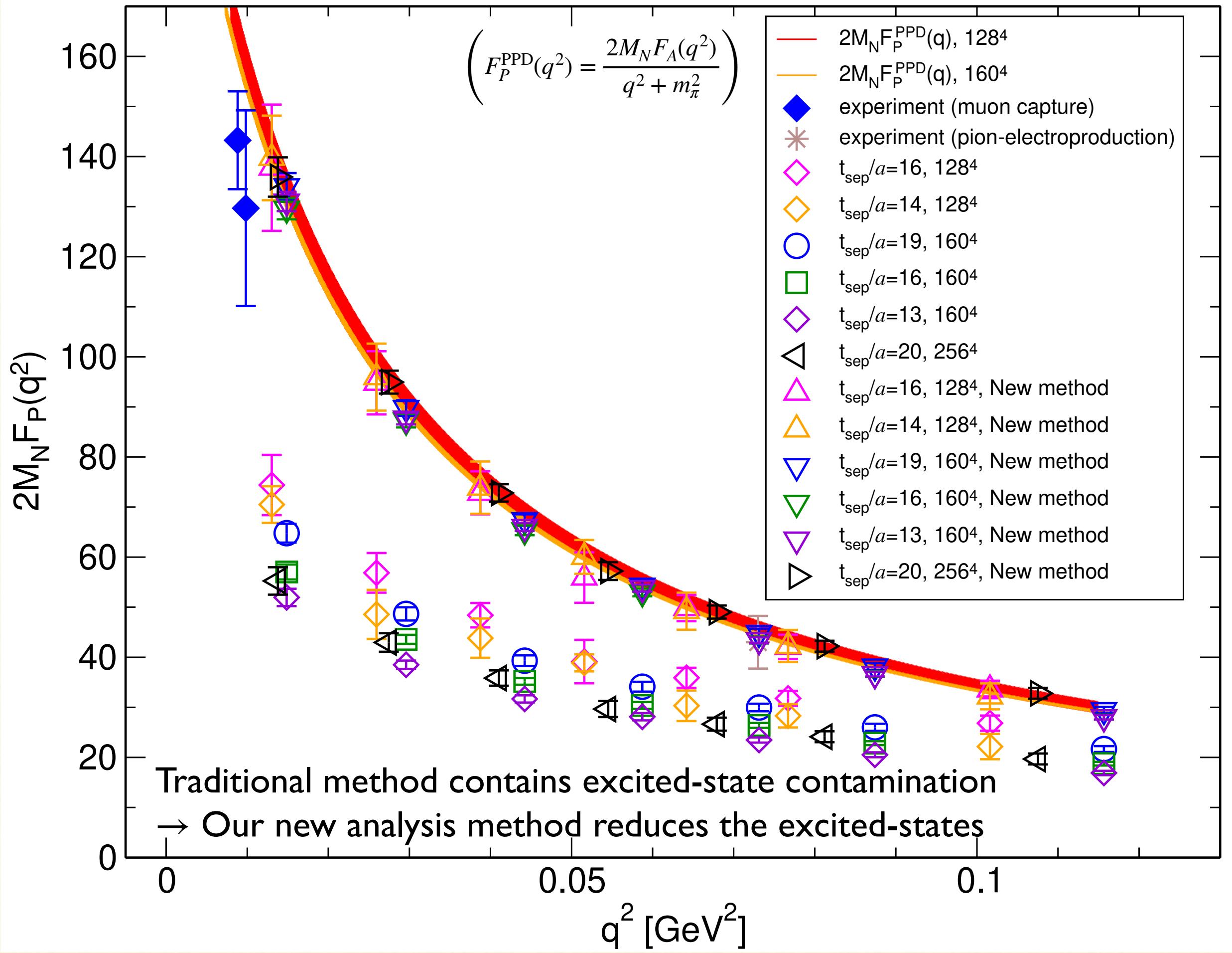
Numerical results II

- Preliminary results for superfine 256^4 lattice

: $F_P, g_P^*, g_{\pi NN}$

! Detail: Talk by S. Sasaki July 29 14:15~!

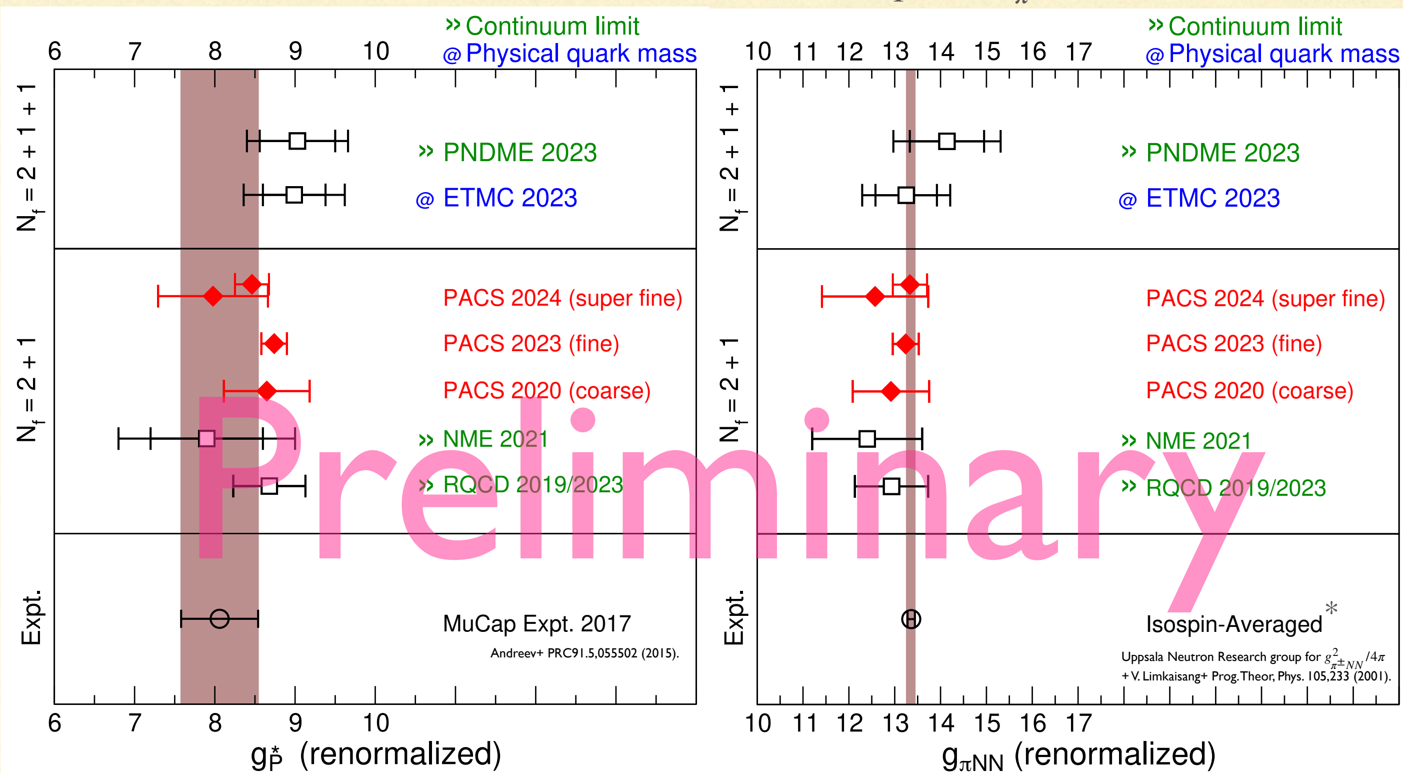
Induced pseudoscalar form factor



Couplings

Induced pseudoscalar coupling $: g_P^* \equiv m_\mu F_P(q^2 = 0.88 m_\mu^2)$

Pion-nucleon coupling $: g_{\pi NN} \equiv \lim_{q^2 \to -m_\pi^2} \frac{m_\pi^2 + q^2}{2\pi} F_P(q^2)$



Summary

Program for Promoting Researches on the Supercomputer Fugaku
Large-scale lattice QCD simulation and development of AI technology

Summary

- Physical point simulation → No chiral extrapolation
- Large-volume simulation → Access low-q^2 region

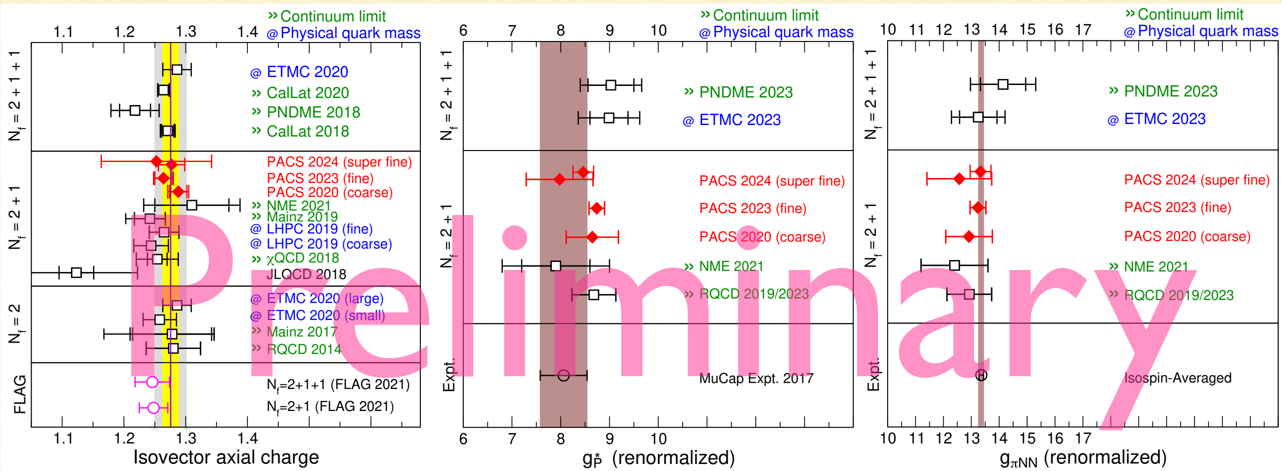
Great advantage!

- *Fully dynamical lattice QCD simulations* towards **continuum limit**

Clarify the nucleon structure in context of QCD

Our results:

- For g_A, both superfine, coarse and fine reproduce PDG within statistical error.
- AWTI is satisfied in the level of the nucleon correlation function.
- F_P from our new method agrees with the PPD model and LQCD results.

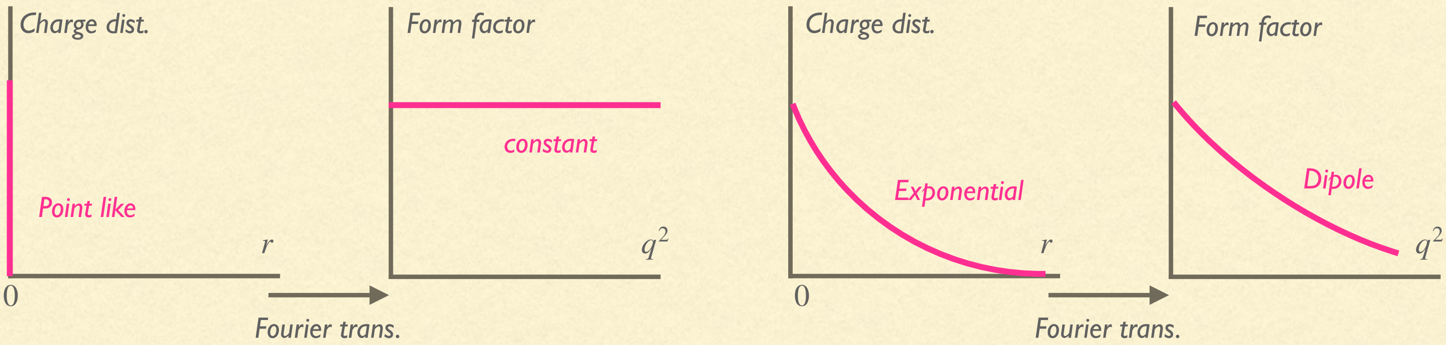


BACKUPS

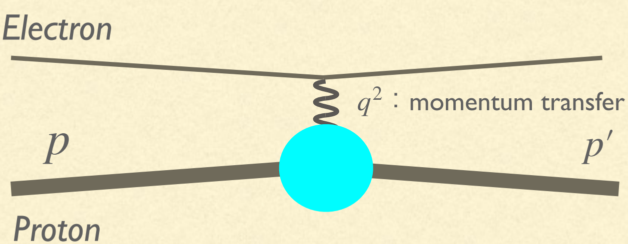
Form factor and cross section

Form factor describes *the internal structure* : $F(q^2) = \int \rho(\boldsymbol{r})\, \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{r}}\, d^3r$

Point particle | *Composite particle*



e.g. Proton-electron elastic scattering



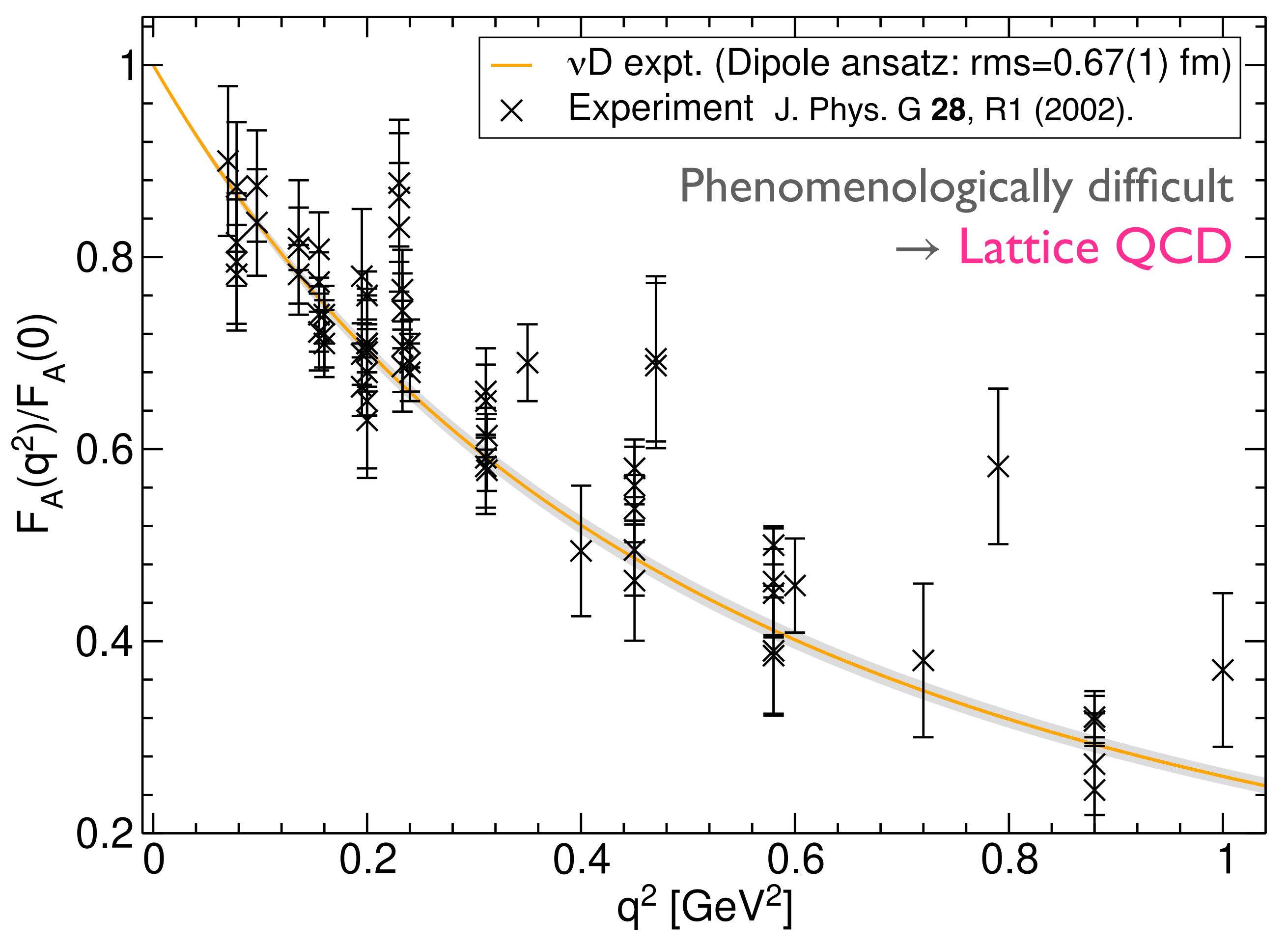
Form factor

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{Point}} \times \frac{G_E^2(q^2) + \frac{\tau}{\epsilon} G_M^2(q^2)}{1+\tau}$$

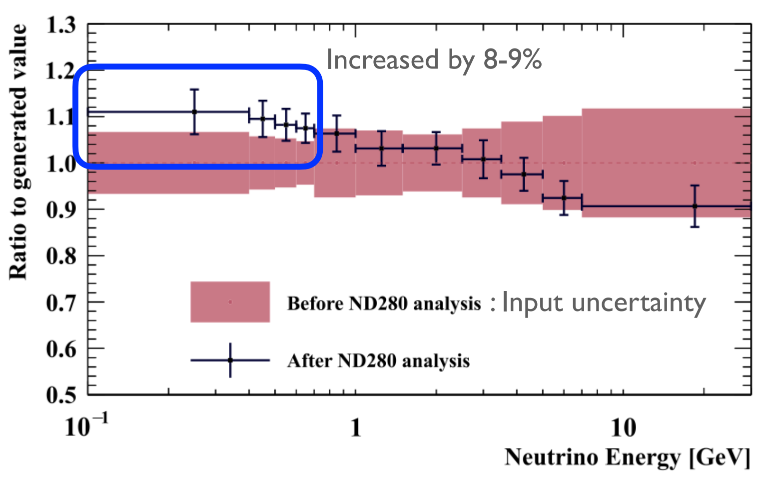
"Internal structure"

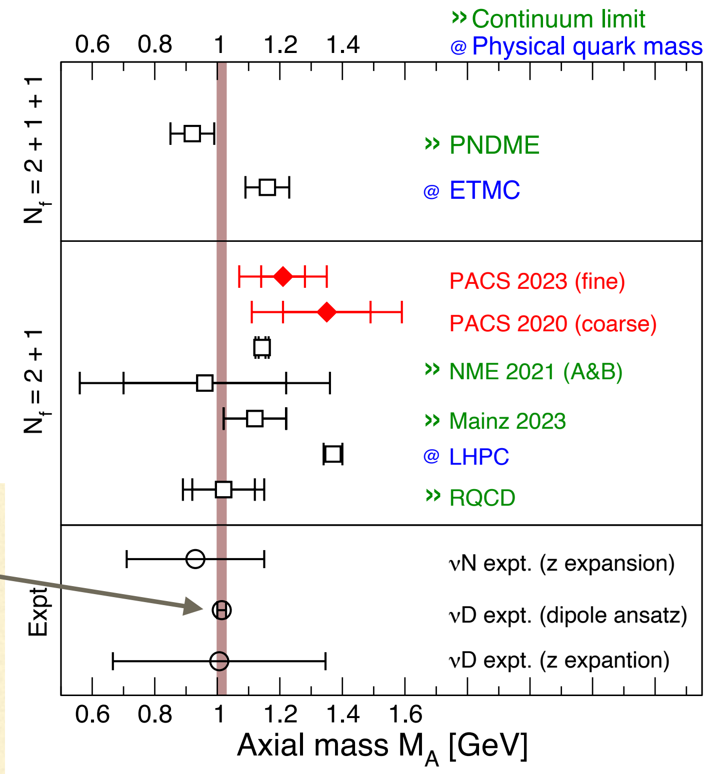
The Rosenbluth formula, $\tau = -q^2/4M_N^2$ and ϵ is the polarization of the exchanged virtual photon

Axial form factor -experiment



Problem in T2K





Ancient('70s) expt. gives $M_A = 1.014(14)$

But, T2K ND expt. features $M_A \sim 1.1$

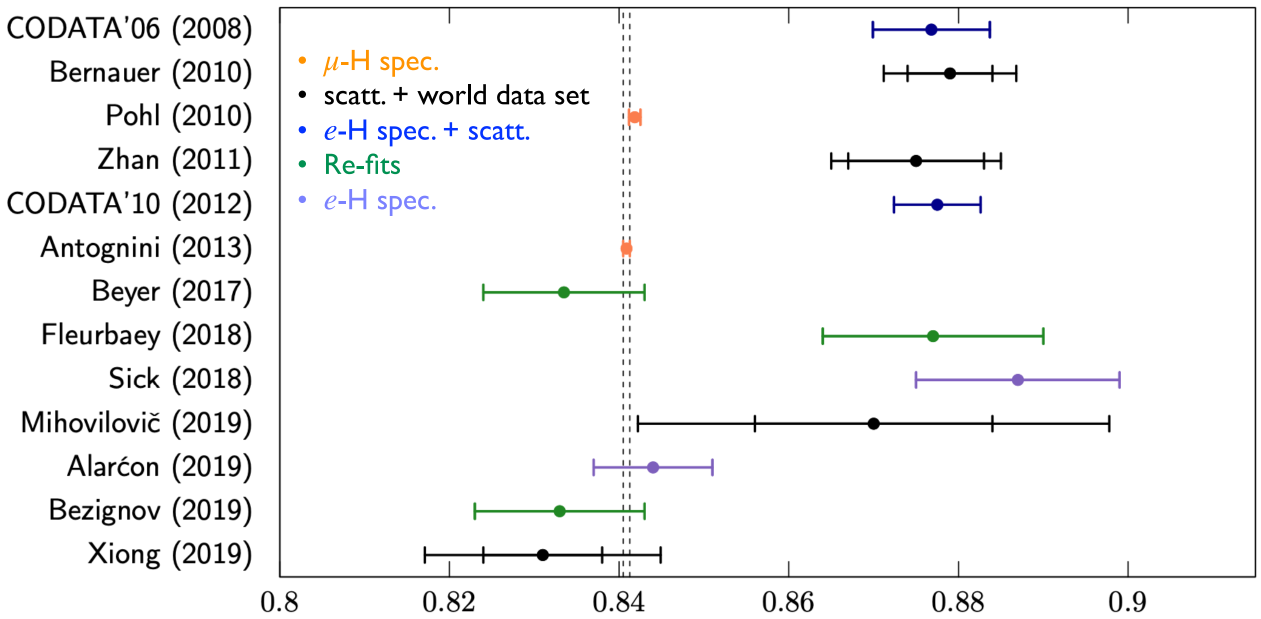
indeed, there are many systematic uncertainties…

What is the problem?

Dipole form: $F_A(q^2) = g_A / \left(1 - q^2/M_A^2\right)^2$

※Old expts. (90s-00s) give larger dipole mass $M_A = 1.1 - 1.2$
But the targets are O or C, while deuteron is used in '70s…

Proton radius puzzle (2010~) (expt. vs expt.)



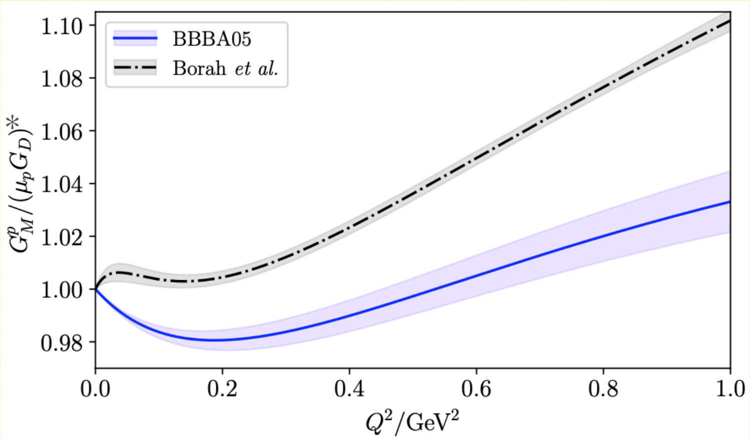
[1] Taken from Bernauer, EPJ Web Conf. 234 (2020)

Recent perspectives

- Experiments with similar kinematics as the earlier ones
- Muon vs Electron → Scattering vs Spectroscopy
- Our understandings are lacking something and more?

LQCD: NOT enough precision → A precent level is needed

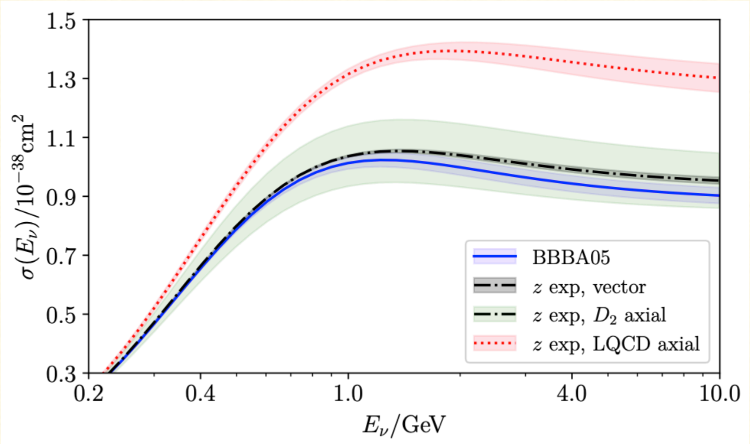
Magnetic and Axial Form Factor



- **Magnetic FF** (expt. vs expt.)

Different parameterizations exhibits clear discrepancy over all $Q^2 > 0$

LQCD: NOT enough precise
→ A percent level precision is needed



- **Axial FF** (expt. $\overset{?}{\text{vs}}$ lat.)

Less known $F_A(q^2)$ behavior causes large uncertainties on νN cross section

LQCD: enough precise
→ Theoretical prediction

* dipole ansatz with a dipole mass of 0.84 GeV

[1] Taken fromAaron A. S. Mayer et al., arXiv:2201.01839 (2022).

Nucleon form factor from lattice QCD

Uncertainties in the calculation

- Statistical error
- Excited-state contamination
- Model-dependence in the analysis
- Chiral-Continuum-Finite-size extrapolation

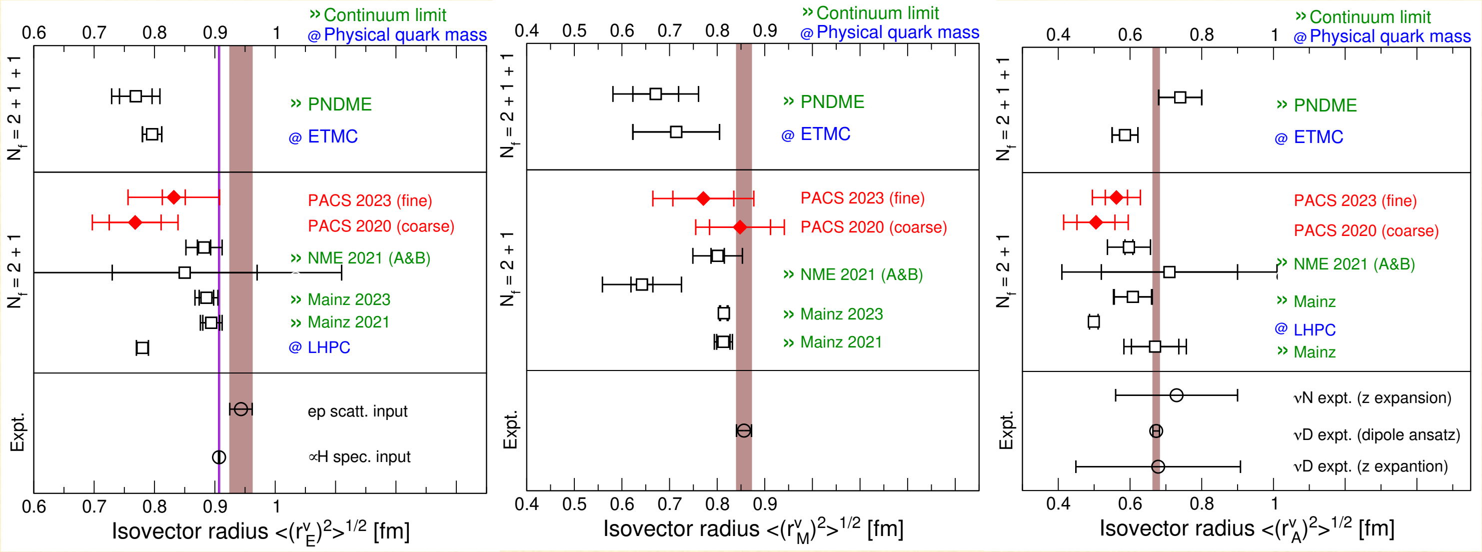
PACS2020 + PACS2024:

✔ Improvement by AMA

✔ Tuning the smearing

✔ Model-independent method

- Large-volume at physical point



Investigate the continuum limit of our configurations!

Isovector quantities

If the strange contributions is ignored under the exact isospin symmetry

Proton-electron:

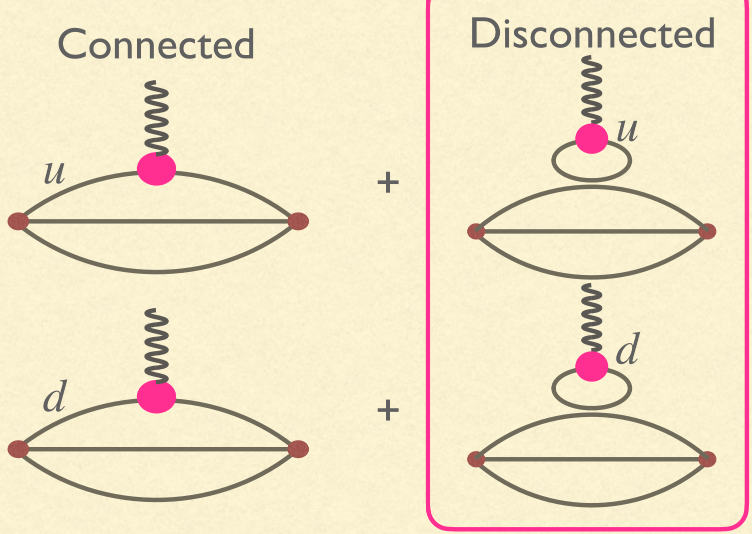
$$\langle p | j_\alpha^{\rm em} | p \rangle = 2/3 \langle p | \bar{u} \gamma_\alpha u | p \rangle - 1/3 \langle p | \bar{d} \gamma_\alpha d | p \rangle$$

Isovector:

$$\langle p | \bar{u} \gamma_\alpha d | n \rangle = \langle p | \bar{u} \gamma_\alpha u - \bar{d} \gamma_\alpha d | p \rangle = \langle p | j_\alpha^{\rm em} | p \rangle - \langle n | j_\alpha^{\rm em} | n \rangle$$

$\langle p | \bar{u} \gamma_\alpha u | p \rangle =$

$\langle p | \bar{d} \gamma_\alpha d | p \rangle =$



Canceled in isovector under the exact isospin sym.

Isovector quantities

$$j_\alpha^{\rm em} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d = \frac{1}{2}\underbrace{\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right)}_{\text{Isovector}} + \frac{1}{6}\underbrace{\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right)}_{\text{Isoscalar}} = j_\alpha^V + \frac{1}{3}j_\alpha^S$$

Proton: $\langle p | j_\alpha^{\rm em} | p \rangle = \langle p | j_\alpha^V | p \rangle + \frac{1}{3}\langle p | j_\alpha^S | p \rangle$

Neutron: $\langle n | j_\alpha^{\rm em} | n \rangle = \langle n | j_\alpha^V | n \rangle + \frac{1}{3}\langle n | j_\alpha^S | n \rangle$

Isospin symmetry: $\langle p | j_\alpha^S | p \rangle = \langle n | j_\alpha^S | n \rangle$, $\langle p | j_\alpha^V | p \rangle = -\langle n | j_\alpha^V | n \rangle$

$$\langle p | j_\alpha^{\rm em} | p \rangle - \langle n | j_\alpha^{\rm em} | n \rangle = 2\langle p | j_\alpha^V | p \rangle = \underbrace{\langle p | \left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right) | p \rangle}_{\text{Isovector}}$$

$$= \langle p | \bar{u}\gamma_\mu d | n \rangle$$ Weak process

Large-volume lattice QCD

Large volume simulation (L > 6 fm) is required in FF studies

1. The finite size effect on nucleon observables

$$(L - 2R) \gg \frac{1}{m_\pi} \to L > 3 \text{ fm}$$

$\star\ R \equiv \sqrt{\langle r_E^2 \rangle} \sim 0.85$ fm

$r_\pi \sim 1.4$ fm

$r_{\rm cut} = 3.2$ fm

2. The small momentum transfer

$$q_{\rm min} = \frac{2\pi}{L} < 2m_\pi \to L > 4.5 \text{ fm}$$

3. Exponential falls of the spatial charge distribution

$$L > 2r_{\rm cut} = 6.4R \to L > 6 \text{ fm}$$

Especially, the exponential falls of the spatial charge distribution, which is coming from the dipole form factor, is crucial for the high-precision nucleon form factor studies

Large-volume lattice QCD

3. Exponential falls of the spatial charge distribution

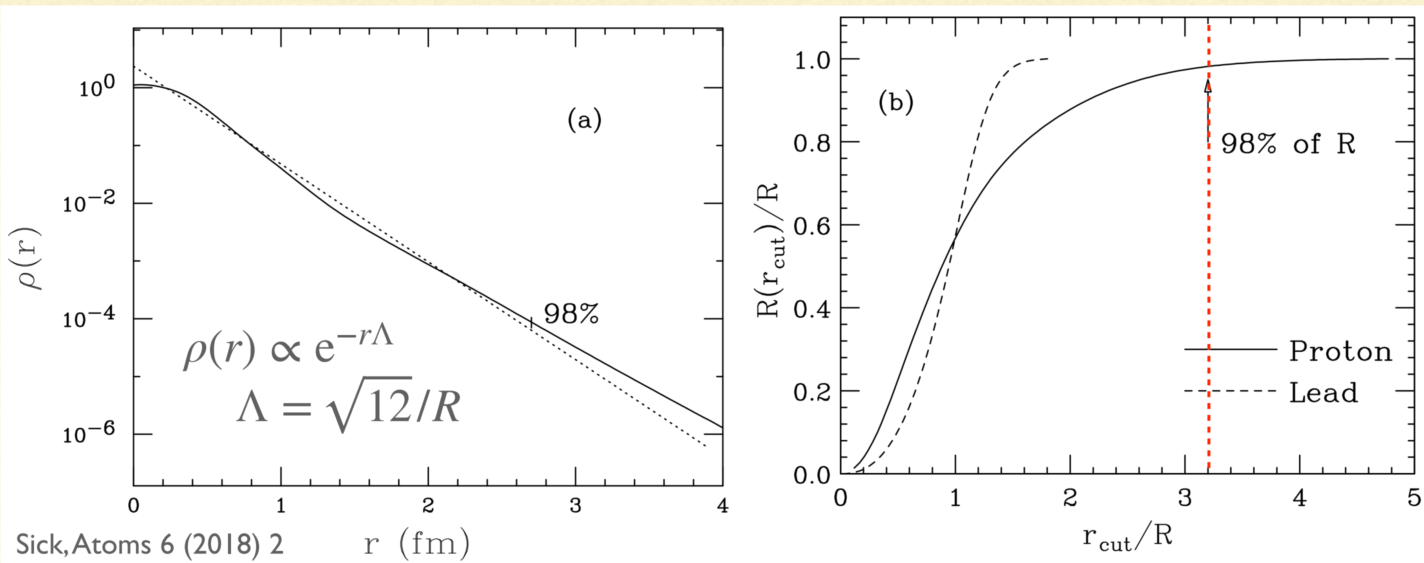
$$L > 2r_{\rm cut} = 6.4R \rightarrow L > 6 \text{ fm}$$

Electric Root-Mean-Square radius: $R^2 = 4\pi \int_0^\infty \rho(r) r^4 dr$

$$\rightarrow R(r_{\rm cut})/R = \sqrt{\left[\int_0^{r_{\rm cut}} \rho(r) r^4 dr \Big/ \int_0^\infty \rho(r) r^4 dr\right]}$$

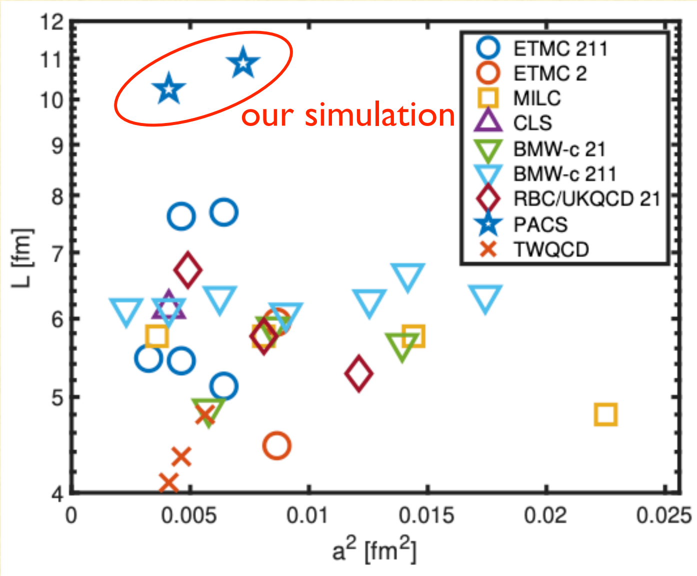
Integration up to $r_{\rm cut} = 3.2R$

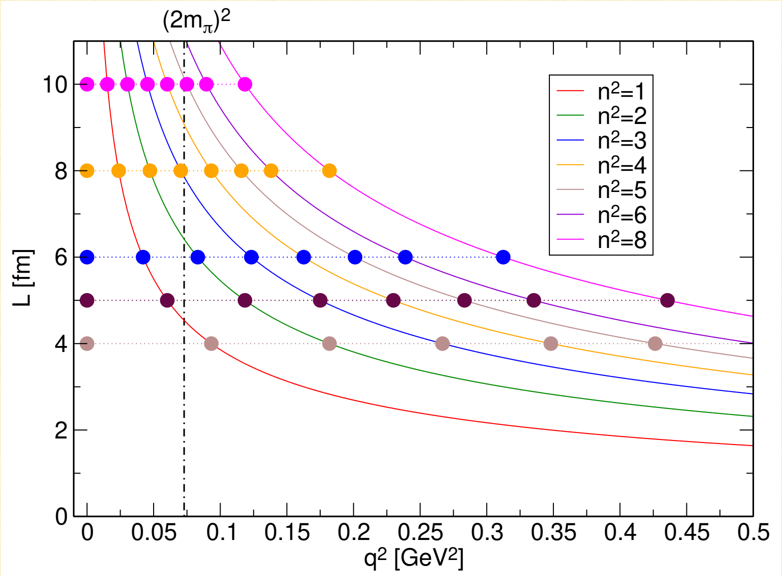
$\rightarrow$98% of R in infinite volume



Sick, Atoms 6 (2018) 2

Large-volume lattice QCD





The momentum is discretized as $q^2 = \left(\frac{2\pi}{L}\right)^2 \times |\boldsymbol{n}|^2$

Low q^2 data are accessible by a large-volume lattice QCD

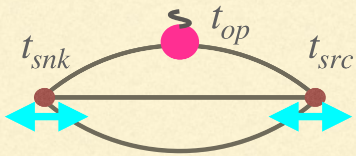
i.e. our simulation is the BEST to seek the low q^2 region!

Excited-states contamination

Major systematic uncertainty in LQCD computation.

Two strategies in this study,

1. Large t_{sep} and the ground-state saturation



$t_{sep} = |t_{snk} - t_{src}|$

$$\frac{\langle N(t_{snk})O(t_{op})N(t_{src})^{\dagger}\rangle}{\langle N(t_{snk})N(t_{src})^{\dagger}\rangle} \rightarrow \langle N|O(0)|N\rangle + \cancel{Ae^{-(E_1-M_N)t_{sep}}} + \cdots$$

2. Checks with the generalized Goldberger-Treiman relation (GGT)

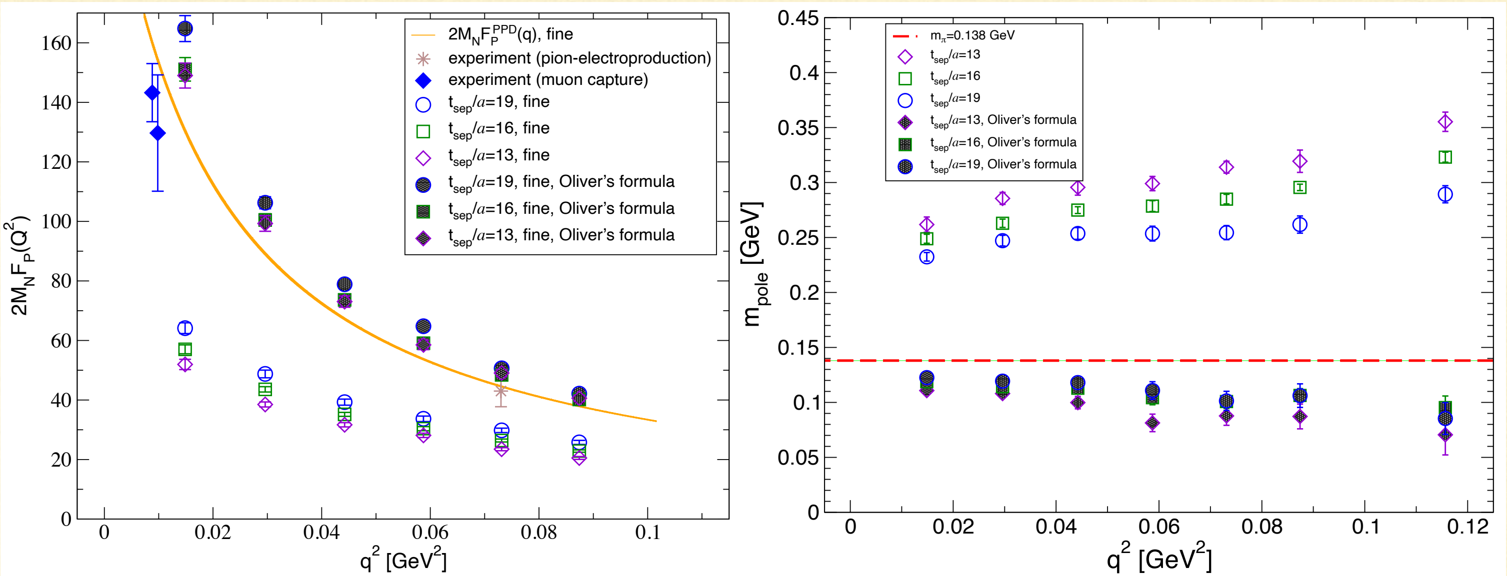
GGT: $2M_N F_A(q^2) - q^2 F_P(q^2) = 2\hat{m} G_P(q^2)$ with quark mass $\hat{m}$

$$\rightarrow m_{\mathrm{AWTI}} \equiv \frac{2M_N F_A(q^2) - q^2 F_P(q^2)}{2G_P(q^2)}$$

The check of $m_{\mathrm{AWTI}} \sim m_{\mathrm{PCAC}}$ should be nontrivial

+ PCAC checks using a notation of LANL

Oliver's formula



$$F_P^{\rm data}(q^2) = F_P(q^2)\left[1 - \exp\left\{-\frac{E_\pi(q)}{2}t_{\rm sep}\right\}\cosh\left\{\frac{q^2}{2M_N}\frac{t_{\rm sep}}{2}\right\}\right]$$: amplify our data

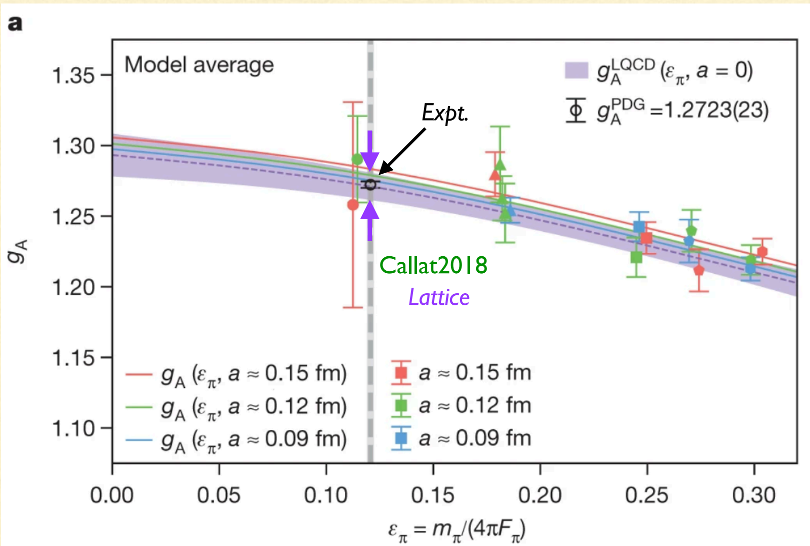
We can *qualitatively* expect that the excited states should be a source of uncertainty.

We KNOW there should be excited-states contamination that are not negligible in our statistical precision, *the analyses are proceeding*

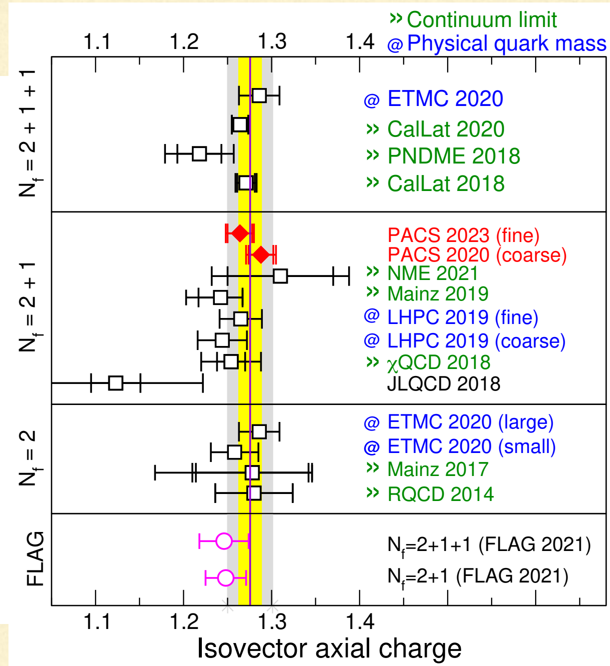
Computational cost roughly increases $\propto a^{-\gamma_0} m^{-\gamma_1} L^{\gamma_2}$ with power laws $\gamma_0, \gamma_1, \gamma_2 > 1$

Physical point simulation

e.g. Axial-vector coupling $F_A(0)$: ***Bench mark***



e.g. Callat Collaboration: C. C. Chang et al., Nature 558, 91 (2018), 1805.12130.



Conventional: Extrapolate the unphysical data into the physical point

= Systematic uncertainty, *Historically underestimated…*

→Ours: Physical point simulation, exclude the systematic uncertainty

Error Budget

Examine the error budget based on HBChPT

- Gap from physical: Heavier mass $m_\pi = 138\ \mathrm{MeV}$ at 160^4 lattice

$$g_A = g_0 \left\{ 1 + \left(\frac{\alpha_2}{(4\pi F)^2} \ln \frac{m_\pi}{\lambda} + \beta_2 \right) m_\pi^2 + \alpha_3 m_\pi^3 \right.$$

1loop: Kambor-Mojzis JHEP 9904, 031 (99)

$$\left. + \left(\frac{\alpha_4}{(4\pi F)^4} \ln^2 \frac{m_\pi}{\lambda} + \frac{\gamma_4}{(4\pi F)^2} \ln^2 \frac{m_\pi}{\lambda} + \beta_4 \right) m_\pi^4 + \alpha_5 m_\pi^5 \right\} + O(m_\pi^6)$$

2loop: Bernard-Meissner PLB639, 278 (06)

→2loop correction is less than 1%

- Finite size effect: L=10 fm is huge but not infinite

$$g_A(\infty) - g_A(L) \propto m_\pi^2 \frac{e^{-m_\pi L}}{\sqrt{m_\pi L}}$$

→L=5 fm & L=10 fm data show the correction is less than 0.1%

Error budget	Stat.	Gap from Physical	Finite size	Discretization	Reno.
$1.264(14)_{\mathrm{stat.}}(3)_{t_{\mathrm{sep}}}$	1.1%	$\lesssim$ 1%	$\lesssim$ 0.1%	?	0.2%

Lattice spacing effect

Error budget	gA	$\sqrt{\langle (r_E^v)^2 \rangle}$	$\sqrt{\langle (r_M^v)^2 \rangle}$	$\sqrt{\langle (r_A^v)^2 \rangle}$
Statistical:	1.9%	5.6%	8.3%	10.5%
Discretization:	1.6%	8.3%	9.0%	11.3%

Statistical error $\lesssim$ Lattice spacing effect

Check

1. Dispersion relation of nucleon

2. $O(a)$ improved current $A_\alpha^{\mathrm{imp}} = A_\alpha + c_A a \partial_\alpha P \to$ PCAC relation

$$m_{\mathrm{PCAC}}^{\mathrm{pion}} \equiv \frac{m_\pi^2 f_\pi}{2\langle 0|P^+(0)|\pi\rangle} \longleftrightarrow m_{\mathrm{PCAC}}^{\mathrm{nucl}} \equiv \frac{\langle N_{\mathrm{snk}} \partial_\mu A_\mu(x) \overline{N}_{\mathrm{src}}\rangle}{2\langle N_{\mathrm{snk}} P(x) \overline{N}_{\mathrm{src}}\rangle}$$

- Pion 2-pt function
- Zero momentum
- Improvement is small

$m_{\mathrm{PCAC}}^{\mathrm{pion}} = (m_{\mathrm{PCAC}}^{\mathrm{pion}})^{\mathrm{imp}}$

$\bar{c}_A$ s.t. $m_{\mathrm{PCAC}}^{\mathrm{pion}} \sim (m_{\mathrm{PCAC}})^{\mathrm{imp}}$

$\to \bar{c}_A \propto m_{\mathrm{AWTI}}^{\mathrm{PCAC}} - (m_{\mathrm{AWTI}}^{\mathrm{PCAC}})^{\mathrm{imp}}$

$\sim m_{\mathrm{AWTI}}^{\mathrm{PCAC}} - m_{\mathrm{PCAC}}$

- Nucleon 3-pt function
- Nonzero momentum
- Improvement works

$(m_{\mathrm{PCAC}}^{\mathrm{nucl}})^{\mathrm{imp}} = m_{\mathrm{PCAC}}^{\mathrm{nucl}} - a c_A q^2/2$

Lattice spacing effect

Error budget	gA	$\sqrt{\langle (r_E^v)^2 \rangle}$	$\sqrt{\langle (r_M^v)^2 \rangle}$	$\sqrt{\langle (r_A^v)^2 \rangle}$
Statistical:	1.9%	5.6%	8.3%	10.5%
Lattice spacing:	1.6%	8.3%	9.0%	11.3%

Statistical error $\lesssim$ Lattice spacing effect

Check

1. What about others? → Dispersion relation of nucleon

2. $O(a)$ improved current $A_\alpha^{\rm imp} = A_\alpha + c_A a \partial_\alpha P$ → PCAC relation

$$m_{\rm PCAC}^{\rm pion} \equiv \frac{m_\pi^2 f_\pi}{2\langle 0 | P^+(0) | \pi \rangle} \longleftrightarrow m_{\rm PCAC}^{\rm nucl} \equiv \frac{\langle N_{\rm snk} \partial_\mu A_\mu(x) \overline{N}_{\rm src} \rangle}{2\langle N_{\rm snk} P(x) \overline{N}_{\rm src} \rangle}$$

- Pion 2-pt function
- Zero momentum
- Improvement is helpless

$m_{\rm PCAC}^{\rm pion} \simeq (m_{\rm PCAC}^{\rm pion})^{\rm imp}$

$\bar{c}_A$ s.t. $m_{\rm PCAC}^{\rm pion} \sim (m_{\rm PCAC})^{\rm imp}$

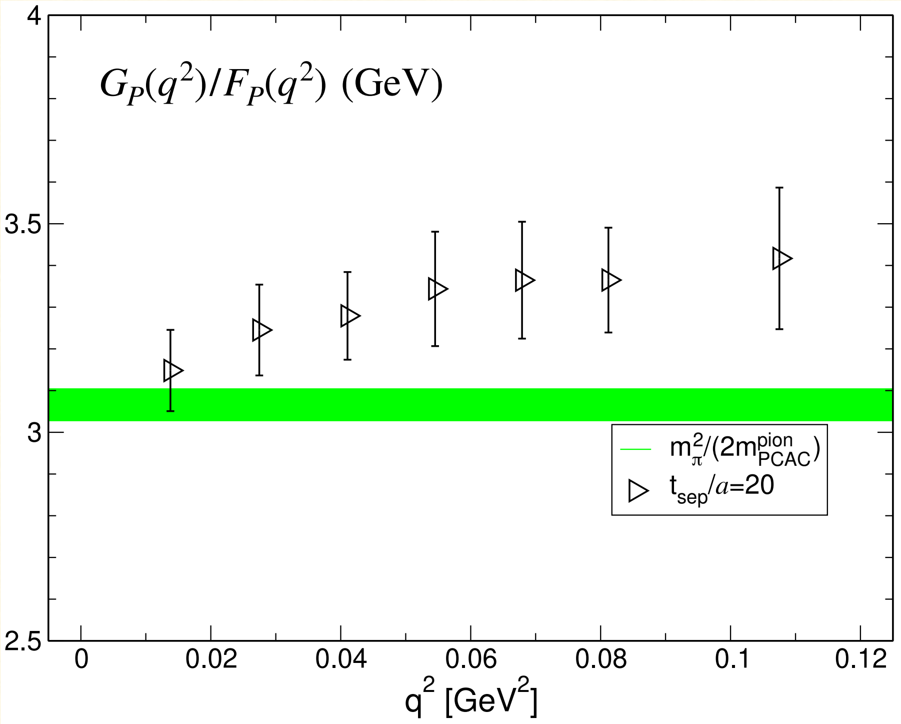
$\to \bar{c}_A \propto m_{\rm AWTI}^{\rm PCAC} - (m_{\rm AWTI}^{\rm PCAC})^{\rm imp}$

$\sim m_{\rm AWTI}^{\rm PCAC} - m_{\rm PCAC}$

- Nucleon 3-pt function
- Nonzero momentum
- Improvement works

$(m_{\rm PCAC}^{\rm nucl})^{\rm imp} = m_{\rm PCAC}^{\rm nucl} - a c_A q^2/2$

Pion-pole dominance (PPD)



Combining PPD and GGT,

$$F_P^{\rm PPD}(q^2) = \frac{2M_N F_A(q^2)}{q^2 + m_\pi^2}$$

$$G_P^{\rm PPD}(q^2) = \frac{m_\pi^2}{2m_{\rm PCAC}} \frac{2M_N F_A(q^2)}{q^2 + m_\pi^2}$$

$$\rightarrow \frac{G_P^{\rm PPD}(q^2)}{F_P^{\rm PPD}(q^2)} = \frac{m_\pi^2}{2m_{\rm PCAC}} \cdots (1)$$

The data shows

✔ Flat q^2-dependence

✔ Agreement with (1)

→ $F_P(q^2)$ and $G_P(q^2)$ are supposed to share the same pion-pole.

Preliminary

Y. C. Jang et al., Phys. Rev. Lett. 124, no.7, 072002 (2020).

Generalized Goldberger-Treiman (GGT)