

High precision calculation of hadronic vacuum polarisation contribution to muon $g-2$: update by **BMW+DMZ collaboration**

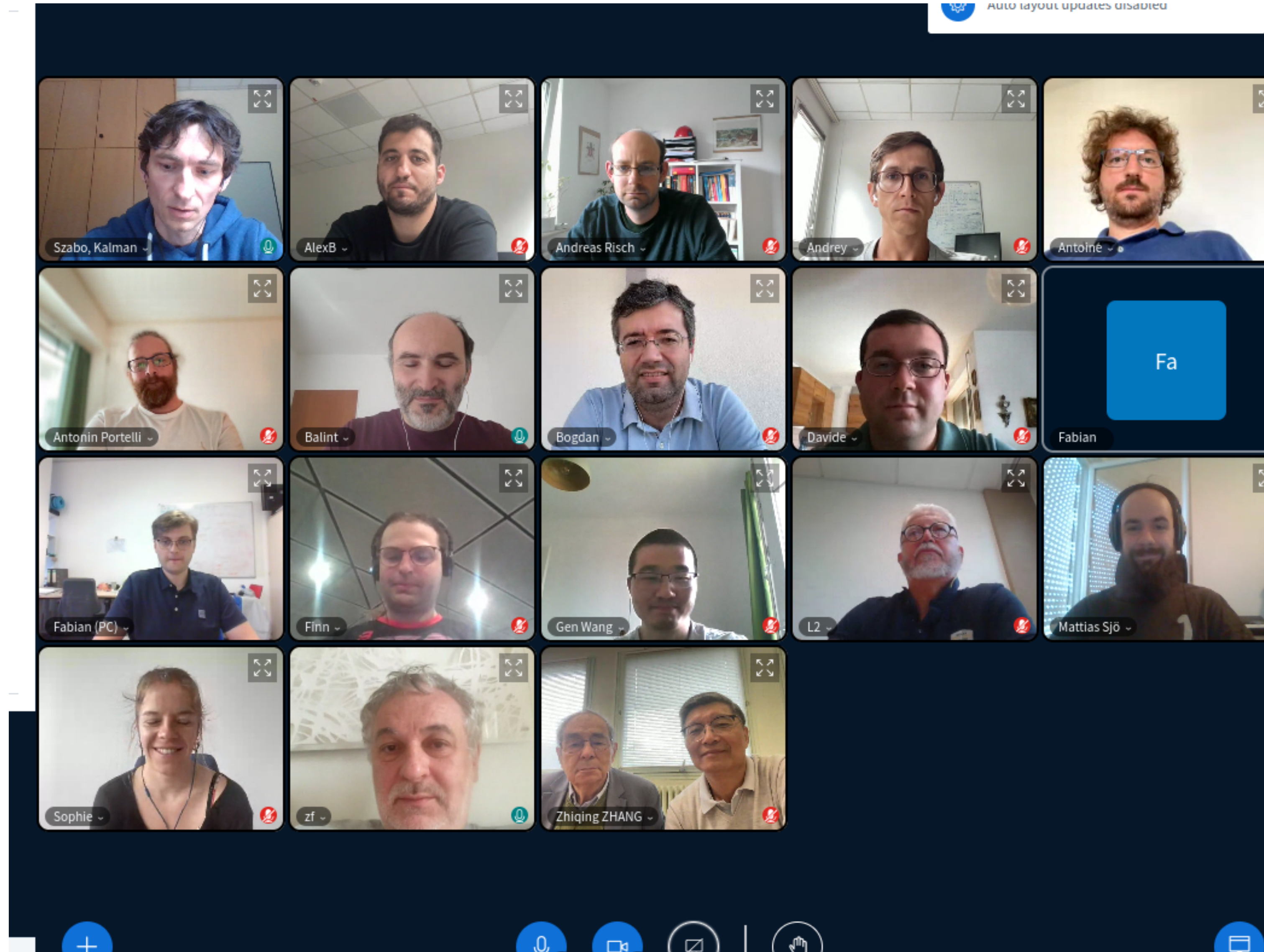
A.Yu. Kotov for the BMW+DMZ collaboration
arXiv: 2407.10913



**A. Boccaletti, Sz. Borsanyi, M. Davier, Z. Fodor, F. Frech, A. Gerardin, D. Giusti,
A.Yu. Kotov, L. Lellouch, Th. Lippert, A. Lupo, B. Malaescu, S. Mutzel, A. Portelli, A. Risch,
M. Sjo, F. Stokes, K.K. Szabo, B.C. Toth, G. Wang, Z. Zhang**

Lattice 2024

Unblinding...

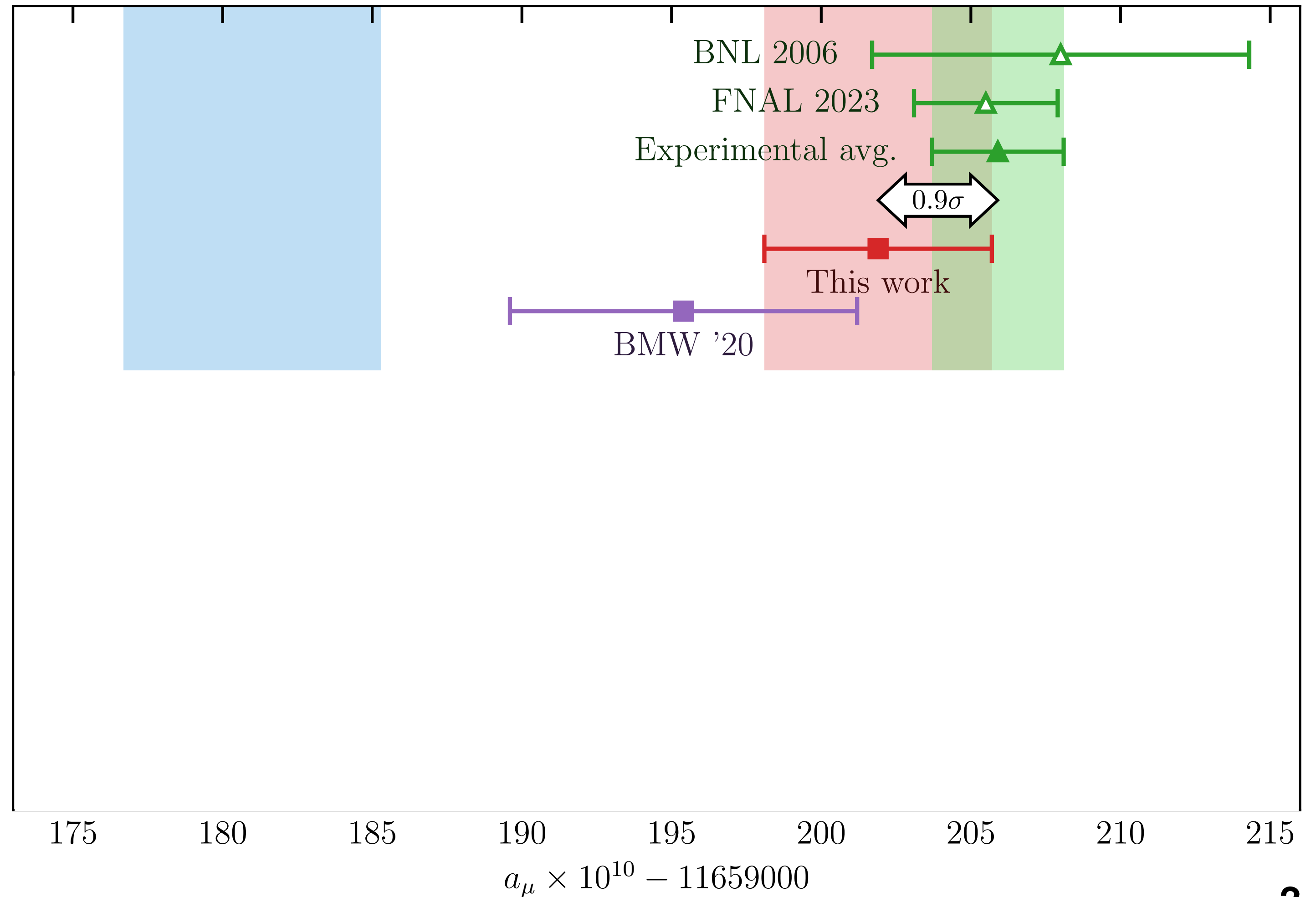


Results

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$$a_{\mu}^{\text{LO-HVP}} = 714.1(2.2)(2.5)[3.3] \times 10^{-10}$$

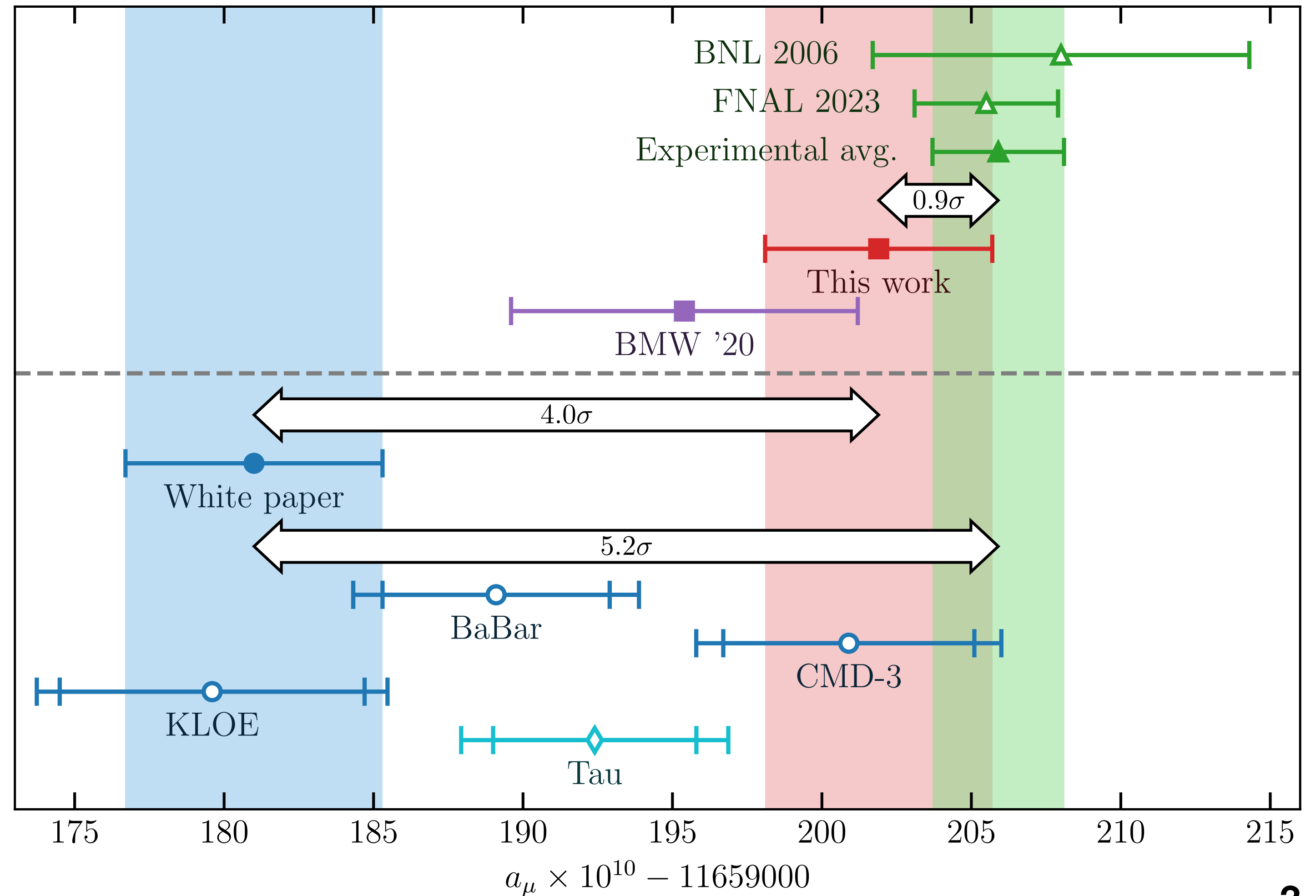
- 0.9σ difference w/experiment



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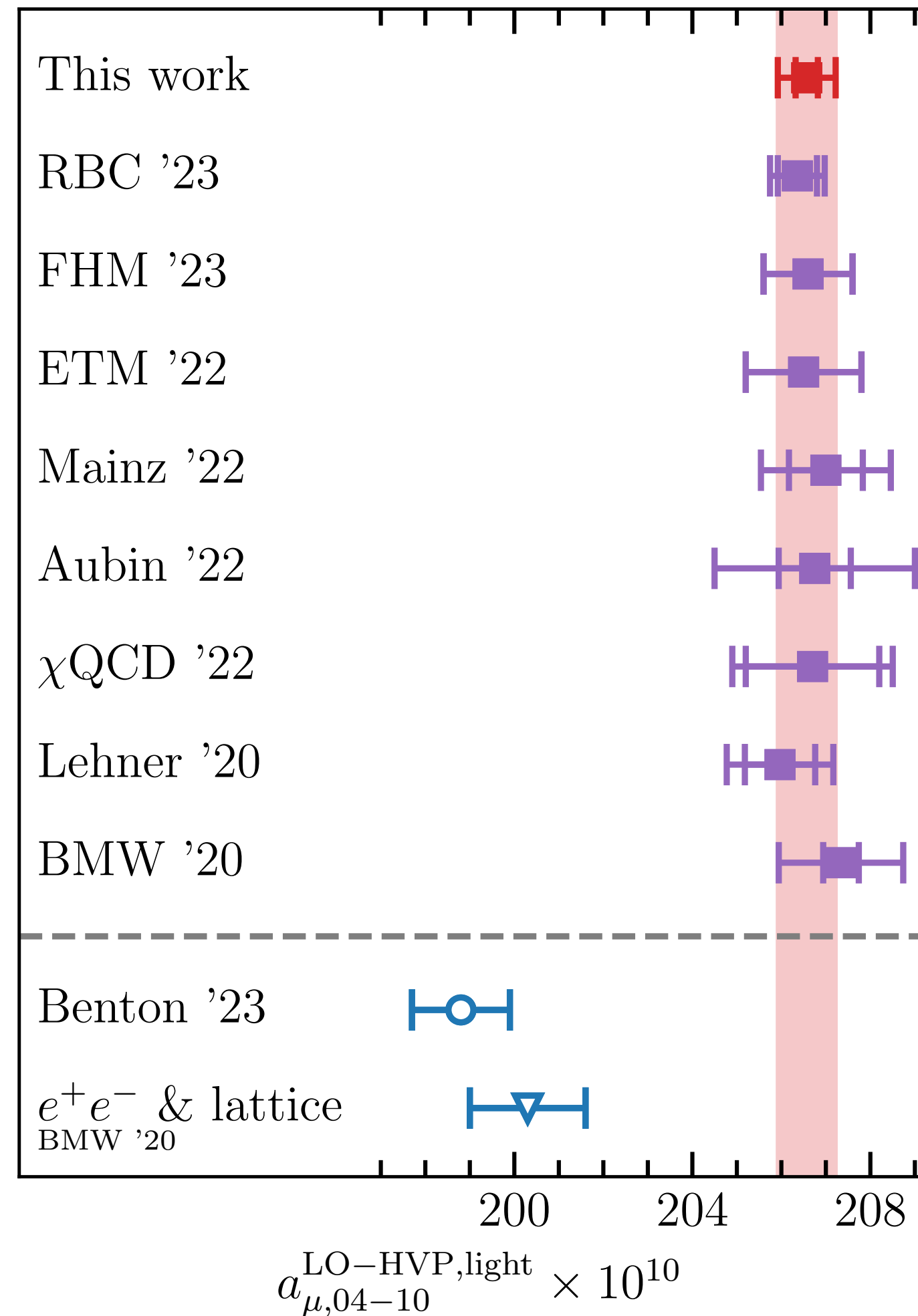
- 0.9σ difference w/experiment
- 4.0σ higher than WP'20



Results'2 (intermediate distance)

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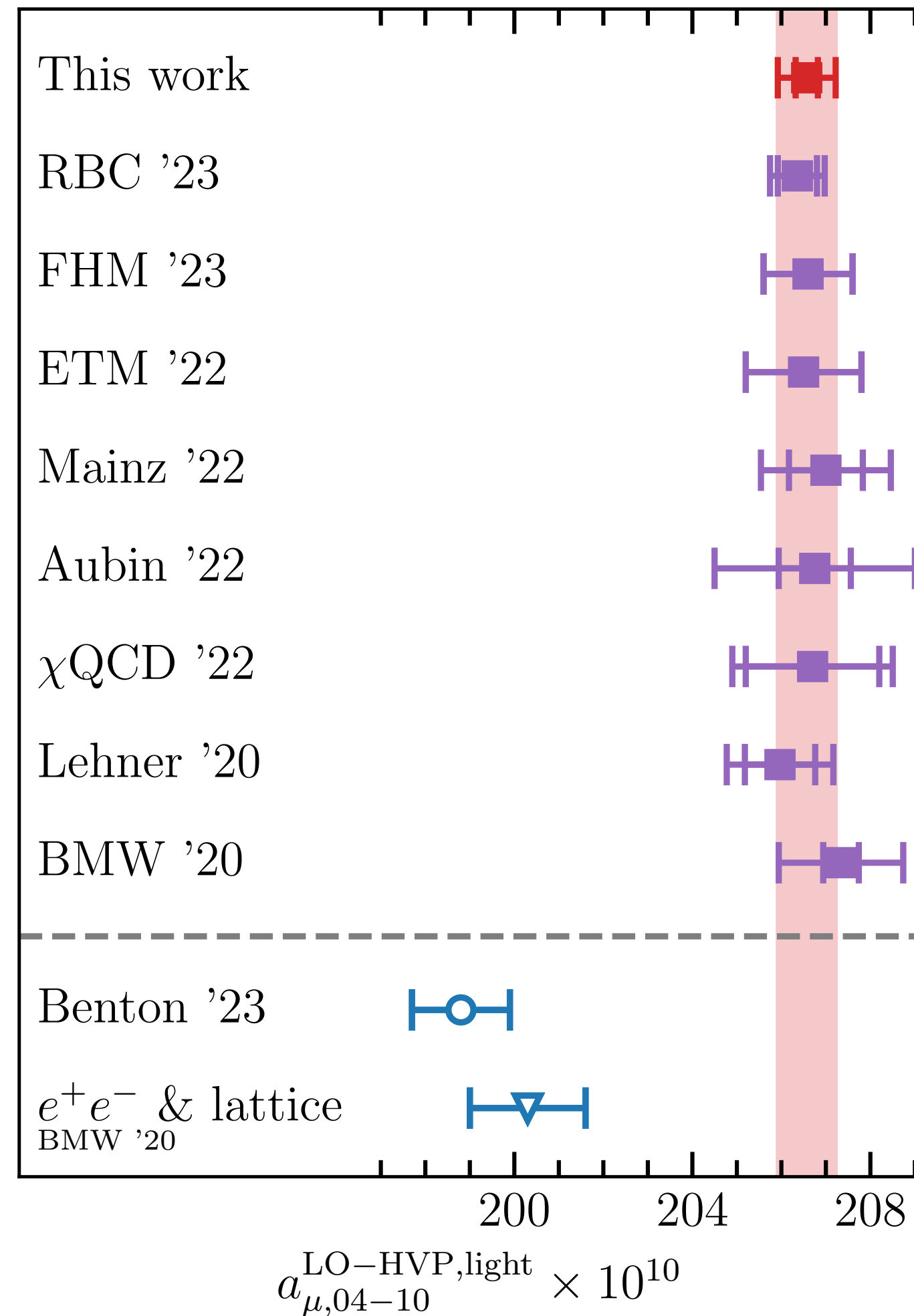
$$a_{\mu}^{\text{LO-HVP,light}} = 206.57(25)(60)[65] \times 10^{-10}$$



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- Light ID, BMW'24 vs R-ratio:
 - $\sim (4 - 5)\sigma$ tension



Results'2 (intermediate distance)

$$a_{\mu}^{\text{LO-HVP,light}} = 206.57(25)(60)[65] \times 10^{-10} \quad a_{\mu}^{\text{LO-HVP}} = 235.94(29)(63)[70] \times 10^{-10}$$

- Light ID, BMW'24 vs R-ratio:

- $\sim (4 - 5)\sigma$ tension

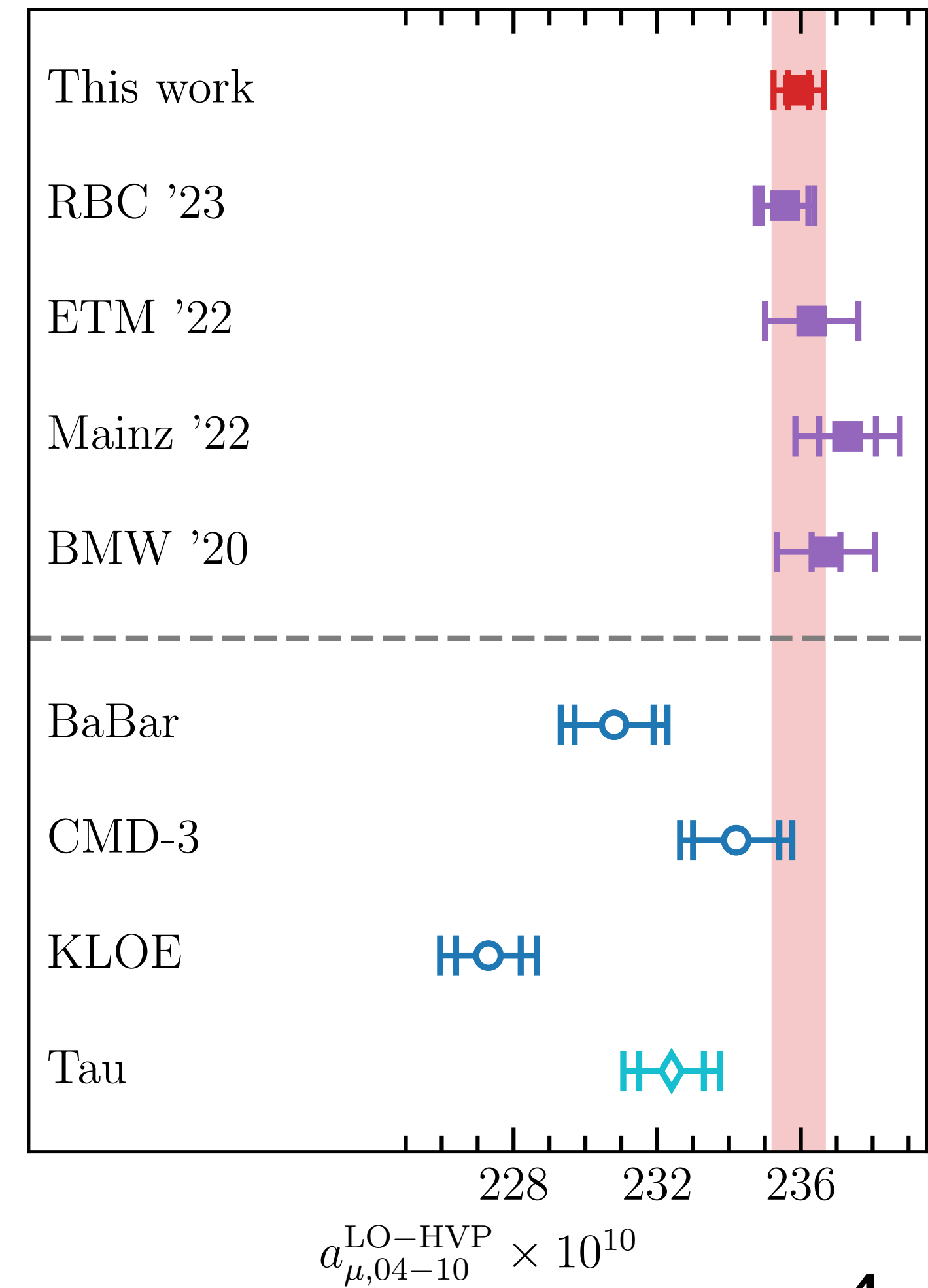
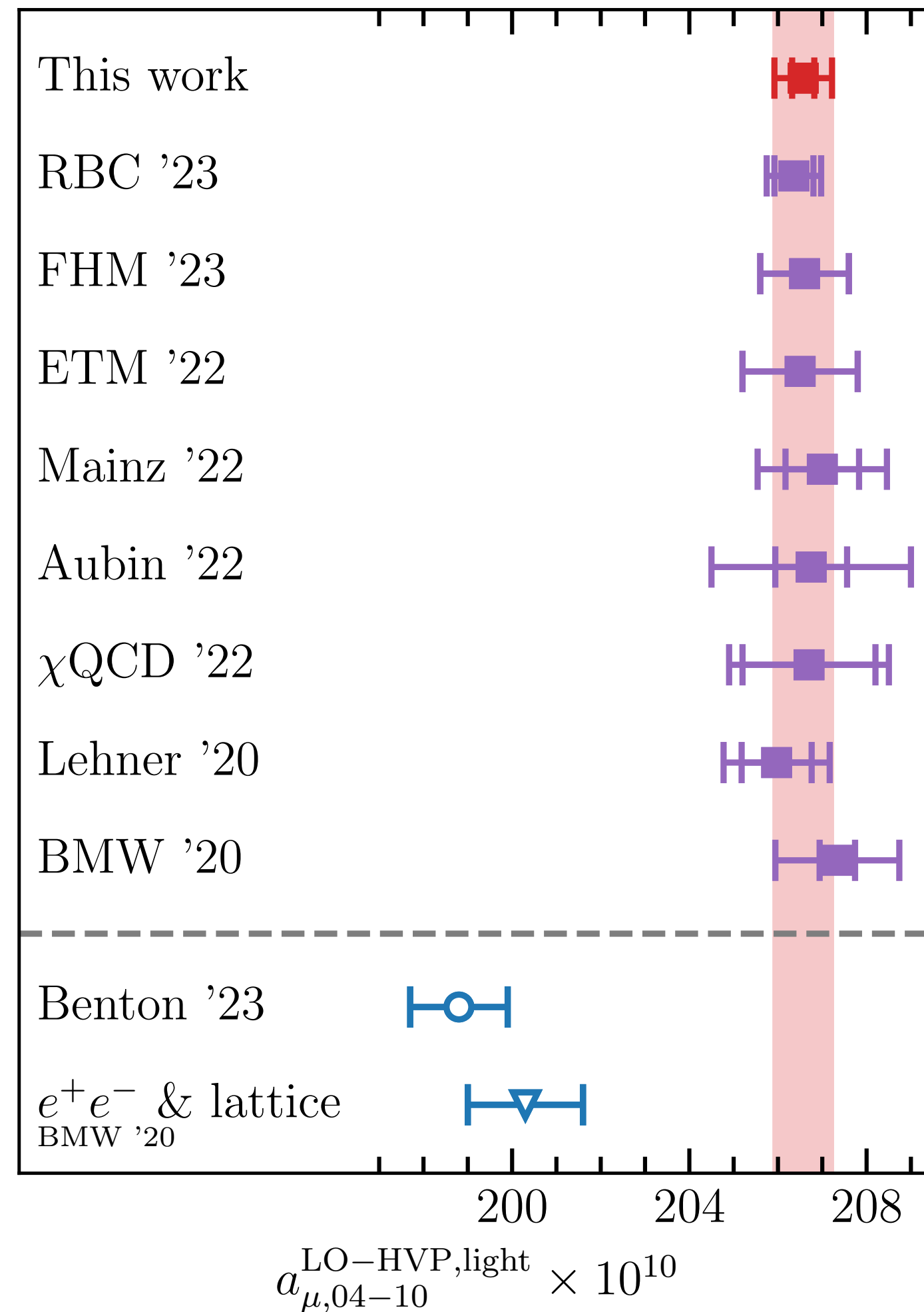
- Full ID, BMW'24:

- KLOE: 5.7σ tension

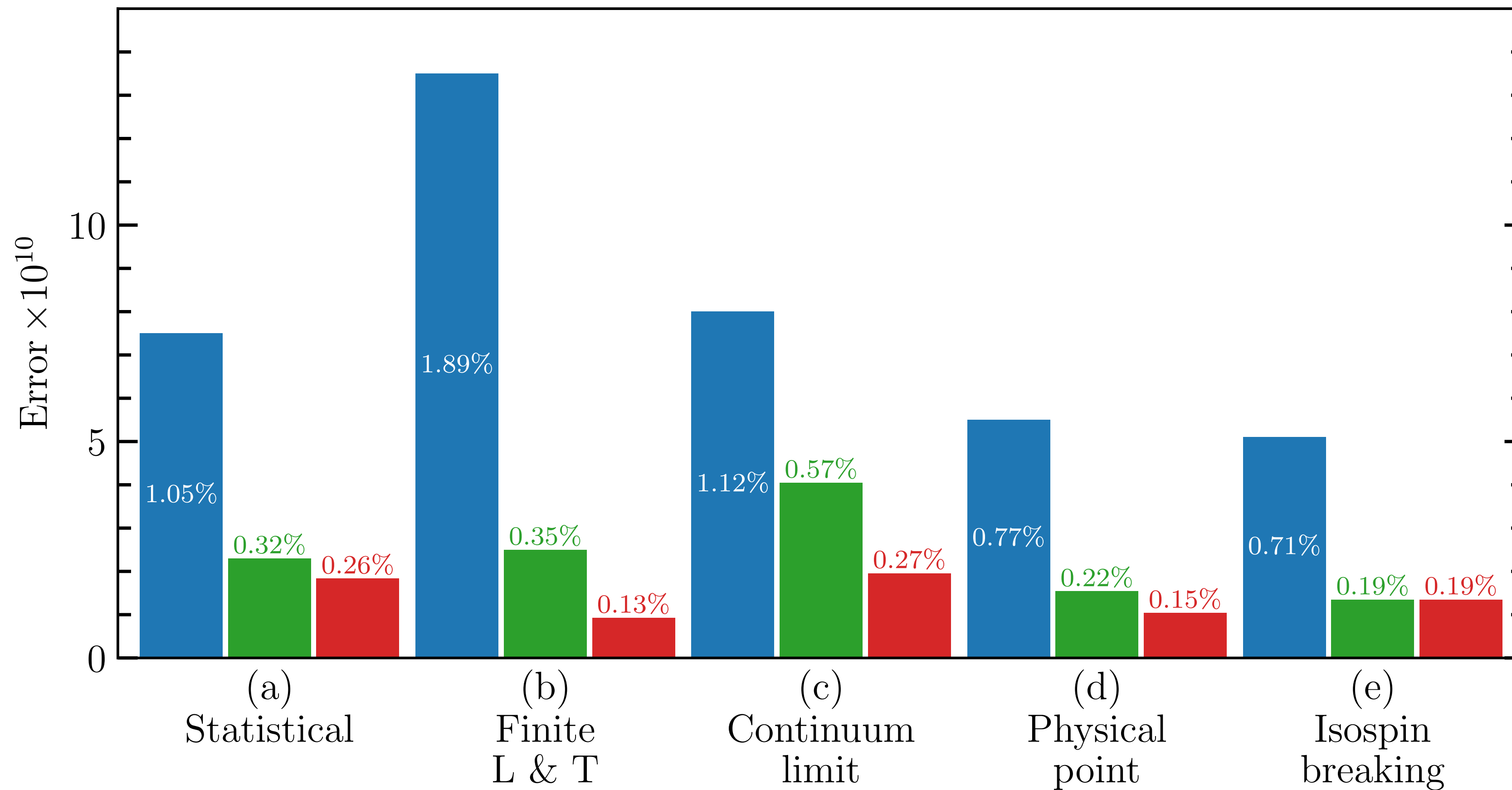
- BaBar: 3.1σ tension

- CMD-3: 1.0σ tension

- τ decays: 2.3σ tension



BMW: 2017 → 2020 → 2024



Overall 40% error reduction since BMW'2020

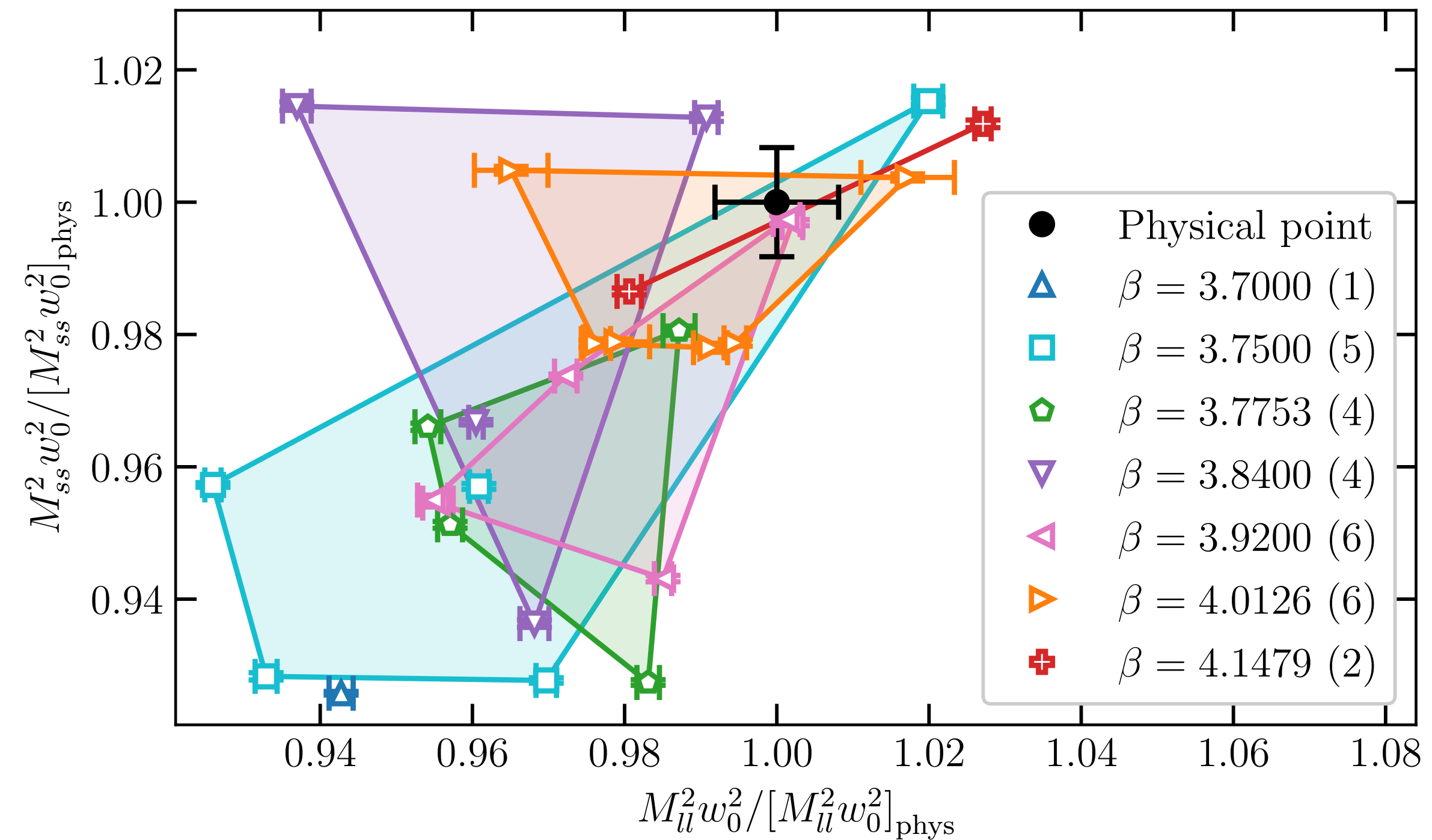
BMW: 2020 → 2024

What is improved

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- Continuum extrapolation: $a = 0.064\text{fm} \rightarrow 0.048\text{fm}$
[A.Risch, Thu, 10.40]
- Increased statistics



BMW: 2020 → 2024

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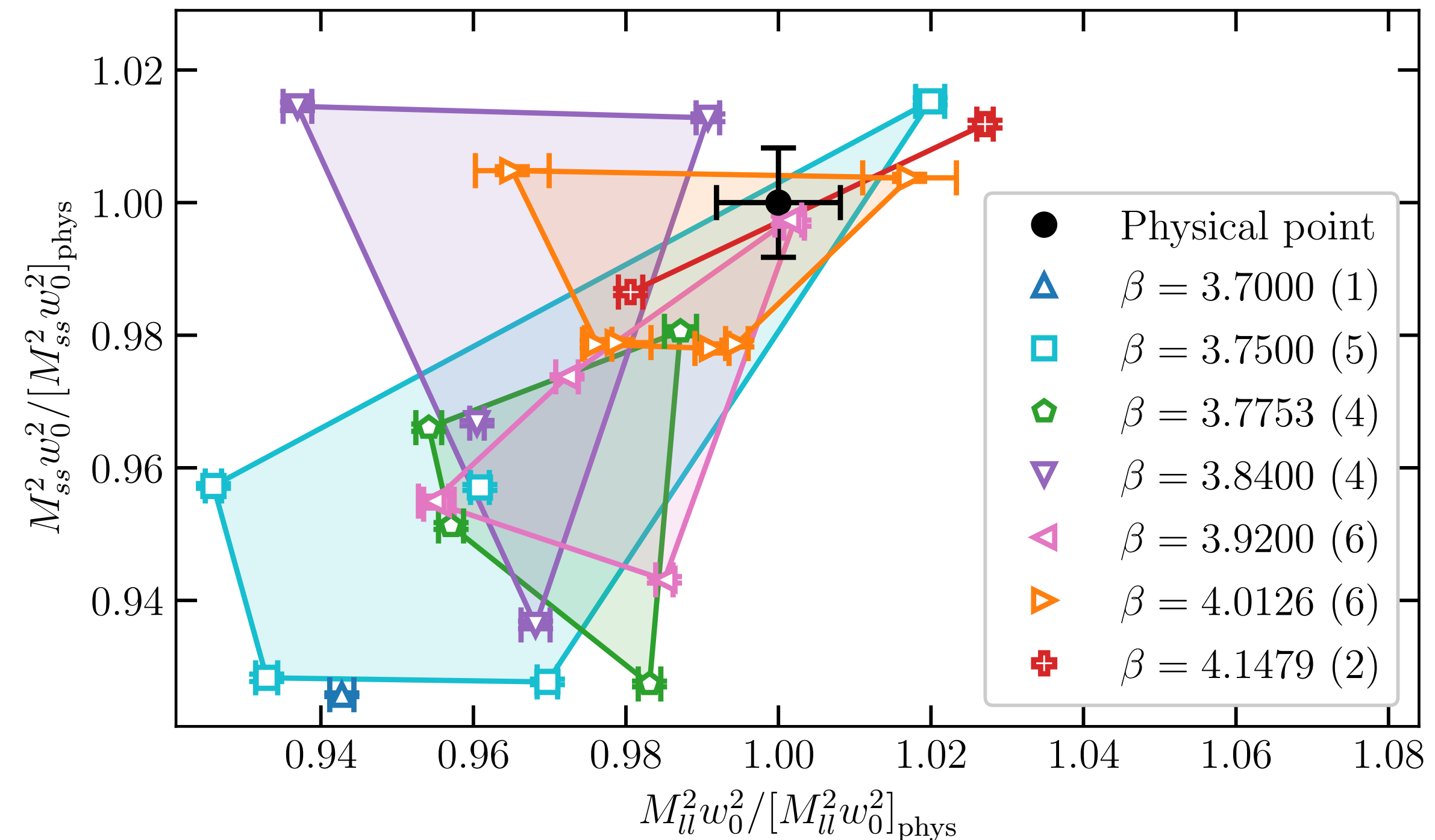
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[A.Risch, Thu, 10.40]

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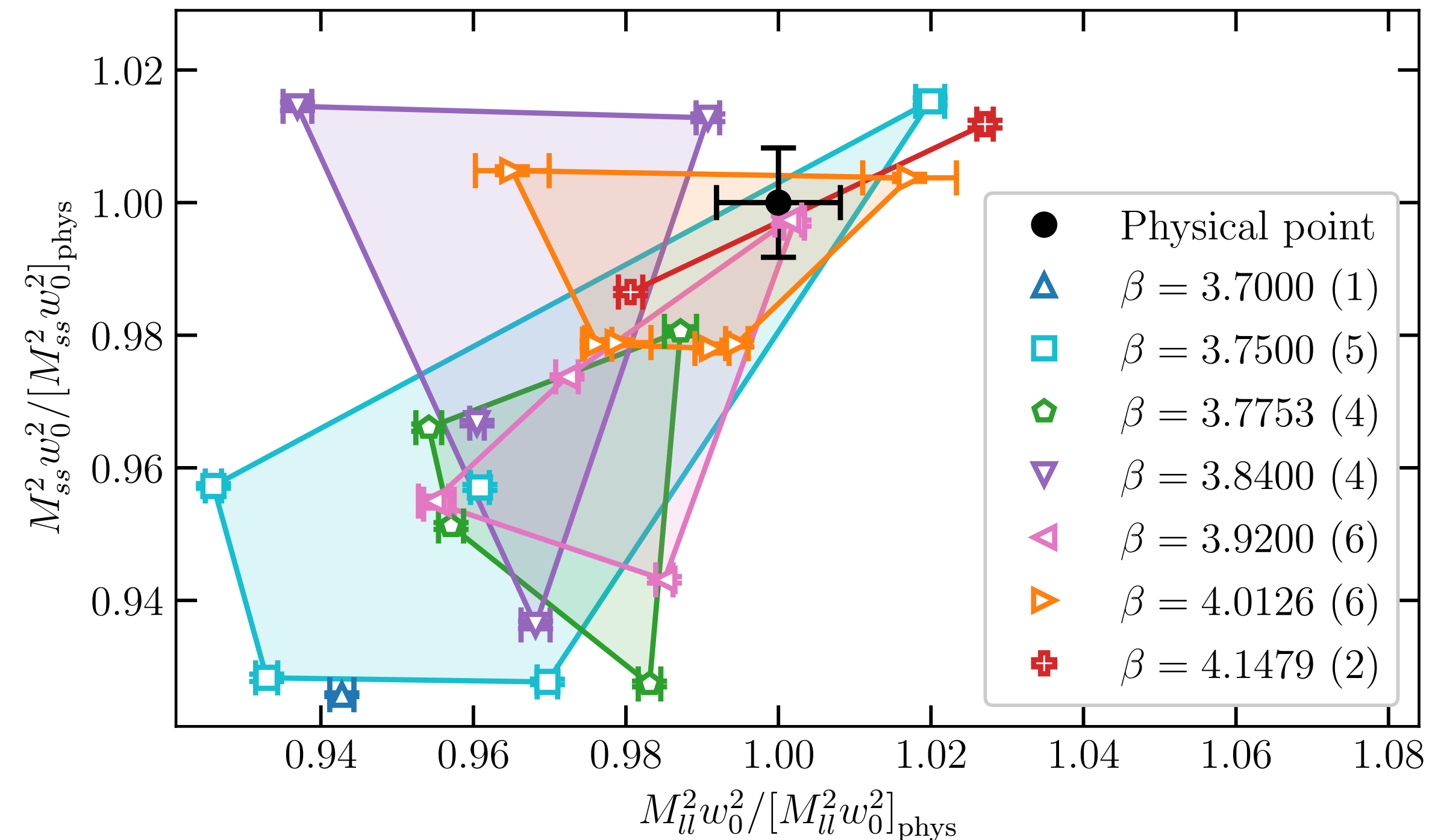
[G.Wang, Fri, 15.55]



BMW: 2020 → 2024

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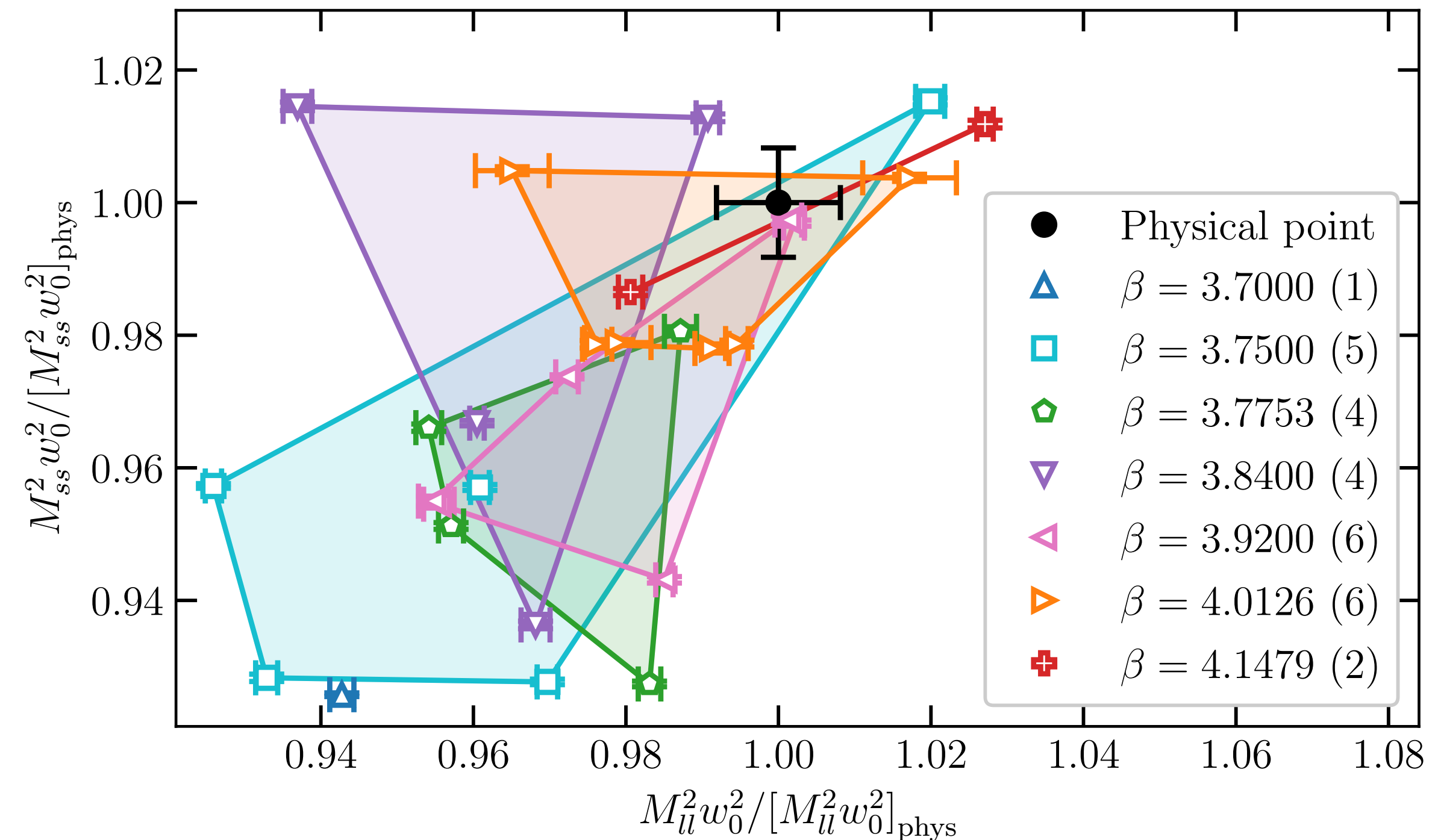
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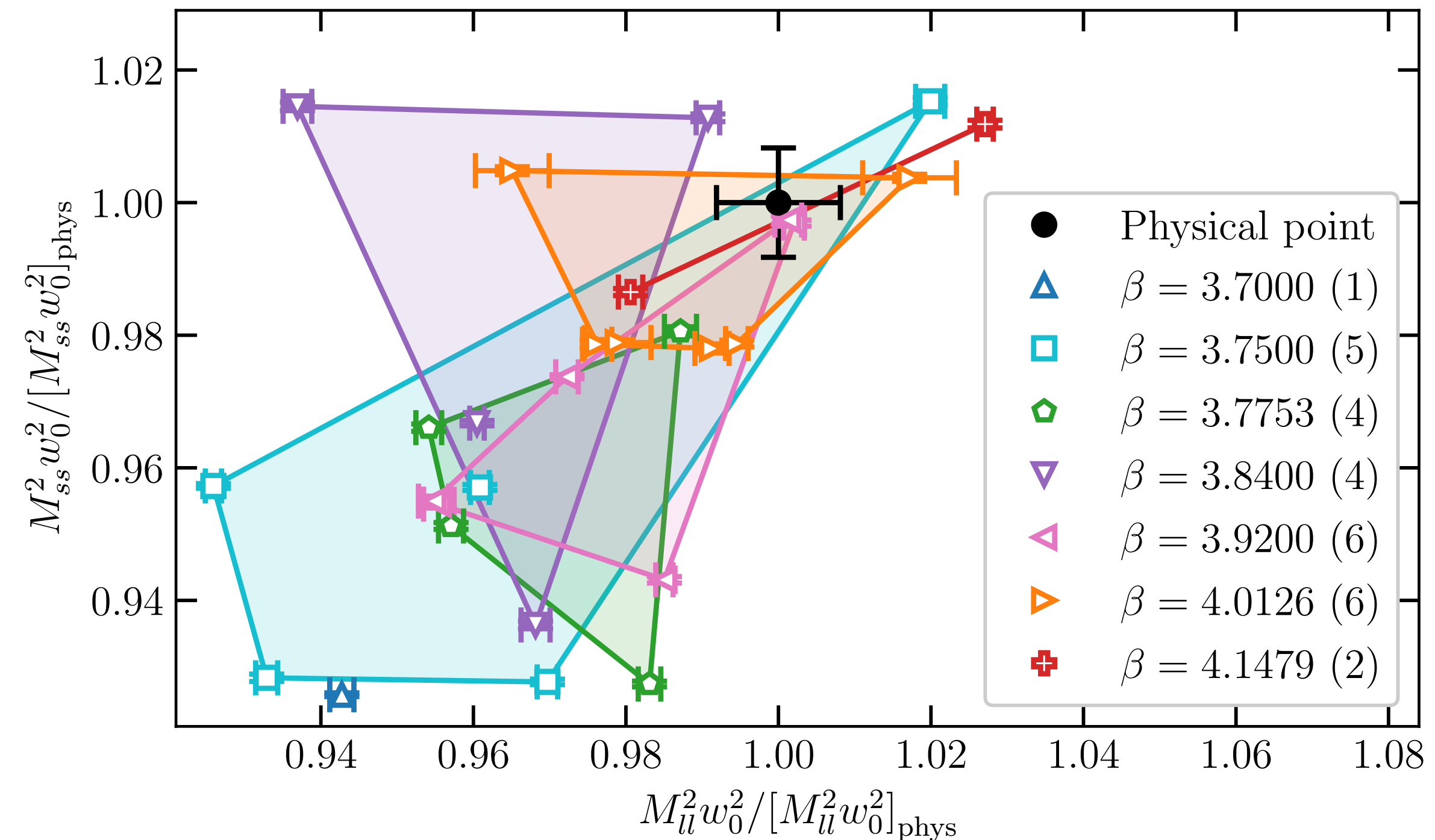
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- Tail from data-driven approach
[Poster, B.Toth]



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[G.Wang, Fri, 15.55]
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- Tail from data-driven approach
[Poster, B.Toth]
- Finite size effects
[A.Lupo, Mon, 15.55]



Analysis

More on windows

$$C(t) = -\frac{1}{3e^2} \sum_{\mu=1,2,3} \int d^3x \langle J_\mu(x,t) J_\mu(0,0) \rangle$$

$$a_\mu^{\text{LO-HVP}} = \alpha^2 \int_0^\infty dt K(tm_\mu) C(t)$$

$$a_\mu^{\text{LO-HVP,win}} = \alpha^2 \int_0^\infty dt K(tm_\mu) C(t) w(t)$$

[RBC/UKQCD'18]

Analysis

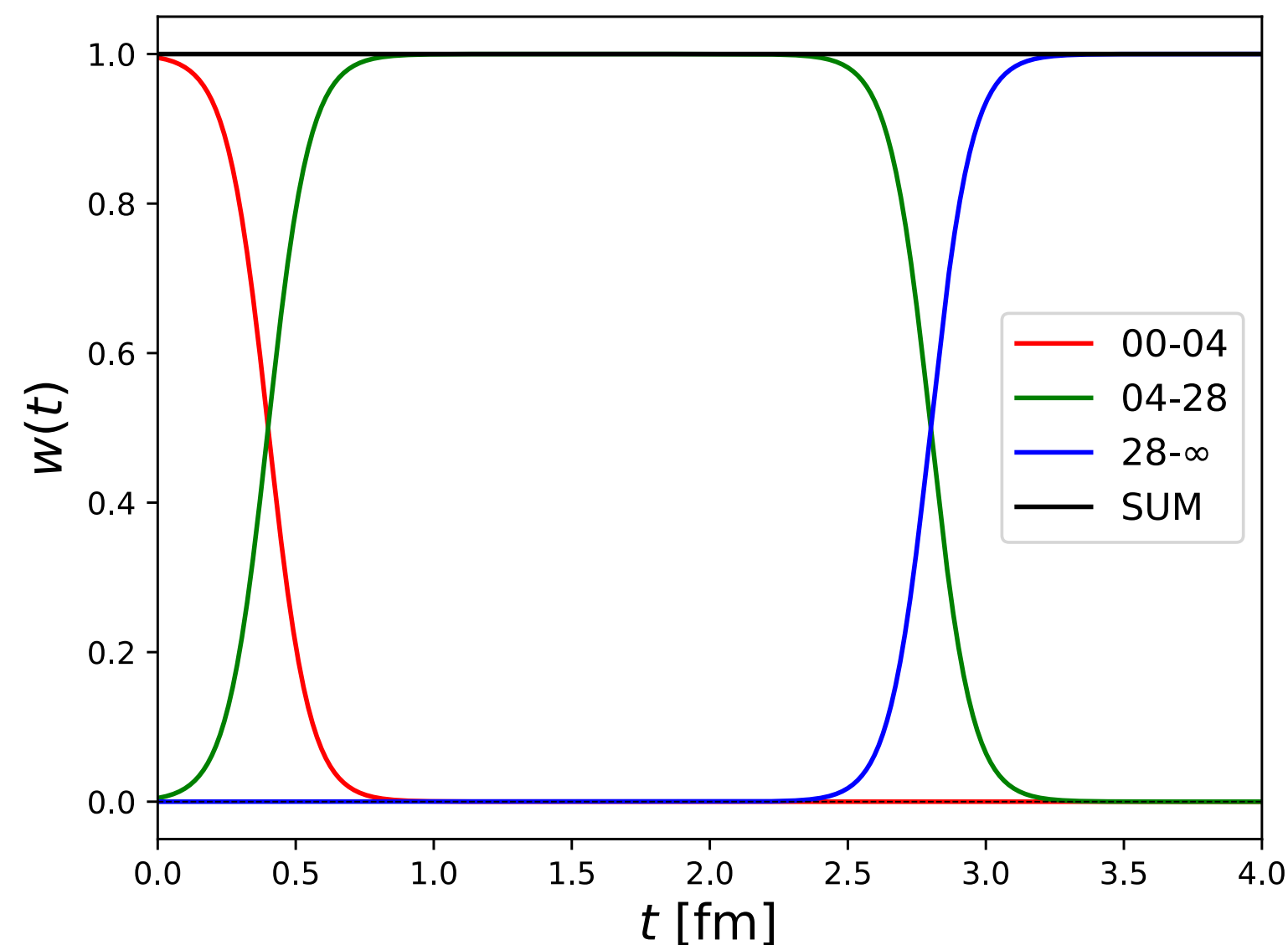
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Different windows: different behaviour

- SD: 00 – 04 fm
- ID and LD: 04 – 28 fm

Lattice

[Poster, B.Toth]

Analysis

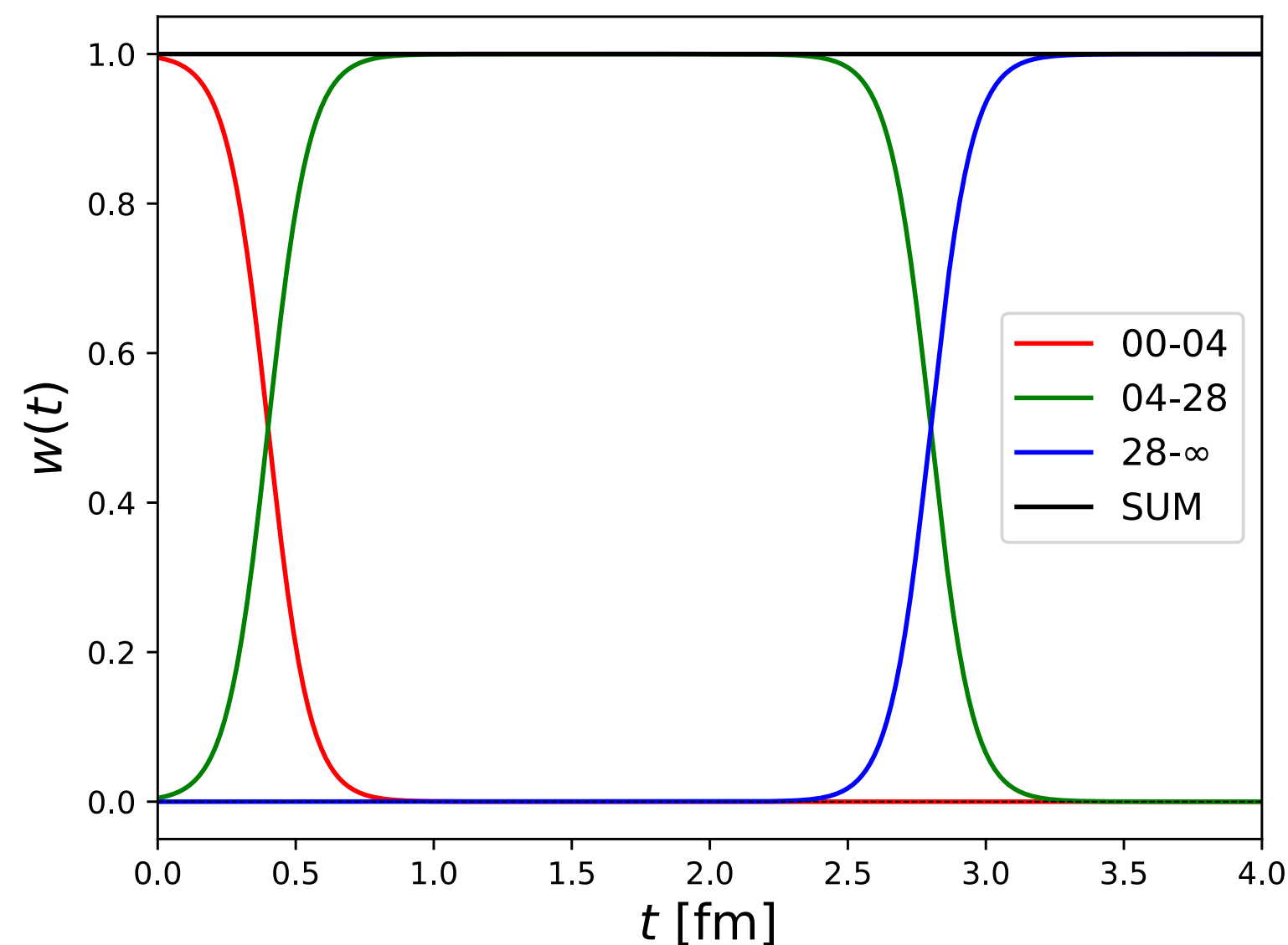
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[Poster, B.Toth]

- Tail: 28 – ∞ fm
- Data-driven approach

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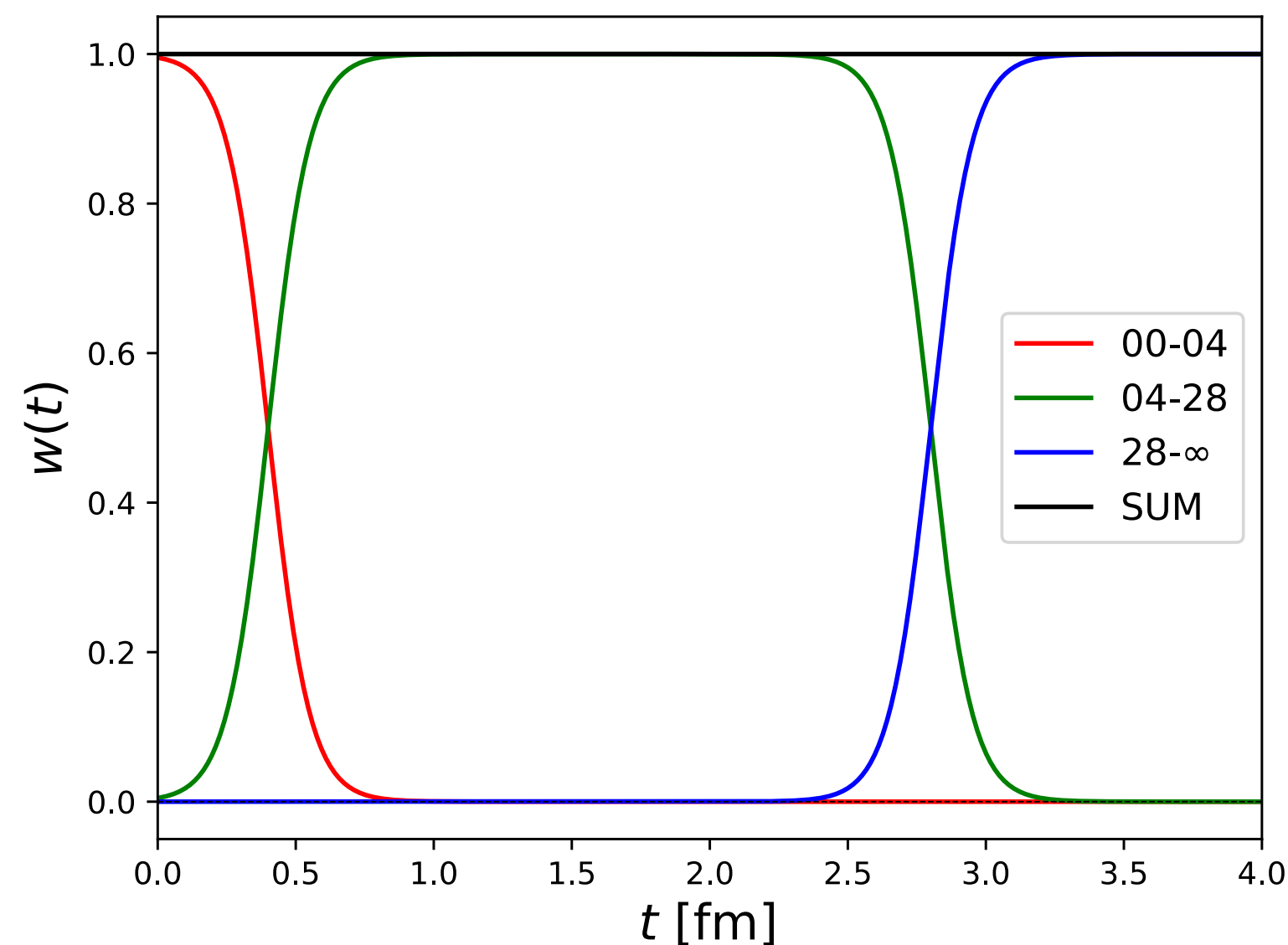
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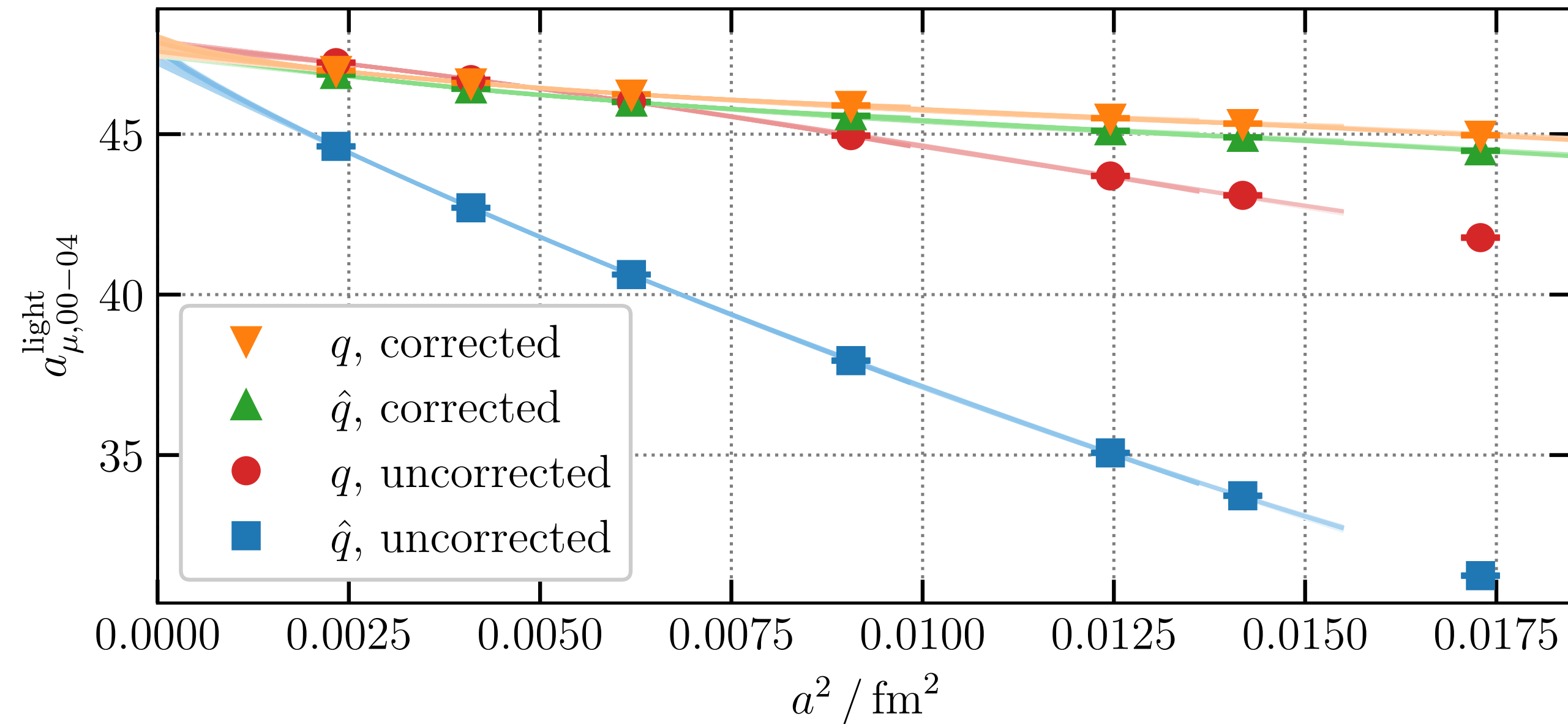
Lattice

- SD: 00 – 04 fm
- ID and LD: 04 – 28 fm

[Poster, B.Toth]

- Tail: 28 – ∞ fm
- Data-driven approach
- $a_\mu = a_\mu^{00-04} + a_\mu^{04-28} + a_\mu^{28-\infty}$

Short distance 00-04 window

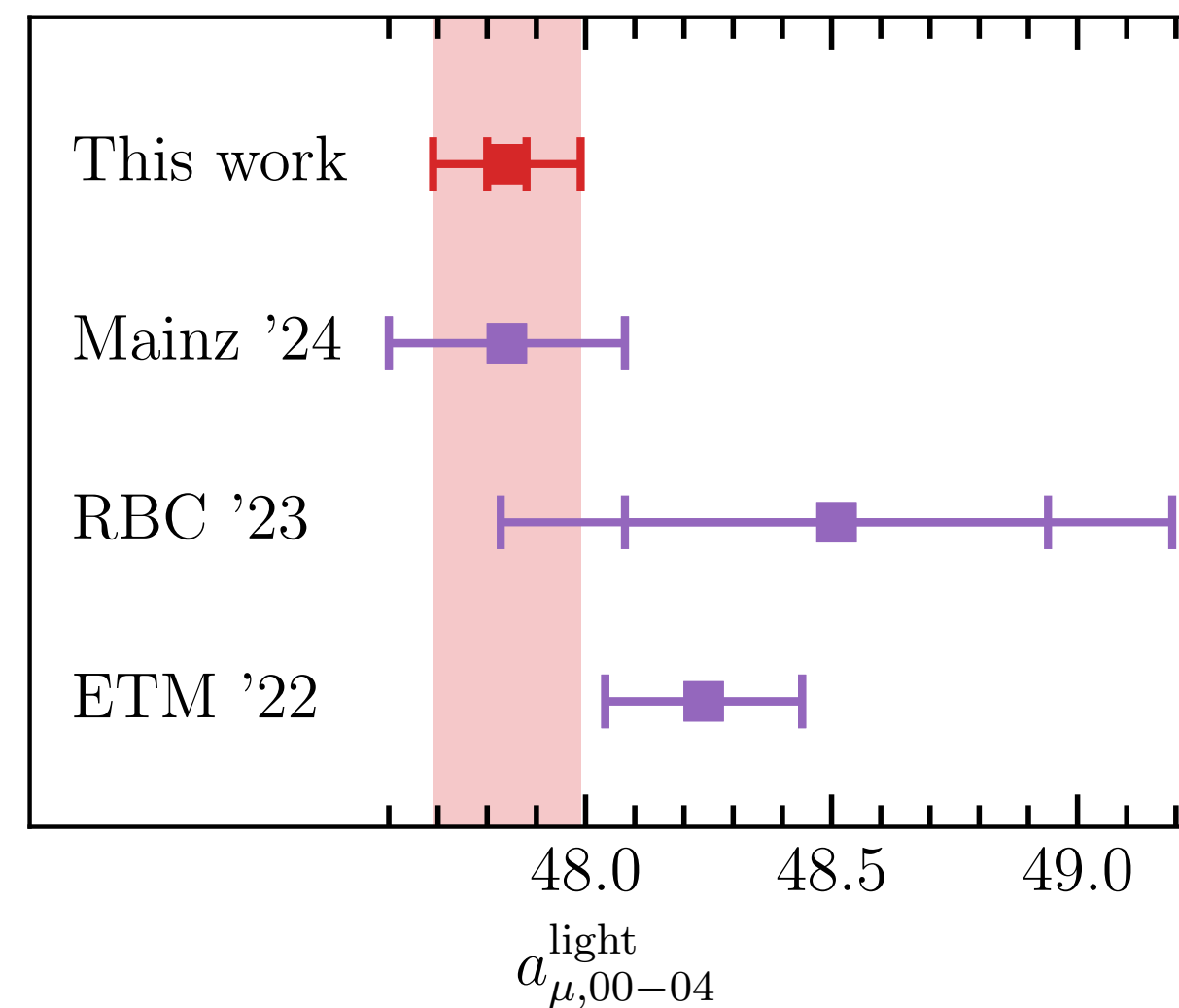
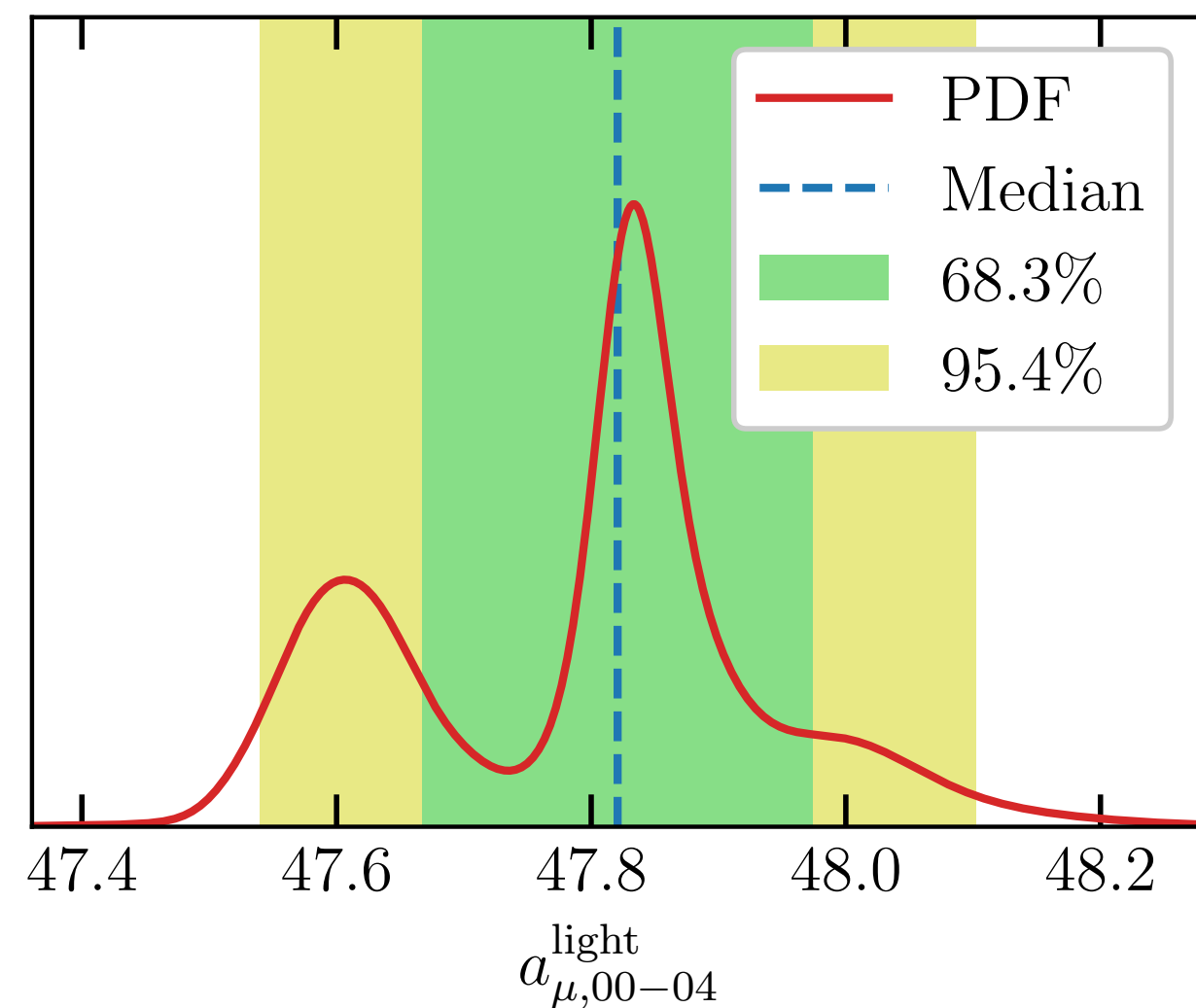


- Contains $a^2 \log(a^2/w_0^2)$

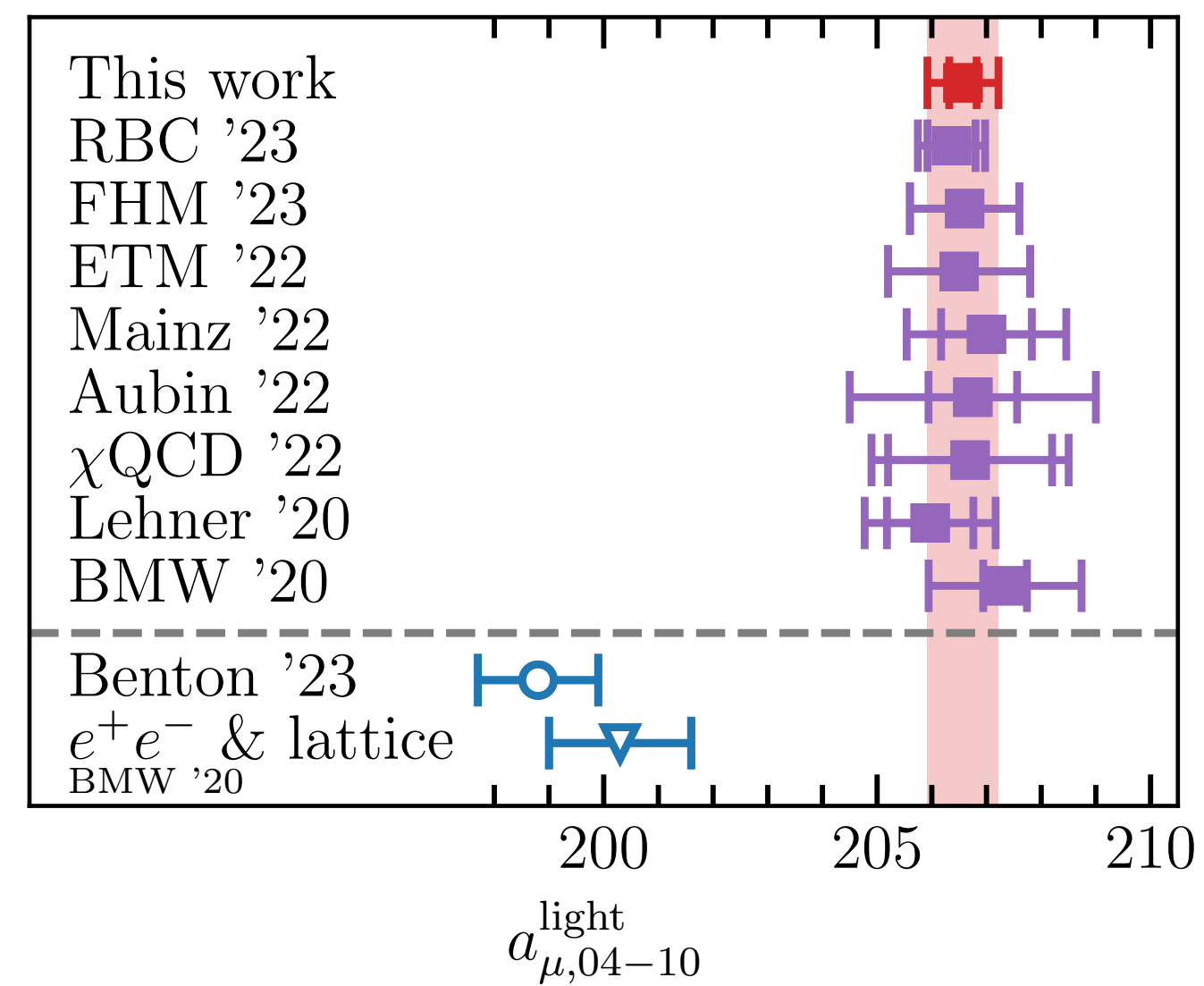
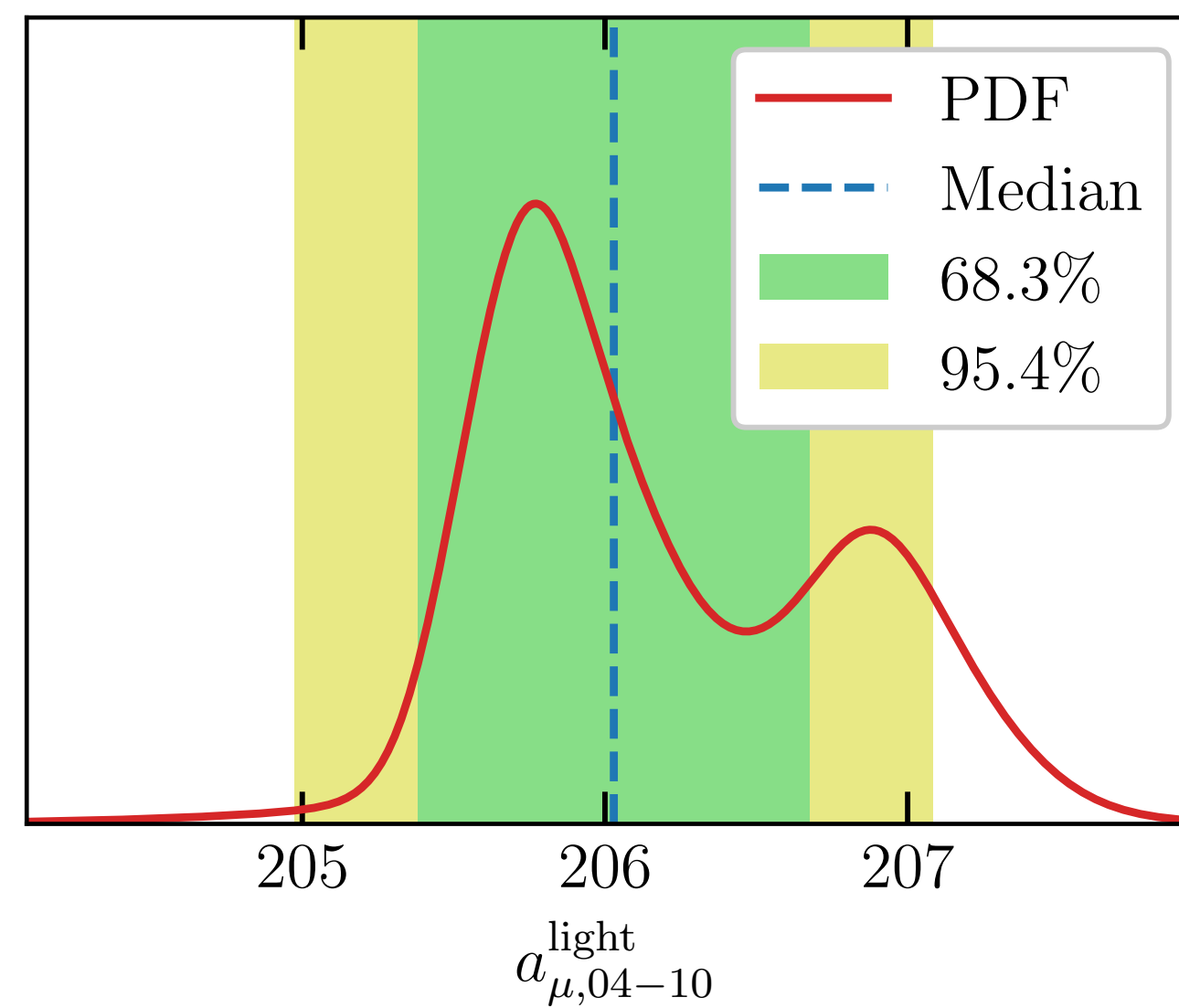
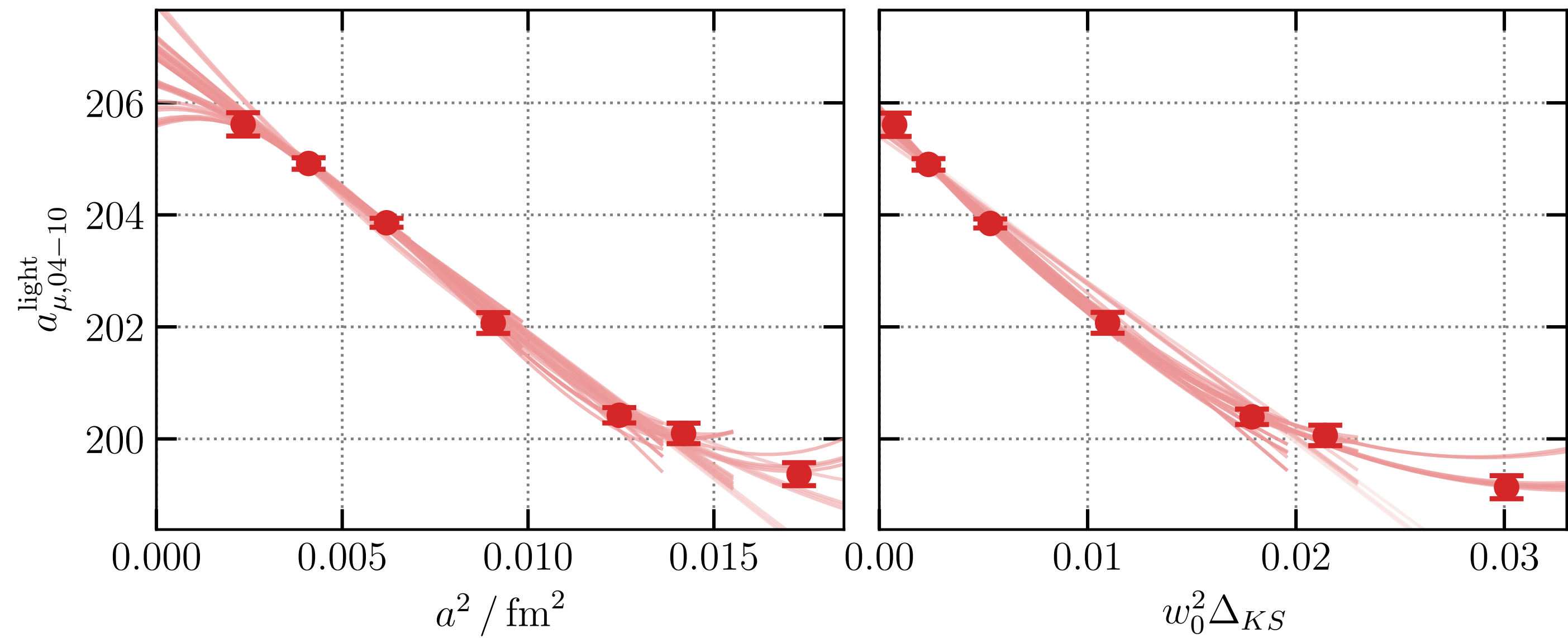
- $\hat{q} = 2 \sin(aq/2)/a$

- Tree-level corrected:

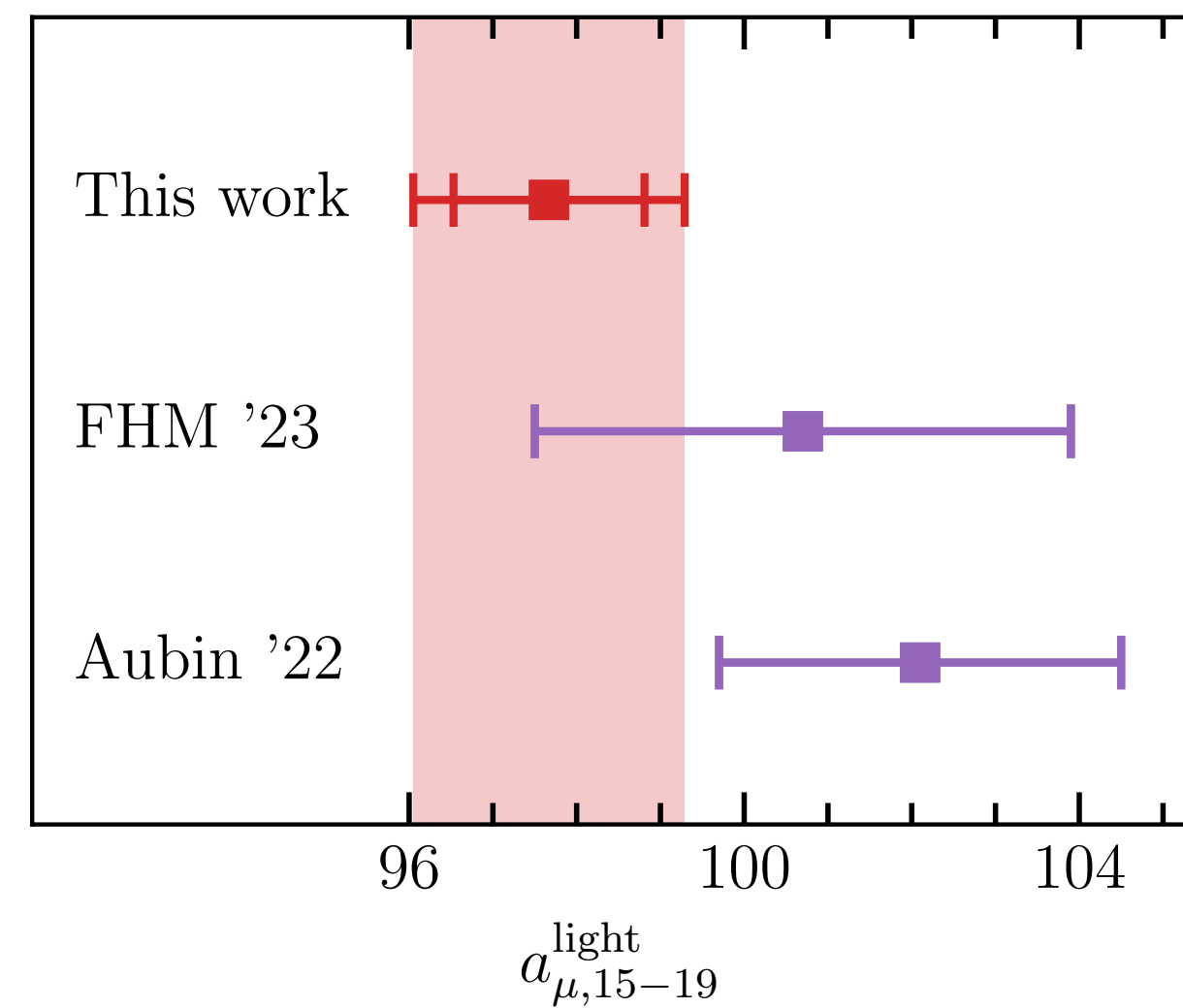
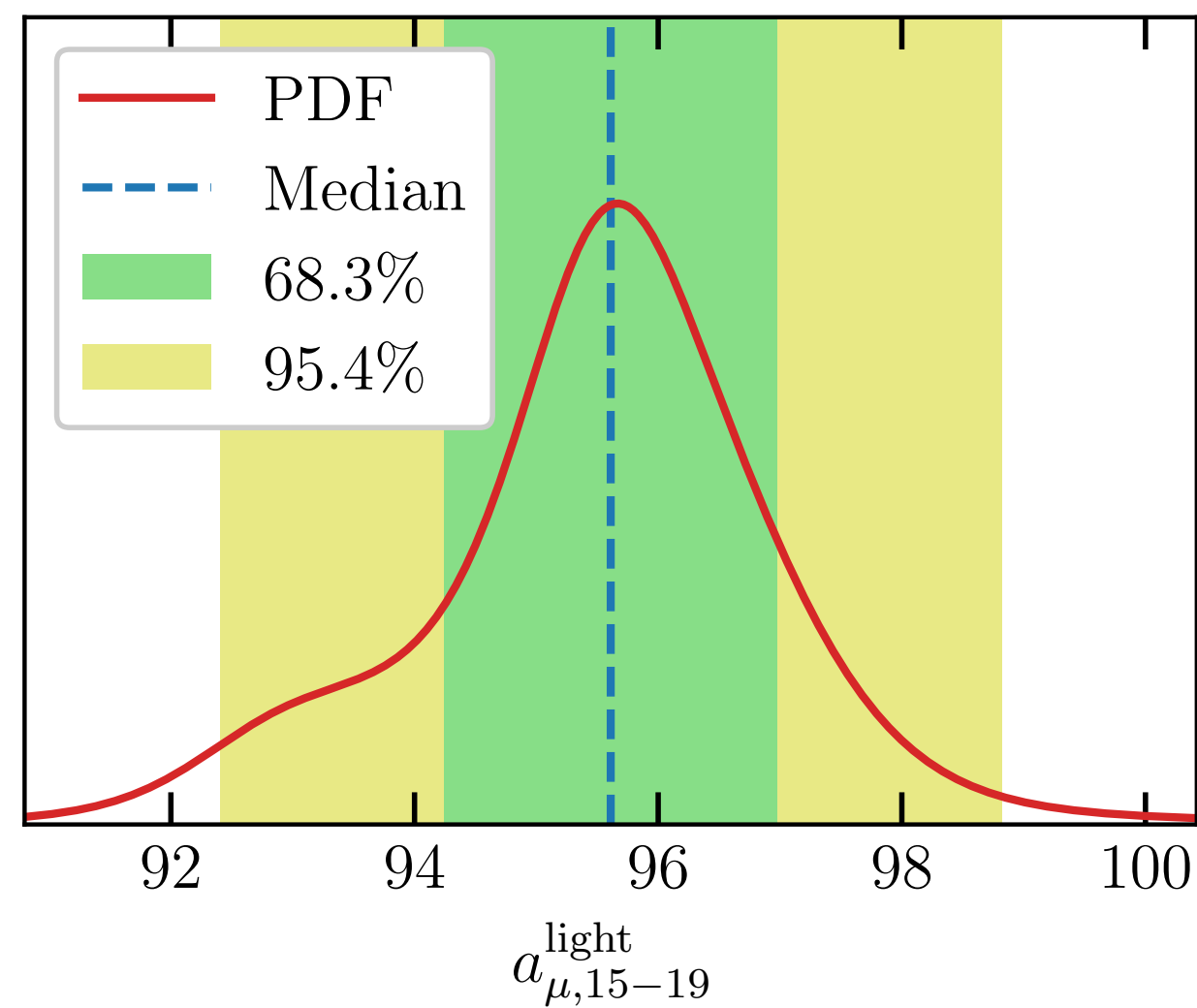
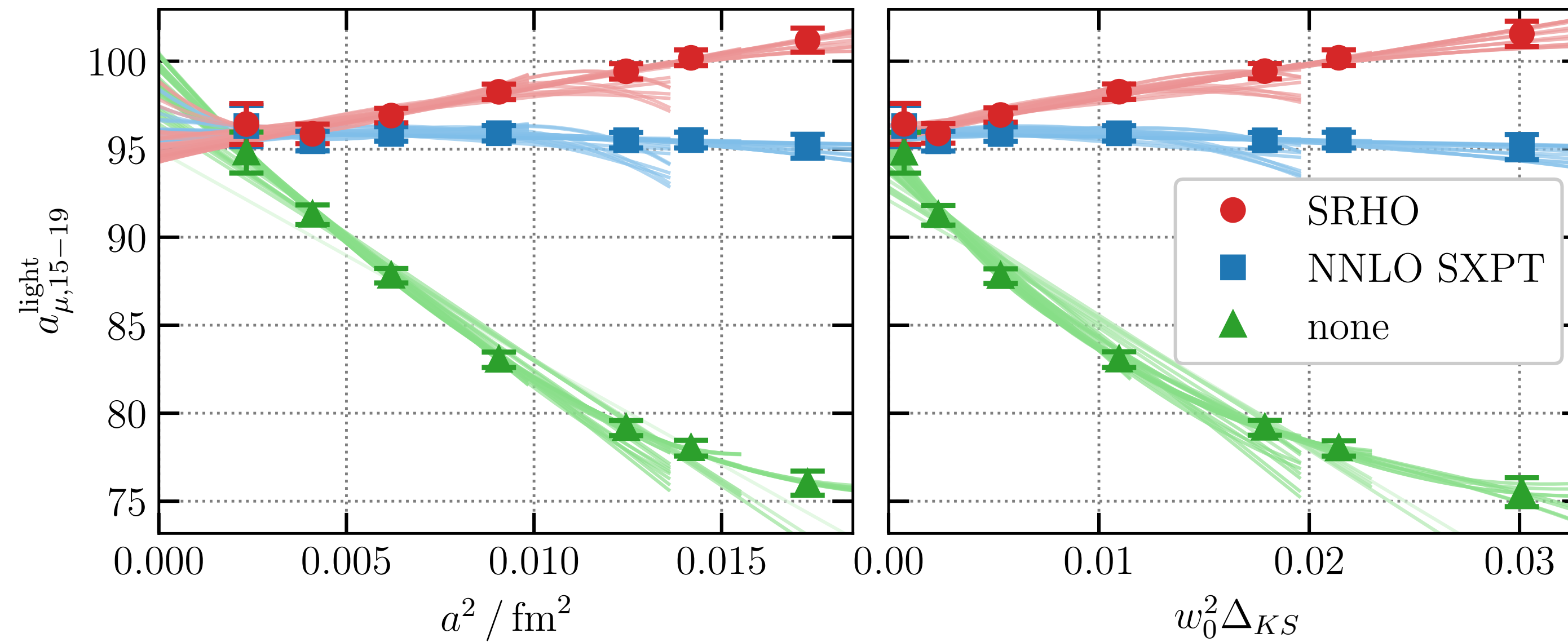
$$a_{\mu,00-04}^{\text{light}} \rightarrow a_{\mu,00-04}^{\text{light}} + a_{\mu,00-04}^{\text{tree}}(a=0) - a_{\mu,00-04}^{\text{tree}}(a)$$



Intermediate distance 04-10 window



15-19 (Aubin et al.'22) window



Analysis'2

Strategy for light and disconnected part

[Poster, B.Toth]

Analysis'2

[Poster, B.Toth]

Strategy for light and disconnected part

$$a_{\mu,00-28}^{\text{light+disc}} = \frac{9}{10} a_{\mu,04-28}^{\text{light}} + a_{\mu,04-28}^{I \approx 0} + a_{\mu,00-04}^{\text{light}} + a_{\mu,00-04}^{\text{disc}}$$

Analysis'2

[Poster, B.Toth]

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$a_{\mu,04-06}^{\text{light}} + a_{\mu,06-12}^{\text{light}} + a_{\mu,12-28}^{\text{light}}$: Joint fit

different windows \Leftrightarrow different functions

Analysis'2

[Poster, B.Toth]

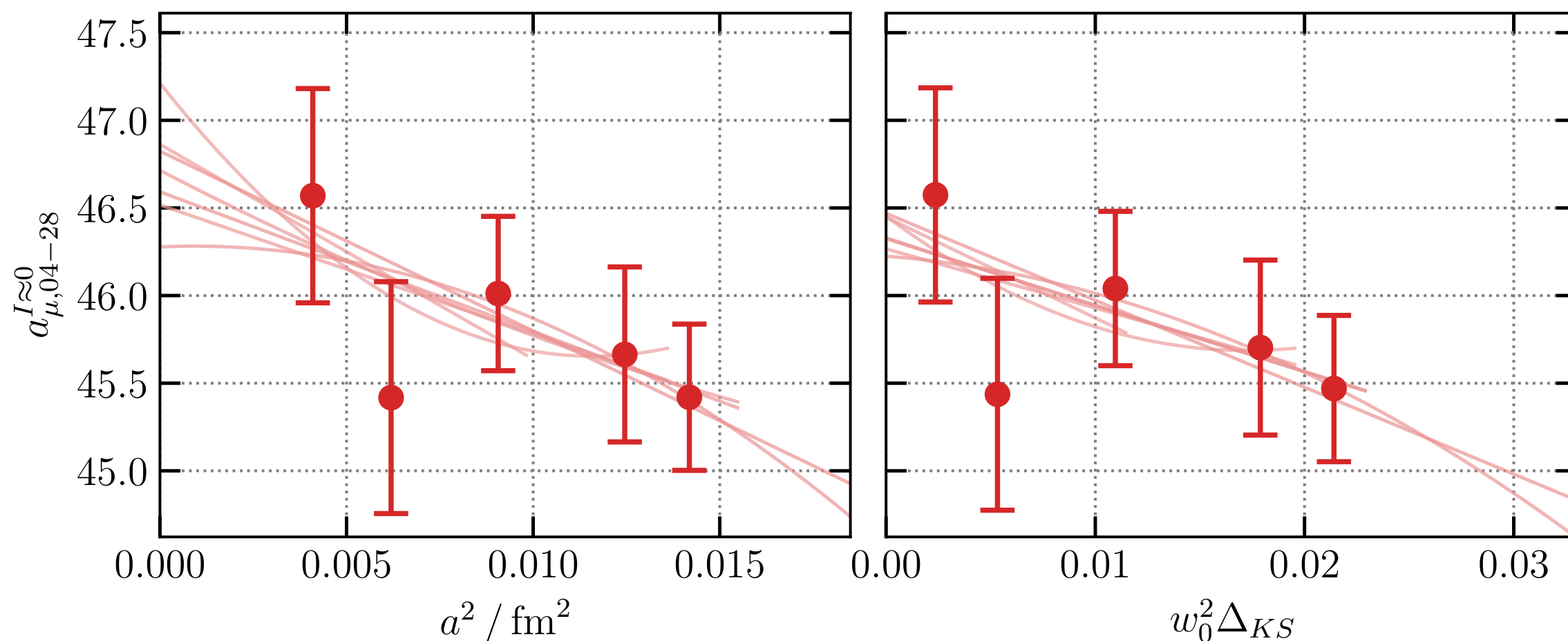
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different windows \Leftrightarrow different functions

Tiny taste-breaking, FV effects



Analysis'2

[Poster, B.Toth]

Strategy for light and disconnected part

Contains $a^2 \log(a^2/w_0^2)$

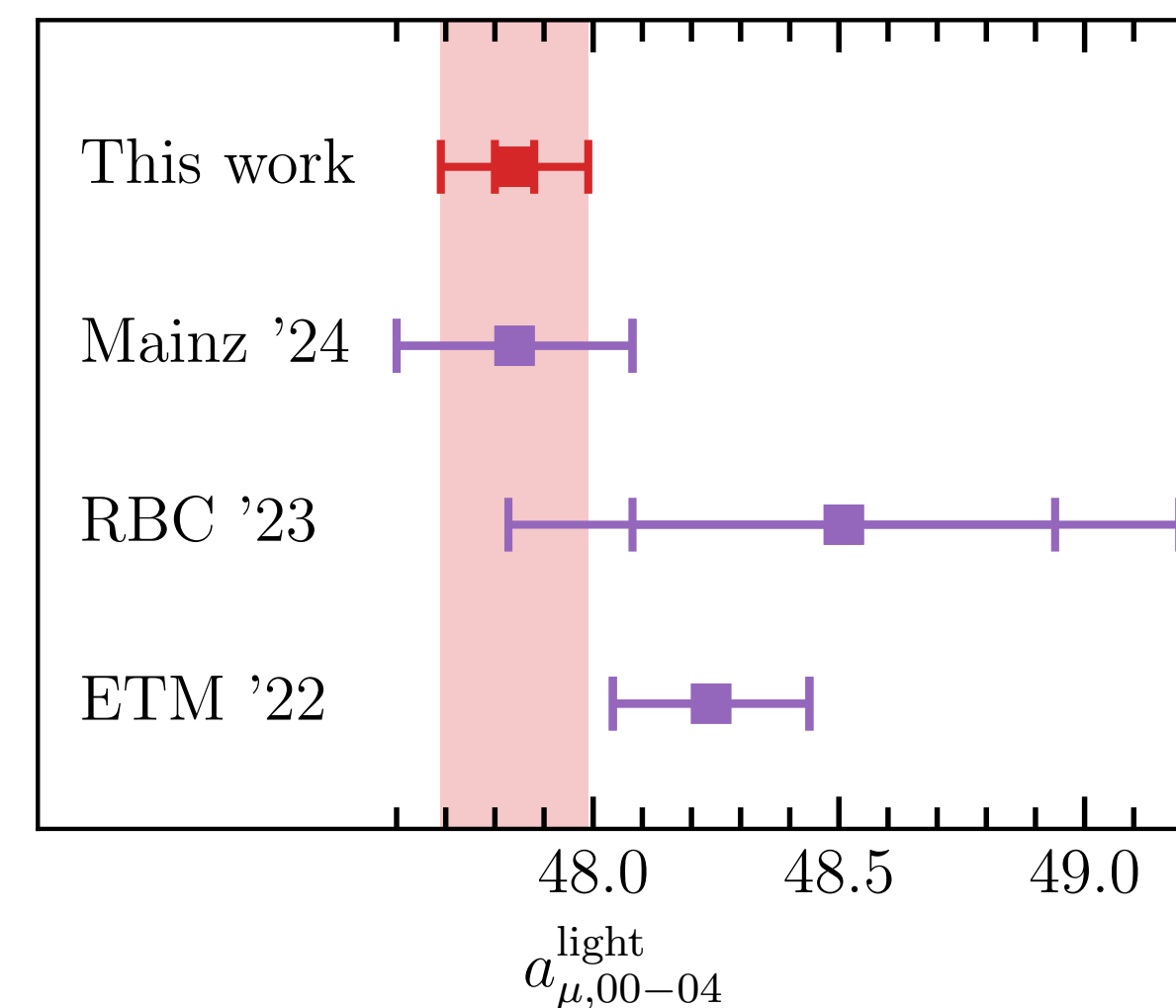
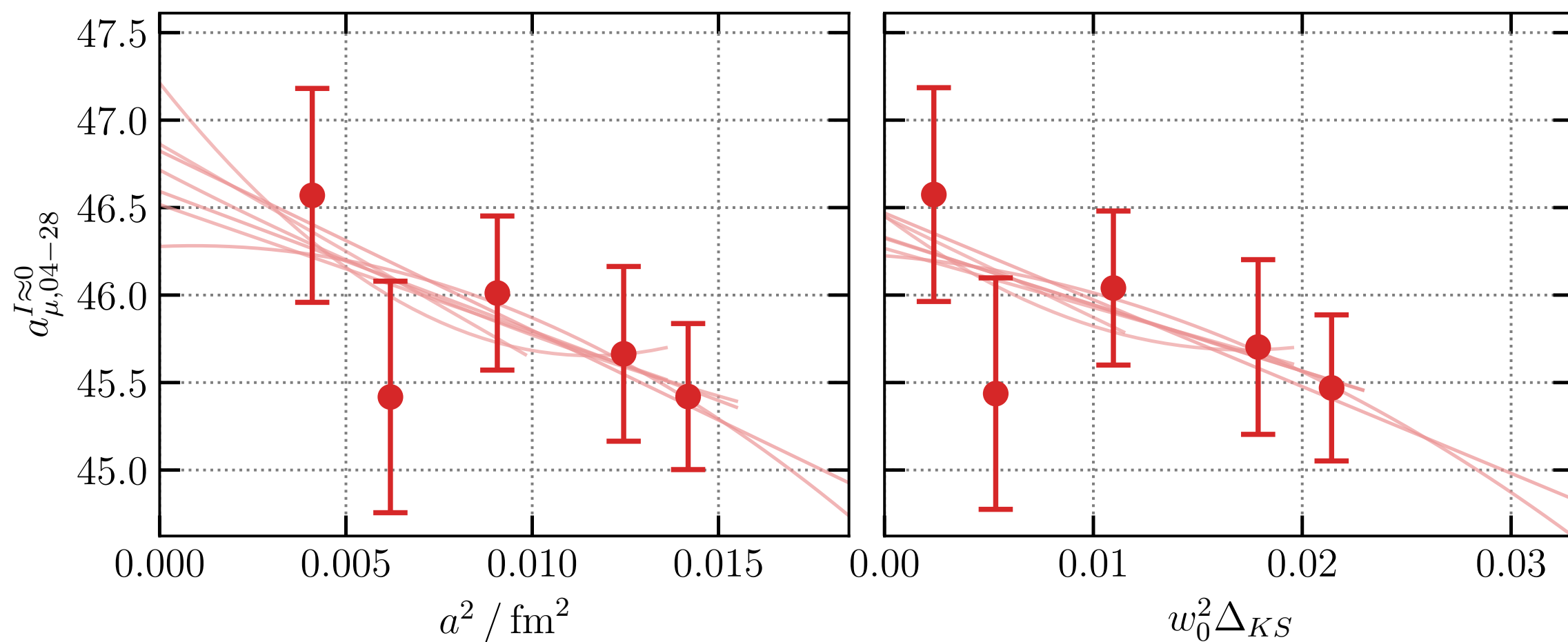
[Cè et al., 2021]

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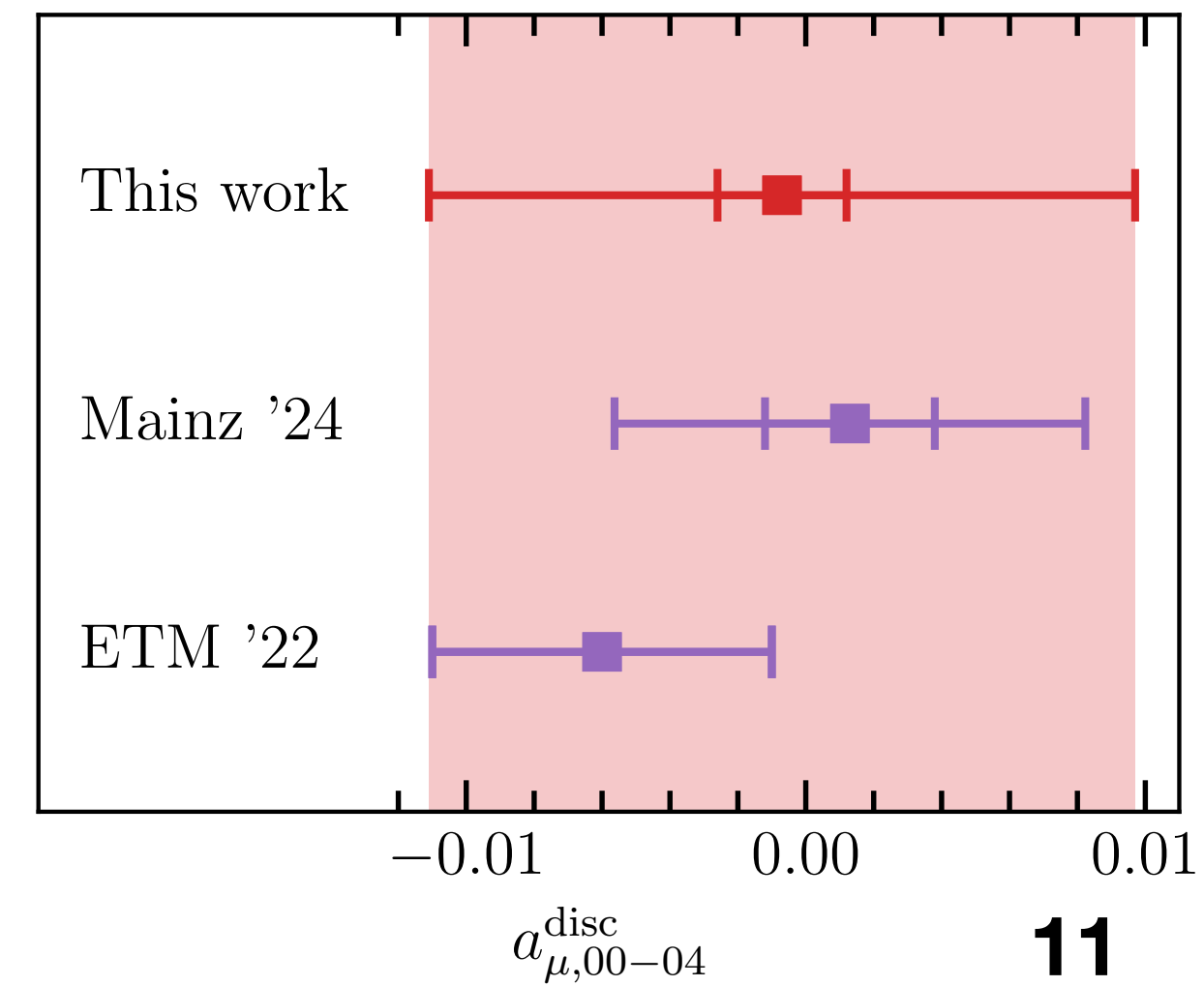
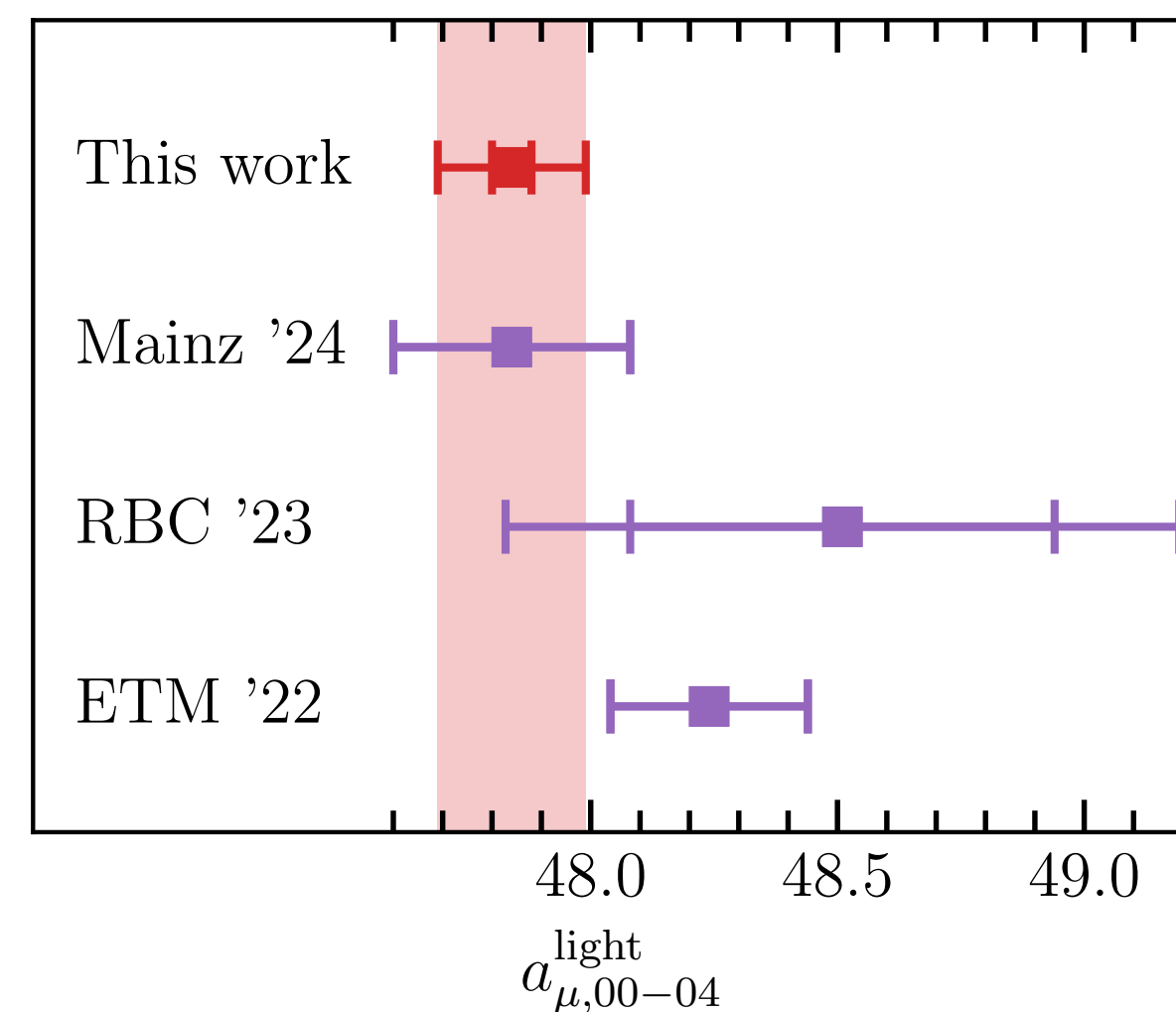
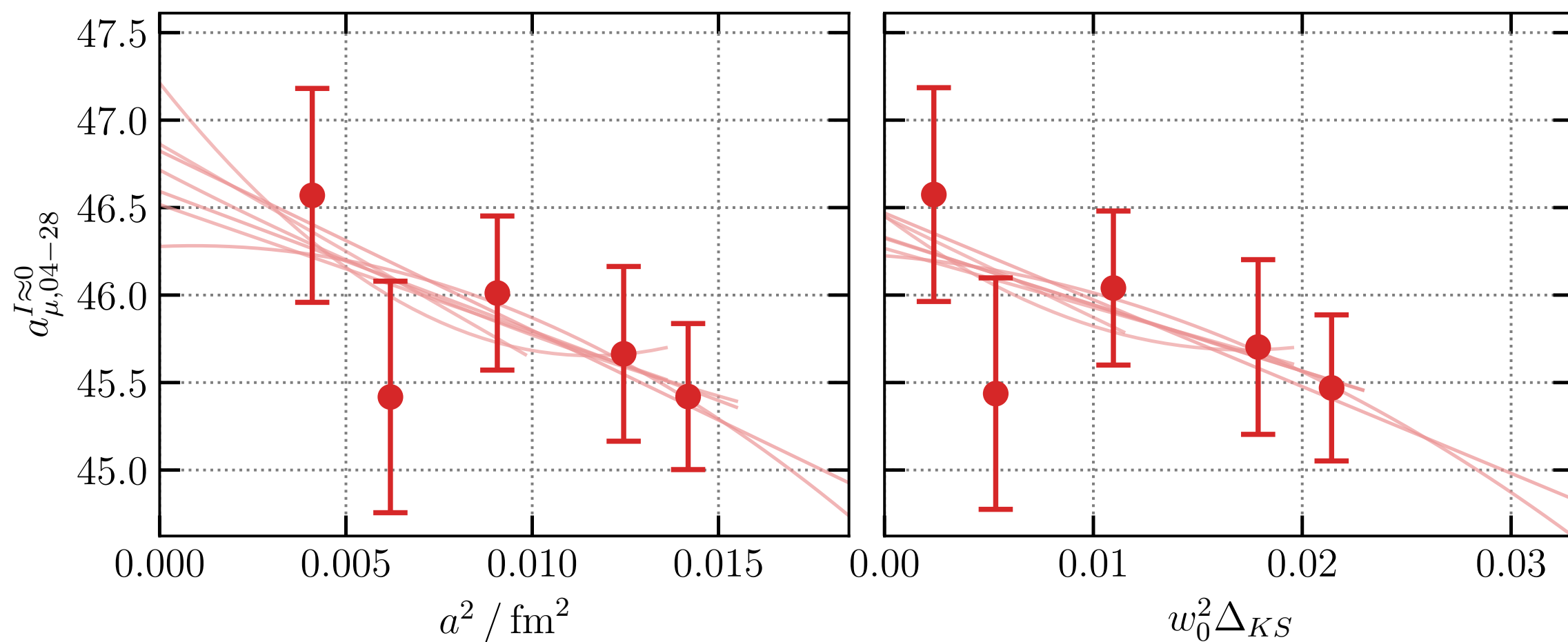
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Less than 0.02

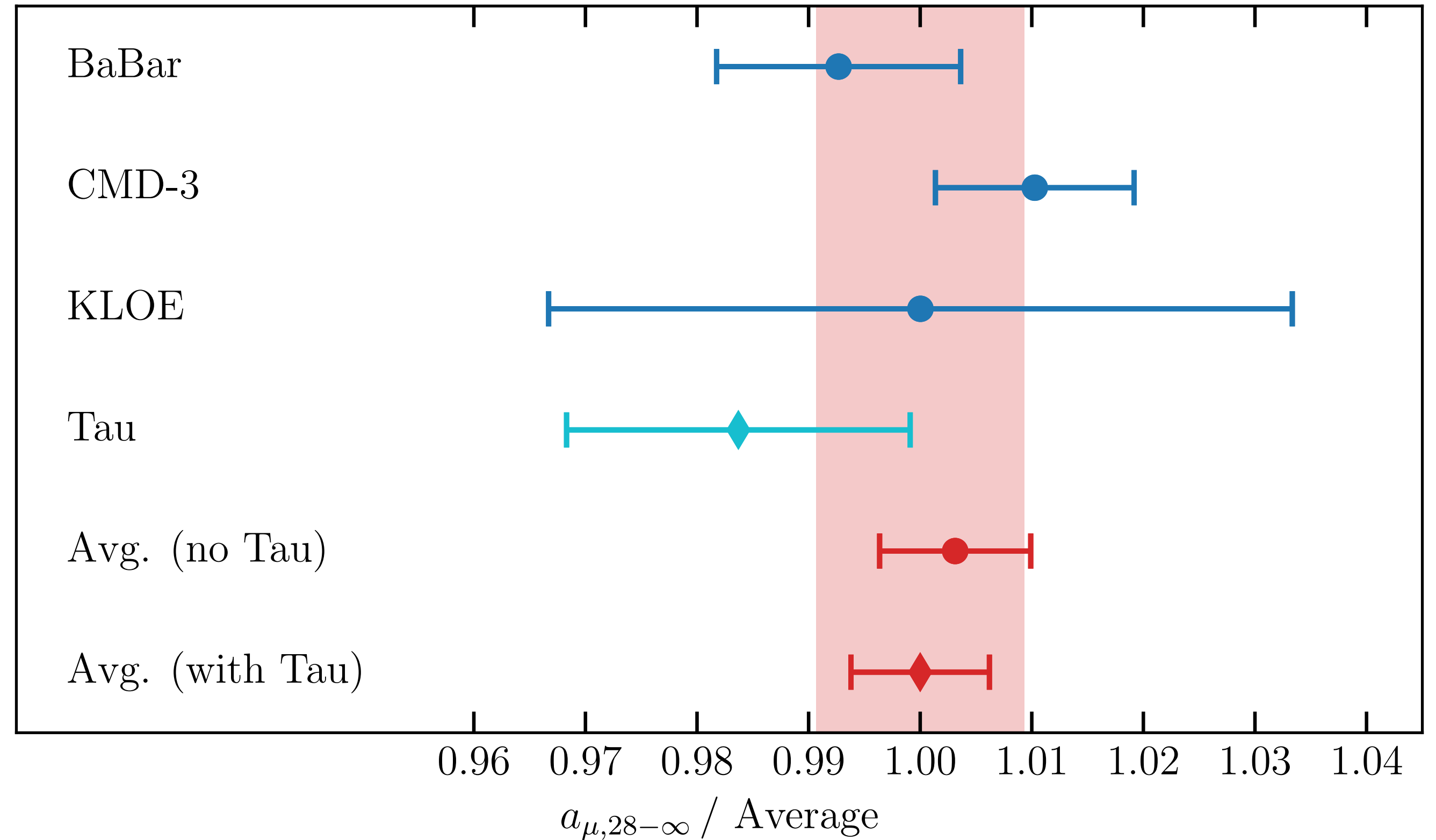
different windows \Leftrightarrow different functions

Tiny taste-breaking, FV effects



Input from data-driven analysis

$$a_{\mu}^{28-\infty} = 27.59(17)(9)[26]$$



[Poster, B.Toth]

Concluding remarks

- HVP contribution to muon g-2:
 - 4.6 ‰ precision
 - 0.9σ difference w/experiment
 - 4.0σ higher than WP'20
- ID window:
 - Agrees within all collaborations
 - Strong tension with data-driven results

