THE UNIVERSITY of ADELAIDE

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Updates on The parity-odd structure function of the nucleon from the Compton amplitude

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Lattice'24, The University of Liverpool, 28 July - 3 August 2024

CSSM/QCDSF/UKQCD Talks

- Gerrit Schierholz @ Mon 14:15 [Vacuum structure and confinement]
  - “Absence of CP violation in the strong interaction”

- Joshua Crawford @ Thur 10:20 [Structure of Hadrons and Nuclei]
  - “Transverse Force Distributions in the Proton from Lattice QCD”

- Roger Horsley @ Fri 15:15 [Structure of Hadrons and Nuclei]
  - “Renormalisation Group Equations for 2+1 clover Fermions”

Motivation

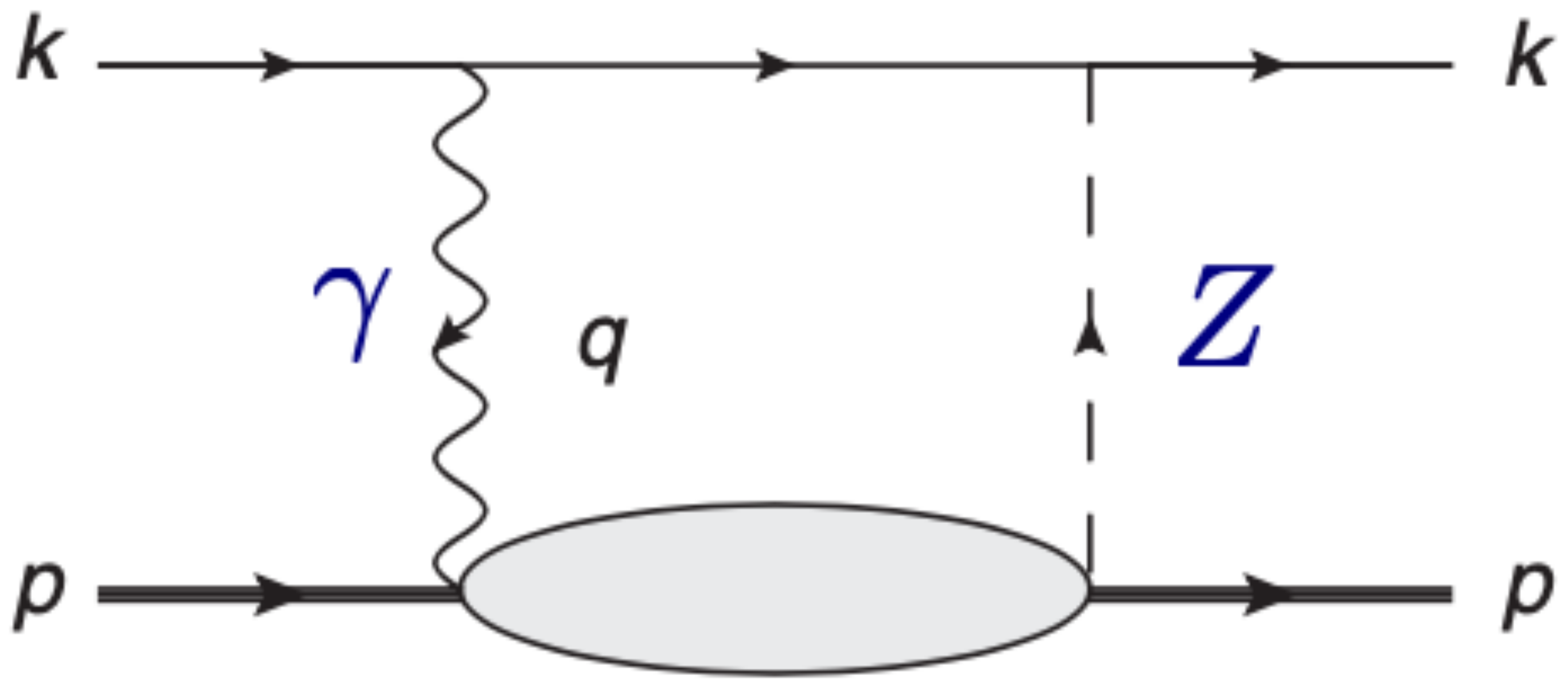
- Leading theoretical uncertainty in:
- Weak charge of the proton,

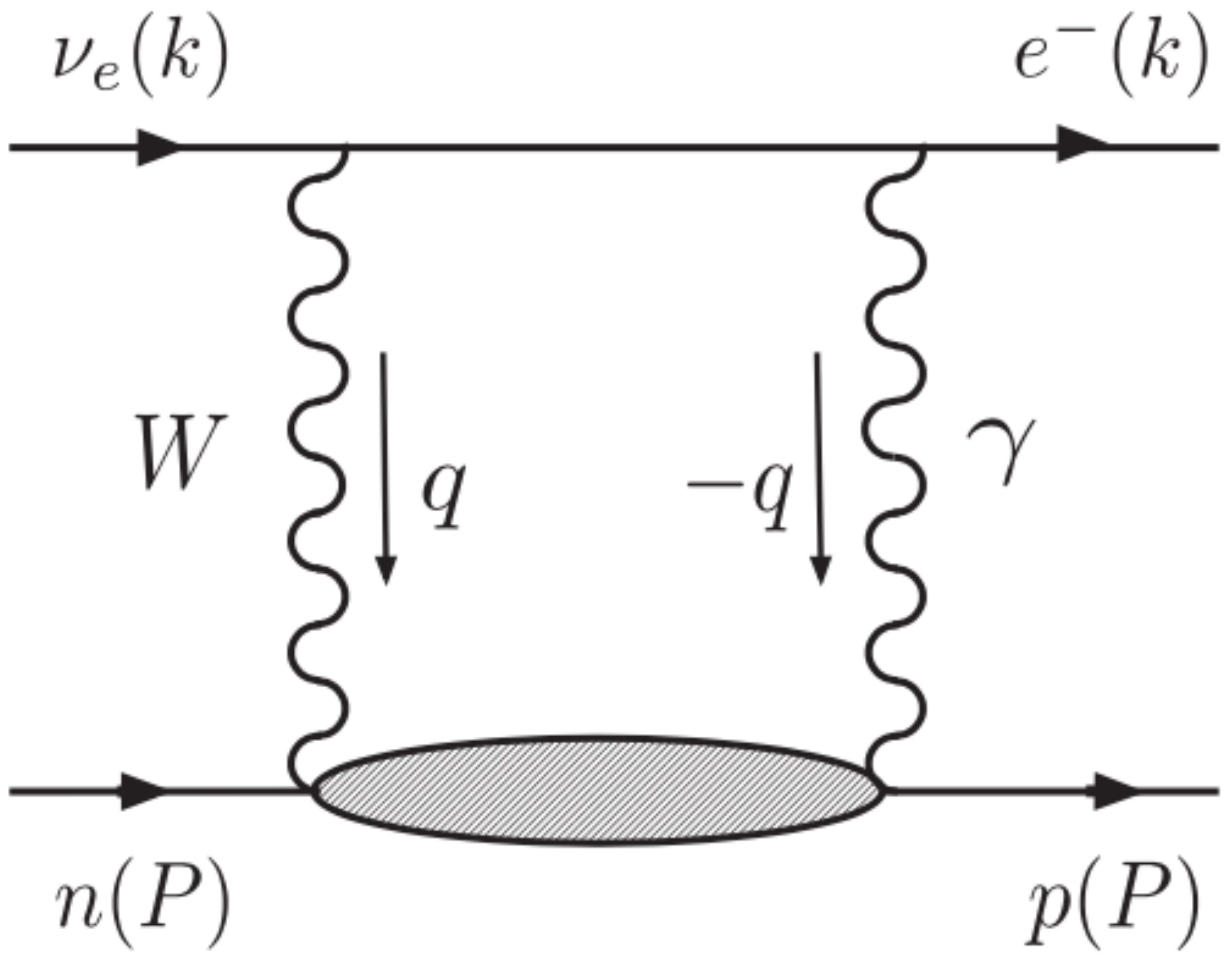
$$Q_W = (1+\Delta_\rho+\Delta_e)(1-4\sin^2\theta_W(0)+\Delta_e') + \Box_{AA}^{WW} + \Box_{AA}^{ZZ} + \Box_{VA}^{\gamma Z}$$

- CKM matrix element extracted from superallowed neutron β decays,

$$|V_{ud}|^2 = \frac{0.97148(20)}{1+\boxed{\Delta_R^V}} \longrightarrow 0.01691 + 2\,\Box_{VA}^{\gamma W}$$

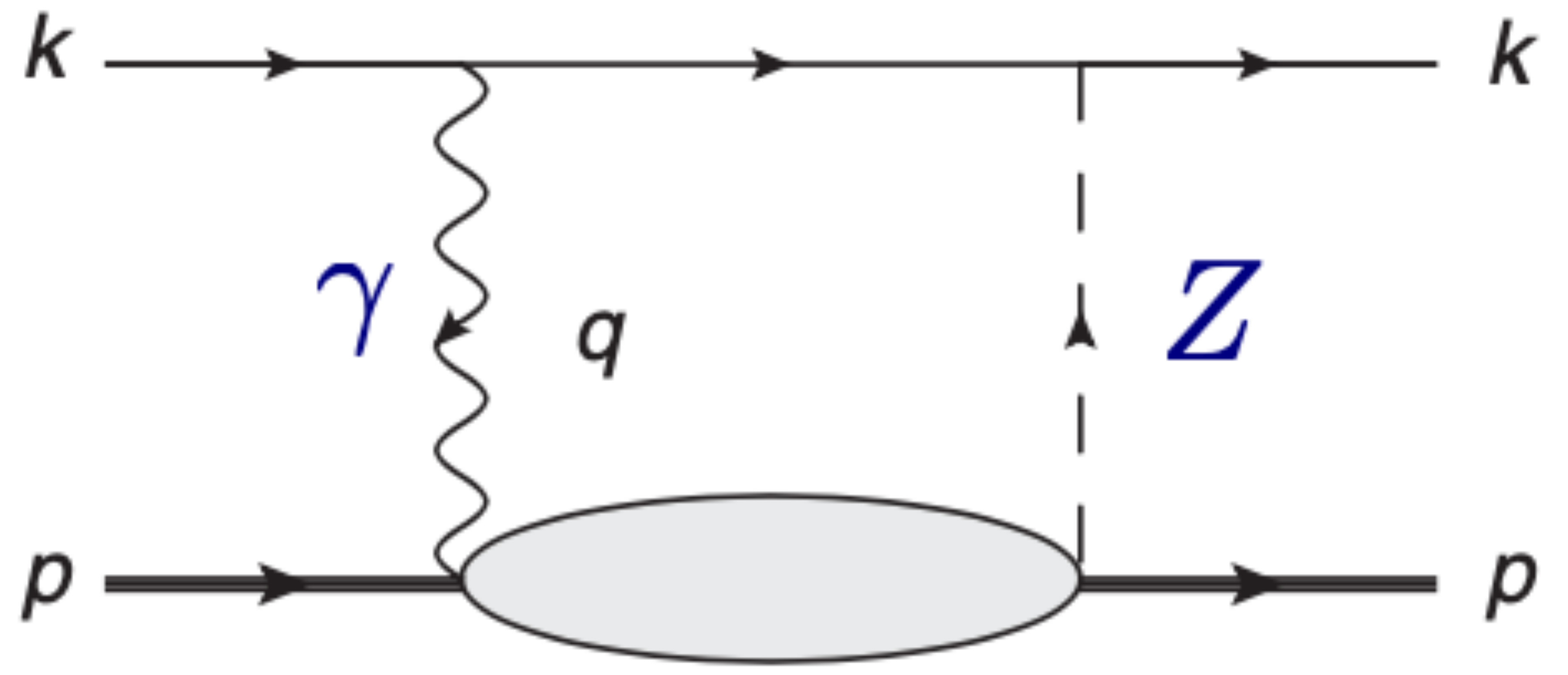
K. Shiells, P.G. Blunden, W. Melnitchouk, PRD104, 033003 (2021) [2012.01580]



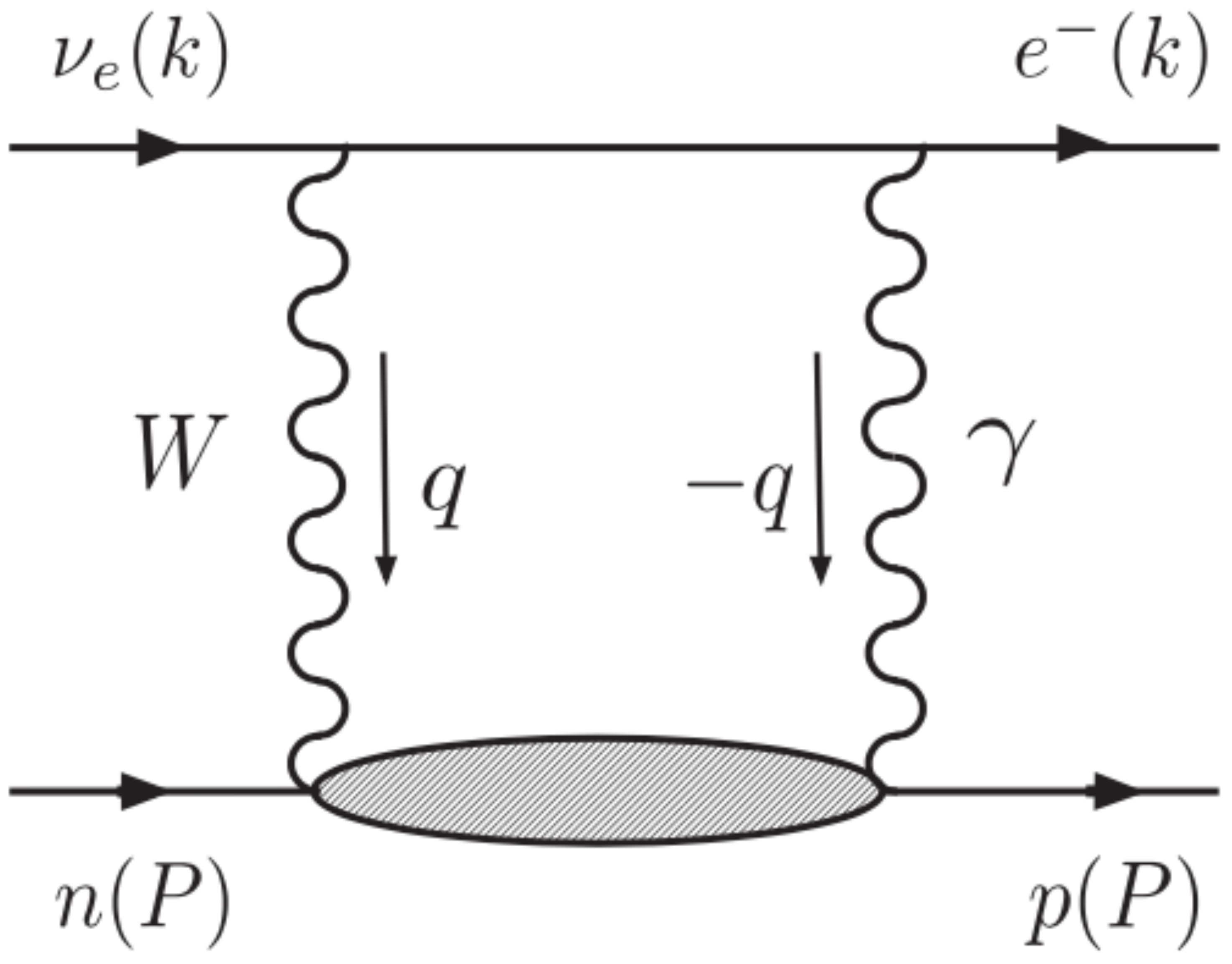


Motivation

$$\Box_A^{\gamma Z} = \nu_e \frac{3\alpha_{EM}}{2\pi} \int_0^\infty \frac{dQ^2}{Q^2} \frac{M_Z^2}{M_Z^2 + Q^2} \underbrace{\int_0^1 dx\, C_N(x, Q^2)\, F_3^{\gamma Z}(x, Q^2)}$$



First Nachtmann moment of F_3



$$\Box_{VA}^{\gamma W} = \frac{3\alpha_{EM}}{2\pi} \int_0^\infty \frac{dQ^2}{Q^2} \frac{M_W^2}{M_W^2 + Q^2} \overbrace{\int_0^1 dx\, C_N(x, Q^2)\, F_3^{(0)}(x, Q^2)}$$

$$F_3^{(0)} = F_3^{\gamma Z,p} - F_3^{\gamma Z,n},$$

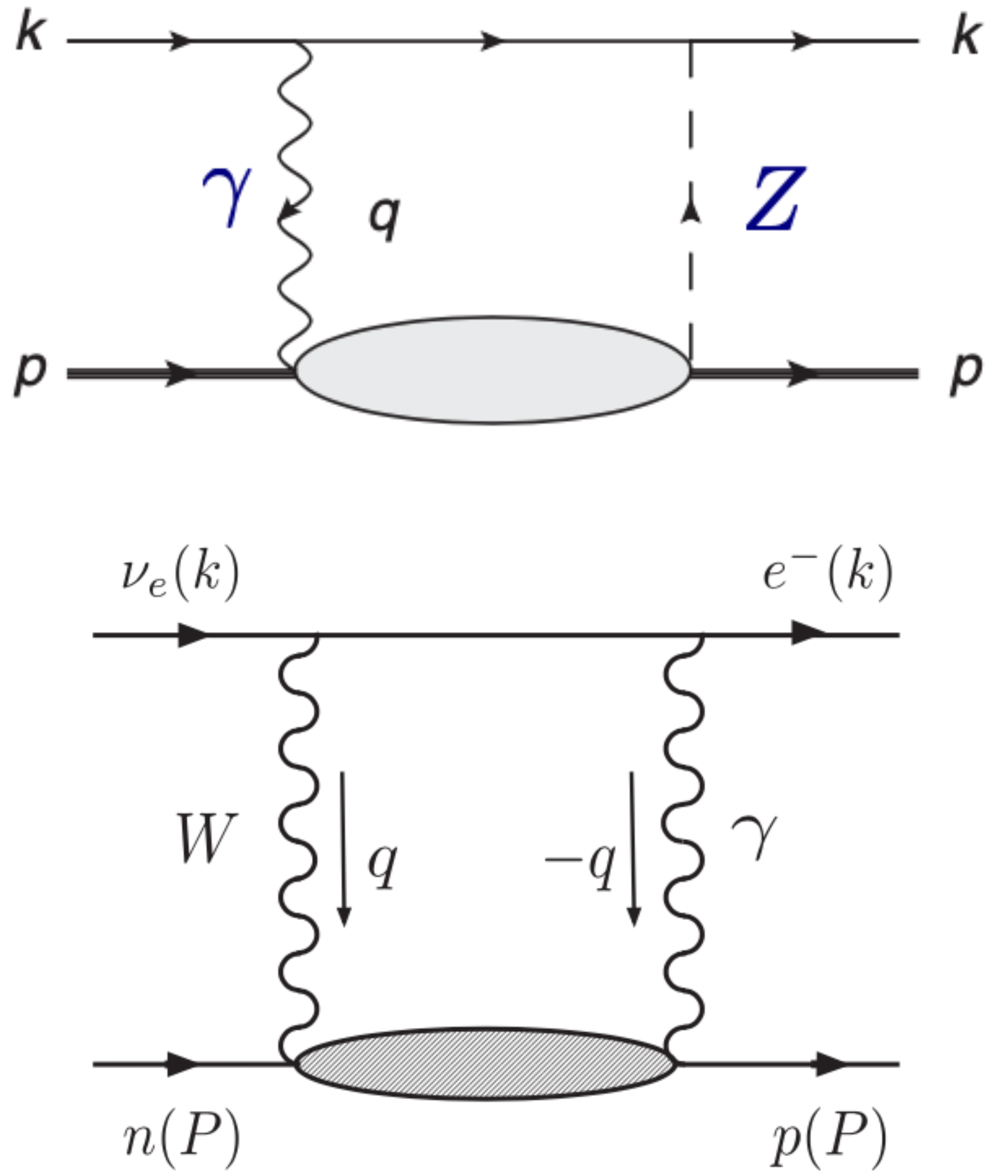
where $C_N(x, Q^2)$ is a known coefficient

Motivation

- Box diagrams proportional to an integral over the whole Q^2 range

$$\Box_A^{\gamma Z/W} \propto \int_0^\infty \frac{dQ^2}{Q^2}\,\mu_1^{(3)}(Q^2)\,(\ldots)$$

- Low-Q^2 (non-perturbative) regime dominates the integral
- F_3 is experimentally poorly determined in low Q^2
- Lattice approach is ideal for a high-precision determination of $\mu_1^{(3)}(Q^2)$ Nachtmann moment



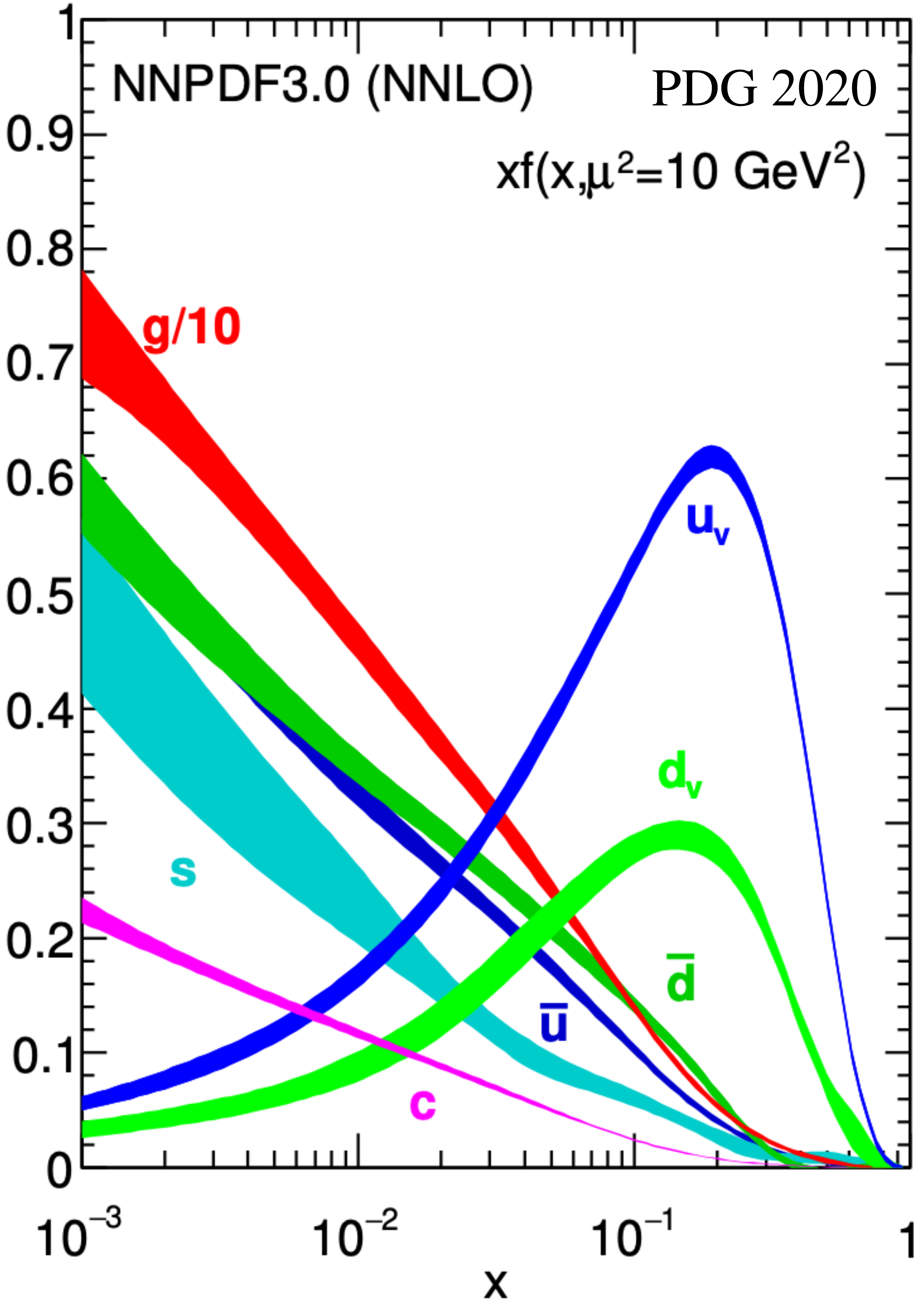
Motivation

- Nucleon structure (leading twist)
  - Structure functions from first principles

- In the parton model

$$F_2 \propto (q + \bar{q})$$

$$F_3^{\gamma Z} \propto (q - \bar{q})$$



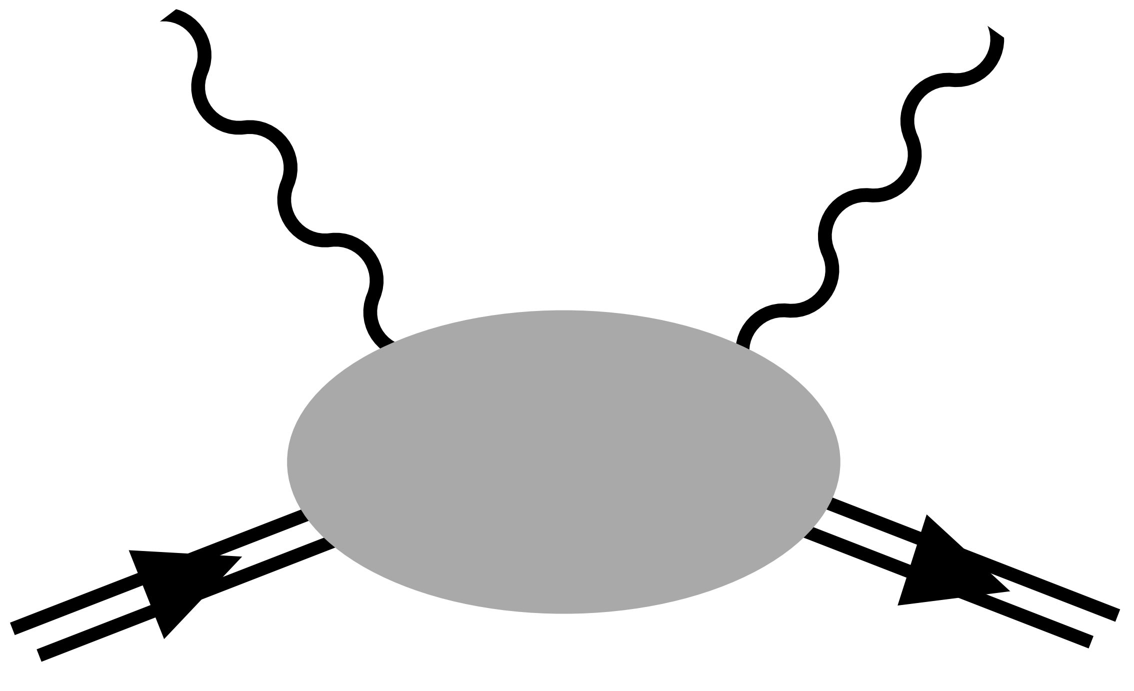
Motivation

- Scaling
  - Q^2 cuts of global QCD analyses
- Power corrections / Higher twist effects
  - Target mass corrections
  - Twist-4 contributions
  - GLS sum rule:

$$S^{GLS} = \int_0^1 dx F_3^{(\nu p + \bar{\nu} p)}(x, Q^2) = 3 \left[1 + \frac{\alpha_s(Q^2)}{\pi} + \ldots \right] - \frac{\Delta^{HT}}{Q^2}$$

  - $\Delta^{HT} \sim 0.15 - 0.5$ see X.-D. Huang et al., NPB969 (2021) 115466 [2101.10922]

Forward Compton Amplitude



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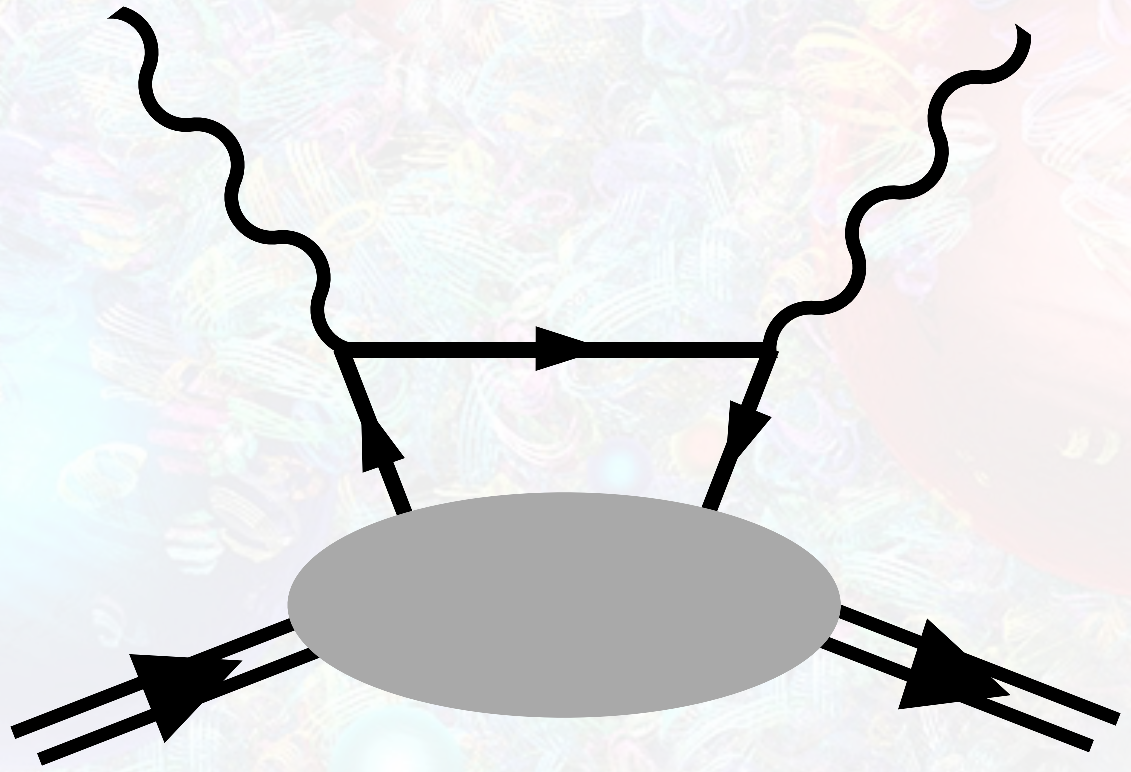


$$+ \ \mathcal{O}\left(\frac{M_N}{Q^2}, \frac{1}{Q^2}\right)$$

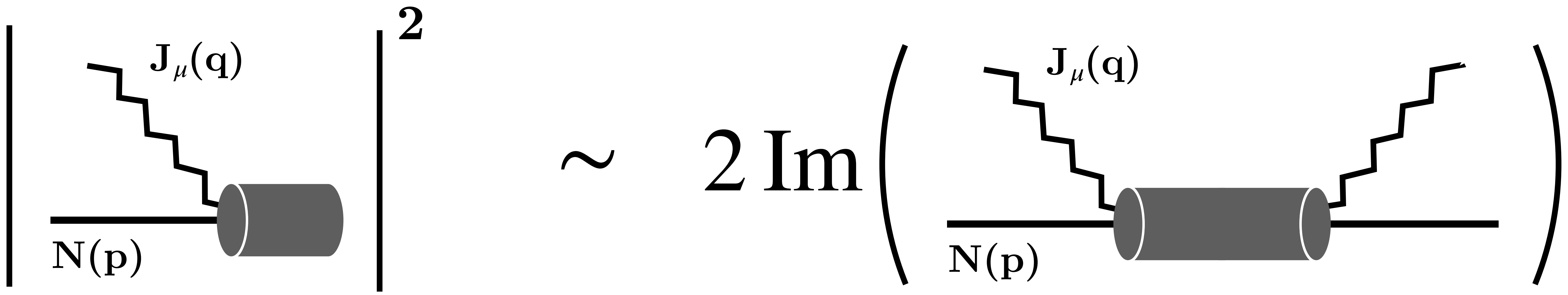
Parity Violating Forward Compton Amplitude

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\,\mathcal{T}\{J_\mu(z)J_\nu(0)\}\,|p,s\rangle \quad \text{, spin avg. } \rho_{ss'}=\frac{1}{2}\delta_{ss'}$$

$$= -g_{\mu\nu}\mathcal{F}_1(\omega,Q^2) + \frac{p_\mu p_\nu}{p\cdot q}\mathcal{F}_2(\omega,Q^2) + i\,\varepsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{2p\cdot q}\mathcal{F}_3(\omega,Q^2)$$

$$+\frac{q_\mu q_\nu}{p\cdot q}\mathcal{F}_4(\omega,Q^2) + \frac{p_{\{\mu}q_{\nu\}}}{p\cdot q}\mathcal{F}_5(\omega,Q^2) + \frac{p_{[\mu}q_{\nu]}}{p\cdot q}\mathcal{F}_6(\omega,Q^2)$$

allowed terms because parity is violated



DIS Cross Section ~ Hadronic Tensor

Forward Compton Amplitude ~ Compton Tensor

$$\omega = \frac{2p\cdot q}{Q^2}$$

$$\varepsilon^{0123} = 1$$

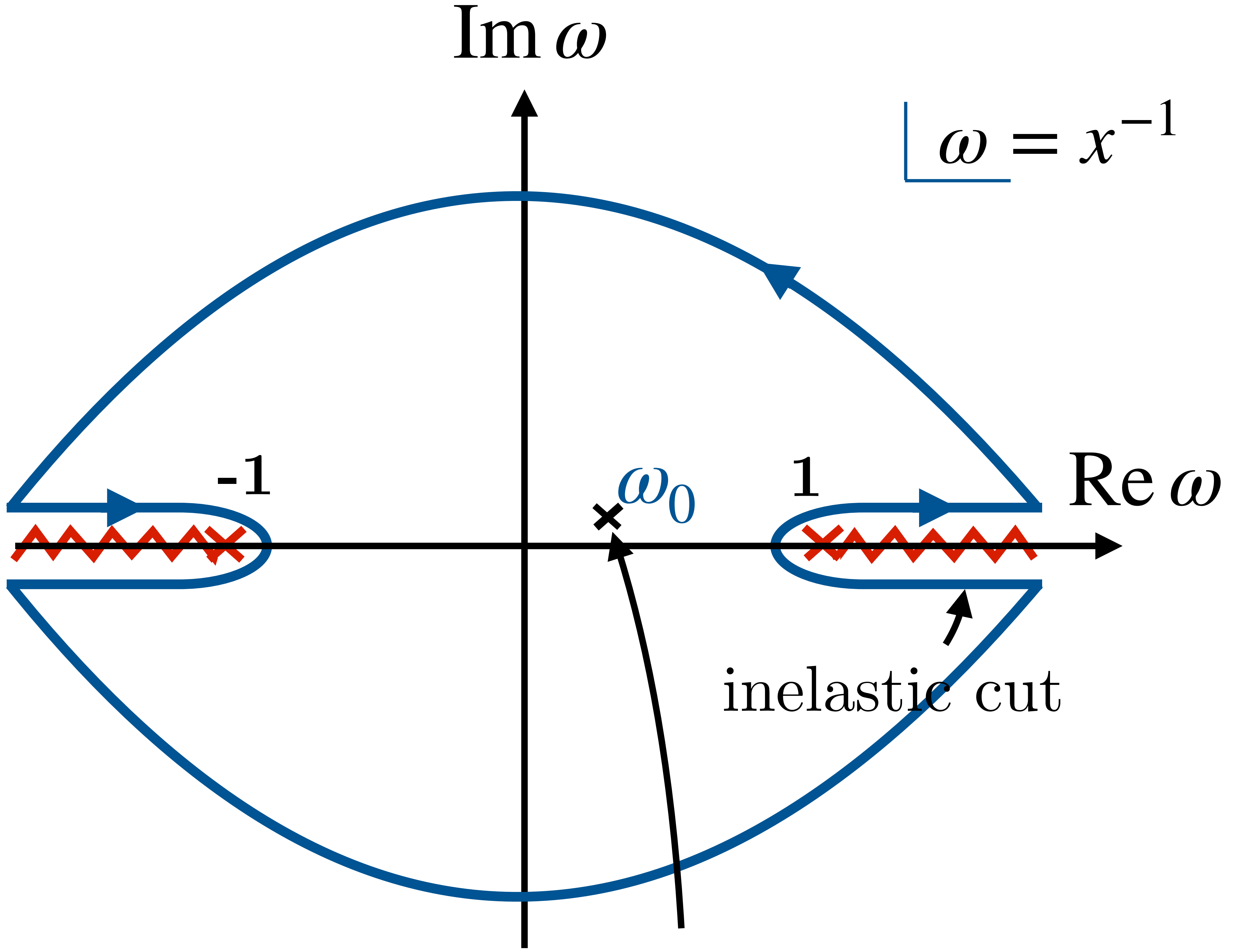
Nucleon Structure Functions

- for $\mu \neq \nu$ and $p_\mu = q_\mu = 0$, and $\beta \neq 0$, we isolate,

$$T_{\mu\nu}(p,q) = i\,\varepsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{2p\cdot q}\mathcal{F}_3(\omega,Q^2)$$

- we can write down dispersion relations and connect Compton SFs to DIS SFs:

$$\mathcal{F}_3(\omega,Q^2) = 4\omega\int dx\frac{F_3(x,Q^2)}{1-x^2\omega^2}$$



Compton Amplitude is an analytic function in the unphysical region $|\omega_0| < 1$

Parity Violating Forward Compton Amplitude

- **The lowest odd Cornwall-Norton (Mellin) moment**

$$M_1^{(3)}(Q^2) = \int_0^1 dx F_3(x, Q^2) = \left.\frac{\mathcal{F}_3(\omega, Q^2)}{4\omega}\right|_{\omega=0}$$

allows for a test of the Gross-Llewellyn-Smith sum rule $(a_s = \alpha_s(Q^2)/\pi)$

$$M_{1,uu}^{(3)}(Q^2) = \int_0^1 dx F_3^{\gamma Z}(x, Q^2) = 2\left(1 + \sum_{i=1}^{4} a_s^i c_i(n_f)\right) + \frac{\Delta_{HT}}{Q^2} + \mathcal{O}\left(\frac{1}{Q^4}\right)$$

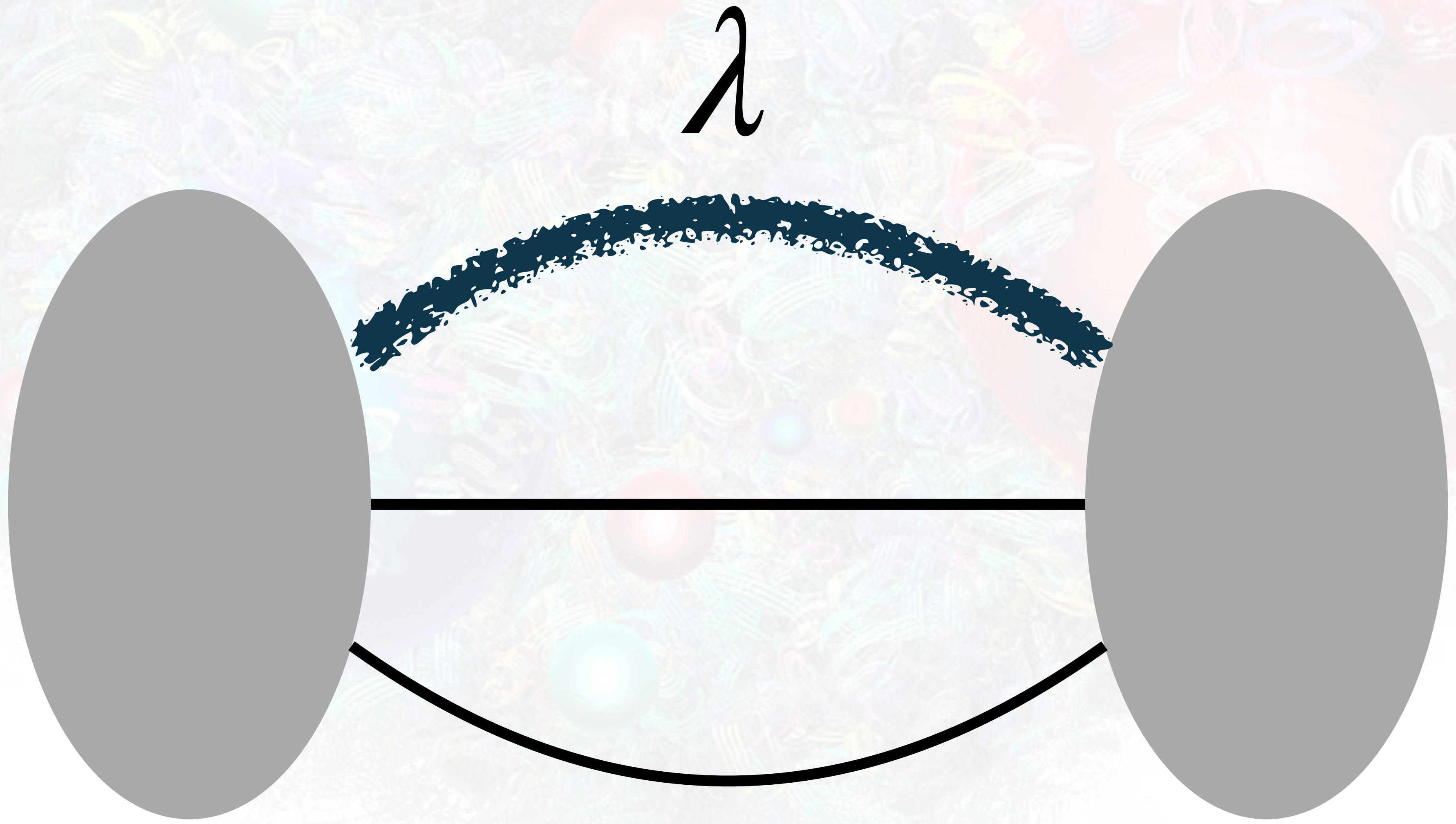
known coeffs.

Higher-twist

- **Nachtmann moment allows for a determination of the box diagrams**

$$\Box_{VA}^{\gamma W/Z} \propto \int_0^\infty \frac{dQ^2}{Q^2}\frac{M_{W/Z}^2}{M_{W/Z}^2 + Q^2}\mu_1^{(3)}(Q^2)$$

Feynman-Hellmann Theorem



FH Theorem at 1st order

in Quantum Mechanics:

$$\frac{\partial E_\lambda}{\partial \lambda} = \langle \phi_\lambda | \frac{\partial H_\lambda}{\partial \lambda} | \phi_\lambda \rangle$$

H_λ: perturbed Hamiltonian of the system

E_λ: energy eigenvalue of the perturbed system

ϕ_λ: eigenfunction of the perturbed system

- **expectation value of the perturbed system is related to the shift in the energy eigenvalue**

in Lattice QCD: energy shifts in the presence of a weak external field

e.g. local bilinear operator

$$S \to S(\lambda) = S + \underset{\nearrow \text{real parameter}}{\lambda} \int d^4x\, \mathcal{O}(x) \longrightarrow \bar{q}(x)\Gamma_\mu q(x) \quad , \Gamma_\mu \in \{\mathbf{1}, \gamma_\mu, \gamma_5\gamma_\mu, \ldots\}$$

@ 1st order

$$\frac{\partial E_\lambda}{\partial \lambda} = \frac{1}{2E_\lambda}\langle 0 | \mathcal{O} | 0 \rangle$$

E_λ → spectroscopy, 2-pt function

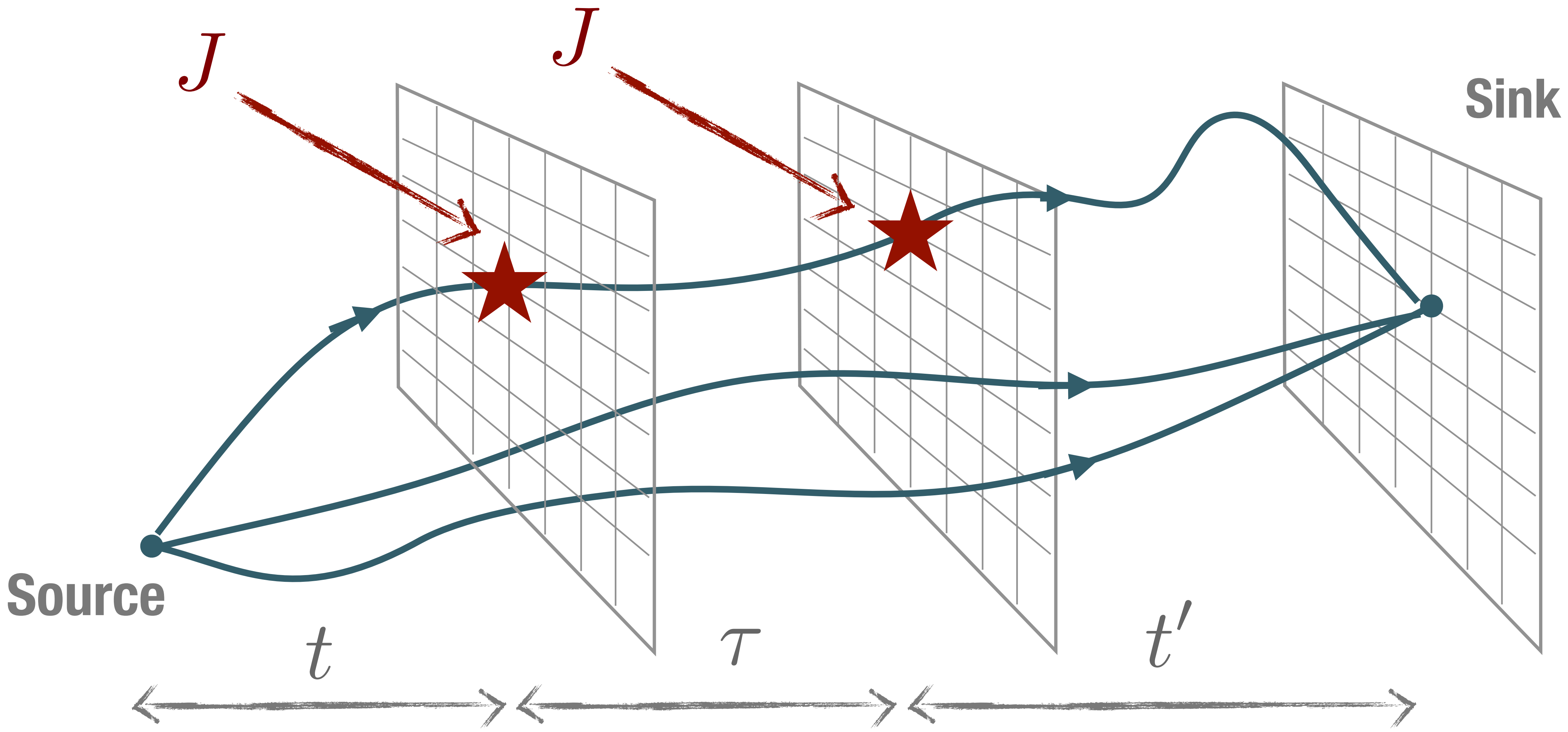
$\langle 0 | \mathcal{O} | 0 \rangle$ → determine 3-pt

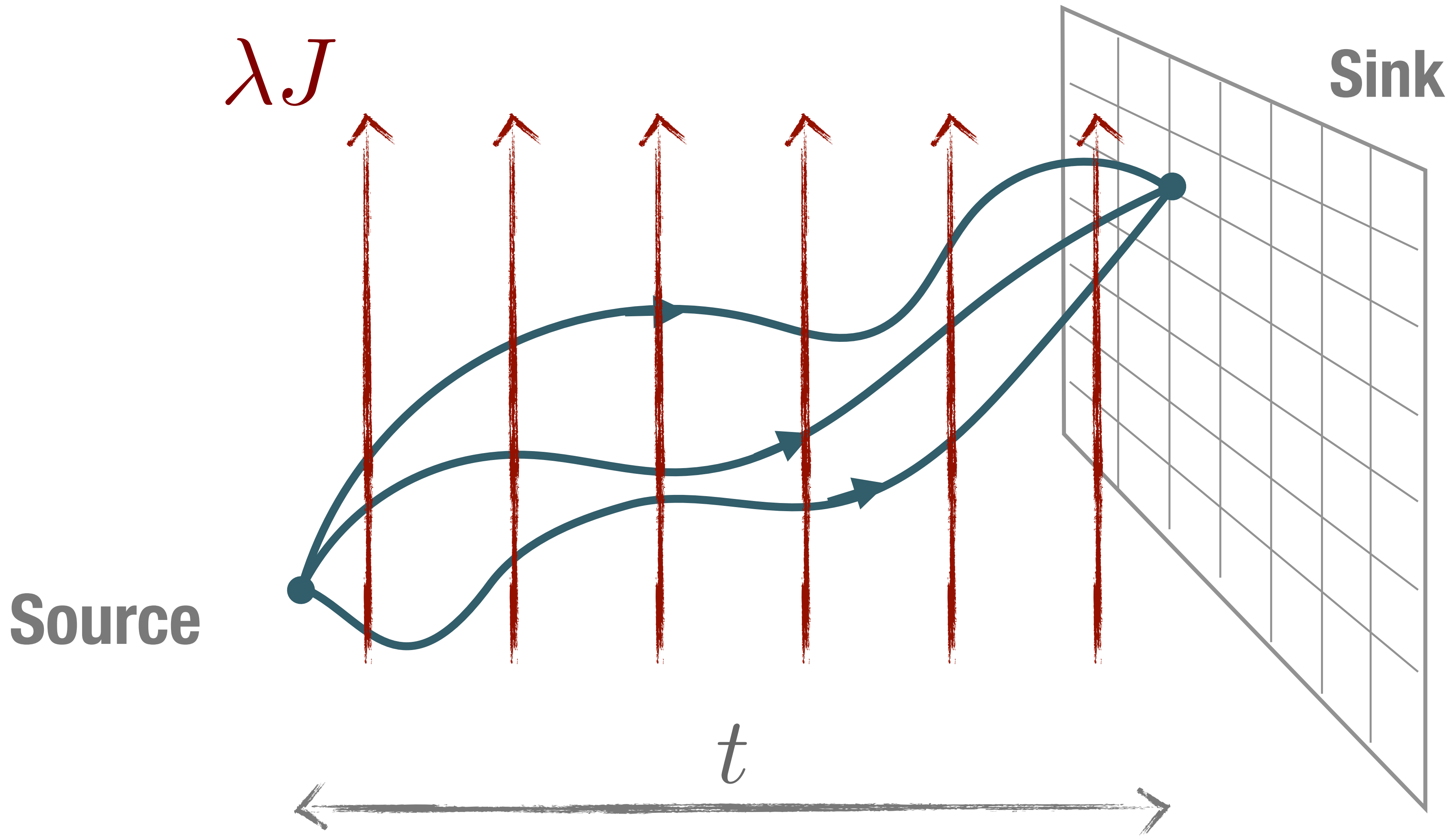
Applications:

- σ - terms
- Form factors

kuc et al. (CSSM/QCDSF/UKQCD) PRD102, 114505 (2020), arXiv:2007.01523 [hep-lat]

Compton amplitude | FH Theorem at 2st order





- **4-pt functions**

$$t, t' \gg \frac{1}{\Delta E}$$

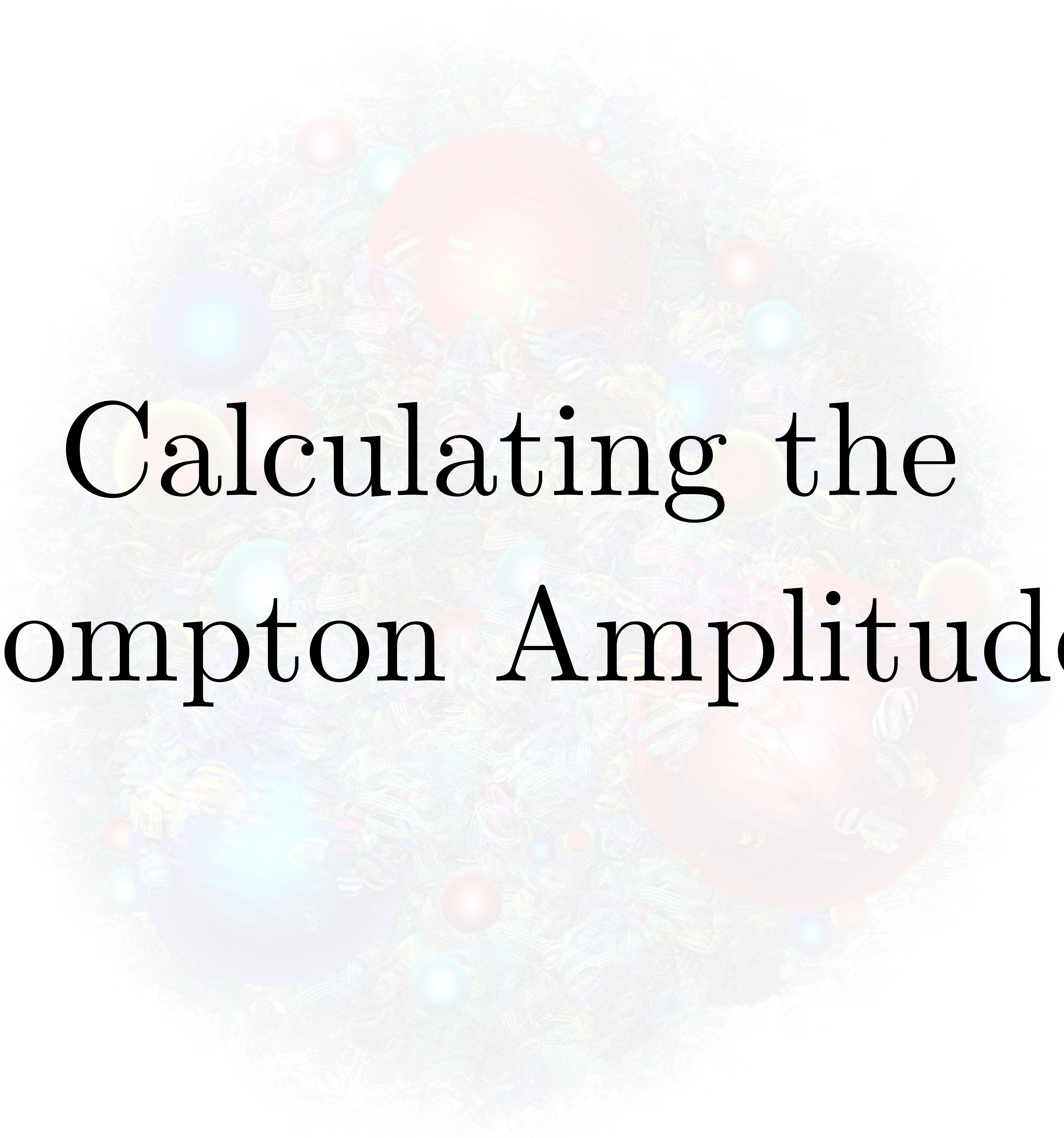
energy gap to the lowest excitation

$$\frac{\langle C_4(t, \tau, t')\rangle}{\langle C_2(t)\rangle\langle C_2(t')\rangle} \propto \langle N | J(\tau_E) J | N \rangle$$

$$\int_0^\infty d\tau_E \to \langle N | JJ | N \rangle$$

- **Feynman—Hellmann**

$$t \gg \frac{1}{\Delta E}, \qquad \left.\frac{\partial^2 E}{\partial \lambda^2}\right|_{\lambda \to 0} \propto \langle N | JJ | N \rangle$$



Calculating the Compton Amplitude

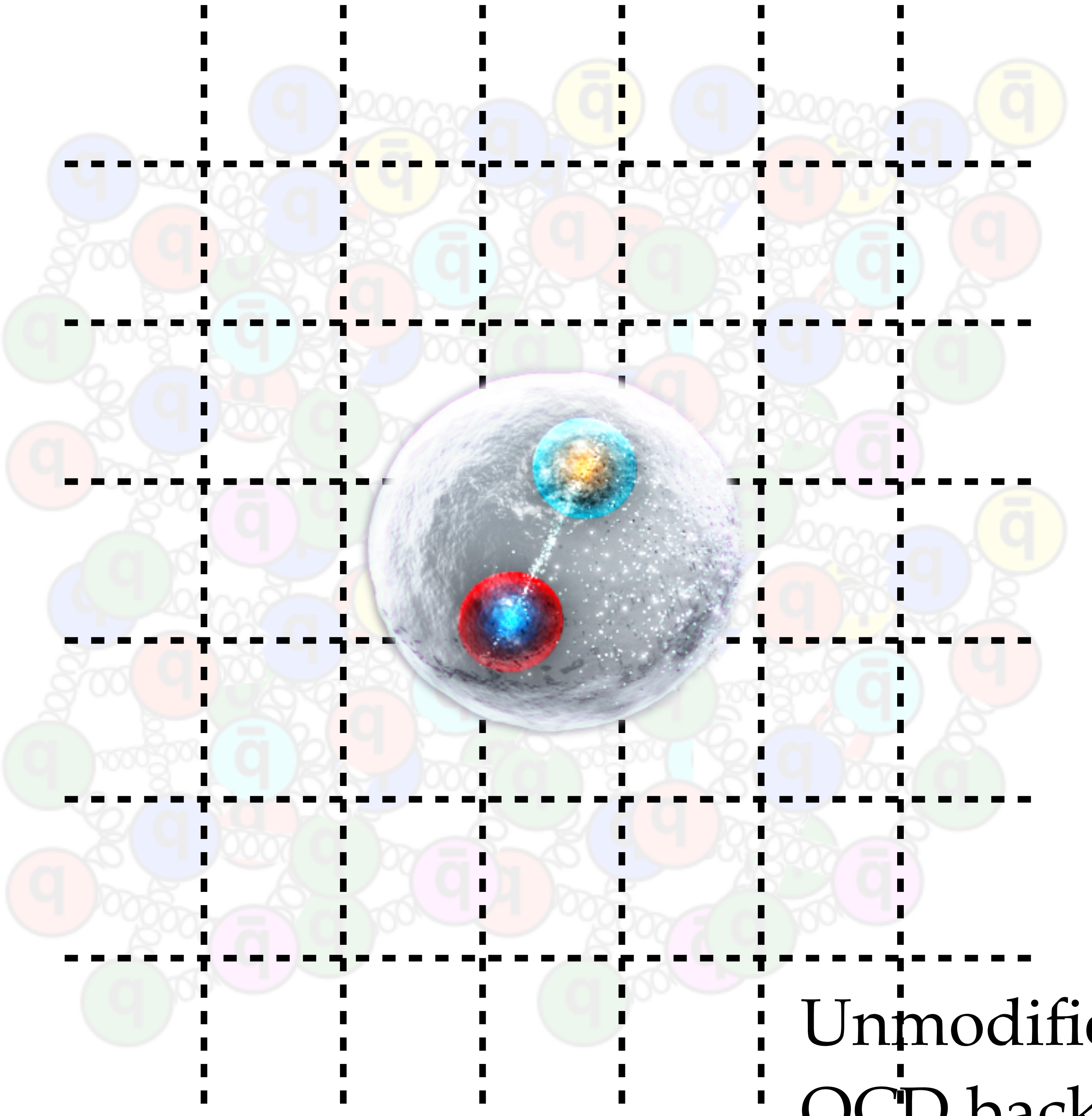
Calculation Details

QCDSF/UKQCD configurations

$48^3 \times 96$, 2+1 flavour (u/d+s)

$\beta = \begin{pmatrix} 5.65 \\ 5.95 \end{pmatrix}$, NP-improved Clover action

PRD 79, 094507 (2009), arXiv:0901.3302 [hep-lat]



$m_\pi \sim 420$ MeV, ~SU(3) sym.

$$m_\pi L \sim \begin{bmatrix} 6.9 \\ 5.3 \end{bmatrix} \qquad a \sim \begin{bmatrix} 0.068 \\ 0.052 \end{bmatrix} \text{ fm}$$

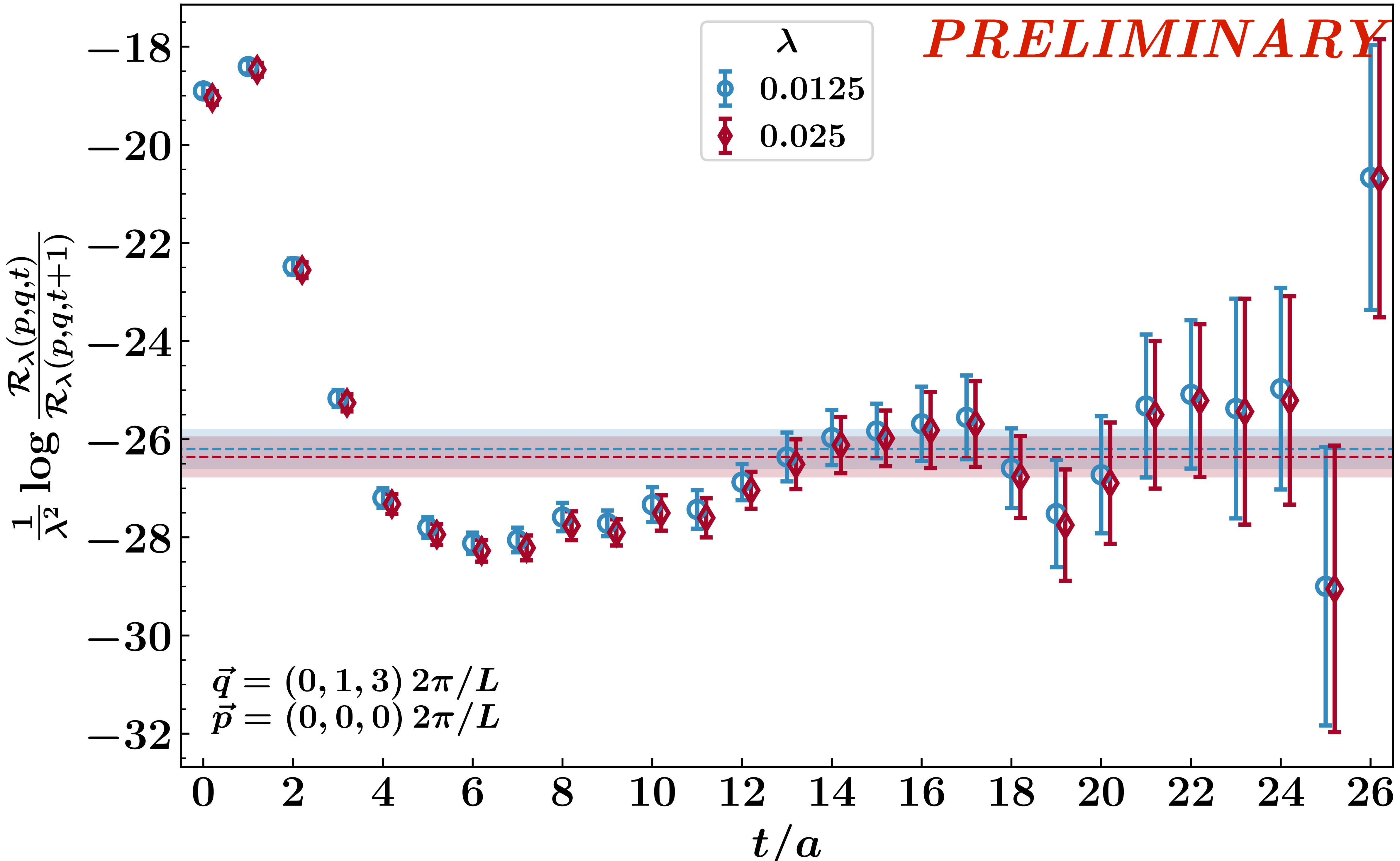
- Local EM and axial current insertion, $J_\mu^{V[A]}(x) = Z_{V[A]}\bar{q}(x)\gamma_\mu[\gamma_5]q(x)$ (valence only)
- 4 Distinct field strengths, $\lambda = [\pm 0.0125, \pm 0.025]$
- Current momenta $0.1 \lesssim Q^2 \lesssim 10$ GeV^2
- Roughly 500 measurements
- Nucleon at rest: $\vec{p} = (0,0,0)$ thus $\omega = 0$, varying $\vec{q}$
- Connected 2-pt only, <u>no disconnected</u> since F_3 is non-singlet

Energy shifts

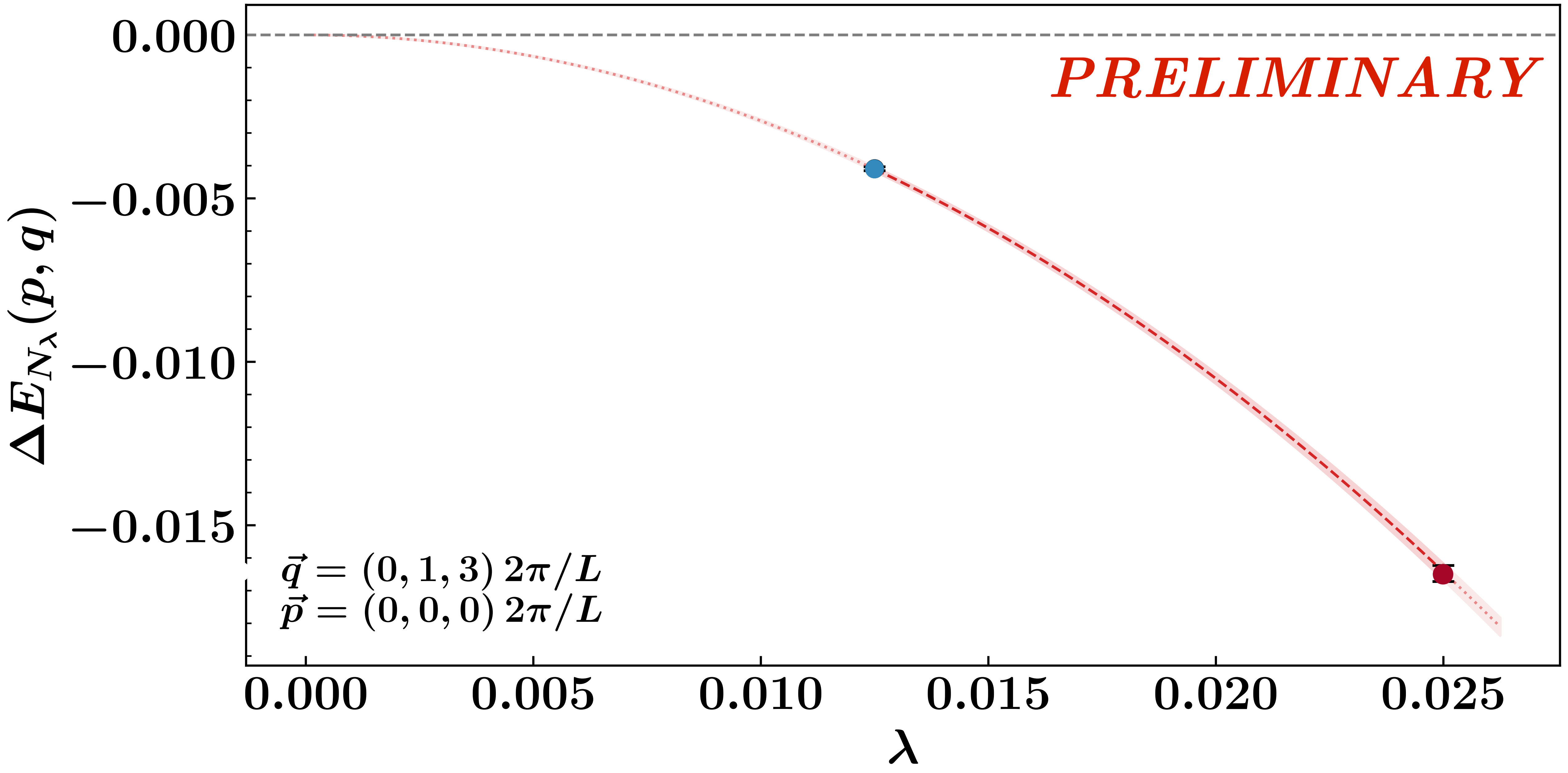
- **Ratio of perturbed 2-pt functions**

$$\mathscr{R}^{qq}_{\lambda}(p,t) \equiv \frac{G^{(2)}_{+\lambda^q_1,+\lambda^q_2}(p,t)\,G^{(2)}_{-\lambda^q_1,-\lambda^q_2}(p,t)}{G^{(2)}_{+\lambda^q_1,-\lambda^q_2}(p,t)\,G^{(2)}_{-\lambda^q_1,+\lambda^q_2}(p,t)} \to A_\lambda e^{-4\Delta E_{N_\lambda}(p)\,t}$$

- **Extract energy shifts for each $|\lambda|$**

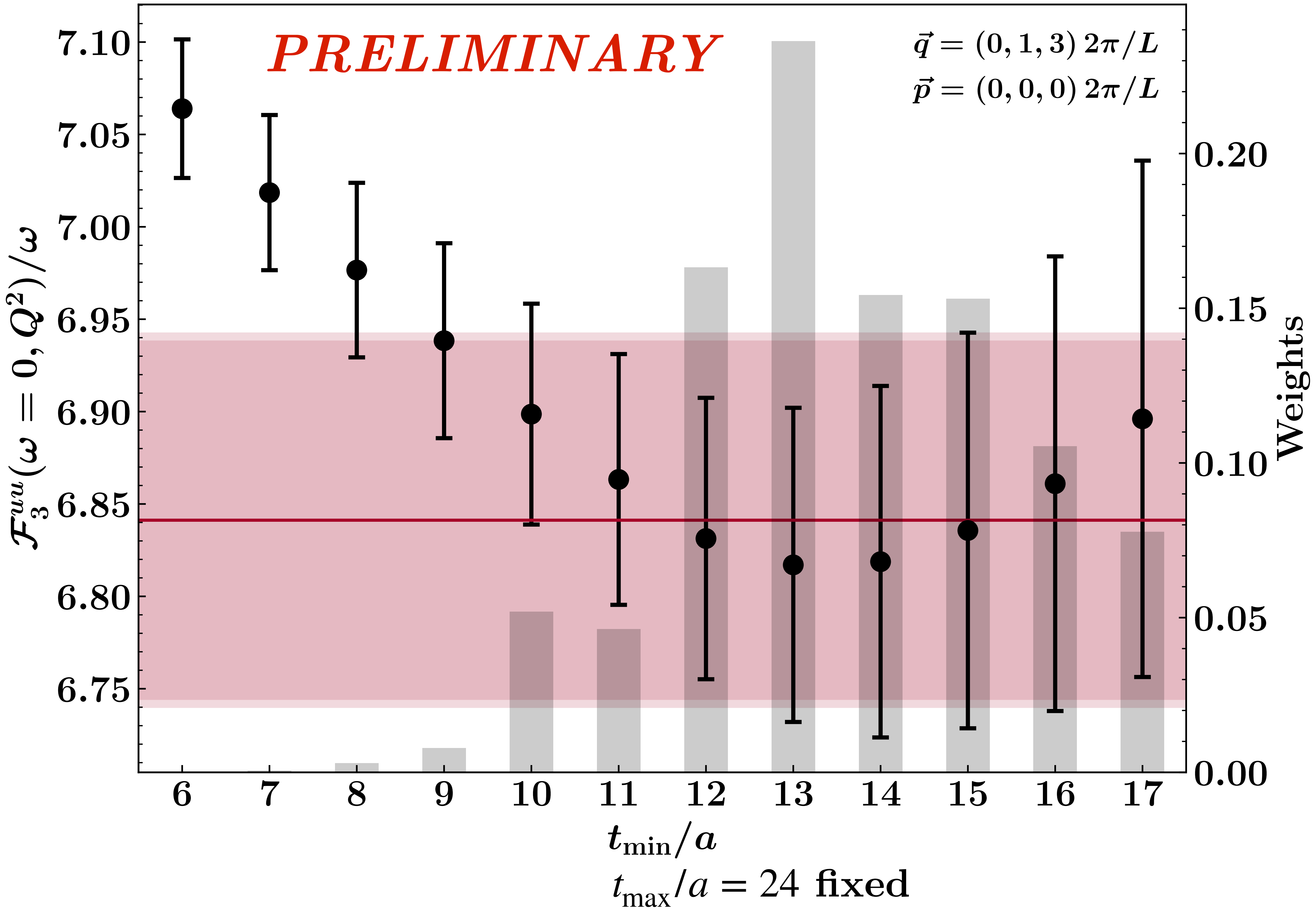


- **Get the 2nd order derivative $\left.\frac{\partial^2 E^\lambda_N(p)}{\partial\lambda_1\partial\lambda_2}\right|_{\lambda=0}$**



Syst. 1: Weighted averaging

$$\frac{\mathcal{F}_3(\omega, Q^2)}{\omega} = \frac{Q^2}{q_2} \frac{\partial^2 E_N^\lambda(p)}{\partial\lambda_1 \partial\lambda_2}\bigg|_{\lambda=0}$$



- **Red line (mean):** $\bar{\mathcal{O}} = \sum_f w^f \mathcal{O}^f$
- **Red band (total uncertainty):**

$$\delta_{\mathbf{stat}}\bar{\mathcal{O}}^2 = \sum_f w^f (\delta\bar{\mathcal{O}}^f)^2$$

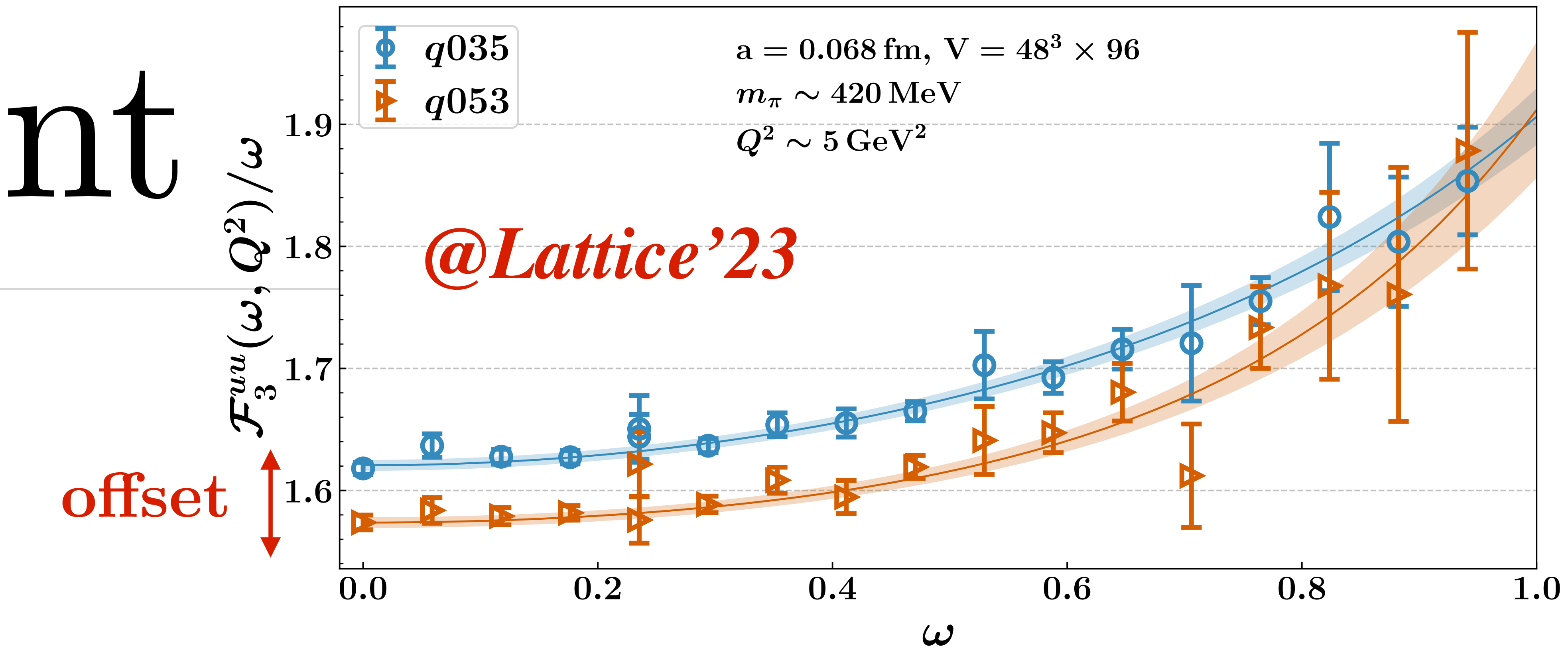
$$\delta_{\mathbf{sys}}\bar{\mathcal{O}}^2 = \sum_f w^f (\mathcal{O}^f - \bar{\mathcal{O}})^2$$

$$\delta\bar{\mathcal{O}} = \sqrt{\delta_{\mathbf{stat}}\bar{\mathcal{O}}^2 + \delta_{\mathbf{sys}}\bar{\mathcal{O}}^2}$$

- **Weights:** $w^f = \dfrac{p_f\,(\delta\mathcal{O}^f)^{-2}}{\sum_{f'} p_{f'}(\delta\mathcal{O}^{f'})^{-2}}$

  where p_f is the one sided p-value of the ratio fits

Syst. 2: LPT improvement

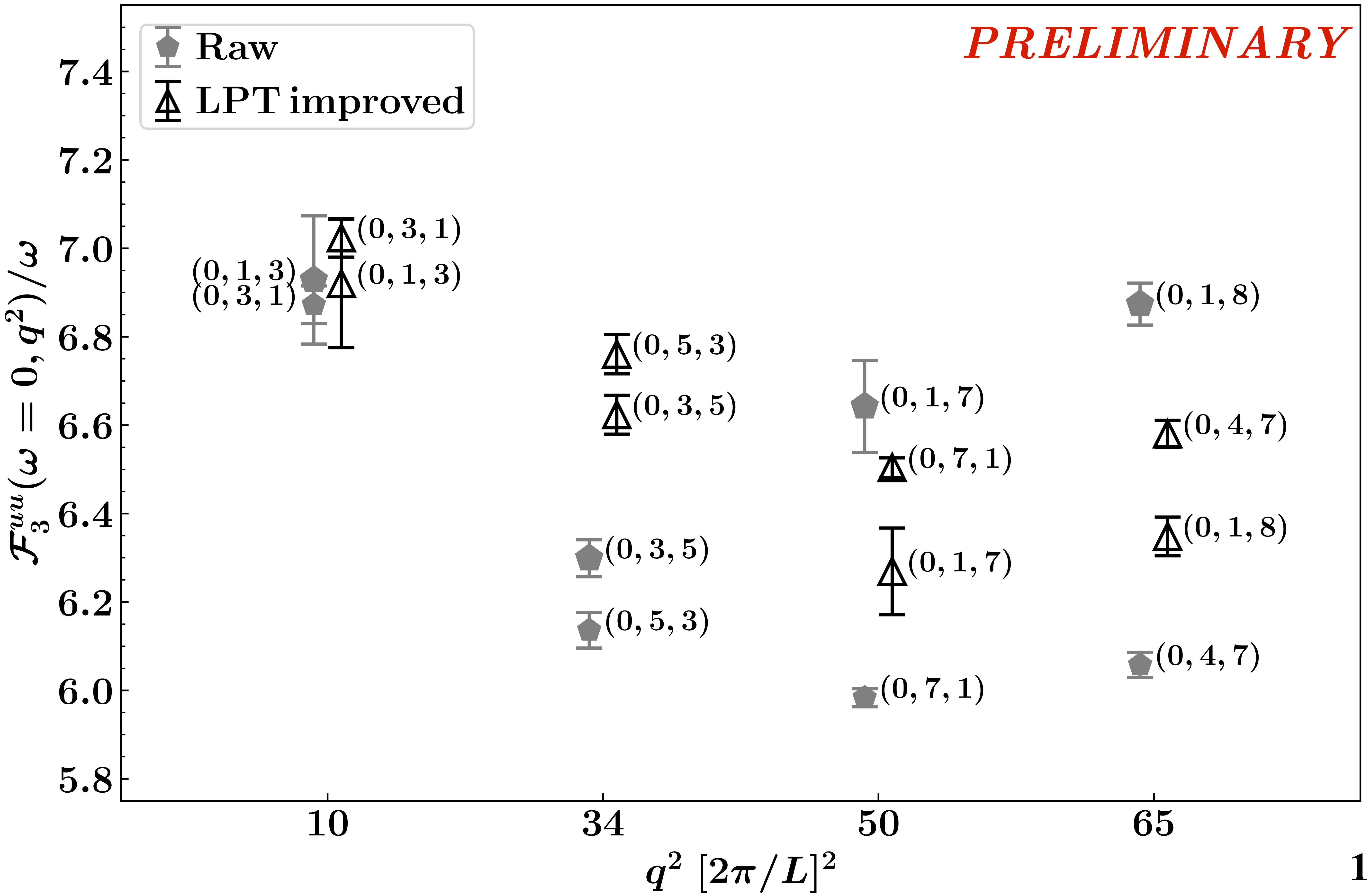


$$\frac{\mathcal{F}_3(\omega, Q^2)}{\omega} = \frac{Q^2}{\boxed{q_2}} \frac{\partial^2 E_N^\lambda(p)}{\partial\lambda_1 \partial\lambda_2}\Bigg|_{\lambda=0}$$

introduces discretisation error due to broken rotational symmetry

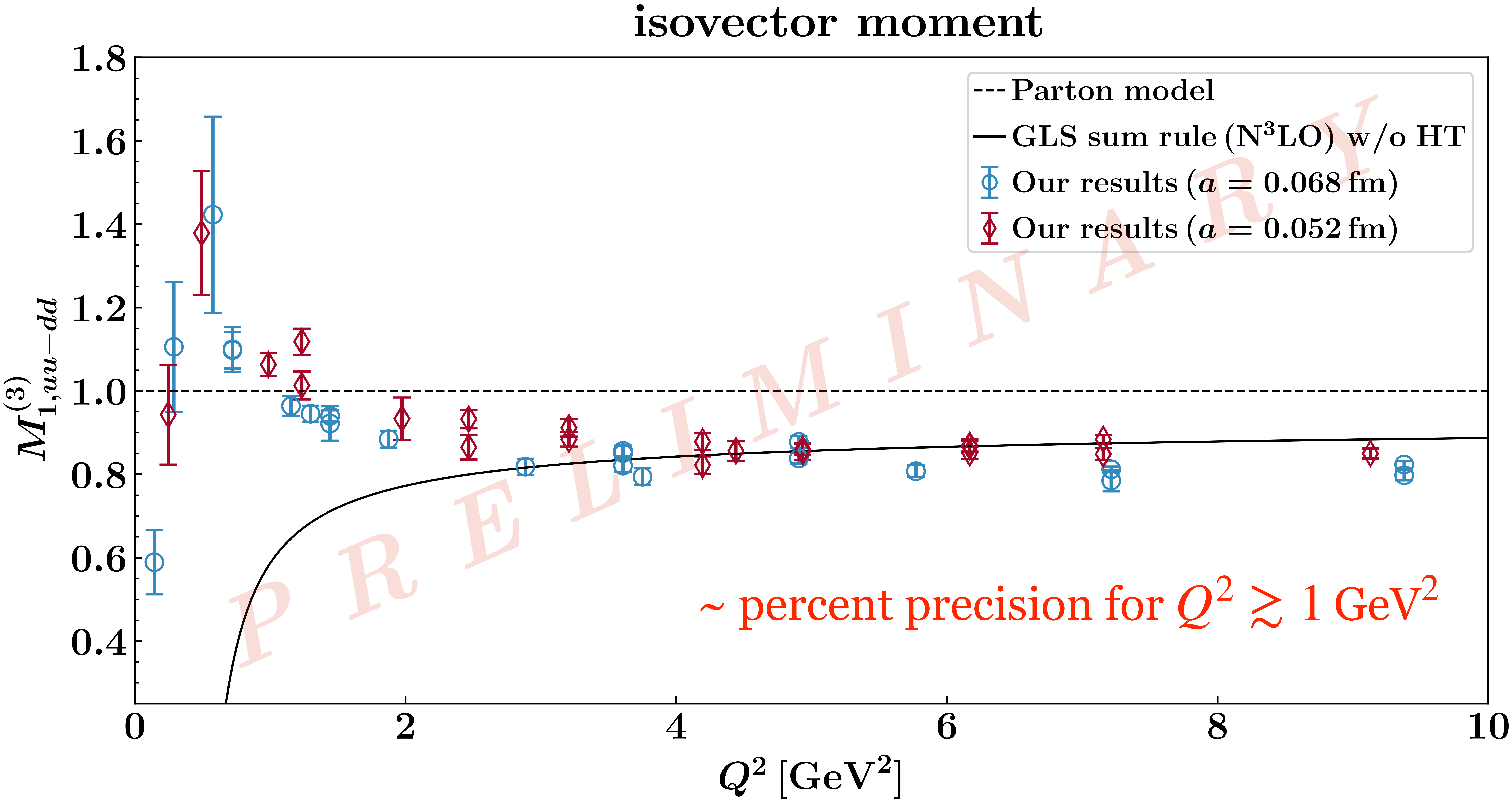
- **Replace the kinematic factor by a lattice perturbation theory motivated factor**

$$\frac{Q^2}{q_2} \rightarrow \frac{\sum_i \sin^2 q_i + \left[\sum_i (1-\cos q_i)\right]^2}{\sin q_2}$$



$\mathscr{F}_3^{\gamma Z}$ | First moment

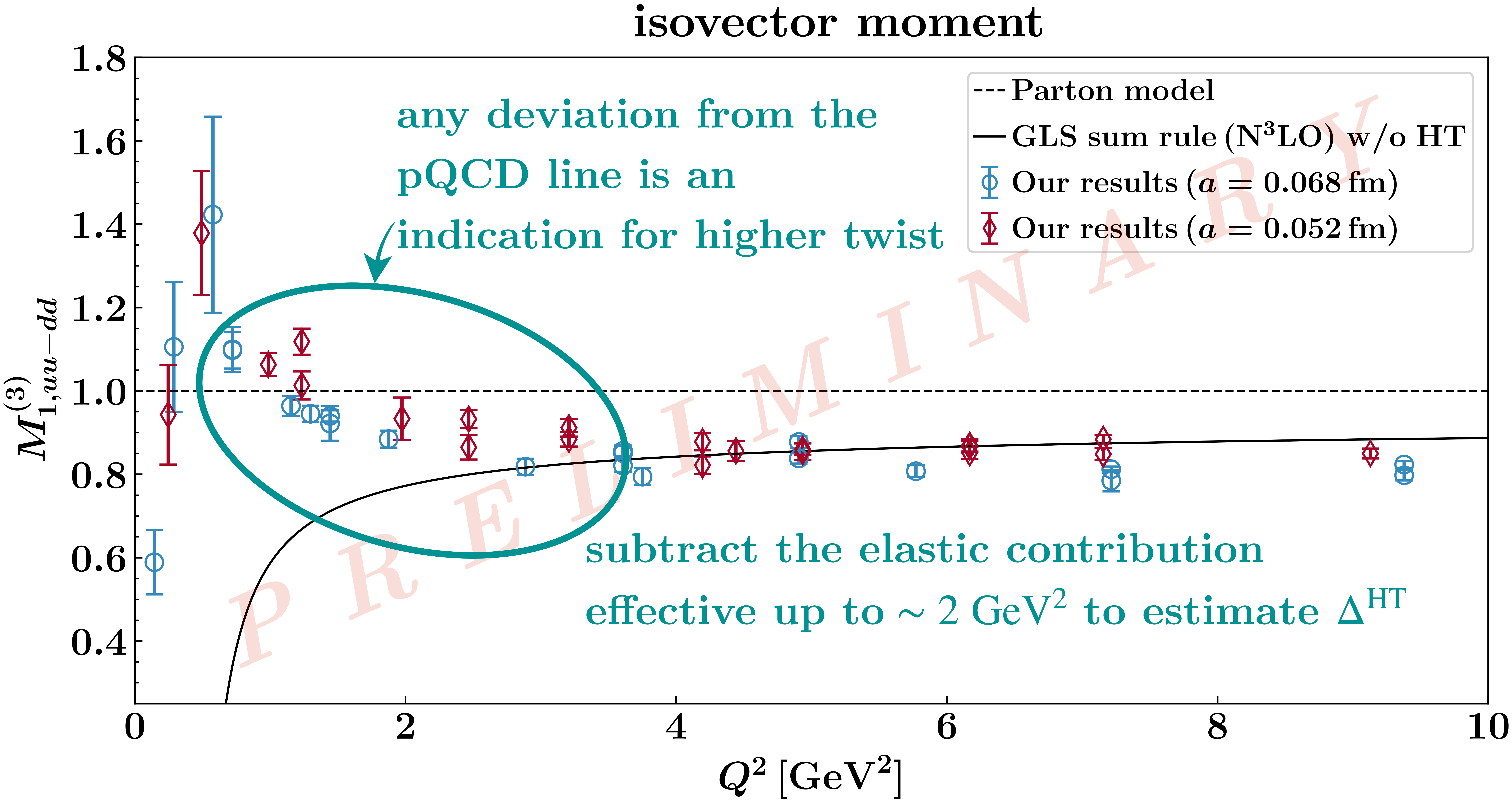
$a = 0.068, 0.052\,\text{fm}$
$m_\pi \sim 410\,\text{MeV}$
48^3x96, 2+1 flavour



~ percent precision for $Q^2 \gtrsim 1\,\text{GeV}^2$

$\mathscr{F}_3^{\gamma Z}$ | Higher-twist

$a = 0.068, 0.052\,\mathrm{fm}$
$m_\pi \sim 410\,\mathrm{MeV}$
48^3x96, 2+1 flavour

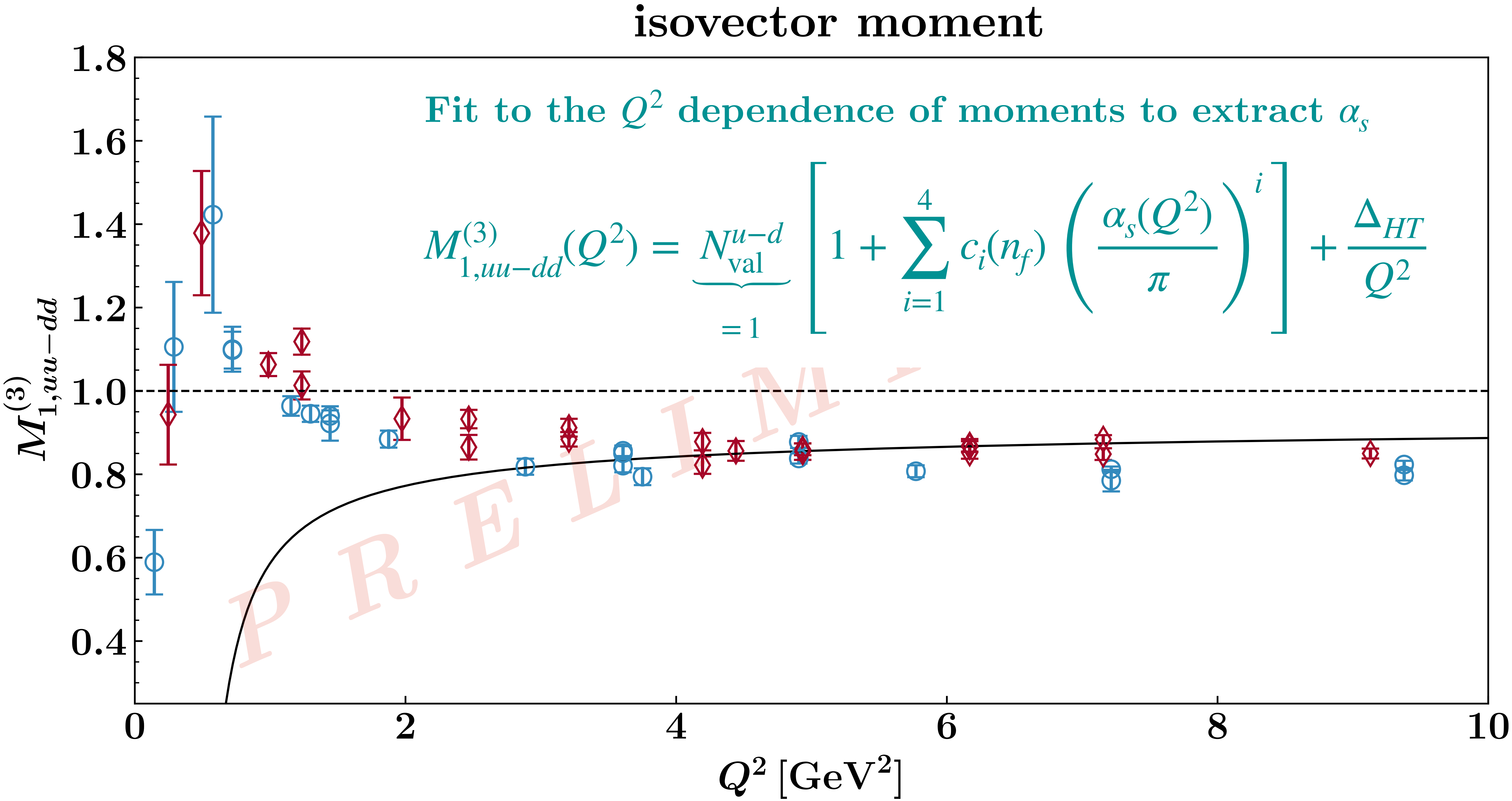


any deviation from the pQCD line is an indication for higher twist

subtract the elastic contribution effective up to $\sim 2\,\mathrm{GeV}^2$ to estimate Δ^{HT}

$\mathcal{F}_3^{\gamma Z}$ | determining α_s

$a = 0.068, 0.052\,\mathrm{fm}$
$m_\pi \sim 410\,\mathrm{MeV}$
48^3x96, 2+1 flavour

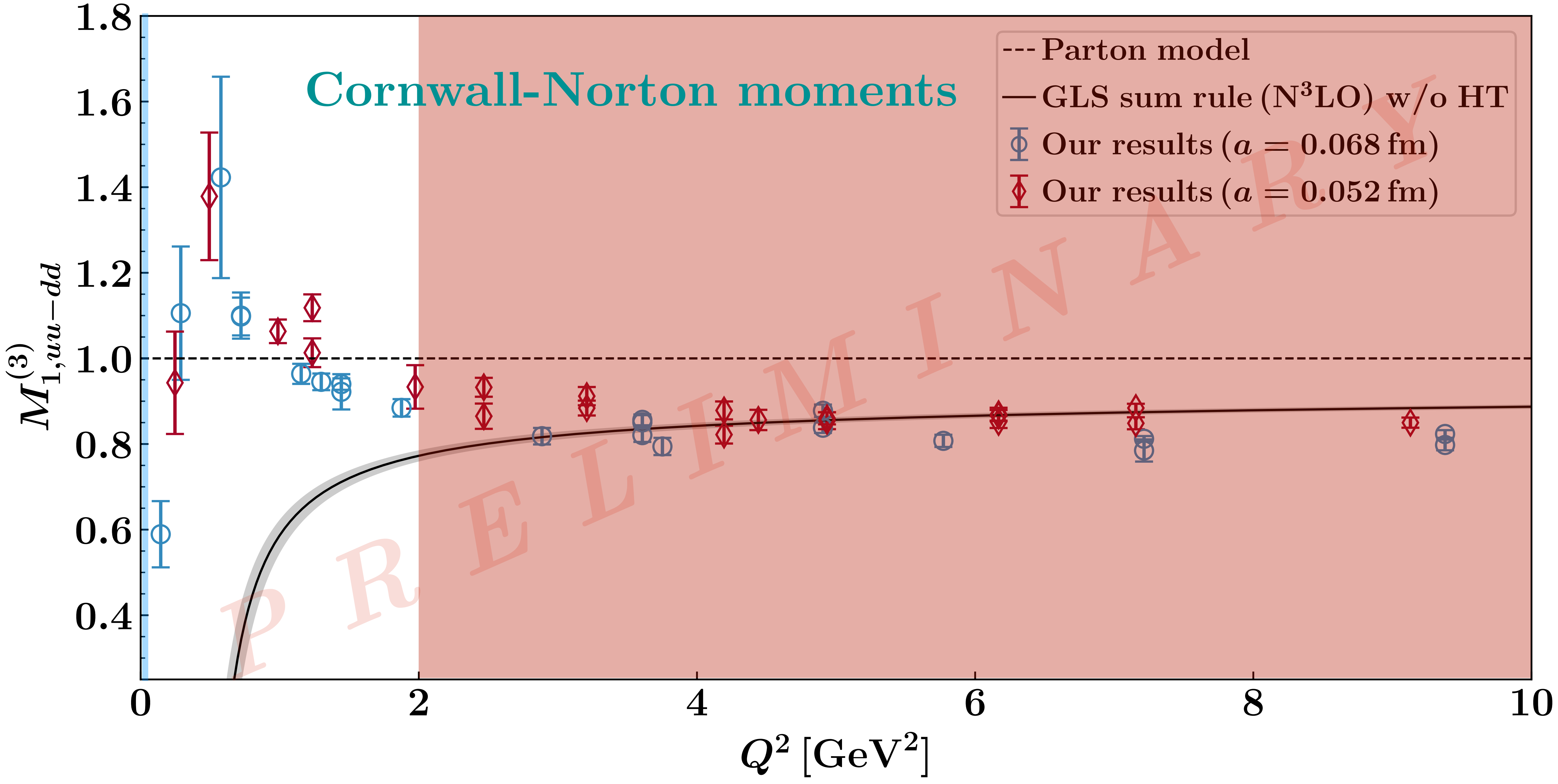


Fit to the Q^2 dependence of moments to extract α_s

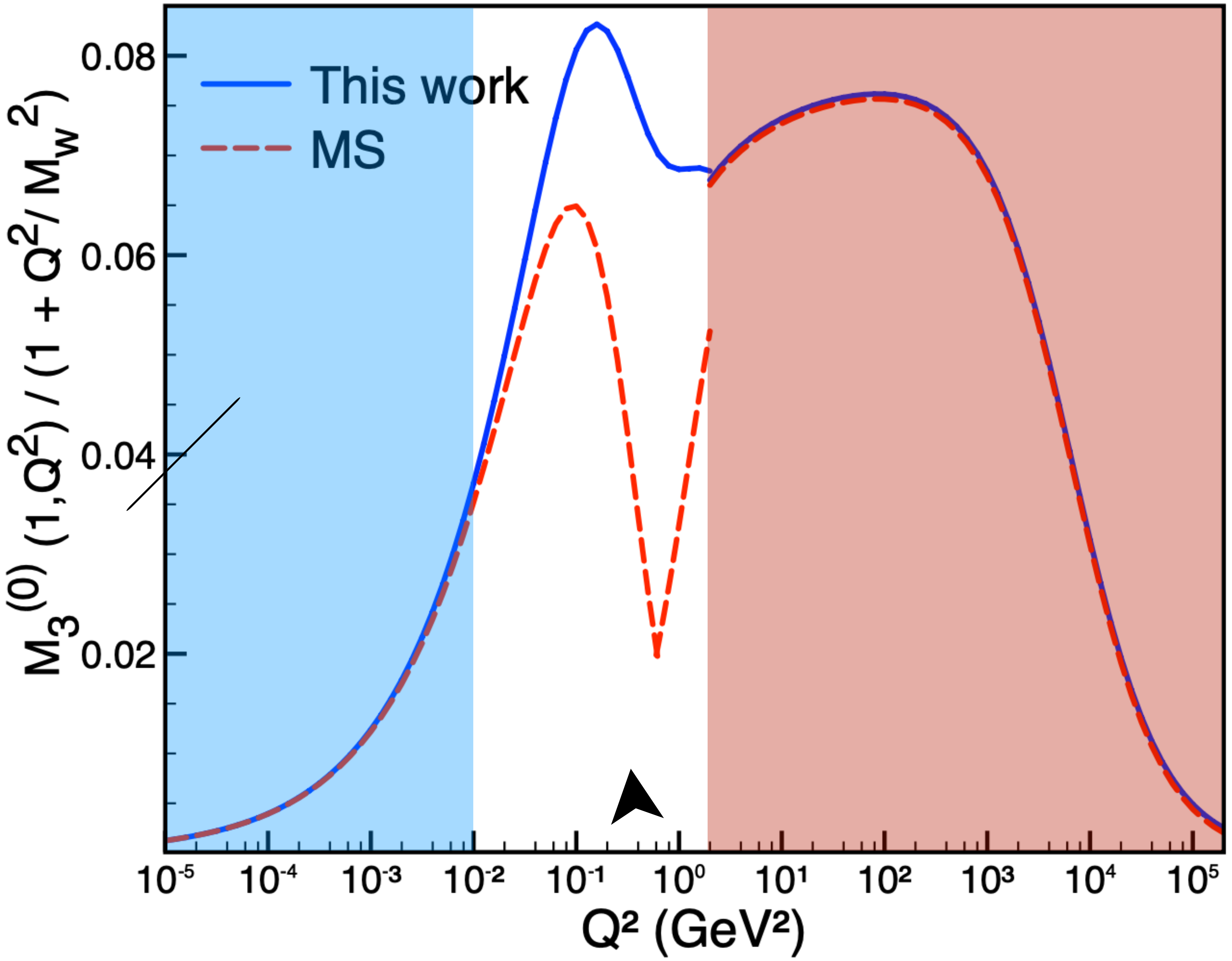
$$M^{(3)}_{1,uu-dd}(Q^2) = \underbrace{N^{u-d}_{\mathrm{val}}}_{=1}\left[1 + \sum_{i=1}^{4} c_i(n_f)\left(\frac{\alpha_s(Q^2)}{\pi}\right)^i\right] + \frac{\Delta_{HT}}{Q^2}$$

$\mathscr{F}_3^{\gamma W}$ | EW box

- Electroweak box diagrams need Nachtmann moments
- We can use lowest 3 Cornwall-Norton moments to reconstruct Nachtmann moments (future work)



C.-Y. Seng, M. Gorchtein, H. H. Patel, and M. J. Ramsey-Musolf, PRL 121, 241804 (2018)



form factors

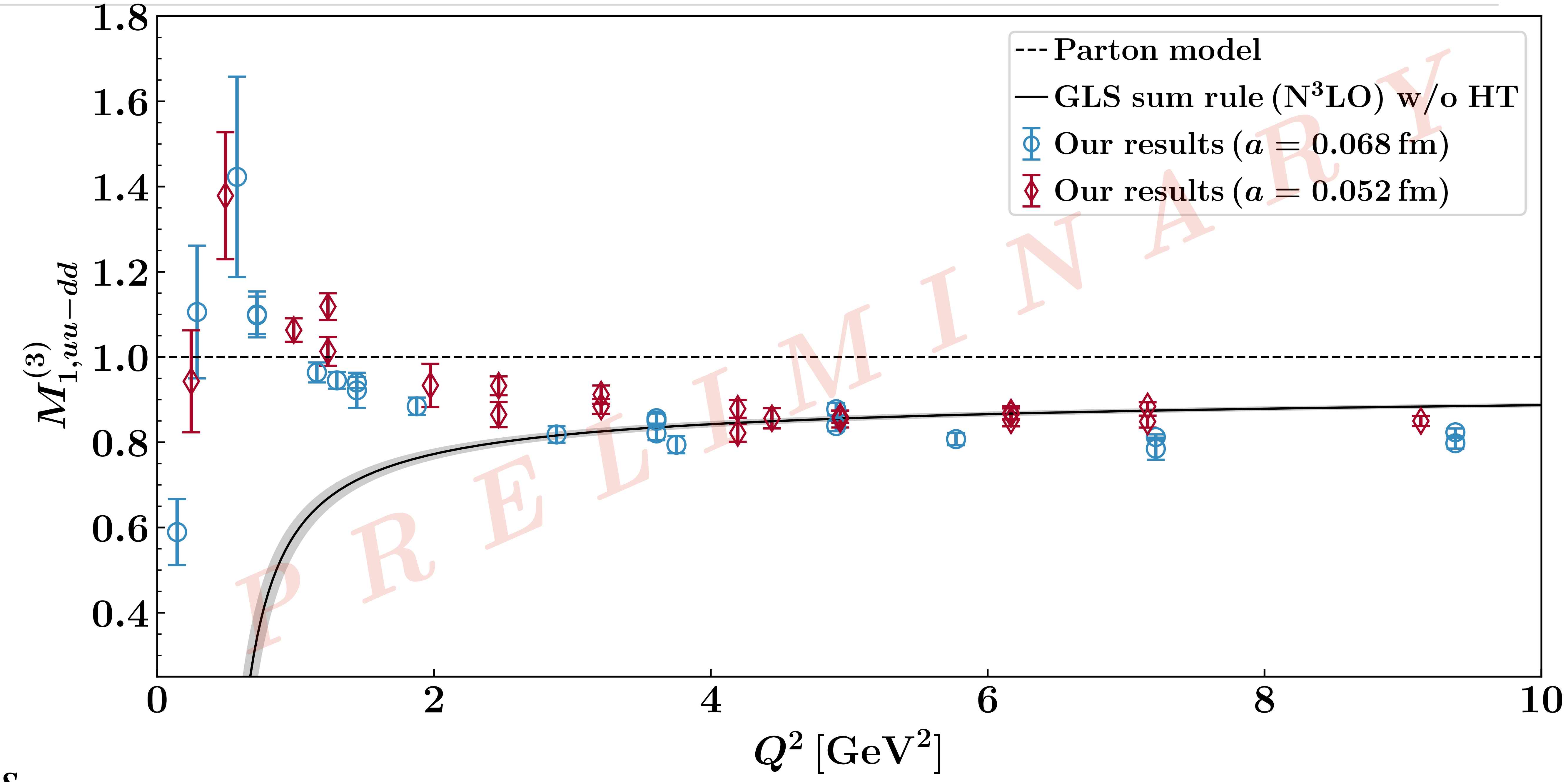
perturbation theory

input from LQCD required

$0.01 \lesssim Q^2 \lesssim 2$ GeV^2

Summary & Outlook

- Lowest moment of $F_3(x, Q^2)$ in a wide range of Q^2
- w/ Good statistical precision, working towards controlled discretisation errors



<u>Outlook</u>

- Utilise GLS sum rule to determine α_s
  - requires continuum extrapolation, $a = 0.082$ fm runs ongoing
- Estimate Nachtmann moments relevant for EW box diagrams, $\square^{\gamma W/Z}_{VA}$
  - requires at least lowest 3 Cornwall-Norton moments
    - we have them for $Q^2 \gtrsim 2\,\text{GeV}^2$
    - need them for phenomenologically interesting region $Q^2 \lesssim 2\,\text{GeV}^2$

Acknowledgements

- The numerical configuration generation (using the BQCD lattice QCD program)) and data analysis (using the Chroma software library) was carried on the
  - DiRAC Blue Gene Q and Extreme Scaling (EPCC, Edinburgh, UK) and Data Intensive (Cambridge, UK) services,
  - the GCS supercomputers JUQUEEN and JUWELS (NIC, Jülich, Germany) and
  - resources provided by HLRN (The North-German Supercomputer Alliance),
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- PELR is supported in part by the STFC under contract ST/G00062X/1.
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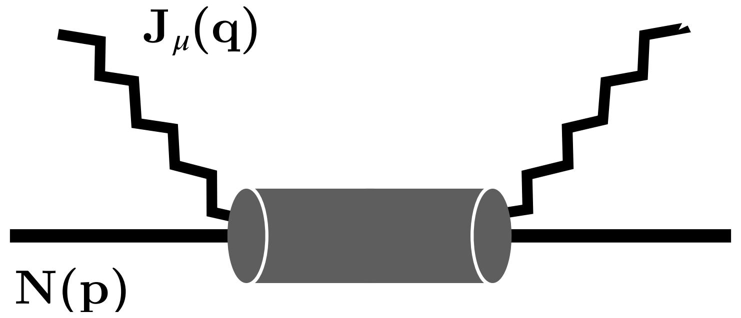
Backup

kuc et al. (CSSM/QCDSF/UKQCD) PRD102, 114505 (2020), arXiv:2007.01523 [hep-lat]

Compton amplitude via the FH relation at 2nd order

- **unpolarised Compton Amplitude**

$$T_{\mu\mu}(p,q) = \int d^4z e^{i\mathbf{q}\cdot\mathbf{z}} \langle N(p) \,|\, \mathscr{T}\{J_\mu(z) J_\mu(0)\} \,|\, N(p)\rangle$$



- **Action modification**

local EM current

$$J_\mu(z) = \sum_q e_q \bar{q}(z)\gamma_\mu q(z)$$

$$S \to S(\lambda) = S + \lambda \int d^4z \left(e^{i\mathbf{q}\cdot\mathbf{z}} + e^{-i\mathbf{q}\cdot\mathbf{z}}\right) J_\mu(z)$$

- **2nd order derivatives of the 2-pt correlator, $G_\lambda^{(2)}(\mathbf{p};t)$, in the presence of the external field**

$$\left.\frac{\partial^2 G_\lambda^{(2)}(\mathbf{p};t)}{\partial\lambda^2}\right|_{\lambda=0} = \left(\frac{\partial^2 A_\lambda(\mathbf{p})}{\partial\lambda^2} - tA(\mathbf{p})\frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial\lambda^2}\right) e^{-E_N(\mathbf{p})t}$$

from spectral decomposition

$$\left.\frac{\partial^2 G_\lambda^{(2)}(\mathbf{p};t)}{\partial\lambda^2}\right|_{\lambda=0} = \frac{A(\mathbf{p})}{2E_N(\mathbf{p})} t e^{-E_N(\mathbf{p})t} \int d^4z \left(e^{iq\cdot z} + e^{-iq\cdot z}\right) \langle N(\mathbf{p})|\mathcal{T}\{\mathcal{J}(z)\mathcal{J}(0)\}|N(\mathbf{p})\rangle$$

from path integral

- **equate the time-enhanced terms:**

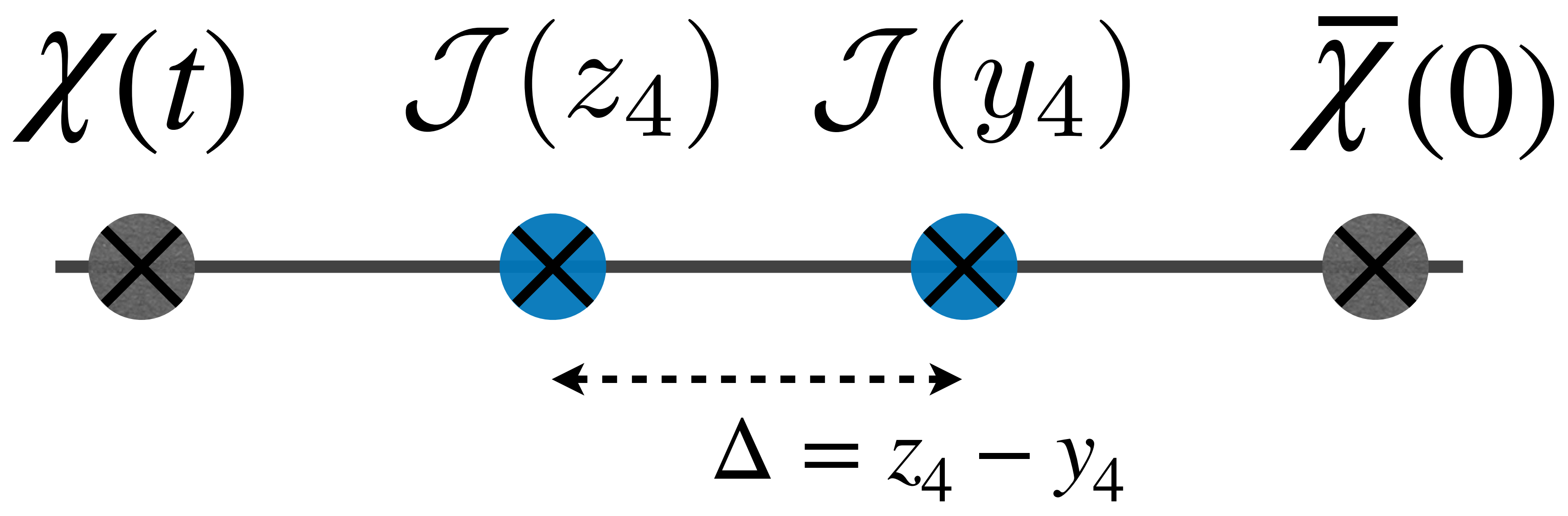
$$\left.\frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial\lambda^2}\right|_{\lambda=0} = -\frac{1}{2E_N(\mathbf{p})} \overbrace{\int d^4z \left(e^{iq\cdot z} + e^{-iq\cdot z}\right) \langle N(\mathbf{p})|\mathcal{J}(z)\mathcal{J}(0)|N(\mathbf{p})\rangle}^{T_{\mu\mu}(p,q)} + (q \to -q)$$

Compton amplitude is related to the second-order energy shift

kuc et al. (CSSM/QCDSF/UKQCD) PRD102, 114505 (2020), arXiv:2007.01523 [hep-lat]

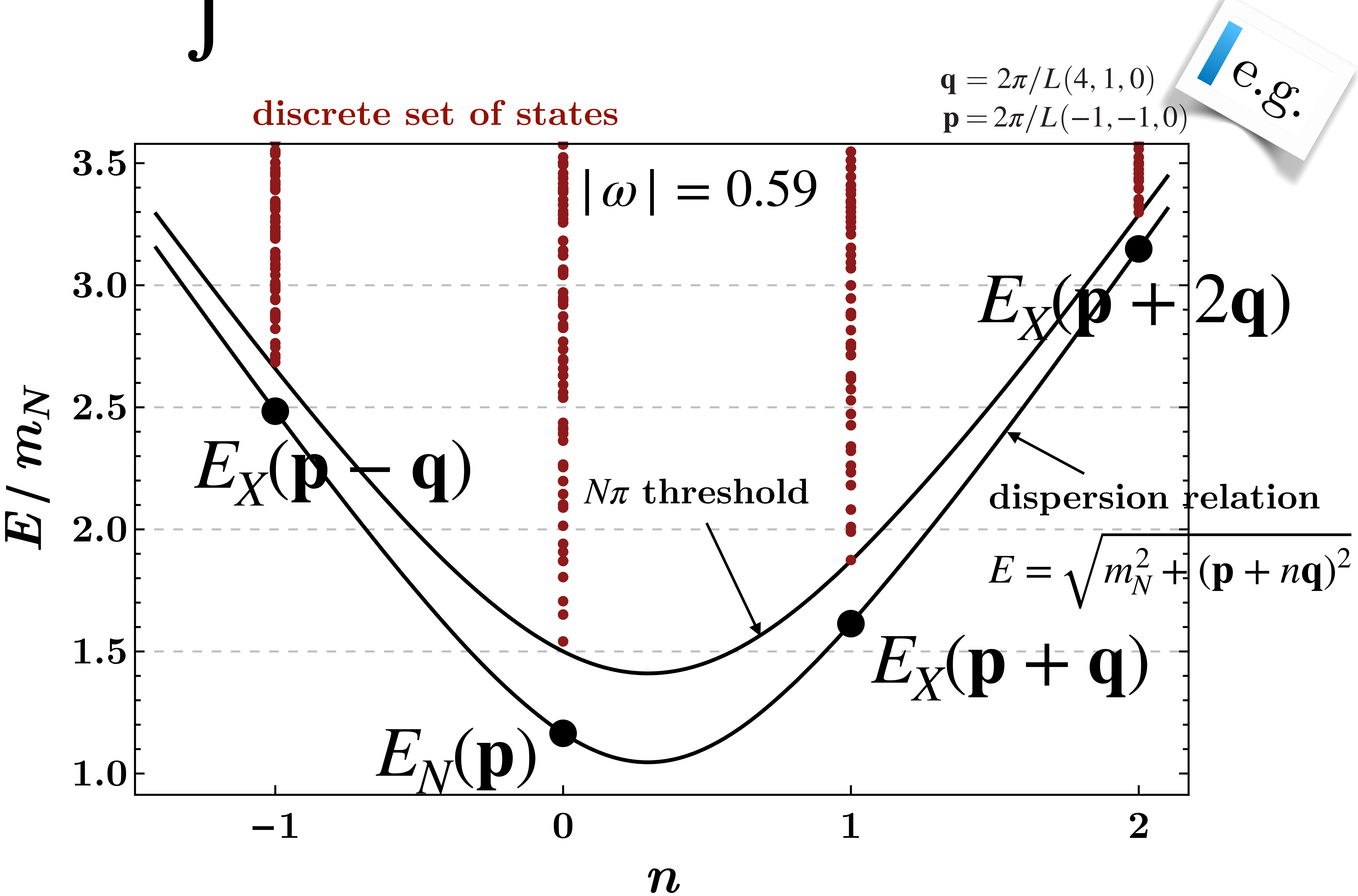
Compton amplitude via the FH relation at 2nd order

- relevant contribution comes from the ordering where the currents are sandwiched



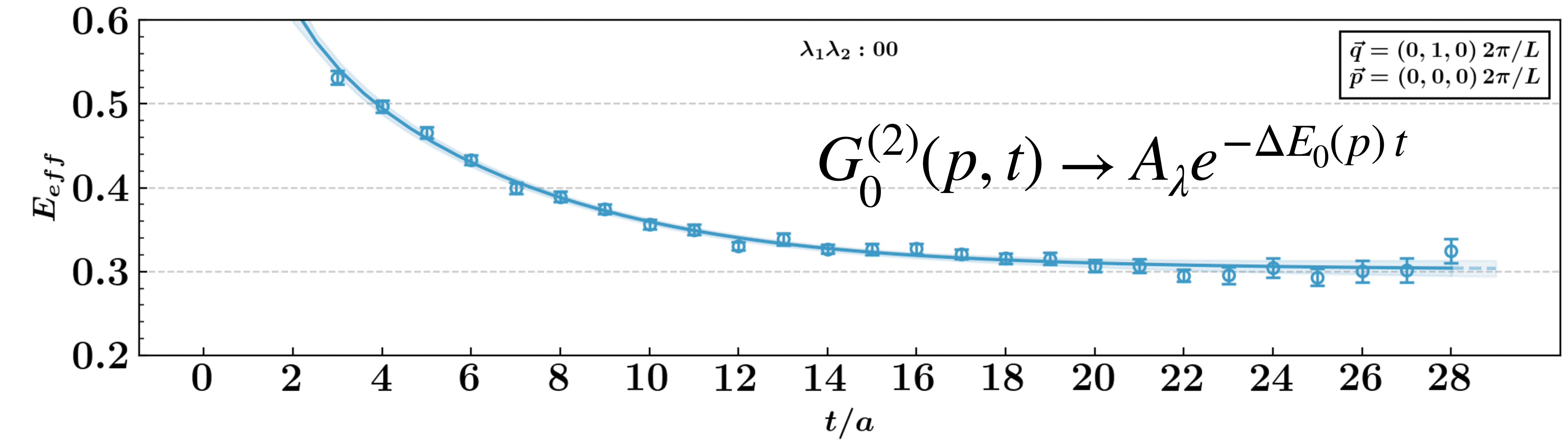
$$\sim e^{-E_N(\mathbf{p})t}\int^t d\Delta\, e^{-\left(E_X(\mathbf{p}+\mathbf{q})-E_N(\mathbf{p})\right)\Delta}\,(t-\Delta)$$

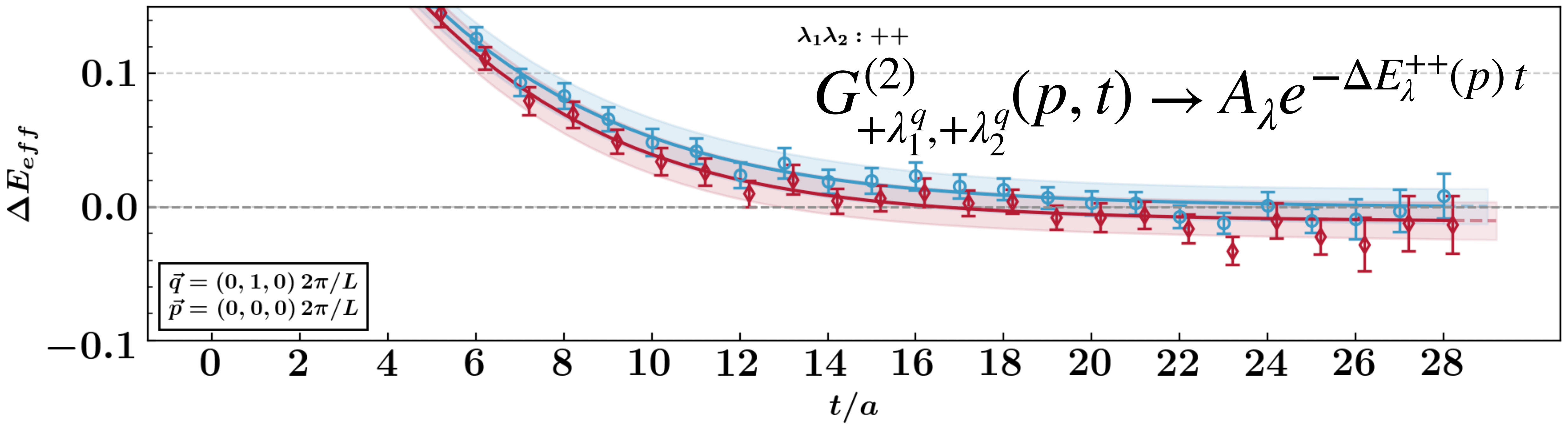
- under the condition $|\omega| < 1$, $E_X(\mathbf{p}+n\mathbf{q}) \gtrsim E_N(\mathbf{p})$, so the intermediate states cannot go on-shell
- ground state dominance is ensured in the large time limit

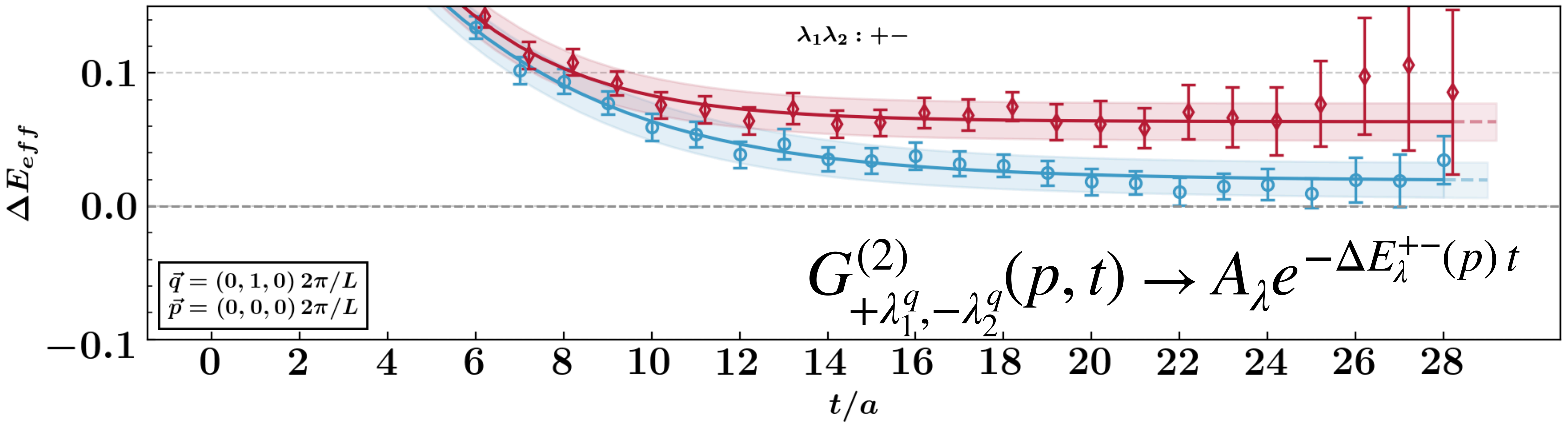


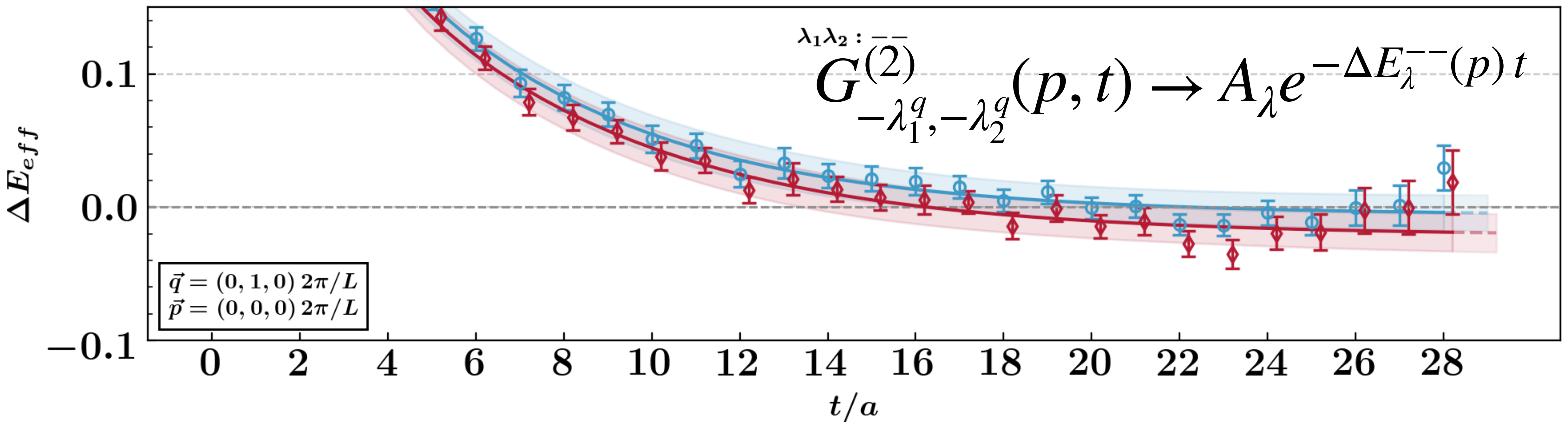
Multi-exp fits ($Q^2 \lesssim 1\ \text{GeV}^2$)

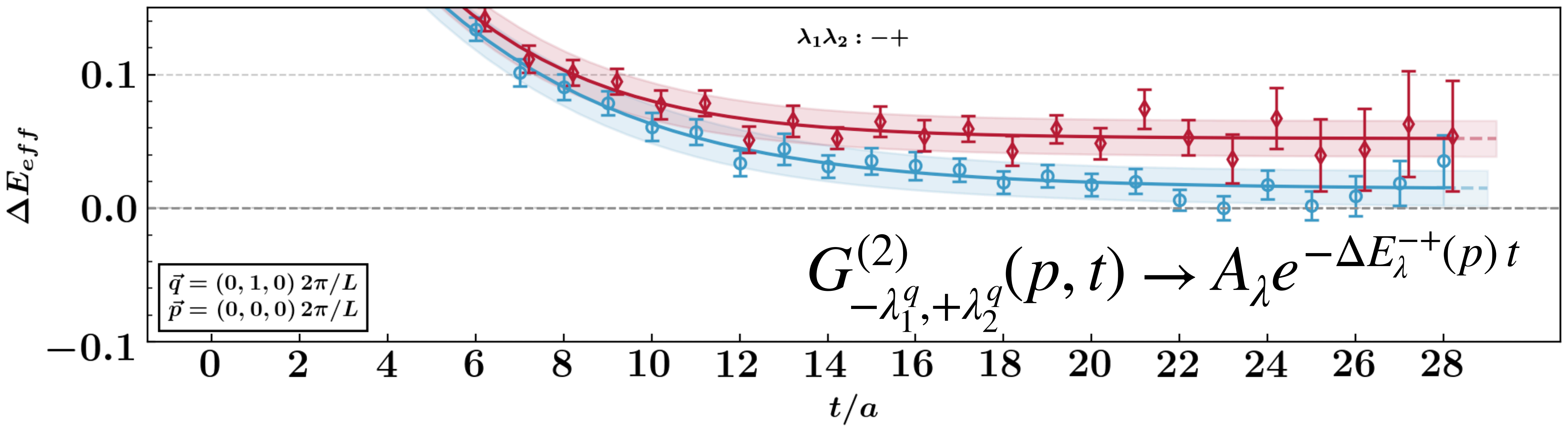
Second order energy shift: $\Delta E_{N_\lambda}(p) = \frac{1}{4}\left[\Delta E_\lambda^{++}(p) + \Delta E_\lambda^{--}(p) - \Delta E_\lambda^{+-}(p) - \Delta E_\lambda^{-+}(p)\right] - E_0(p)$











Future lattices

Currently thermalising/generating

- $64^3 \times 96$, $a = (0.068, 0.052)$ fm, $m_\pi = (220, 270)$ MeV *(completed - early 2024)*
- $80^3 \times 114$, $a = 0.068$ fm, $m_\pi = 150$ MeV *(still thermalising)*
- $96^3 \times 128$, $a = 0.052$ fm, $m_\pi = 140$ MeV *(thermalised + O(50) trajectories)*

Using BQCD [EPJ Web Conf. 175 (2018) 14011]

on

- JUWELS (Jülich, Germany)
- CSD3 (Cambridge, UK)
- Tursa (Edinburgh, UK)