Light-quark connected HVP contribution to muon g-2

Michael Lynch [1]

[1]University of Illinois, Urbana-Champaign

31 July, 2024

Fermilab Lattice, HPQCD, and MILC Collaborations

Lattice 2024, University of Liverpool, Liverpool, UK

Fermilab-HPQCD-MILC

Fermilab Lattice and MILC

- Alexei Bazavov
- Pietro Butti
- David Clarke
- Carleton DeTar
- Aida El-Khadra
- Elvira Gámiz
- Steven Gottlieb
- Anthony Grebe
- Leon Hostetler
- William Jay
- Hwancheol Jeong
- Andreas Kronfeld
- Shaun Lahert
- Michael Lynch
- Andrew Lytle
- Ethan Neil
- Curtis Peterson
- James Simone
- Jacob Sitison
- Ruth Van de Water
- Alejandro Vaquero

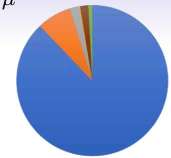
HPQCD

- Christine Davies
- Peter Lepage
- Craig McNeile
- Gaurav Ray

Computing Resources

- ACCESS
- ALCC
- Dirac
- ERCAP
- INCITE
- Indiana U
- LRAC
- USQCD
- XSEDE

$a_\mu^{\mathrm{HVP,LO}}$ from Lattice QCD



$$a_\mu^{\mathrm{HVP,LO}} = 4\alpha^2 \int_0^\infty dt C(t) \tilde{K}(t) = a_\mu^{ll}\,(\mathrm{conn.}) + a_\mu^{ss}\,(\mathrm{conn.}) + \dots$$
$$+ a_\mu\,(\mathrm{disc.}) + \Delta a_\mu\,(\mathrm{SIB}) + \Delta a_\mu\,(\mathrm{QED})$$

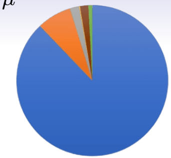
$$C(t) = \frac{1}{3}\sum_i^3 \int d^3x \, \langle J_i(\vec{x},t) J_i(0) \rangle \,,$$

$$J_i(x) = Q_u \bar{u}(x)\gamma_i u(x) + Q_d \bar{d}(x)\gamma_i d(x) + \dots$$

- $a_\mu^{ll}\,(\mathrm{conn.})$ accounts for $\approx 90\%$ of total

$a_\mu^{\mathrm{HVP,LO}}$ (SD+W)	(Shaun Lahert, Wed. 12:15)
$a_\mu^{ll}\,(\mathrm{conn.})$	(this talk)
$a_\mu\,(\mathrm{disc.})$	(David Clarke, Thurs. 9:00)
$\Delta a_\mu\,(\mathrm{SIB})$	(Jake Sitison, Thurs. 9:20)
$\Delta a_\mu\,(\mathrm{QED})$	(Craig McNeile, poster)

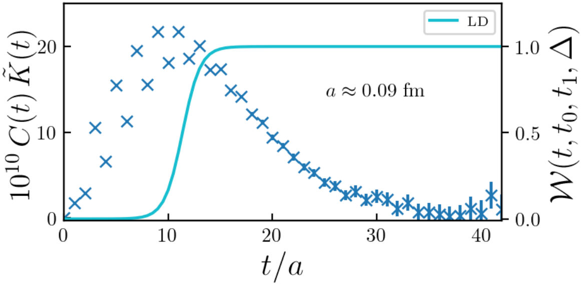
$a_\mu^{\text{HVP,LO}}$ from Lattice QCD



$$a_\mu^{\text{HVP,LO}} = 4\alpha^2 \int_0^\infty dt C(t)\tilde{K}(t) = a_\mu^{ll}\,(\text{conn.}) + a_\mu^{ss}\,(\text{conn.}) + \ldots$$
$$+ a_\mu\,(\text{disc.}) + \Delta a_\mu\,(\text{SIB}) + \Delta a_\mu\,(\text{QED})$$

- $a_\mu^{ll}\,(\text{conn.})$ Suffers from large-t noise
  - Prominent in LD window

$$a_\mu^{\text{LD}} = 4\alpha^2 \int_0^\infty dt C(t)\tilde{K}(t) \times \mathcal{W}(t, 1, \infty, \Delta)$$

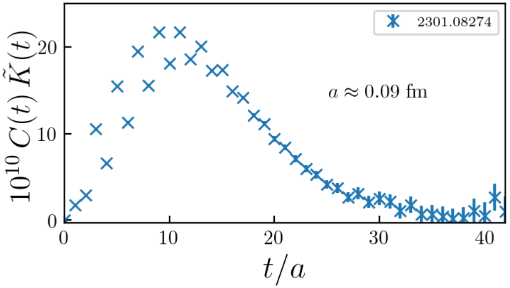


Low-mode averaging

$$C_{\mathrm{RW}}^{\Gamma}(t) = \mathrm{Tr}\left\{M^{-1}\Gamma\eta_i\eta_i^\dagger M^{-1}\Gamma\right\}$$

- Random wall (RW) sources precisely capture $C(t)$ for small t.
  - $\sigma_{\mathrm{RW}}^2 \propto \frac{1}{N_{\mathrm{cfg}}}\frac{1}{N_{\mathrm{src}}}$
- Low eigenmodes precisely capture $C(t)$ for large t

arXiv:hep-lat/0012021 , 1402.0244



$a_\mu^{\mathrm{HVP,LO}} = 4\alpha^2 \int_0^\infty dt C(t)\tilde{K}(t)$

Low-mode averaging

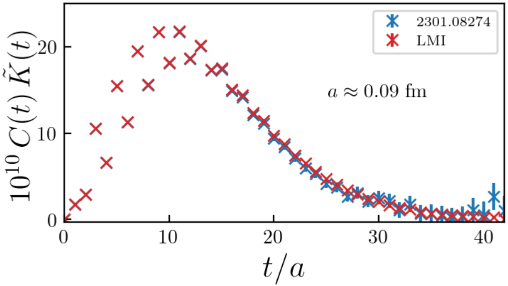
$$C^{\Gamma}_{\mathrm{LMI}}(t) = \mathrm{Tr}\left\{M^{-1}\Gamma\eta_i\eta_i^\dagger M^{-1}\Gamma\right\} - \mathrm{Tr}\left\{M_L^{-1}\Gamma\eta_i\eta_i^\dagger M_L^{-1}\Gamma\right\} + \mathrm{Tr}\left\{M_L^{-1}\Gamma M_L^{-1}\Gamma\right\}$$

$$M_L^{-1} = \sum_n^{N_e} \frac{1}{\lambda_n} v_n v_n^\dagger$$

- Random wall (RW) sources precisely capture $C(t)$ for small t.
  - $\sigma^2_{\mathrm{RW}} \propto \frac{1}{N_{\mathrm{cfg}}}\frac{1}{N_{\mathrm{src}}}$
- Low eigenmodes precisely capture $C(t)$ for large t

  arXiv:hep-lat/0012021 , 1402.0244
  - Increase $N_{\mathrm{eig}} \to$ precision at earlier t
- Eigenvectors also used to deflate CG

  Deflation on HISQ (Leon Hostetler, Thurs. 10:40)



Low-mode All-to-all

$$\mathrm{Tr}\{M_L^{-1}\Gamma M_L^{-1}\Gamma\} = \sum_{t_0}\sum_{n,m}^{N_e} \mathcal{M}^{\Gamma}_{m,n}(t+t_0)\mathcal{M}^{\Gamma}_{n,m}(t_0)$$

- All-to-all using modified Grid/Hadrons Libraries
  - + A2A GPU support
  - + Checkerboarded A2A contractions
  - + Checkerboarded deflation
  - + `class StagGamma` for general local/one-link spin-taste ops

Low-mode all-to-all meson fields

$$\mathcal{M}^{\Gamma}_{n,m}(t) \equiv \sum_{\vec{x}} w_n^{\dagger}(\vec{x},t)\Gamma u_m(\vec{x},t),$$

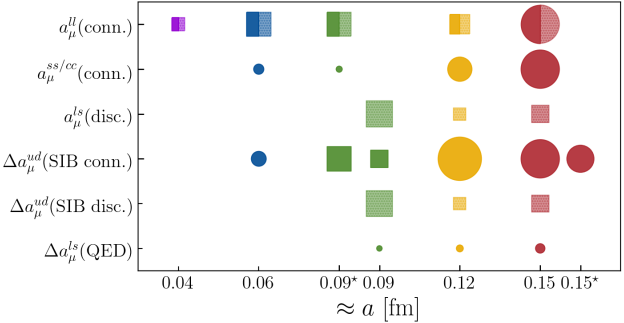
$$w_n \in \{v_1/\lambda_1, \ldots, v_{N_e}/\lambda_{N_e}\},$$

$$u_n \in \{v_1, \ldots, v_{N_e}\}$$

10.1016/j.cpc.2005.06.008

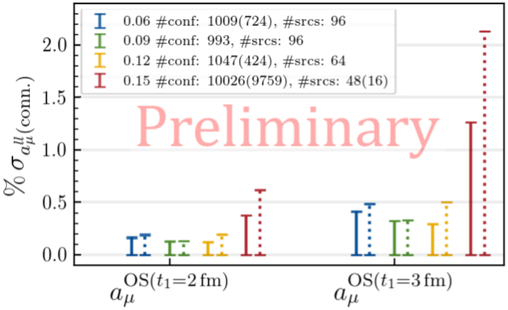
$a_\mu^{\mathrm{HVP,LO}}$ HISQ ensembles

- MILC HISQ gauge field ensembles
- Solid/hatched indicate different discretizations for vector current
- a_μ^{ll} (conn.) square ensembles (top row) are LMA-improved with 2k eigenvector pairs
- Maximize MC time spacing on large phys. MILC ensembles to avoid autocorrelation



Statistical error result

- Stat. error from one-sided (OS) window $[0, t_1]$
  - No noise reduction procedure applied
- Solid(dashed) lines show (non-)local vector currents
- $\sim 0.5\%$ stat. error out to 3 fm



Analysis workflow

- Mult. blinding factor applied to all data before analysis
- Use fit/bounding method to constrain LMA data at large t.
- Add finite volume, pion mass mistuning, (and taste-breaking) corrections

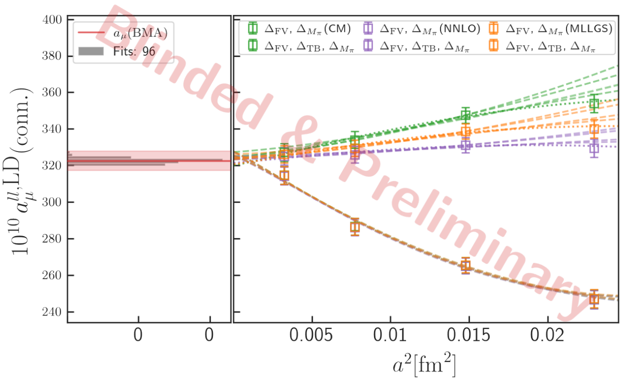
$$a_\mu^{ll}(\mathrm{conn.})\left(L_\infty, M_{\pi_{\mathrm{phys}}}\right) = a_\mu^{ll}(\mathrm{conn.})\left(L_{\mathrm{latt}}, M_{\pi_{\mathrm{lat},\xi_1}}, \cdots, M_{\pi_{\mathrm{lat},\xi_{16}}}\right) + \Delta_{\mathrm{FV}} + \Delta_{M_\pi}\,(+\Delta_{\mathrm{TB}})$$

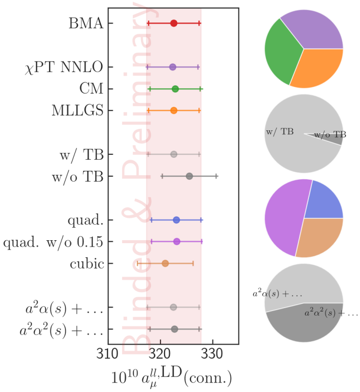
- Extrapolate to the continuum limit

$$a_\mu^{ll}(\mathrm{conn.})\left(a, \{m_f\}\right) = a_\mu^{ll}(\mathrm{conn.})\left(1 + C_{a^2,n}a^2\alpha_s^n + C_{a^4}a^4 + \cdots + C_{\mathsf{sea}}\sum_{f=l,l,s}\delta m_f\right)$$

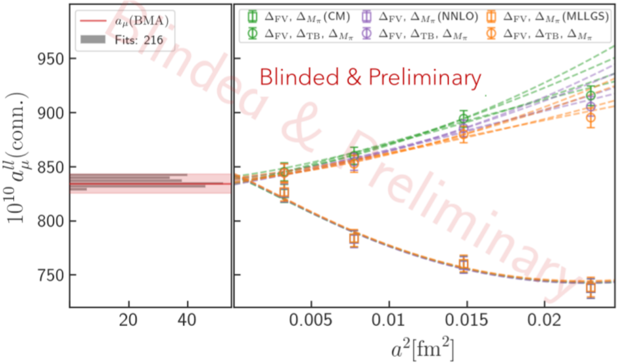
- Systematic error estimated using Bayesian Model Averaging

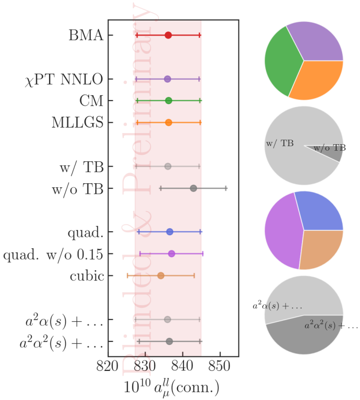
a_μ^{ll} (conn.) - LD window





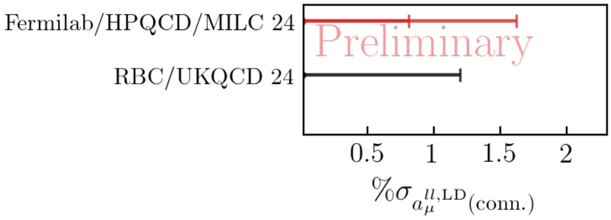
a_μ^{ll} (conn.) - Full

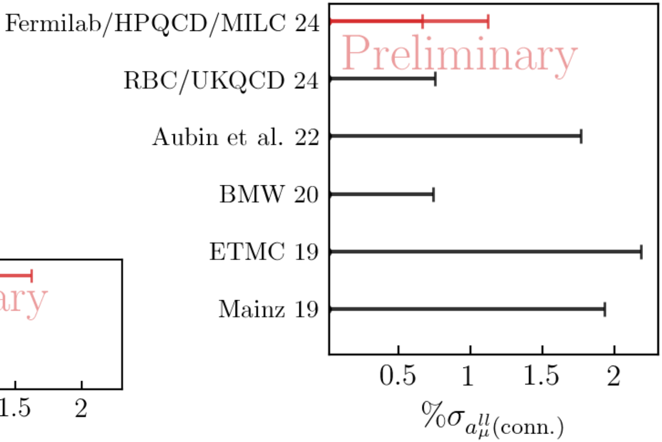




Total error result

- Inner error w/o scale setting (w_0 fm) uncertainty
- Scale setting is now dominant error contributor.



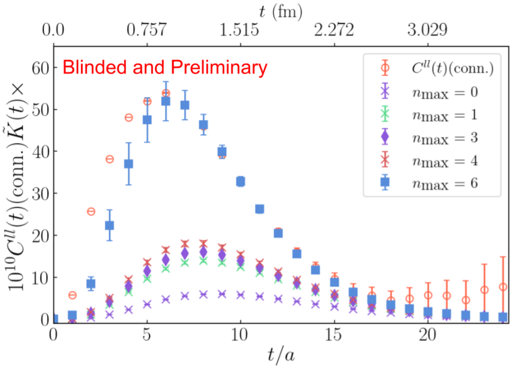


Two-pion study

- First lattice calculation with staggered two-pion operators
- Obtain low-lying FV spectrum from study of staggered two-pion states

$$C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$$

- Use (A_n, E_n) to reconstruct correlation function at large t
  - $C(t > t_c) = \sum_n^{n_{\max}} |A_n|^2 e^{-E_n t}$
- Yields ~2.5x reduction in stat. error (compared with bounding method)



Outlook

We expect further improvements in stat., sys. uncertainties from...

- Generation of correlator data at a lattice spacing of 0.04 fm is in progress.
- Improved scale setting via M_Ω.
- Joint fit analysis with multiple vector current discretizations
- Direct finite volume study: $L \sim 5.5 \text{ fm} \to L \sim 11 \text{ fm}$ (at $a = 0.09 \text{ fm}$) to replace EFT-based FV error estimates.
- Calculation of two-pion contributions to vector-current correlation functions at finer lattice spacings.

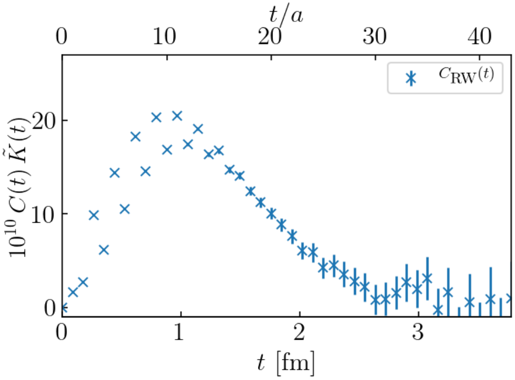
Back-up Slides

a_μ^{ll} (conn.) long-distance noise

Approx. $C(t)$ using "Random Wall" sources, η_i

$$C^{\Gamma}_{\rm RW}(t) = \sum_i \mathrm{Tr}\left\{ S_i^{\rm RW} \Gamma \gamma^5 S_i^{\rm RW\dagger} \gamma^5 \Gamma \right\}, \quad S_i^{\rm RW}(x) = M^{-1}(x;y)\eta_i(y)$$

- $\mathrm{Var}(C(t))$ overlaps with low-energy two-pion states
  - Exp. noise growth at large t
- **Solution:** Low eigenmodes capture *both signal and noise* at large t.



a_μ^{ll} (conn.) long-distance noise

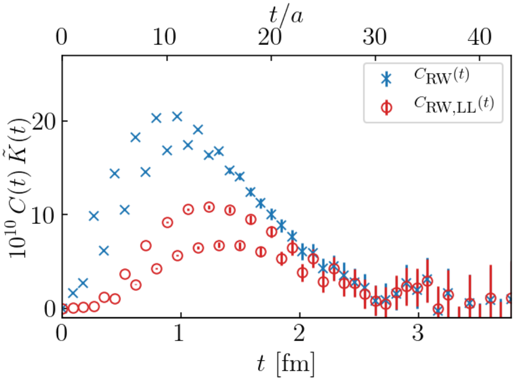
Low eigenmode contribution to RW result

$$C^{\Gamma}_{\mathrm{RW,LL}}(t) = \sum_i \mathrm{Tr}\left\{ S_i^{\mathrm{RW,L}} \Gamma \gamma^5 S_i^{\mathrm{RW,L}\dagger} \gamma^5 \Gamma \right\}, \quad S_i^{\mathrm{RW,L}}(x) = M_L^{-1}(x;y)\eta_i(y)$$

Low-mode propagator

$$M_L^{-1} = \sum_n^{N_e} \frac{1}{\lambda_n} v_n v_n^\dagger,$$

$$\lambda_n = \begin{cases} m + i\tilde{\lambda}_n & (n \bmod 2 = 0) \\ m - i\tilde{\lambda}_n & (n \bmod 2 = 1) \end{cases}$$



a_μ^{ll} (conn.) long-distance noise

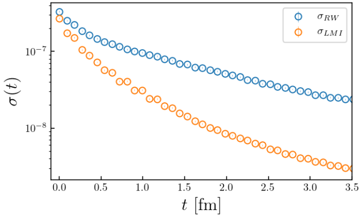
Low-mode–improved random wall estimator

$$C^{\Gamma}_{\rm LMI}(t) = C^{\Gamma}_{\rm RW}(t) - C^{\Gamma}_{\rm RW,LL}(t) + C^{\Gamma}_{\rm LL}(t)$$

Exact low-mode contribution

$$C^{\Gamma}_{\rm LL}(t) = \sum_i {\rm Tr}\{M_L^{-1}\Gamma M_L^{-1}\Gamma\}$$

- $\sim$100x error reduction at large t
- LMI data generated for 3 of our finest ensembles
- $a \approx 0.04$ fm LMI data currently in production

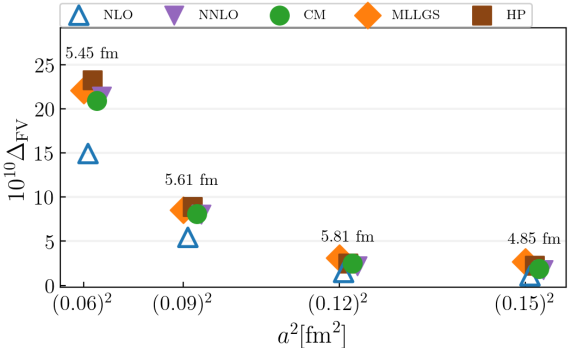


EFT Schemes

$$a_\mu^{ll}(\text{conn.})(a,\{m_f\}) = a_\mu^{ll}(\text{conn.})\Big(1 + F^{\text{lat}}(a) + F^{m}(\{\delta m_f\})\Big)$$

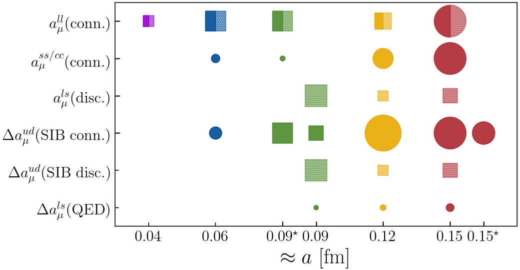
$$F^{\text{lat}}(a) = C_{a^2,n}\left[(a\Lambda)^2\alpha_s^n\right] + C_{a^4}(a\Lambda)^4 + C_{a^6}(a\Lambda)^6, \quad F^{m}(\{\delta m_f\}) = C_{\text{sea}}\sum_{f=l,l,s}\delta m_f/\Lambda$$

- (N)NLO Chiral Perturbation Theory
  - Theory of pions + LECs
- Chiral Model (CM)
  - χPT + dynamical rho meson
- MLLGS
  - Inf. Vol. scattering amplitude $\Leftrightarrow$ Fin. Vol. energies and overlap amplitudes
- HP
  - Non-perturbative resummation



Lattice ensembles

$\approx a/\mathrm{fm}$	L/fm	$N_s^3 \times N_t$	$am_l^{\mathsf{sea}}/am_s^{\mathsf{sea}}/am_c^{\mathsf{sea}}$
0.15	4.85	$32^3 \times 48$	$0.002426/0.0673/0.84$
0.12	5.83	$48^3 \times 64$	$0.001907/0.05252/0.6$
0.09	5.62	$64^3 \times 96$	$0.001326/0.03636/0.4$
0.06	5.46	$96^3 \times 192$	$0.0008/0.022/0.260$



Constraining $C(t)$ at large t

- Bounding Method: Average over points where bounds meet
  - $E_{\pi\pi}$: Lowest energy spin-1 2π state in free field
  - E_{eff}: Single exp. fit to data

Fit Method

$$C_{\text{fit}}(t) = \sum_{n}^{N_{\text{states}}} \left[Z_n^2 e^{-E_n t} + (-1)^t Z_{n,\text{osc}}^2 e^{-E_{n,\text{osc}} t} \right]$$

Bounding Method

$$C(t_{\text{min}}) \frac{\phi_{E_{\text{eff}}}(t)}{\phi_{E_{\text{eff}}}(t_{\text{min}})} \leq C(t) \leq C(t_{\text{min}}) \frac{\phi_{E_{\pi\pi}}(t)}{\phi_{E_{\pi\pi}}(t_{\text{min}})}$$

$$\phi_E(t) = e^{-Et}$$