Short- and intermediate-distance HVP contributions to the muon g-2

Shaun Lahert

on behalf of the Fermilab Lattice, HPQCD & MILC collaboration

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Computing Resources

- ACCESS
- ALCC
- Dirac
- ERCAP
- INCITE
- Indiana U
- LRAC
- USQCD
- XSEDE

Lattice HVP details

- Lattice a_μ^{HVP} calculations are typically performed in the (Euclidean)time-momentum rep.[1]

$$a_\mu^{\mathrm{HVP,LO}} = 4\alpha^2 \int_0^\infty dt \tilde{K}(t) C(t), \quad C(t) = \frac{1}{3}\sum_i^3 \int d^3x \, \langle J_i(x) J_i(0) \rangle \ \text{(F.T. of HVP)}$$

$$J_i(x) = \sum_f Q_{q_f} \bar{q}_f(x) \gamma_i q_f(x), \quad Q_u = +\frac{2}{3},\ Q_d = -\frac{1}{3},\ Q_s = -\frac{1}{3}, \ldots$$

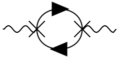
$C(t)$ from the lattice.

[1] D. Bernecker and H. B. Meyer, Eur. Phys. J. A, 47, 148 (2011).

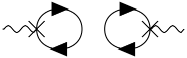
HVP Contributions[3]

- Calculation broken down by flavor, Wick contraction & isospin symmetric/breaking

$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{ll}(\mathrm{conn.}) + a_\mu^{ss}(\mathrm{conn.}) + a_\mu^{cc}(\mathrm{conn.}) + \ldots$$

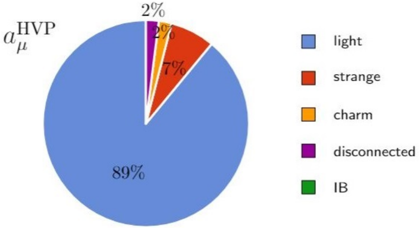
$\sum_f$  q_f

$$+\ a_\mu(\mathrm{disc.})$$

$\sum_{f,f'}$  q_f $q_{f'}$

$$+\,\Delta a_\mu^{ud}(\mathrm{SIB}) + \Delta a_\mu(\mathrm{QED})$$

- Integrand broken into Euclidean windows[2]:
  SD + W + LD = Full

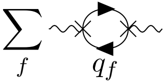


[2] T. Blum et al., Phys. Rev. Lett. 121.2, 022003 (2018).
[3] C. Davies et al., Phys. Rev. D, 101.3, 034512 (2020).

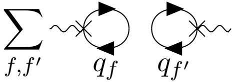
HVP Contributions[5]

- Calculation broken down by flavor, Wick contraction & isospin symmetric/breaking

$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{ll}(\mathrm{conn.}) + a_\mu^{ss}(\mathrm{conn.}) + a_\mu^{cc}(\mathrm{conn.}) + \ldots$$

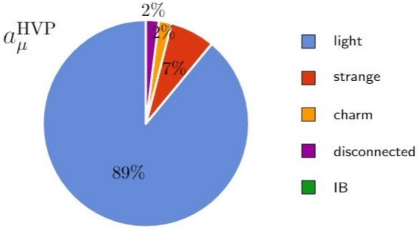


$$+ \; a_\mu(\mathrm{disc.})$$



$$+ \Delta a_\mu^{ud}(\mathrm{SIB}) + \Delta a_\mu(\mathrm{QED})$$

- Integrand broken into Euclidean windows[4]:
  SD + W + LD = Full



[4] T. Blum et al., Phys. Rev. Lett. 121.2, 022003 (2018).
[5] C. Davies et al., Phys. Rev. D, 101.3, 034512 (2020).

Calculation on 2+1+1 HISQ physical-mass ensembles

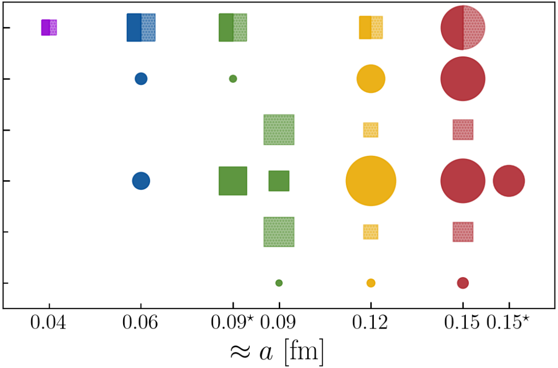
This talk (SD+W), Next: Michael Lynch (LD) → a_μ^{ll}(conn.)

This talk (SD+W) → $a_\mu^{ss/cc}$(conn.)

David Clarke (Thursday 9:00) → a_μ^{ls}(disc.)

Jake Sitison (Thursday 9:20) → Δa_μ^{ud}(SIB conn.), Δa_μ^{ud}(SIB disc.)

Craig McNeile (Poster) → Δa_μ^{ls}(QED)



Scale setting: Alexei Bazavov (Friday 15:15)

- Solid color (local current) hatched (one-link)
- Squares: low-mode improved.
- Size ∼ statistics

HVP Calculation Overview

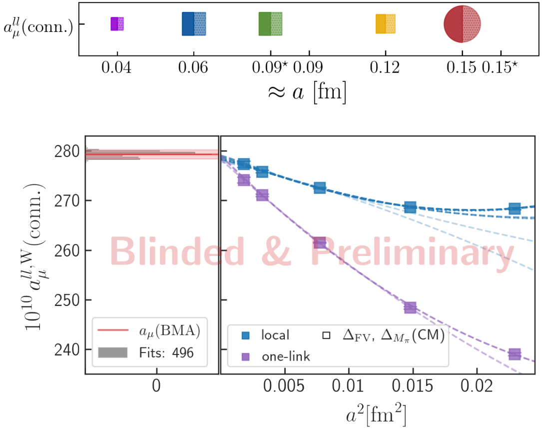
- HISQ ensembles:
  - 5 spacings: 0.151 fm - 0.042 fm
  - All within $\approx 0.5\%$ of the physical masses[6].
  - $M_\pi L = 3.7 - 4.1$[7].
- Analyses are all independently blinded.
  - Multiplicative blinds for all observables, additional additive blinds for ratios.
- Systematic uncertainties from Bayesian Model Averaging.
  - Correction scheme variation (light quarks): χPT, MLLGS, CM, HP
  - Continuum fit variations.
- Global bootstrap(+BMA) for correlations, stat. & parameters, between obs.

[6] Except 0.09

[7] Except 0.15 fm: $M_\pi L = 3.4$

Intermediate-distance window

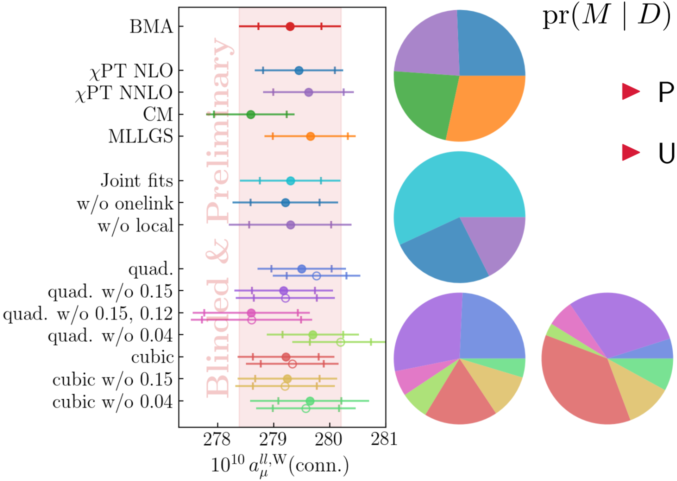
Light-quark connected



Previous calculation: 2301.08274

- New:
  - Exact low-modes everywhere except 0.15 fm
  - Second current: 'local' + 'one-link'.
  - Retuned 0.088 fm ensemble (MILC+CalLat).
  - 0.042 fm physical mass ensemble.
- Fits to both currents simultaneously (correlated).
- BMA for analysis systematics, ≈ 500 models.

Light-quark connected BMA breakdown

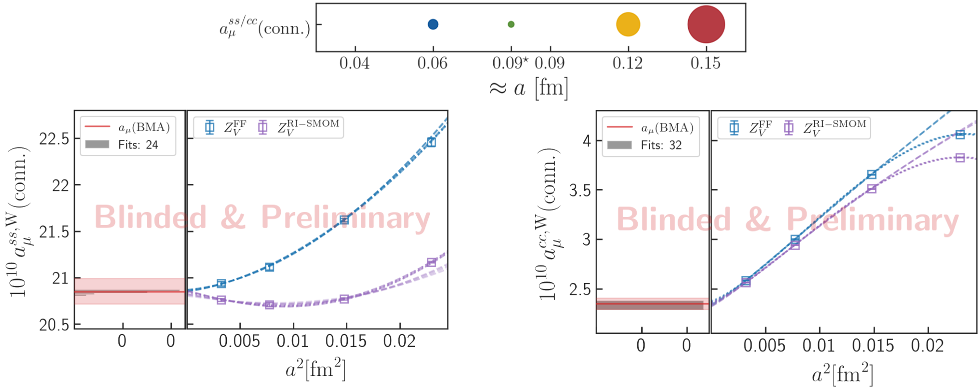


Filled & unfilled pairs: local & one-link variations

$$\mathrm{pr}(M \mid D) \equiv \exp\left[-\tfrac{1}{2}\left(\chi^2_{\mathrm{data}}\left(\mathbf{a}^\star\right) + 2k + 2N_{\mathrm{cut}}\right)\right]$$

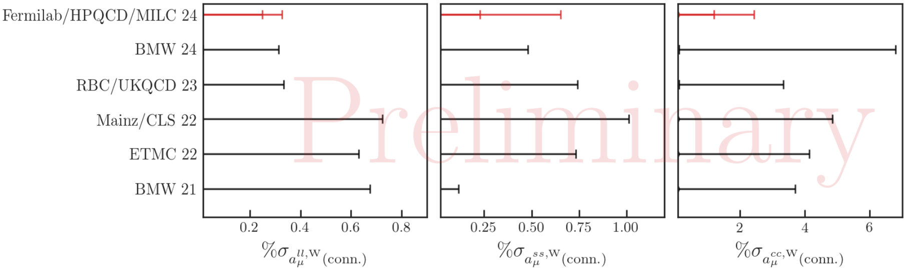
- Pie charts: relative probabilities in BMA.
- Uncertainty driven by:
  - Stat: scale setting (w_0 fm)
  - Sys: Model tension

Strange & Charm



- Local current: two different renormalization schemes (separate fits).
- Large charm discretization effects: a^6 fits required.
- Uncertainty dominated by scale-setting for both.

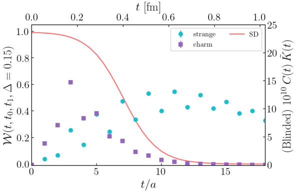
W uncertainty summary



- Scale setting (w_0 fm) is significant uncertainty in all contributions (Inner error bar: no abs. scale setting uncertainty.).

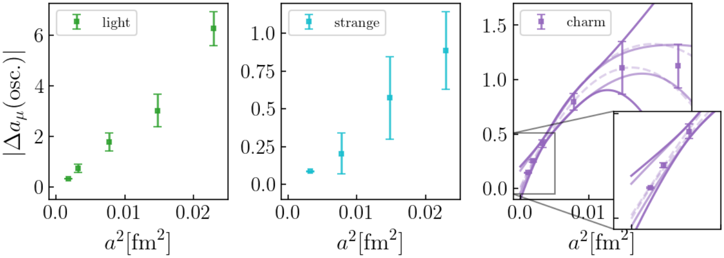
Short-distance window

SD analysis overview



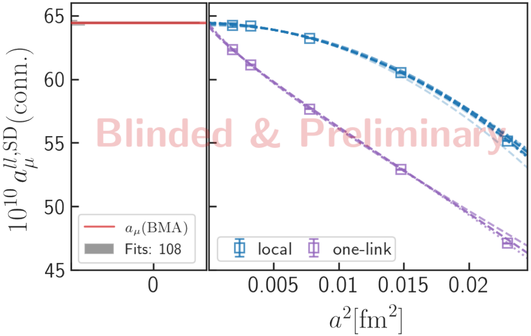
- **Staggered oscillations**
  Verify no impact on continuum.
- **Log-enhancement**
  Account for this in fit function.
  HISQ local current 'protected' (leading $a^2\alpha_s$), one-link is not (leading a^2).
- **pQCD cross-checks**
  $\sim$RBC/UKQCD strategy (complementary windows, pQCD+latt.) for all flavors.

Oscillations



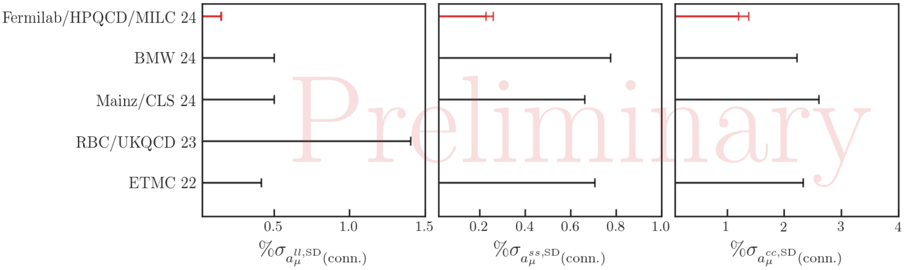
- Fit correlators over SD: $t_{\mathrm{min}}/a = 2$, $t_{\mathrm{max}} = 0.7$ fm.
- Construct: $C_{\text{no osc.}}(t) \Rightarrow \Delta a_\mu(\mathrm{osc.}) \equiv \int dt\, K(t)\mathcal{W}_{\mathrm{SD}}(t)\left[C(t) - C_{\text{no osc.}}(t)\right]$
- Charm data @ 0.04 fm, 0.03 fm unphysical light mass. HPQCD:2005.01845
- Verified: oscillating contribution falls off faster than $a_\mu(a)$

Light-quark connected

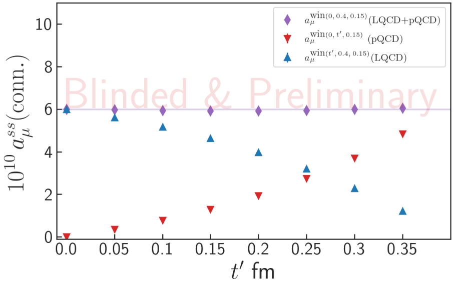


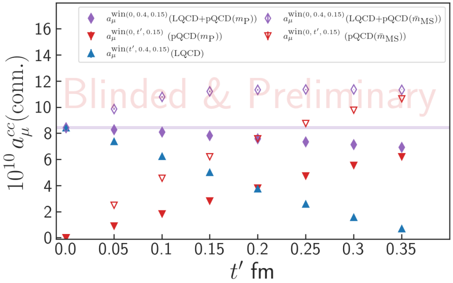
- Same setup as W (no EFT corrections)
- Leading $a^2 \log(a)$ noticeable in one-link current.
- Can't discern between leading a^2 vs $a^2\alpha_s$ in local current.

SD uncertainty summary



- Competitive uncertainties for all flavors.
- HISQ local-current mitigating log-enhancement.
- Light: reduced scale setting and EFT model dependence in SD shows strength of new dataset.





- $a_\mu^{\mathsf{SD}} = a_\mu^{\mathrm{pQCD}}(0,t',\Delta) + a_\mu^{\mathsf{latt.}}(t',0.4,\Delta)$[8]
- Using rhad (4-loop R-ratio).
- Good agreement for massless strange result from pQCD.
- Two options for massive charm: fixed m_{pole} or running $\bar{m}_{\mathrm{ms}}(\mu=\sqrt{s})$.

[8] T. Blum et al., Phys. Rev. D, 108.5, 054507 (2023).

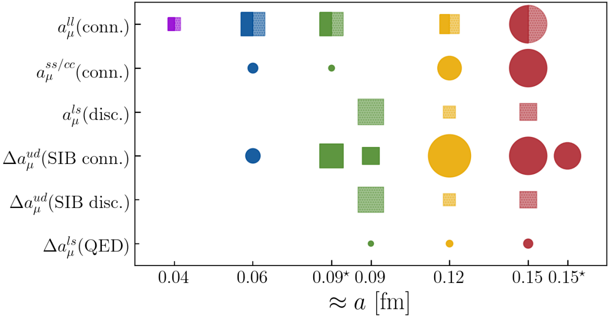
Outlook

- Plan: unblind SD & W before g-2 Theory Initiative meeting
  - Including disconnected, SIB and QED results (see other talks/posters)
  - Complete HVP windows from global bootstrap + BMA approach.
- Leading source of uncertainty in W: scale setting (w_0 fm).
  - Talk by Alexei Bazavov on Friday
  - Will update SD, W results.
  - Follow up with LD/Full. (next talk).

Thank you

Backup Slides

Datasets



Light-quark connected

$\approx a/\mathrm{fm}$	Local		One-link		
	N_{conf}	N_{src}	N_{conf}	N_{src}	$N_{\mathrm{eig.}}$
0.15	10019	$48^\dagger$	9759	$16^\dagger$	-
0.12	1060	64	9885	$16^\dagger$	2000
$0.09^\star$	993	96	993	96	2000
0.06	1009	96	724	96	2000
0.04	313	144	256	144	2000

Strange- and charm-quark connected

$\approx a/\mathrm{fm}$	N_{conf}	N_{src}
0.15	10019	48
0.12	2985	64
$0.09^\star$	252	48
0.06	1424	24

Continuum Fits

$$a_\mu^{qq}(a,\{M_A\}) = a_\mu^{qq}\left(1 + F^{\text{disc.}}(a) + F^M(\{M_A\})\right), \tag{1}$$

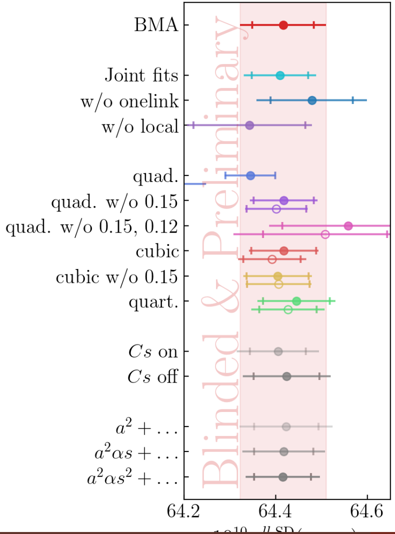
where

$$F^{\text{disc.}}_{\text{local}}(a) = \left\{C_{a^2}(a\Lambda)^2,\ C_{a^2,n}\left[(a\Lambda)^2\alpha_s^n\right]\right\} + \sum_{k=2}^{4} C_{a^{2k}}(a\Lambda)^{2k} \tag{2}$$

$$F^{\text{disc.}}_{\text{one-link}}(a) = \left\{C_{a^2}(a\Lambda)^2\log(a\Lambda),\ C_{a^2}(a\Lambda)^2\right\} + \sum_{k=2}^{4} C_{a^{2k}}(a\Lambda)^{2k} \tag{3}$$

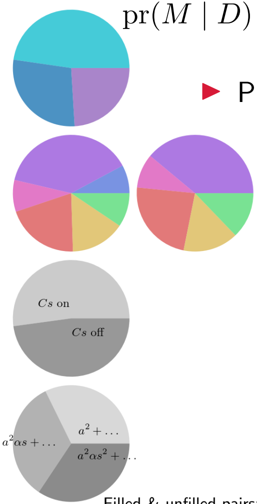
$$F^M(\{M_A\}) = C_{\text{sea}} \sum_{A=\pi,K,D_s} \delta M_A^2\,, \qquad \delta M_A^2 = \frac{M^2_{A,\,\text{phys.}} - M^2_{A,\,\text{latt.}}}{M^2_{A,\,\text{phys.}}}. \tag{4}$$

SD: Light-quark connected BMA breakdown



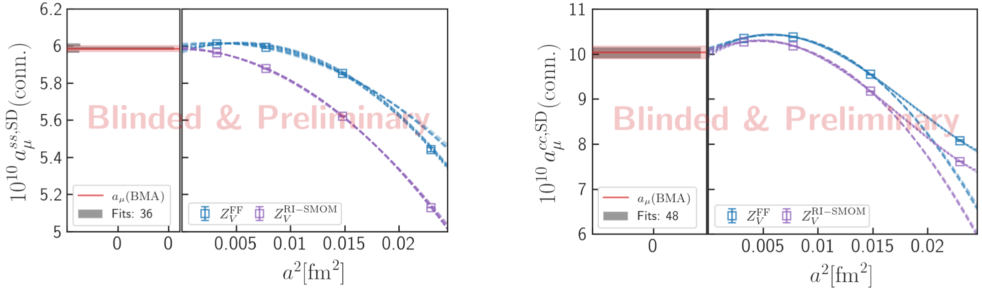
$$\mathrm{pr}(M \mid D) \equiv \exp\left[-\frac{1}{2}\left(\chi^2_{\mathrm{data}}\left(\mathbf{a}^\star\right) + 2k + 2N_{\mathrm{cut}}\right)\right]$$

- Pie charts: relative probabilities in BMA.



Filled & unfilled pairs: local & one-link variations

SD: Strange & Charm



- Local current: two different renorm. schemes (separate fits).
- Again, any log-enhancement is seemingly suppressed by HISQ.
- Dominant uncertainties:
  - Strange: Renormalization & continuum extrap
  - Charm: Scale setting