Hadronic vacuum polarization contribution to the muon $g-2$ at short and long distances

Simon Kuberski for the Mainz lattice group

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Lattice 2024 Liverpool

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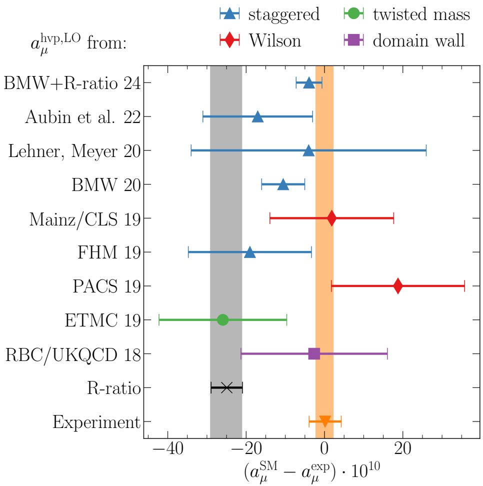
$a_\mu^{\rm hvp}$ FROM LATTICE QCD



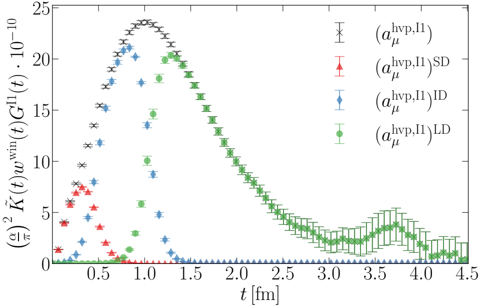
[BNL $g-2$, hep-ex/0602035]
[FNAL $g-2$, 2104.03281, 2308.06230]

Community goal

Several ab initio results at $< 0.5\%$ precision.

- Use windows in the time-momentum representation to compute [Blum et al., 1801.07224]

$$a_\mu^{\rm hvp} = (a_\mu^{\rm hvp})^{\rm SD} + (a_\mu^{\rm hvp})^{\rm ID} + (a_\mu^{\rm hvp})^{\rm LD}$$



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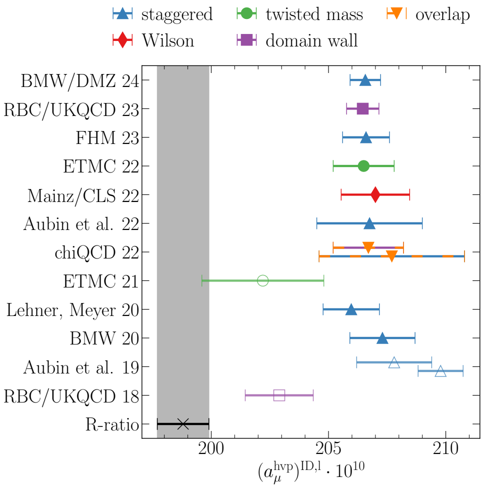

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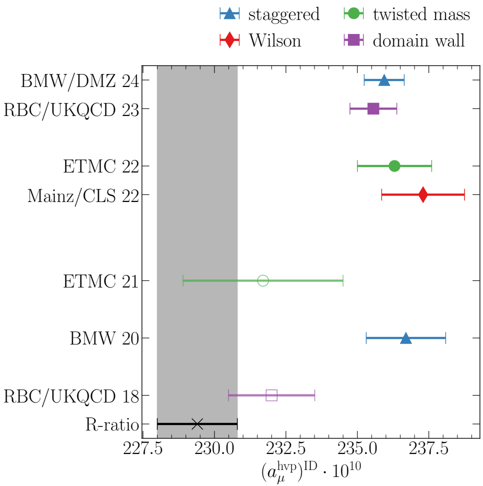

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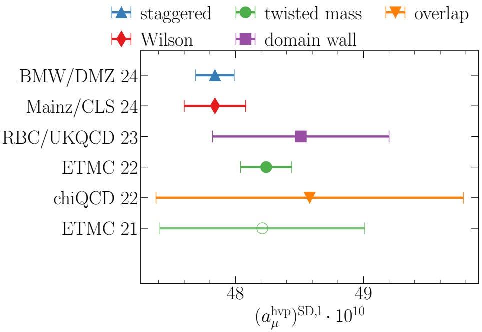

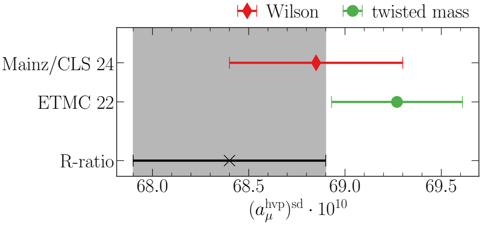

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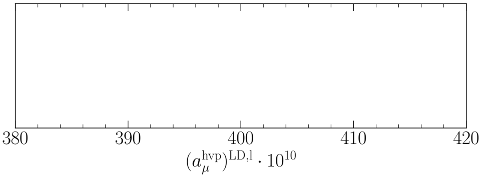
a_μ^{hvp} FROM LATTICE QCD



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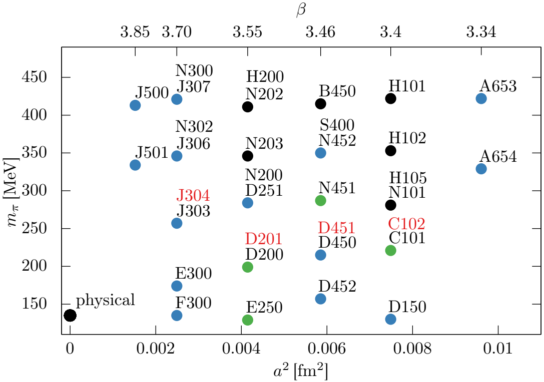
  - Long distance (. . . , this talk): noise and finite-volume effects

The Mainz/CLS setup

a_μ^{hvp} from $2+1$ flavors of $\mathrm{O}(a)$ improved Wilson-clover fermions

$2+1$ FLAVOR CLS ENSEMBLES



- Six values of $a \in [0.039, 0.099]$ fm.
- Open boundary conditions in the temporal direction.
- $a\mathrm{Tr}[M_\mathrm{q}] = 2am_\mathrm{l} + am_\mathrm{s} = \mathrm{const.}$ and $m_\mathrm{s} \approx m_\mathrm{s}^{\mathrm{phys}}$ to stabilize the strange-quark interpolation.

● New ensemble / ● significantly improved statistics since [Gérardin et al., 1904.03120].

- Generating a third ensemble with $m_\pi \approx m_\pi^{\mathrm{phys}}$: F300 with 256×128^3 at $0.05\,\mathrm{fm}$, $\rightarrow$ increase precision and further constrain $(am_\pi)^2$ effects.

Computational setup

- $\mathrm{O}(a)$ improved correlation functions with
  - local-local (LL) and local-conserved (LC) vector currents
  - two different lines of constant physics for the improvement (set 1/ set 2)

  that all differ by $\mathrm{O}(a^2)$.

- Finite-volume correction via spacelike [Hansen and Patella, 1904.10010, 2004.03935] and timelike [Meyer, 1105.1892] [Lellouch and Lüscher, hep-lat/0003023] pion formfactor.

- Scale setting via t_0^{phys}: uncertainty propagates significantly into $(a_\mu^{\mathrm{hvp}})^{\mathrm{LD}}$.

The short distance contribution

[SK et al., 2401.11895]

$a_\mu^{\rm hvp}$ AT SHORT DISTANCES

- Cutoff effects are the main concern at short distances, especially those of $\mathrm{O}(a^2\log(a))$ [Della Morte et al., 0807.1120][Cè et al., 2106.15293] [Sommer et al., 2211.15750]:
  - removal via perturbative QCD in the spacelike regime at high energies Q^2.

Starting from the well-known formula [Bernecker and Meyer, 1107.4388]

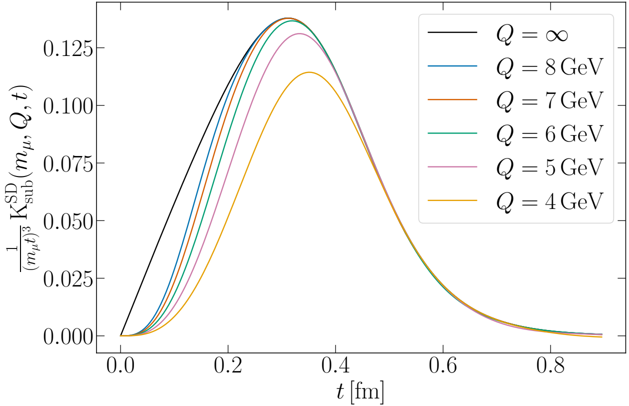
$$(a_\mu^{\rm hvp})^{\rm SD} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}t\, w^{\rm SD}(t)\widetilde{K}(t)G(t)\,,$$

with the short-distance window $w^{\rm SD}(t)$, we change to a modified QED kernel via

$$w^{\rm SD}(t)\tilde{K}(t) \to \mathrm{K}^{\rm SD}_{\rm sub}(Q,t) = w^{\rm SD}(t)\tilde{K}(t) - w^{\rm SD}(0)\frac{16\pi^2 m_\mu^2}{9Q^2} f(Q,t)$$

where $f(Q,t) = \frac{16}{Q^2}\sin^4\left(\frac{Qt}{4}\right)$ is the kernel to compute

$$\Pi(Q^2) - \Pi((Q/2)^2) = \int_0^\infty \mathrm{d}t\, f(Q,t)G(t)\,.$$



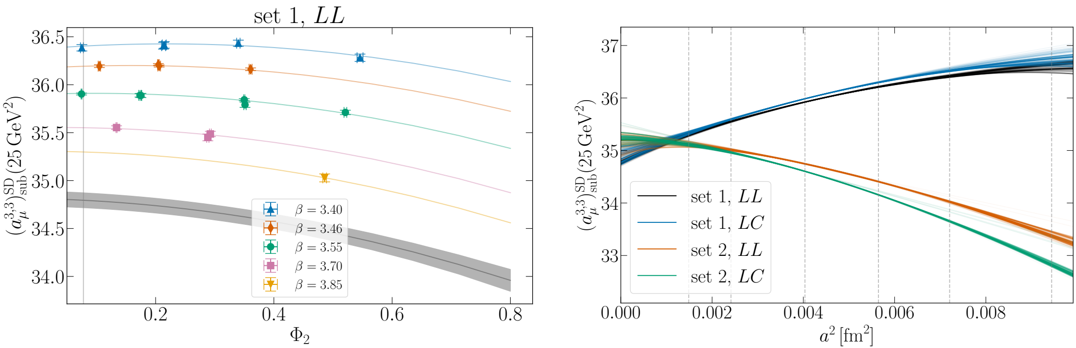
- Remove term $\propto t^4$ in kernel and thus the $a^2 \log(a)$ effects.
- New slope is Q dependent.
- We focus on $Q = 5\,\mathrm{GeV}$.
- Relevant scale for perturbation theory is $Q/2$.

- Based on the Adler function $D(Q^2)$, we evaluate [Baikov et al., 0801.1821, 1001.3606],

$$\Pi(Q^2) - \Pi((Q/2)^2) = \frac{\pi^2}{12} \int_{(Q/2)^2}^{Q^2} \frac{\mathrm{d}Q'^2}{Q'^2} D(Q'^2)$$

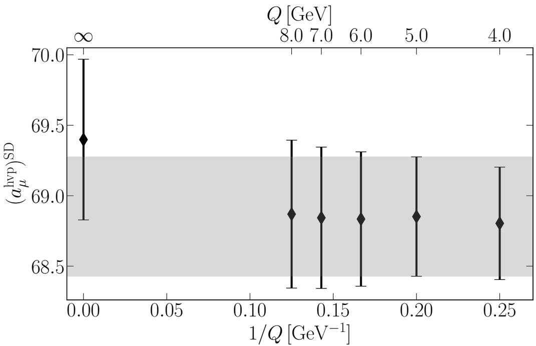
and expect good convergence of the perturbative series [Jegerlehner, 2020].

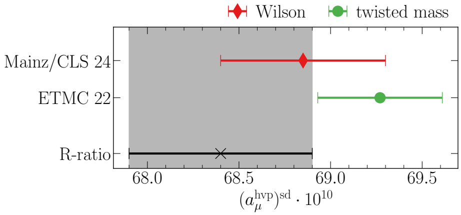
$(a_\mu^{\rm hvp})^{\rm SD}$ IN THE ISOVECTOR CHANNEL



- Tiny uncertainties, benign chiral dependence, significant cutoff effects.
- Use tree-level improvement to reduce the cutoff effects.
- Combine with strange, disconnected, charm and valence connected isospin-breaking contributions for the full $(a_\mu^{\rm hvp})^{\rm SD}$.

Full result for $(a_\mu^{\rm hvp})^{\rm SD}$





- Stability under variation of the modification scale Q.
- Small but noticeable shift when $a^2 \log(a)$ effects are not removed ($1/Q = 0$).
- Final uncertainty dominated by systematics from the continuum limit.

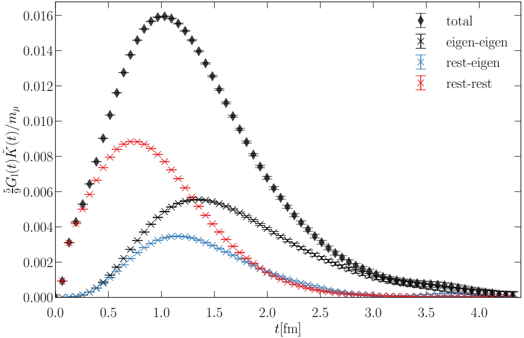
The long distance contribution

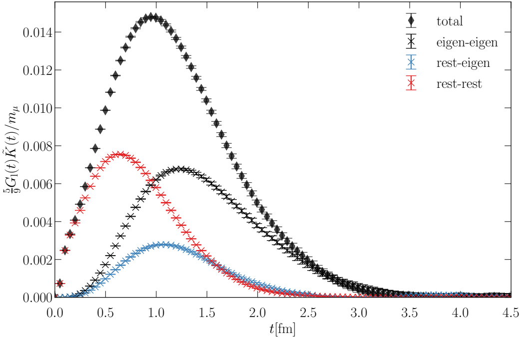
Preliminary, blinded

Our goal

Determine with $(a_\mu^{\mathrm{hvp}})^{\mathrm{LD}}$ the last building block for the full a_μ^{hvp}.

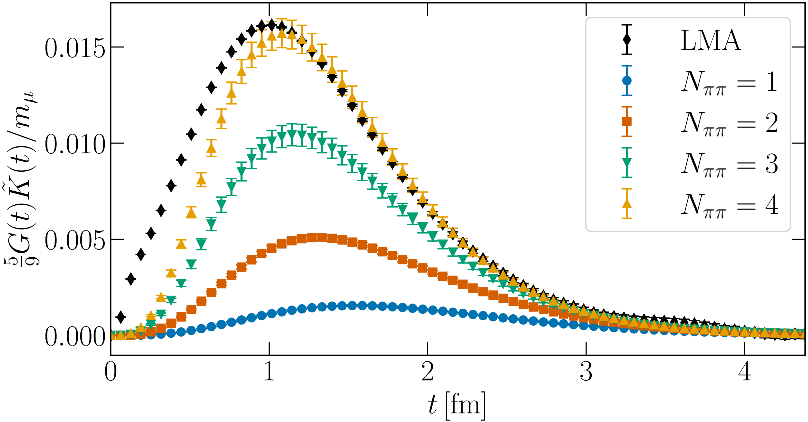
- Noise reduction techniques to get to sub-percent precision for $I = 1$:
  - **Low-mode averaging** (LMA).
  - Spectral reconstruction.
- Finite-volume effects are sizable.
- Expect cutoff effects to be less relevant for Wilson quarks.
- Everything is **blinded**: Analyze multiple TMR kernels with
  - multiplicative offsets,
  - artificial cutoff effects,
  - ...?





- Use low-mode averaging for all ensembles where $m_\pi < 280\,\mathrm{MeV}$.
  - Left: $m_\pi = 132\,\mathrm{MeV}$, $a = 0.064\,\mathrm{fm}$ (E250)
  - Right: $m_\pi = 177\,\mathrm{MeV}$, $a = 0.049\,\mathrm{fm}$ (E300)
- Autocorrelation becomes a limiting factor at fine lattice spacing.

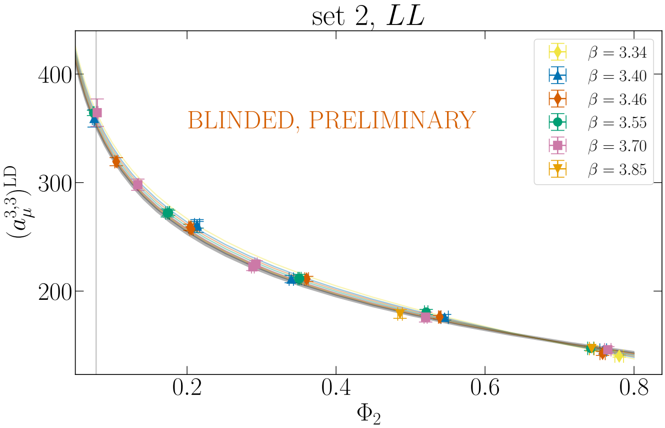
Noise reduction: spectral reconstruction



Careful extraction of energies and overlaps: [Nolan Miller, Tue 16:15]

- Spectral reconstruction of the isovector correlation function on E250 at m_π^{phys}.
- Solves the signal-to-noise problem, but LMA is more precise for $t < 2.5\,\text{fm}$.
- Inclusion reduces the uncertainty on this ensemble by a factor of 2.

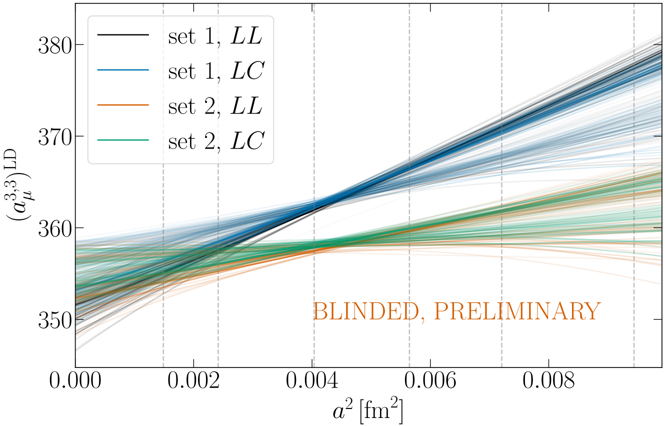
$(a_\mu^{\mathrm{hvp}})^{\mathrm{LD}}$ IN THE ISOVECTOR CHANNEL: CHIRAL DEPENDENCE



- Dependence of $(a_\mu^{3,3})^{\mathrm{LD}}$ on $\Phi_2 = 8t_0m_\pi^2$.
- Data is corrected for finite-size effects.
- Weak dependence on the cutoff.
- Mass-dependent cutoff effects noticeable.

- Chiral dependence well constrained across the range of pion masses. Need to include a term that is divergent in the chiral limit for good fit quality.

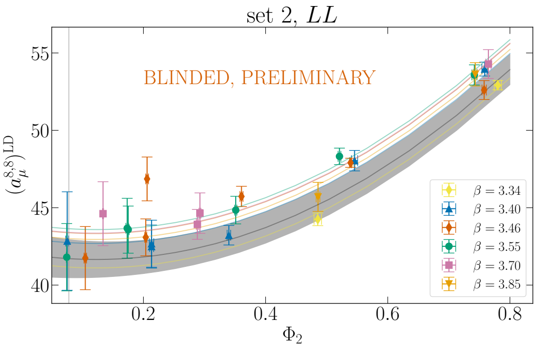
$(a_\mu^{\text{hvp}})^{\text{LD}}$ IN THE ISOVECTOR CHANNEL: CUTOFF DEPENDENCE

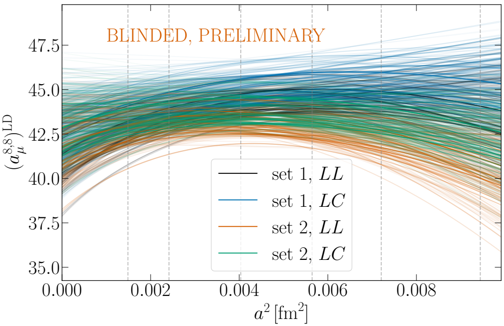


- Dependence of $(a_\mu^{3,3})^{\text{LD}}$ on a^2 at physical quark masses.
- Four sets of data (colors) differ by $\mathrm{O}(a^2)$.
- Each line represents a fit in the model average.
- Include terms à la $[\alpha_s(1/a)]^{0.395}\, a^2$ [Husung, 2401.04303].

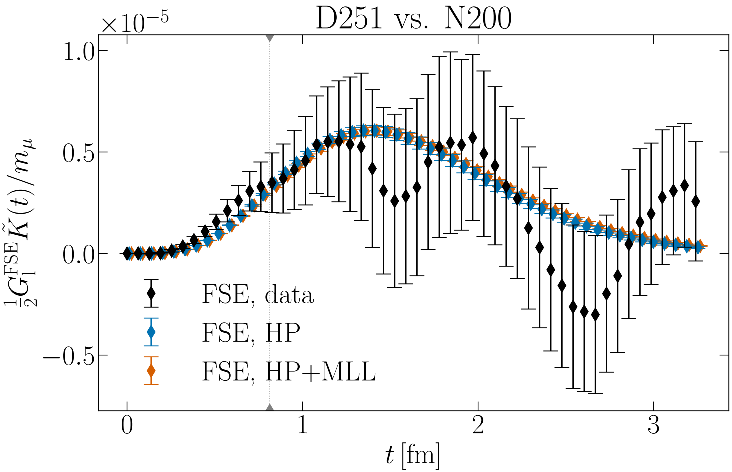
- Contains **artificial cutoff effects from the blinding** procedure.
- Higher order cutoff effects have a small weight in the model average.

$(a_\mu^{\rm hvp})^{\rm LD}$ IN THE ISOSCALAR CHANNEL





- Quark-disconnected diagram contributes significantly to noise in the isoscalar channel, despite using multiple noise reduction techniques [Cè et al., 2203.08676].
- Bounding method in the isoscalar channel to tame the long-distance tail.
- Leading finite-size effects of light-connected and disconnected cancel.



- $I = 1$ channel
- $m_\pi = 286\,\mathrm{MeV}$
- L: $3\,\mathrm{fm} \to 4.1\,\mathrm{fm}$
- $m_\pi L$: $4.4 \to 5.9$
- $a = 0.064\,\mathrm{fm}$

- Compare finite-size effects in the data with the two model predictions.
- Excellent agreement (with large statistical uncertainties).

$(a_\mu^{\mathrm{hvp}})^{\mathrm{LD}}$: Status and outlook

Achievements

- High statistical precision at m_π^{phys} and excellent control of the m_π dependence.
- Large span of lattice spacings to control the continuum extrapolation.

Challenges

- Scale setting remains a dominant source of uncertainty.
  The global status of gradient flow scales is unsatisfactory [FLAG23].
- Autocorrelation hinders precise estimates at very fine lattice spacing.
- Isospin breaking effects need to be computed accurately.
  [Julian Parrino, Thu 9:40] [Dominik Erb, Thu 10:00]

Advertisement

- Stay tuned for our unblinded result for $(a_\mu^{\mathrm{hvp}})^{\mathrm{LD}}$!

- Related work of the Mainz group at Lattice 2024:
  - The timelike pion form factor and other applications of $I = 1\pi\pi$ scattering [Nolan Miller, Tue 16:15]
  - The hadronic contribution to the running of α and the electroweak mixing angle [Alessandro Conigli, Thu 9:40]
  - Machine-learning techniques as noise reduction strategies in lattice calculations of the muon $g-2$ [Hartmut Wittig, Wed 11:35]
  - UV-finite QED correction to the hadronic vacuum polarization contribution to $(g-2)_\mu$ [Julian Parrino, Thu 9:40]
  - The isospin-violating part of the hadronic vacuum polarisation [Dominik Erb, Thu 10:00]