Checks on QED and strong-isospin breaking corrections to a_μ^{HVP}

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arXiv: 2407.10913

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Thursday 1st August, 2024

Talk by **Alessandro Lupo, Mo 29.07., 03:55 PM, Quark and lepton flavour physics**
Poster by **Balint Toth, Tue 30.07., 6:15 PM, Poster session**
Talk by **Andrey Kotov, Wed 31.07., 11:55 AM, Quark and lepton flavour physics**
Talk by **Gen Wang, Fri 02.08., 3:55 PM, Standard Model parameters**

$\mathrm{QCD}_{\mathrm{iso}}$ Ensembles: Landscape

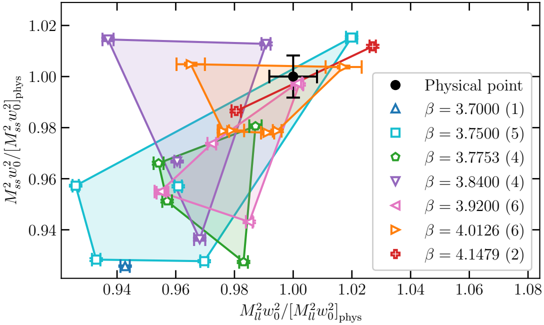
β	a [fm]	$L/a \times T/a$
3.7000	0.1315	48×64
3.7500	0.1191	56×96
3.7753	0.1116	56×84
3.8400	0.0952	64×96
3.9200	0.0787	80×128
4.0126	0.0640	96×144
4.1479	0.0483	128×192

Landscape of ensembles. Lattice spacings and volumes.

- Tree-level improved Symanzik gauge action[1], stout smeared[2] staggered fermion action ($n_{\mathrm{step}} = 4$, $\rho = 0.125$)
- $2+1+1$ dynamical quark flavours with $m_u = m_d = m_l$
- Ensembles with 7 lattice spacings from 0.1315 fm to 0.0483 fm
- Added finer lattice spacing with $a = 0.0483$ fm compared to previous work[3]

[1] Luscher and Weisz 1985.
[2] Morningstar and Peardon 2004.
[3] Borsanyi 2021.

QCD_{iso} Ensembles: Landscape



Landscape of ensembles. Black point: isospin-symmetric physical point.

- M_{qq}^2 mass of quark-connected pseudo-scalar meson[4]
- m_l and m_s scatter around physical point
- m_c from ratio[5] $m_c/m_s = 11.85$ within one per-cent of FLAG value[6]

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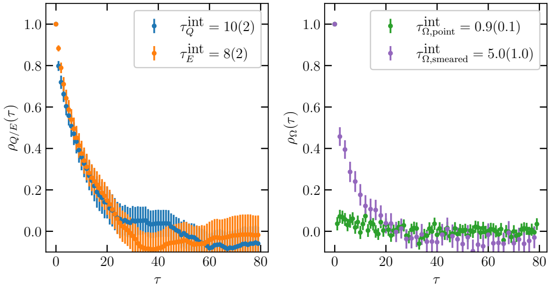
[4] Borsanyi 2013.
[5] McNeile et al. 2010.
[6] Aoki 2022.

$\mathrm{QCD_{iso}}$ Ensembles: Autocorrelation analysis



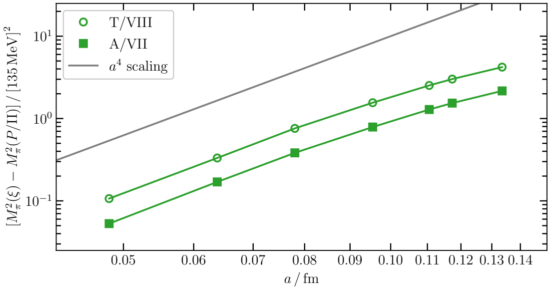
Normalised autocorrelation functions $\rho(\tau)$ and integrated autocorrelation times τ^{int} in units of configurations for the finest lattice with $\beta = 4.1479$

Autocorrelation analysis:

- Topological charge Q and energy density E evaluated at gradient flow time $t_{\mathrm{f}} = w_0^2$ corresponding to $r_{\mathrm{smear}} \approx 0.5\,\mathrm{fm}$
- Ω-baryon 2-pt function $C(t \approx 1.5\ \mathrm{fm})$ (plateau region of m_{eff}) for point and smeared sources
- $\tau^{\mathrm{int}} \leq 10$ for considered observables

$\Rightarrow$ Jackknife resampling with 48 blocks, block length $B \approx 5 \cdot \tau_{\mathrm{Q}}^{\mathrm{int}}$ at $\beta = 4.1479$

$\mathrm{QCD_{iso}}$ Ensembles: Taste violations



Taste violations as a function of lattice spacing a for axial-vector A and tensor T tastes.

Lattice artefacts related to taste symmetry violations of staggered fermions:

- Masses of pions $M_\pi^2(\xi) = M_{ll}^2 + \Delta_{KS}(\xi)$ computed at average valence quark mass $\frac{1}{3}(2m_l + m_s)$, $\xi \in \{P, A, T, V, I\}$ sixteen meson tastes of the taste $SU(4)$ group
- $M_{ll}^2 = M_\pi^2(P)$ squared mass of pseudo-Goldstone pion $\Rightarrow \Delta_{KS}(P) = 0$
- Taste violations decrease with approx. with a^4, consistent with $\alpha_s^3 a^2$
- $M_\pi(T) \approx 142\,\mathrm{MeV}$ at $\beta = 4.1479$

Physical point and isospin breaking decomposition

QCD+QED:

- Physical point defined by M_{π^0}, M_{K^0}, M_{K^+}, M_{Ω^-}, α
- Matching scheme: $\hat{M}^2 = \frac{1}{2}(M_{\mathrm{uu}}^2 + M_{\mathrm{dd}}^2)$, $\Delta M^2 = M_{\mathrm{dd}}^2 - M_{\mathrm{uu}}^2$, M_{ss}^2, w_0, α

  ΔM^2 measure for strong isospin breaking

Matching scheme allows to decompose observables into isospin symmetric and isospin breaking contributions:

- For isospin breaking decomposition $[O]_{\mathrm{phys}} = [O]_{\mathrm{iso}} + [O]_{\mathrm{sib}} + [O]_{\mathrm{qed}}$ study functional dependence of observable $O(\hat{M}w_0, M_{ss}w_0, \Delta M^2 w_0^2, e)$
- Physical value $[O]_{\mathrm{phys}} = O\left([\hat{M}w_0]_{\mathrm{phys}}, [M_{ss}w_0]_{\mathrm{phys}}, [\Delta M^2 w_0^2]_{\mathrm{phys}}, e\right)$
- Isospin-symmetric contribution: $[O]_{\mathrm{iso}} = O\left([\hat{M}w_0]_{\mathrm{phys}}, [M_{ss}w_0]_{\mathrm{phys}}, 0, 0\right)$
- Pure QCD contribution: $[O]_{\mathrm{qcd}} = O\left([\hat{M}w_0]_{\mathrm{phys}}, [M_{ss}w_0]_{\mathrm{phys}}, [\Delta M^2 w_0^2]_{\mathrm{phys}}, 0\right)$.
- Strong isospin breaking contribution $[O]_{\mathrm{sib}} = [O]_{\mathrm{qcd}} - [O]_{\mathrm{iso}}$
- Electromagnetic contribution: $[O]_{\mathrm{qed}} = [O]_{\mathrm{phys}} - [O]_{\mathrm{qcd}}$

Physical values of $M_{\rm uu}$, $M_{\rm dd}$, $M_{\rm ss}$ and w_0

- In partially-quenched χPT + photons[7]: $\hat{M}^2 = \frac{1}{2}(M_{\rm uu}^2 + M_{\rm dd}^2) = M_{\pi^0}^2 + \text{NLO in IB}$

$$[\hat{M}]_{\rm phys} = 134.9768(5)\ \text{MeV}.$$

- Physical values of meson masses[8]:

$$[\Delta M^2]_{\rm phys} = 13170(320)(270)[420]\ \text{MeV}^2 \qquad [M_{ss}]_{\rm phys} = 689.89(28)(40)[49]\ \text{MeV}$$

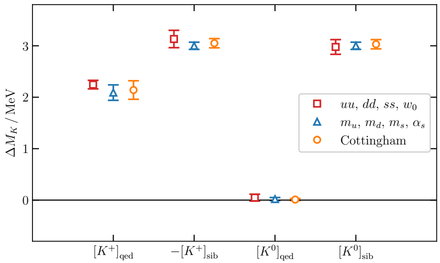
- Update on gradient-flow scale w_0 adding the 0.048 fm lattice spacing :

$$[w_0]_{\rm phys} = 0.17245(22)(46)[51]\ \text{fm}$$

More details → Talk by **Gen Wang, Friday 02.08., 3:55 PM, BSM Session**

[7] Bijnens and Danielsson 2007.
[8] Borsanyi 2021.

Matching scheme ambiguity: Kaon mass decomposition



Decomposition of the neutral and charged kaon masses in three different schemes.

$$
\begin{aligned}
[M_{K^{0/+}}]_{\text{iso}} &= 494.55(31)\ \text{MeV}\\
[M_{K^0}]_{\text{sib}} &= +2.98(14)\ \text{MeV}\\
[M_{K^0}]_{\text{qed}} &= 0.05(7)\ \text{MeV}\\
[M_{K^+}]_{\text{sib}} &= -3.13(17)\ \text{MeV}\\
[M_{K^+}]_{\text{qed}} &= 2.25(8)\ \text{MeV}
\end{aligned}
$$

Decomposition in the $\{M_{\text{uu}}, M_{\text{dd}}, M_{\text{ss}}, w_0\}$ scheme.

Agreement on decomposition of M_K indicates equivalence of schemes ($\Delta M_\pi \propto \alpha$ at LO):

- This work[9]: $\{M_{\text{uu}}, M_{\text{dd}}, M_{\text{ss}}, w_0\}$
- Gasser-Rusetsky-Scimemi (GRS) scheme[10]: $\{m_{\text{u}}, m_{\text{d}}, m_{\text{s}}, \alpha_{\text{s}}\}$ in $\overline{\text{MS}}$ at $\mu = 2\,\text{GeV}$
- Cottingham-formula based decomposition[11]: Relates electromagnetic self-energy to forward Compton tensor

$\Rightarrow$ Good agreement with GRS and Cottingham-formula based schemes

[9] Borsanyi 2021.
[10] Di Carlo et al. 2019.
[11] Stamen et al. 2022.

Verification of IB contributions: Previous setup

Computation of strong-isospin breaking (SIB) and the valence QED contributions to a_μ^{light}:

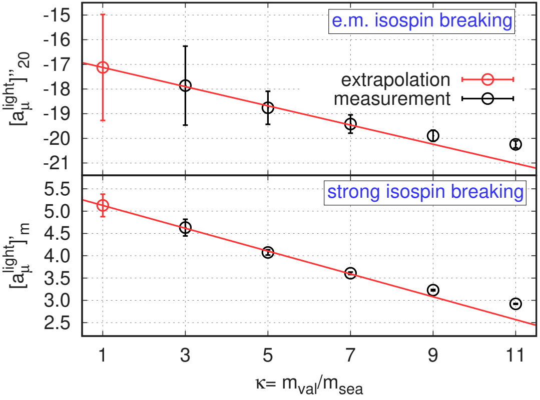
$$[a_\mu^{\text{light}}]'_m \equiv m_l \left. \frac{\partial [a_\mu^{\text{light}}]}{\partial\, \delta m} \right|_{\delta m = 0}$$

$$[a_\mu^{\text{light}}]''_{20} \equiv \frac{1}{2} \left. \frac{\partial^2 [a_\mu^{\text{light}}]}{\partial e_v^2} \right|_{e_v = 0}$$

$\delta m \equiv m_d - m_u$
$m_l \equiv \frac{1}{2}(m_u + m_d)$
e_v charge of valence quarks

Previous setup[12] based on chiral extrapolation:

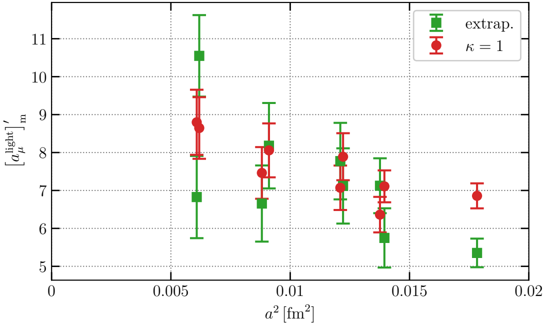
- Measurements at valence quark mass $\kappa \cdot m_l$ with $\kappa = 3, 5, 7, 9, 11$
- Linear chiral extrapolation to $\kappa = 1$ based on $\kappa = 3, 5, 7$



Extrapolation procedure for $[a_\mu^{\text{light}}]'_m$ and $[a_\mu^{\text{light}}]''_{20}$ for $\beta = 3.7000$

[12] Borsanyi 2021.

Verification of IB contributions: Light connected SIB



Comparison between $[a_\mu^{\text{light}}]'_m$ based on chiral extrapolation and LMA

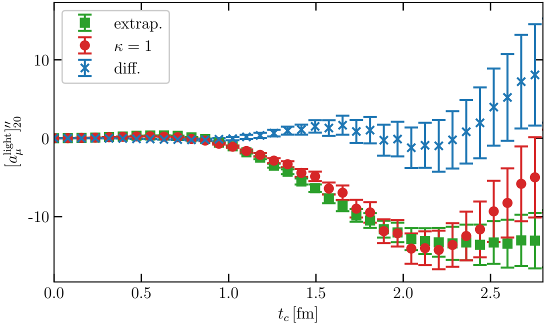
Computation of $[a_\mu^{\text{light}}]'_m$ with exact mass derivative:

- Previous work[13]: Chiral extrapolation
- Now: Low Mode Averaging (LMA) technique at $\kappa = 1$

$\Rightarrow$ New computation confirms previous results

[13] Borsanyi 2021.

Isospin breaking effects: Light connected valence QED



$[a_\mu^{\text{light}}]''_{20}$ on a single configuration at $a = 0.0787\,\text{fm}$ as function of the upper limit of integration t_c.

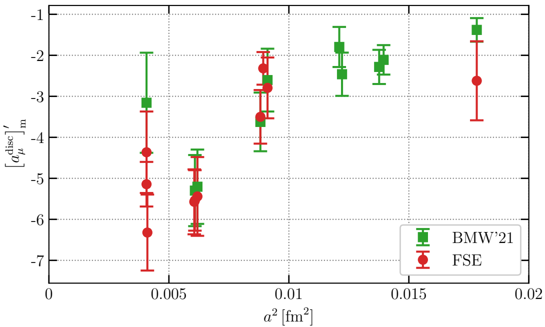
Computation of $[a_\mu^{\text{light}}]''_{20}$ based on discrete derivative ($e_v = +\frac{1}{3}e, 0, -\frac{1}{3}e$):

- Previous work[14]: Chiral extrapolation
- Now: Low-mode averaging (LMA) at $\kappa = 1$

$\Rightarrow$ Checked extrapolation procedure on single QCD and QED configuration at $a = 0.0787\,\text{fm}$

[14] Borsanyi 2021.

Verification of IB contributions: Disconnected SIB



Comparison between standard stochastic estimator and FSE of SIB to $(a_\mu^{\rm disc})_{\rm sib}$.

Computation of $(a_\mu^{\rm disc})_{\rm sib}$:

- Previous work[15]: Discrete derivative $(1.0 \cdot m_l$ and $0.9 \cdot m_l)$ with 3000 random source vectors
- Now: Exact mass derivative, one-end trick (split-even estimator)[16] with 128 random source vectors

$\Rightarrow$ Comparable results at reduced cost

[15] Borsanyi 2021.

[16] Boucaud 2008.

Summary

- Update of previous work[17]
- Added two ensembles with lattice spacing $a = 0.0483\,\text{fm}$
- Autocorrelation analysis
- Isospin breaking decomposition of M_K consistent with GRS and Cottingham formula based schemes
  $\Rightarrow$ strong indicator of equivalence of schemes (at current level of precision)
- Improved and verified computation of leading isospin breaking corrections to a_μ^{HVP}

[17] Borsanyi 2021.

References I

Y. Aoki et al. "FLAG Review 2021". In: *Eur. Phys. J. C* 82.10 (2022), p. 869. DOI: 10.1140/epjc/s10052-022-10536-1. arXiv: 2111.09849 [hep-lat].

Johan Bijnens and Niclas Danielsson. "Electromagnetic Corrections in Partially Quenched Chiral Perturbation Theory". In: *Phys. Rev. D* 75 (2007), p. 014505. DOI: 10.1103/PhysRevD.75.014505. arXiv: hep-lat/0610127.

Sz. Borsanyi et al. "Isospin splittings in the light baryon octet from lattice QCD and QED". In: *Phys. Rev. Lett.* 111.25 (2013), p. 252001. DOI: 10.1103/PhysRevLett.111.252001. arXiv: 1306.2287 [hep-lat].

Sz. Borsanyi et al. "Leading hadronic contribution to the muon magnetic moment from lattice QCD". In: *Nature* 593.7857 (2021), pp. 51–55. DOI: 10.1038/s41586-021-03418-1. arXiv: 2002.12347 [hep-lat].

Philippe Boucaud et al. "Dynamical Twisted Mass Fermions with Light Quarks: Simulation and Analysis Details". In: *Comput. Phys. Commun.* 179 (2008), pp. 695–715. DOI: 10.1016/j.cpc.2008.06.013. arXiv: 0803.0224 [hep-lat].

M. Di Carlo et al. "Light-meson leptonic decay rates in lattice QCD+QED". In: *Phys. Rev. D* 100.3 (2019), p. 034514. DOI: 10.1103/PhysRevD.100.034514. arXiv: 1904.08731 [hep-lat].

M. Luscher and P. Weisz. "On-shell improved lattice gauge theories". In: *Commun. Math. Phys.* 98.3 (1985). [Erratum: Commun.Math.Phys. 98, 433 (1985)], p. 433. DOI: 10.1007/BF01205792.

C. McNeile et al. “High-Precision c and b Masses, and QCD Coupling from Current-Current Correlators in Lattice and Continuum QCD”. In: *Phys. Rev. D* 82 (2010), p. 034512. DOI: 10.1103/PhysRevD.82.034512. arXiv: 1004.4285 [hep-lat].

Colin Morningstar and Mike J. Peardon. “Analytic smearing of SU(3) link variables in lattice QCD”. In: *Phys. Rev. D* 69 (2004), p. 054501. DOI: 10.1103/PhysRevD.69.054501. arXiv: hep-lat/0311018.

Dominik Stamen et al. “Kaon electromagnetic form factors in dispersion theory”. In: *Eur. Phys. J. C* 82.5 (2022), p. 432. DOI: 10.1140/epjc/s10052-022-10348-3. arXiv: 2202.11106 [hep-ph].