A high-precision continuum limit study of the HVP short-distance window

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Introduction

- Theoretical uncertainty of a_μ is dominated by HVP contribution
- Euclidean window approach (RBC/UKQCD, 2018) allows separation of LO HVP into three parts:
  - **short-distance (SD)**: severe discretization effects and relies on perturbative input
  - window (W): tractable statistical and systematic errors
  - long-distance (LD): statistical noise and finite volume effects
- **Goal**: scrutinize continuum limit of SD contribution from first principles Lattice QCD without perturbative input
- **Ansatz**:
  - perform precise quenched continuum limit using many (18) cheap ensembles ($a^{-1} = 1.6\,\mathrm{GeV} - 6.1\,\mathrm{GeV}$)
  - use quenched results as input for dynamical continuum limit using only a few (4) expensive ensembles ($a^{-1} = 1.7\,\mathrm{GeV} - 3.5\,\mathrm{GeV}$)

Dynamical dataset

- Consists of 4 existing ensembles which were produced in part for arXiv:2301.08696 (RBC/UKQCD, 2023)
- Iwasaki gauge action
- Möbius domain-wall fermion $N_f = 2 + 1$ sea quarks and valence quarks
- Lattice spacings range from $a^{-1} = 1.7\,\text{GeV}$ to $a^{-1} = 3.5\,\text{GeV}$
- Pion masses roughly 150 MeV above physical point

ID	a^{-1}/GeV	$L^3 \times T \times L_s/a^4$	BC	M_π/MeV	M_K/MeV
4	1.7312(28)	$24^3 \times 48 \times 24$	periodic	274.8(2.5)	530.1(3.1)
9	2.3549(49)	$32^3 \times 64 \times 12$	periodic	278.79(62)	530.98(75)
F	2.6920(67)	$48^3 \times 96 \times 12$	periodic	283.2(1.0)	519.3(1.4)
E	3.53(01)	$48^3 \times 192 \times 12$	open	289.5(2.1)	540.0(2.5)

Table 1: Dynamical ensembles

Quenched dataset

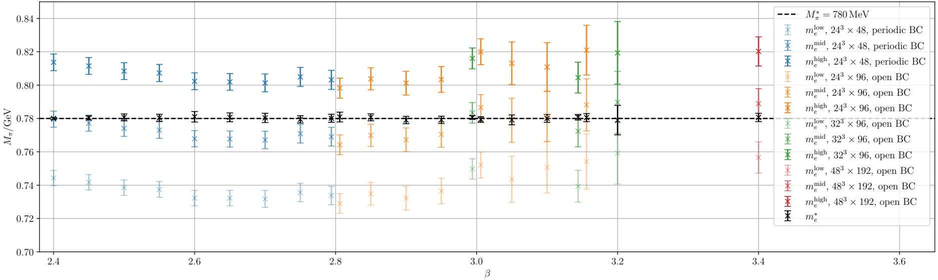
- Consists of 21 ensembles → details in arXiv:24XX.XXXXX
- Iwasaki gauge action and Möbius domain-wall fermion valence quarks
- Lattice spacings range from $a^{-1} = 1.6\,\text{GeV}$ to $a^{-1} = 6.1\,\text{GeV}$ (determined via $\sqrt{t_0}$)
- Values indicated by an asterisk $(^*)$ are expected to have larger errors than specified
  → No problem for a_μ^{SD} which exhibits significantly smaller autocorrelations
- For duplicate inverse couplings β, we used:
  - open BC ensemble for $\beta = 2.8$
  - $32^3 \times 92$ ensembles for $\beta = 3.0, 3.15$

β	a^{-1}/GeV	$L^3 \times T/a^4$	BC
2.40	1.5677(95)	$24^3 \times 48$	periodic
2.45	1.691(11)	$24^3 \times 48$	periodic
2.50	1.824(12)	$24^3 \times 48$	periodic
2.55	1.966(13)	$24^3 \times 48$	periodic
2.60	2.109(13)	$24^3 \times 48$	periodic
2.65	2.265(14)	$24^3 \times 48$	periodic
2.70	2.425(16)	$24^3 \times 48$	periodic
2.75	2.601(17)	$24^3 \times 48$	periodic
2.80	2.781(18)	$24^3 \times 48$	periodic
2.80	2.780(18)	$24^3 \times 96$	open
2.85	2.969(20)	$24^3 \times 96$	open
2.90	3.184(22)	$24^3 \times 96$	open
2.95	3.394(24)	$24^3 \times 96$	open
3.00	3.650(26)	$24^3 \times 96$	open
3.05	3.901(30)	$24^3 \times 96$	open
3.10	4.142(32)	$24^3 \times 96$	open
3.15	4.462(43)*	$24^3 \times 96$	open
3.00	3.635(24)	$32^3 \times 96$	open
3.15	4.433(31)	$32^3 \times 96$	open
3.20	4.737(39)*	$32^3 \times 96$	open
3.40	6.099(43)*	$96^3 \times 192$	open

Table 2: Quenched ensembles

Mass interpolation for quenched ensembles

- Compute pion correlators for three different quark masses and determine corresponding pion masses
- Select single physical pion mass $M_\pi^* = 780$ MeV to lie within the ranges defined by the three quark masses for all of the 21 ensembles
- Quadratic interpolation for each quenched ensemble e to find quark masses m_e^* corresponding to selected physical point
- Interpolate vector correlators to same physical point



Vector currents

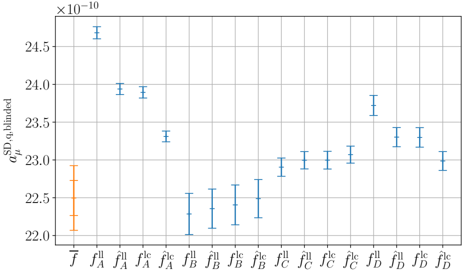
- We study the vector correlators

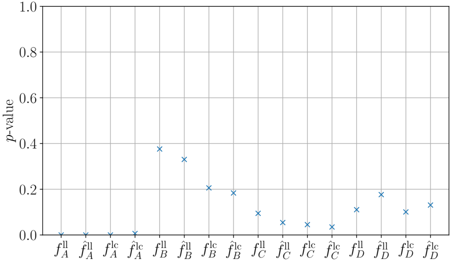
$$C^{ab}(t) = \frac{1}{3}\sum_{\vec{x}}\sum_{j=0,1,2}\langle J_j^b(\vec{x},t)J_j^a(0)\rangle$$

  in their local-local (C^{ll}) and local-conserved (C^{lc}) versions using local and conserved vector currents: J_μ^l and J_μ^c
- Consider up and down quark-connected contribution of $C(t)$ in isospin symmetric limit
- Renormalization factors Z_V are determined via plateau fit to the ratio $C^{\mathrm{lc}}/C^{\mathrm{ll}}$
- Employed blinding procedure before the final datasets were generated
- Separate analysis by both authors with common blinding factor applied to the correlators

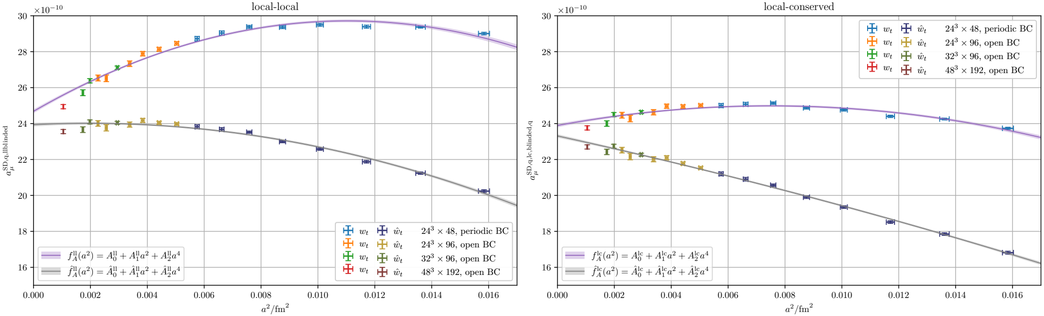
Continuum limit of $a_\mu^{\text{SD,q}}$ - quenched approximation

- Perform blinded quenched continuum fits of $a_\mu^{\text{SD,q,i}}$ with $i \in \{\text{ll}, \text{lc}\}$ and weights $w_t, \hat{w}_t$ (different discretization of photon and muon subdiagram) using 18 distinct ensembles
- Fit ansaetze:
  - $f_A^i(a^2) = A_0^i + A_1^i a^2 + A_2^i a^4$
  - $f_B^i(a^2) = B_0^i + B_1^i a^2 + B_2^i a^4 + B_3^i a^2 \log a^2$
  - $f_C^i(a^2) = C_0^i + C_1^i a^2 + C_2^i a^2 \log a^2$
  - $f_D^i(a^2) = D_0^i + D_1^i a^2 + D_2^i a^4 + D_3^i a^6$
- Compute model average of all 16 continuum results using AIC



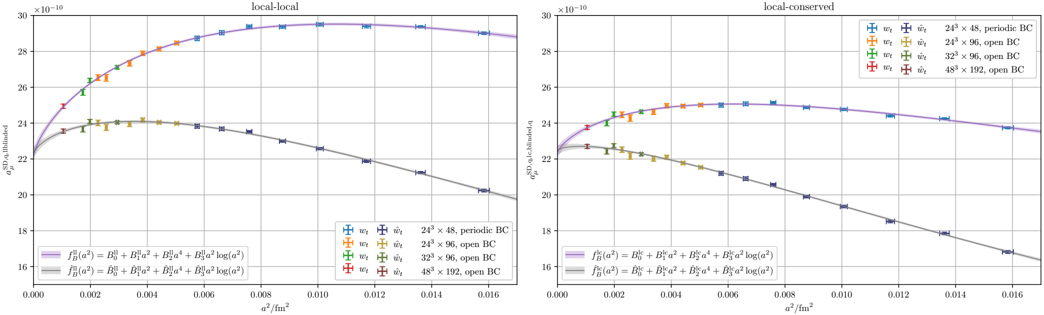


Continuum limit of $a_\mu^{\mathrm{SD,q}}$ - quenched approximation



Continuum fits for blinded quenched $a_\mu^{\mathrm{SD,q}}$ using fit model $f_A^i(a^2) = A_0^i + A_1^i a^2 + A_2^i a^4$.

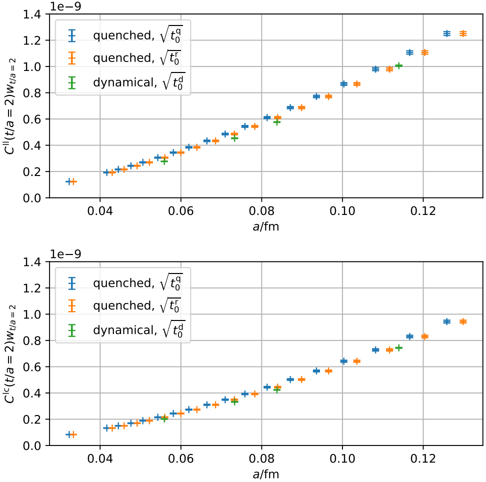
Continuum limit of $a_\mu^{\mathrm{SD,q}}$ - quenched approximation



Continuum fits for blinded quenched $a_\mu^{\mathrm{SD,q}}$ using fit model $f_B^i(a^2) = B_0^i + B_1^i a^2 + B_2^i a^4 + B_3^i a^2 \log a^2$.

$\rightarrow a^2 \log\left(a^2\right)$-term necessary to obtain consistent continuum results

Continuum limit of a_μ^{SD} - matching of the scales



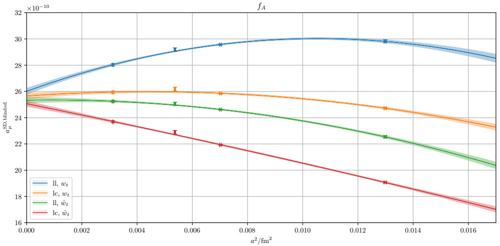
- Optimize matching of quenched and dynamical scales by matching $t=2$ of vector current
  $\rightarrow$ Leads also to effective matching for other short-distance times
- Rescale quenched scale $\sqrt{t_0^{\mathrm{q}}}$ to match dynamical scale $\sqrt{t_0^{\mathrm{d}}} \longrightarrow \sqrt{t_0^{\mathrm{r}}}$

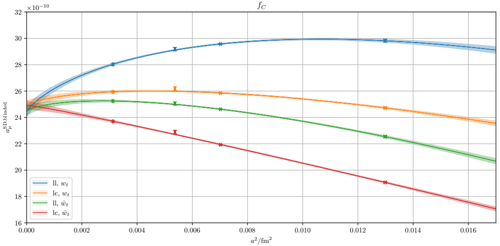
$\rightarrow$ Summands $C(t)w_t$ which mainly contribute to a_μ^{SD} $(t=2,3)$ appear to be matchable in terms of the scale as $a \rightarrow 0$

$\rightarrow$ Model parameters for dynamical and rescaled quenched scale may be similar

- Difference of discretization errors is suppressed (two-loop pQCD effect)

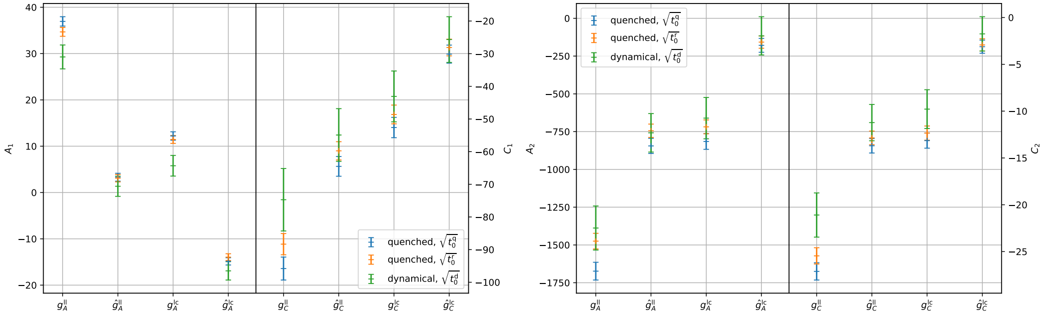
Continuum limit of a_μ^{SD} - continuum fits





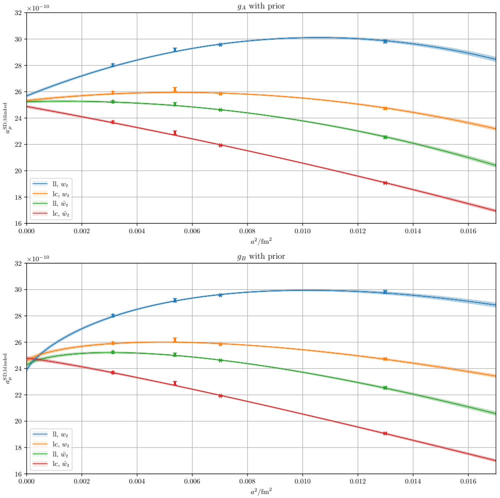
- 4 ensembles $\rightarrow$ can only fit models with 3 parameters: f_A and f_C
- Both models yield reasonable p-values between 0.1 and 0.6, but the variance of parameters is in few-percent to 10/100 ballpark
- **Idea:** constrain all but the offset parameter of dynamical fits by quenched parameters with prior $\sum_i \frac{(p_i^{\mathrm{d}} - p_i^{\mathrm{q}})^2}{\mathrm{var}(p_i^{\mathrm{q}})^2}$
- Product ansatz gave best results:
  - $g_A^i(a^2) = A_0^i(1 + A_1^i a^2 + A_2^i a^4)$
  - $g_C^i(a^2) = C_0^i(1 + C_1^i a^2 + C_2^i a^2 \log a^2)$

Continuum limit of a_μ^{SD} - parameter comparison



Observation: quenched fit parameters for scale $\sqrt{t_0^r}$ for almost all cases in good agreement with dynamical fit parameters

Continuum limit of a_μ^{SD} - constrained continuum fits

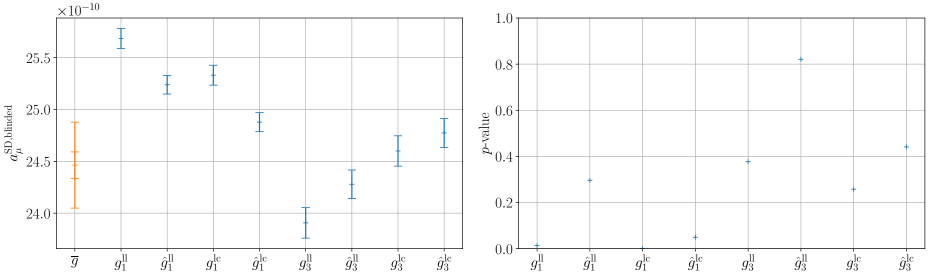


- Error on constrained continuum results is reduced roughly to the same order of magnitude as for the quenched case
- Spread of fits used in model average for error estimate
- Can also consider fits for models with more parameters:
  - $g_B^i(a^2) = B_0^i(1 + B_1^i a^2 + B_2^i a^4 + B_3^i a^2 \log a^2)$
  - $g_D^i(a^2) = D_0^i(1 + D_1^i a^2 + D_2^i a^4 + D_3^i a^6)$

$\rightarrow$ **However:** vanishing p-values

$\rightarrow$ Details in arXiv:24XX.XXXXX

Continuum limit of a_μ^{SD} - dynamical result

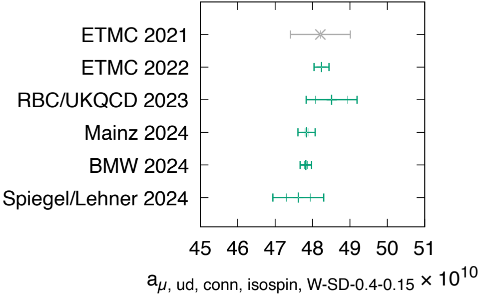


- Fits yield p-values ≥ 0.05 for all but two fits
- Compute model average of all 8 continuum results using AIC
- Correction from the $\approx 280\,\mathrm{MeV}$ pion mass of the dynamical ensembles has to be added (RBC/UKQCD, 2023): $\Delta a_\mu^{\mathrm{SD,mass}} = 0.41(4) \times 10^{-10}$
- Unblinded result:

$$a_\mu^{\mathrm{ud,\ iso,\ SD}} = 47.62(0.32)_{\mathrm{stat.}}(0.60)_{\mathrm{syst.}} \times 10^{-10}$$

Conclusion and outlook

- Consistency with previous results for the up and down quark-connected contribution to a_μ^{SD}
- Error of the result is dominated by systematic error
- Continuing efforts:
  - Increase statistics
  - Check results against perturbative result



Backup slides

Introduction

- Anomalous magnetic moment of the muon: $a_\mu = \frac{g_\mu - 2}{2}$
  - $\rightarrow$ probes interactions with all known and unknown particles
  - $\rightarrow$ one of the most precisely measured quantities in physics
- Latest experimental result provided by FNAL Muon g-2 Collaboration has a precision of 0.19 ppm
- Standard model theory result is work in progress
- First-principles lattice QCD result at per-mille level precision desirable

Methodology - Time-momentum representation

- Leading-order HVP contribution to a_μ may be computed as (Meyer-Bernecker, 2011)

$$a_\mu^{\text{HVP LO}} = \sum_{t=0}^{\infty} w_t C(t),$$

where $C(t) = \frac{1}{3}\sum_{\vec{x}}\sum_{j=0,1,2}\langle J_j(\vec{x},t)J_j(0)\rangle$ with $J_\mu(x) = i\sum_f Q_f\bar{\psi}_f(x)\gamma_\mu\psi_f(x)$

- Consider up and down quark-connected contribution of $C(t)$
- Weights w_t capture photon and muon parts of the HVP diagrams
- Use two different weight definitions to investigate the continuum limit:

$$w_t = 8\alpha^2\int_0^\infty \mathrm{d}Q^2\left(\frac{\cos(Qt)-1}{Q^2}+\frac{1}{2}t^2\right)f(Q) \quad\text{and}\quad \hat{w}_t = 8\alpha^2\int_0^\infty \mathrm{d}Q^2\left(\frac{\cos(Qt)-1}{(2\sin(Q/2))^2}+\frac{1}{2}t^2\right)f(Q),$$

where

$$f(Q) = \frac{m_\mu^2 Q^2 Z^3(Q)\left(1-Q^2 Z(Q)\right)}{1+m_\mu^2 Q^2 Z^2(Q)} \quad\text{with}\quad Z(Q) = \frac{\sqrt{Q^4+4Q^2 m_\mu^2}-Q^2}{2m_\mu^2 Q^2}$$

Methodology - Euclidean windows

- Utilize Euclidean window approach (RBC/UKQCD, 2018) to separate the contributions from different time slices into well-defined individual contributions:

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

- Short-distance contribution is given by

$$a_\mu^{\mathrm{SD}}(t_0, \Delta) = \sum_{t=0}^{\infty} C(t) w_t [1 - \Theta(t, t_0, \Delta)]$$

  with smearing kernel $\Theta(t, t', \Delta) = [1 + \tanh[(t - t')/\Delta]]/2$

- For our analysis, we use the values $t_0 = 0.4$ fm and $\Delta = 0.15$ fm

Scale setting

- Compute the gradient flow scales t_0 and w_0 defined by:

$$t^2 \langle E(t) \rangle \big|_{t=t_0} = 0.3 \quad \text{and} \quad t\frac{d}{dt}\left(t^2 \langle E(t) \rangle\right) \big|_{t=w_0} = 0.3,$$

where $E(t)$ denotes the Wilson flow smeared energy density at flow time t

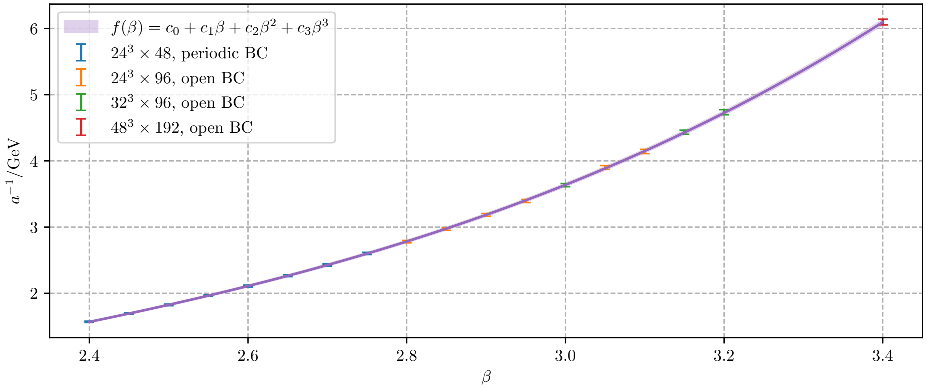
- Lattice spacing determined using scale t_0 with physical input (arXiv:1311.5585 , M.Bruno and R.Sommer):

$$\sqrt{t_0} = 0.1638(10)\,\mathrm{fm}$$

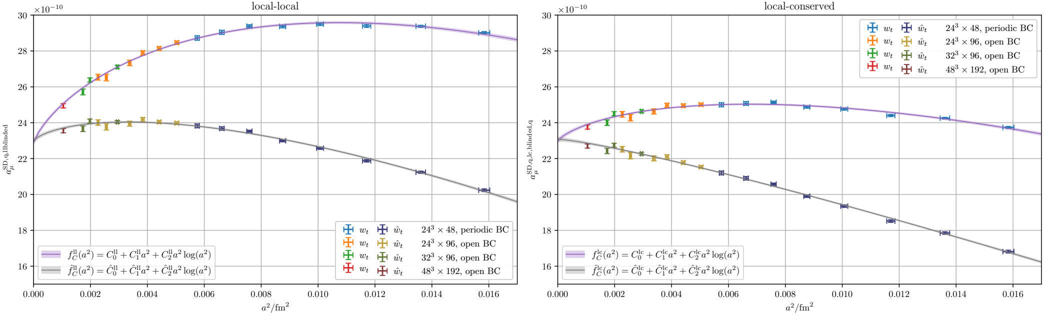
- This choice defines how dimensionful quantities are translated to dimensionless units in the quenched world
- **However:** our final results are corrected to include dynamical sea quarks and are independent of this choice

Scale setting

- Interpolation of a^{-1}/GeV derived from $\sqrt{t_0}$ versus β using cubic fit model $f(\beta) = c_0 + c_1\beta + c_2\beta^2 + c_3\beta^3$ has p-value 0.38:

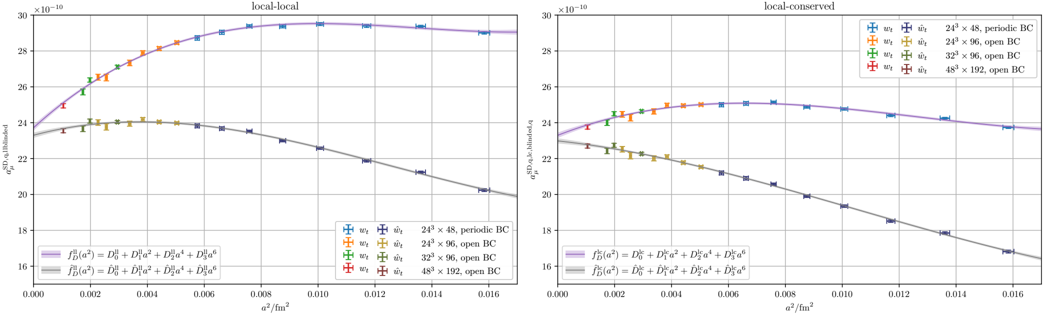


Continuum limit of $a_\mu^{\mathrm{SD,q}}$ - quenched approximation



Continuum fits for blinded quenched $a_\mu^{\mathrm{SD,q}}$ using fit model $f_C^i(a^2) = C_0^i + C_1^i a^2 + C_2^i a^2 \log a^2$.

Continuum limit of $a_\mu^{\mathrm{SD,q}}$ - quenched approximation



Continuum fits for blinded quenched $a_\mu^{\mathrm{SD,q}}$ using fit model $f_D^i(a^2) = D_0^i + D_1^i a^2 + D_2^i a^4 + D_3^i a^6$.