The connected isospin-violating part of the hadronic vacuum polarisation

Dominik Erb
Volodymyr Biloshytskyi, Antoine Gerardin, Franziska Hagelstein, Harvey Meyer, Julian Parrino, Vladimir Pascalutsa

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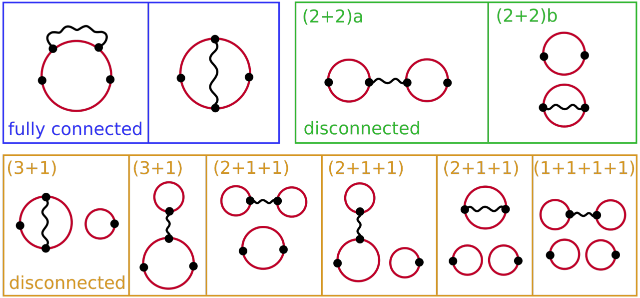
QED Corrections to the HVP

$$a_\mu^{HVP,NLO} = -\frac{e^2}{2}\int_{x,y,z} H_{\lambda\sigma}(z)\delta_{\mu\nu}\,[G(x-y)]_\Lambda \,\langle j_\lambda(z) j_\mu(x) j_\nu(y) j_\sigma(0)\rangle + c.t. \qquad (1)$$

Covariant coordinate-space (CCS) kernel: $H^{TL}_{\lambda\sigma}(z) = \left(-\delta_{\lambda\sigma} + 4\frac{z_\lambda z_\sigma}{z^2}\right)\mathcal{H}_2(|z|)$

Pauli-Villars (PV) regulated photon propagator: $[G(y)]_\Lambda = \frac{1}{4\pi^2|y|^2} - \frac{\Lambda K_1\left(\Lambda\frac{|y|}{\sqrt{2}}\right)}{2\sqrt{2}\pi^2|y|} + \frac{\Lambda K_1(\Lambda|y|)}{4\pi^2|y|}$



The connected isospin-violating Part

$$a_\mu^{HVP,NLO,38} = -\frac{e^2}{2}\int_{x,y,z} H_{\lambda\sigma}(z)\delta_{\mu\nu}\left[G(x-y)\right]_\Lambda \langle j_\lambda^3(z) j_\mu^{em}(x) j_\nu^{em}(y) j_\sigma^8(0)\rangle + c.t. \quad (2)$$

$j_\mu^3(z)$: Isovector current, $j_\mu^8(z)$: Isoscalar current

$\langle j_\lambda^3(z) j_\mu^{em}(x) j_\nu^{em}(y) j_\sigma^8(0)\rangle = -2\mathcal{Q}\,\mathrm{Re}[\,2\mathrm{Tr}($  $) + \mathrm{Tr}($  $)\,]$ (3)

$$c.t. = -\frac{\Delta m_K^{em} - \Delta m_K^{phys}}{\langle K^+|\bar{u}u - \bar{d}d|K^+\rangle}\frac{\partial a_\mu^{HVP}}{\partial m_l} \quad (4)$$

Here the charge factor $\mathcal{Q} = 1/36$ and $\Delta m_K^{phys} = -3.934$ MeV

All calculations are done at the SU(3) symmetric point

$\rightarrow$ For final result only (2+2)a diagram needs to be added

Used CLS Ensembles

	N300	N202	H200	B450	H101
β	3.70	3.55	3.55	3.46	3.40
size	$48^3 \cdot 128$	$48^3 \cdot 128$	$32^3 \cdot 96$	$32^3 \cdot 64$	$32^3 \cdot 96$
a (fm)	0.04981	0.06426	0.06426	0.07634	0.08636
m_π (MeV)	421(5)	412(5)	416(5)	417(5)	416(4)
$m_\pi L$	5.1	6.4	4.3	5.2	5.8
L (fm)	2.4	3.1	2.1	2.4	2.8

Overview over Calculations

- Connected LbL Contribution
  - Crosscheck with QED
  - QCD Calculations
- The Counterterm
  - The Kaon Mass Splitting with a PV Cutoff Λ
    - Computational Strategy
    - Large PV-mass Behavior
  - Light-quark mass Derivative of the Kaon Mass and HVP
- Continuum/ PV-mass Extrapolation of $a_\mu^{HVP,NLO,38}$ (connected Part)

$$-\frac{e^2}{2}\int_{x,y,z} H_{\lambda\sigma}(z)\delta_{\mu\nu}\left[G(x-y)\right]_\Lambda \langle j^3_\lambda(z) j^{em}_\mu(x) j^{em}_\nu(y) j^8_\sigma(0)\rangle \tag{5}$$

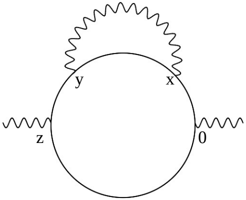


Fig. 2.a)
Self-energy diagram

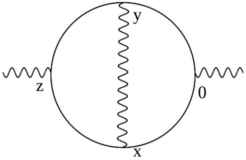


Fig. 2.b)
2-Loop diagram

$$j^l_\mu(z) = \bar{q}_z \gamma_\mu q_z \tag{6}$$

$$j^c_\mu(z) = \frac{1}{2}\left[\bar{q}_z(\gamma_\mu + 1)U^\dagger_{\mu,z}\ q_{z+\hat{\mu}} + \bar{q}_{z+\hat{\mu}}(\gamma_\mu - 1)U_{\mu,z}\ q_z\right] \tag{7}$$

Crosscheck with QED

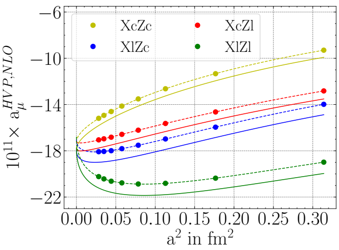


Fig. 3.a)
Self-energy

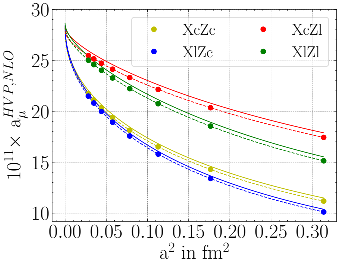


Fig. 3.b)
2-Loop

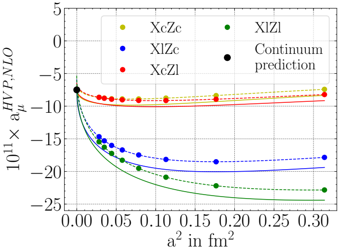


Fig. 3.c)
Total=2×SE+2Loop

Fit function: $f_{fit}(\mathbf{a}) = b + c \cdot \mathbf{a} + d \cdot \mathbf{a}^2 + e \cdot \mathbf{a}^3$, PV-mass: $\Lambda = 3 \cdot m_\mu$

Table: The values are given in units of 10^{-11}. The expected value is -7.5 $\cdot 10^{-11}$.

	XlZl	XcZl	XlZc	XcZc
Total	-6.90	-7.36	-7.44	-7.56

QCD Calculation

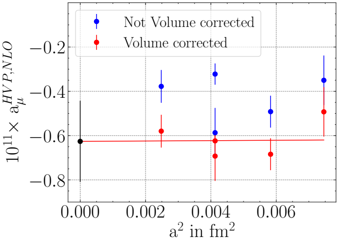


Fig. 4.a)
Self-energy

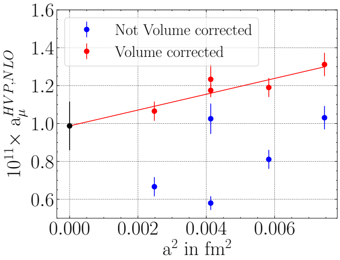


Fig. 4.b)
2-Loop

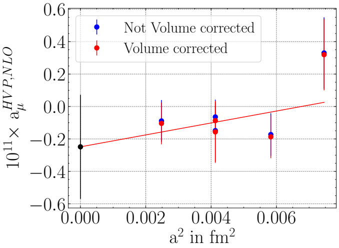


Fig. 4.c)
Total=2×SE+2Loop

Fit function: $f_{fit}(\mathbf{a}, m_\pi L) = b + c \cdot \mathbf{a}^2 + d \cdot e^{-\frac{m_\pi L}{2}}$, PV-mass: $\Lambda = 16 \cdot m_\mu$

Continuum values from fit:

Self-Energy:	2-Loop:	Total:
$(-0.63 \pm 0.19) \cdot 10^{-11}$	$(0.99 \pm 0.13) \cdot 10^{-11}$	$(-0.25 \pm 0.33) \cdot 10^{-11}$

The Kaon Mass Splitting with a PV Cutoff Λ

$$\Delta m_K^{em}(\Lambda) = (m_{K^+} - m_{K^0})(\Lambda)$$

At SU(3) point only two diagrams contribute:

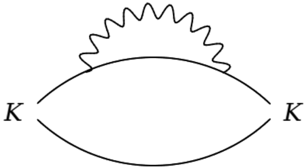


Fig. 5.a)
Asymmetric diagram

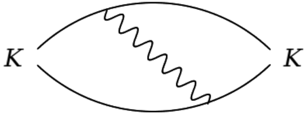


Fig. 5.b)
Symmetric diagram

Known analytic large distance (elastic) behavior
$\rightarrow$ Use lattice data only for short distance part

Computational Strategy

1. Compute diagrams for different source-sink separation times
2. Restrict lattice data to short distance part
3. Extrapolate to infinite separation times and zero lattice spacing
4. Add long-distance part using the kaon e.m. form factor
5. Repeat for different PV-masses ($\Lambda/m_\mu \in [3, 5, 8, 10, 16, 20, 32, 50, 64]$)

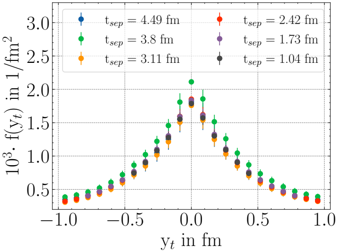


Fig. 6.a)
Data limited to $y_t < 1$ fm

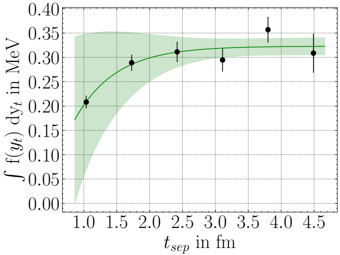


Fig. 6.b)
Extrapolation of left data

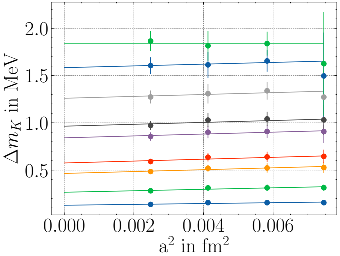


Fig. 6.c)
Continuum extrapolation

Large PV-Mass Behavior

Use Operator Product Expansion [2209.02149] to predict divergent terms for $\Lambda \to \infty$:

$$(m_{K^+} - m_{K^0})_{QED}(\Lambda) \approx \frac{3\alpha}{2\pi} \log\left(\frac{\Lambda}{\mu_{IR}}\right)(Q_u^2 - Q_d^2) m_l \frac{\partial m_K}{\partial m_l} \quad (8)$$

$$= \mathcal{C} \log\left(\frac{\Lambda}{\mu_{IR}}\right), \ \mathcal{C} \approx 0.12 \text{ MeV}$$

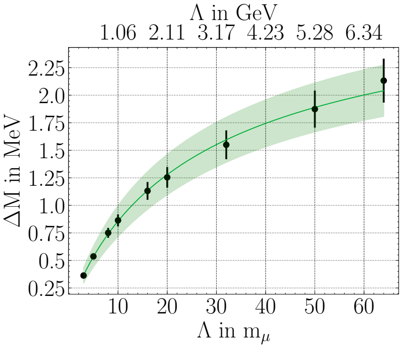


Fig. 7.a)
PV-mass extrapolation

Fit function:

$$f_{fit}(\Lambda) = a \cdot \frac{\Lambda}{\Lambda + d} + \mathcal{C} \cdot \log\left(\frac{\Lambda + b}{b}\right) \quad (9)$$

Reproduces expected behavior for $\Lambda \to \infty$ and $\Lambda \to 0$

Average χ^2 of 0.055

Light-quark mass Derivative of the Kaon Mass and HVP (Counterterm)

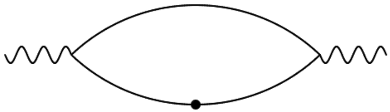


Fig. 8.a)
HVP mass insertion

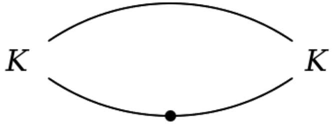


Fig. 9.a)
Kaon propagator mass insertion

$$\frac{\partial a_\mu^{HVP}}{\partial m_l} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_t\; w(x_t) G(x_t)$$

With the TMR kernel $w(x_t)$

This calculation uses stochastic wall sources

Get matrix element $\langle K^+|\bar{u}u - \bar{d}d|K^+\rangle$ with constant fit method

LO ChPT prediction:
$m_l\langle K^+|\bar{u}u - \bar{d}d|K^+\rangle \approx \frac{m_\pi^2}{4m_K} \approx 104$ MeV
Lattice result: (102 ± 1) MeV

Continuum/ PV-Mass Extrapolation of $a_\mu^{HVP,NLO,38}$ (connected Part)

Fit function:
$f_{fit}(\mathbf{a}, m_\pi L) = b + c \cdot \mathbf{a}^2 + d \cdot e^{-\frac{m_\pi L}{2}}$

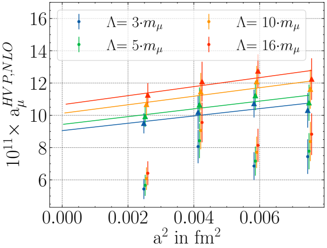


Fig. 10.a)
Continuum extrapolation

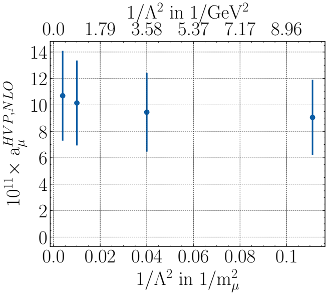


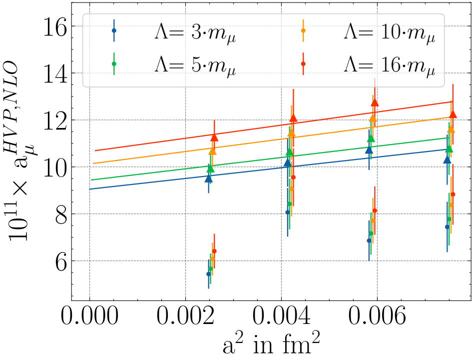
Fig. 10.b)
PV-mass extrapolation

Λ in m_μ	3	5	10	16
$1/\Lambda^2$ in $1/m_\mu^2$	0.11	0.04	0.01	0.004
$10^{11} \cdot a_\mu^{HVP,NLO,38}(\Lambda)$	9.07 ± 2.86	9.52 ± 2.99	10.12 ± 3.23	10.48 ± 3.44

Summary and Outlook

Complete SU(3) Calculation:

Connected:	$-0.05(30) \cdot 10^{-11}$
Disconnected:	$-0.56(12) \cdot 10^{-11}$ (N202 only)
Counterterm:	$10.42(2.34) \cdot 10^{-11}$
Result:	$9.74(2.36) \cdot 10^{-11}$



Outlook:

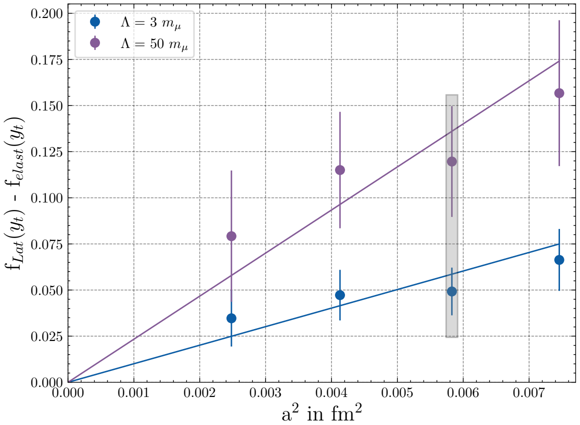
- H200 lattice for other PV-masses
- Additional (bigger) PV-mass
- Additional (smaller) lattice spacing
- Going to the physical point

Additional Slides

	N300	N202	H200	B450	H101
β	3.70	3.55	3.55	3.46	3.40
size	$48^3 \cdot 128$	$48^3 \cdot 128$	$32^3 \cdot 96$	$32^3 \cdot 64$	$32^3 \cdot 96$
a (fm)	0.04981	0.06426	0.06426	0.07634	0.08636
a (1/GeV)	0.252	0.326	0.326	0.390	0.438
m_π (MeV)	421(5)	412(5)	416(5)	417(5)	416(4)
$m_\pi L$	5.1	6.4	4.3	5.2	5.8
L (fm)	2.4	3.1	2.1	2.4	2.8

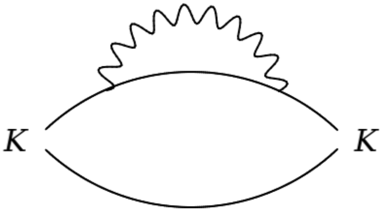
$16 \cdot m_\mu \cdot a_{H101} = 0.736$

Difference between Elastic Part and Lattice Data



Kaon mass splitting values

Λ	Lattice part in MeV	$I(t_c, \Lambda)$ in MeV	Total in MeV
3	0.128 ± 0.014	0.1174	0.363 ± 0.014
5	0.266 ± 0.027	0.1355	0.537 ± 0.027
8	0.465 ± 0.046	0.1430	0.751 ± 0.046
10	0.575 ± 0.055	0.1443	0.864 ± 0.055
16	0.841 ± 0.081	0.1450	1.131 ± 0.081
20	0.965 ± 0.094	0.1450	1.255 ± 0.094
32	1.260 ± 0.131	0.1450	1.550 ± 0.131
50	1.584 ± 0.169	0.1450	1.874 ± 0.169
64	1.842 ± 0.200	0.1450	2.132 ± 0.200



$$V_\mu(x)V_\nu(0) \sim \frac{1}{|x|^6} + \frac{\mathcal{O}_3}{|x|^3} + \frac{\mathcal{O}_4}{|x|^2} + \dots$$

- First term irrelevant for this kind of correlator
- Second term only appears because of lattice
- Third term is first relevant term for continuum value

$$\int d^3\vec{x}\ V_\mu(x_0,\vec{x})V_\nu(0)\cdot\delta_{\mu,\nu}G_\Lambda(x_0,\vec{x}) \overset{x_0\to 0}{\to} C_1 - C_2\ln(x_0)$$

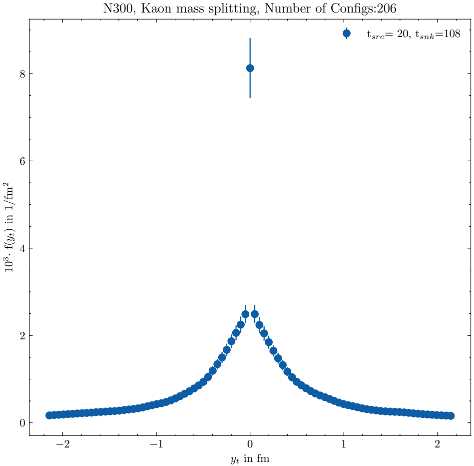


Fig. 14.a)
$y_t = 0$ from lattice calculation.

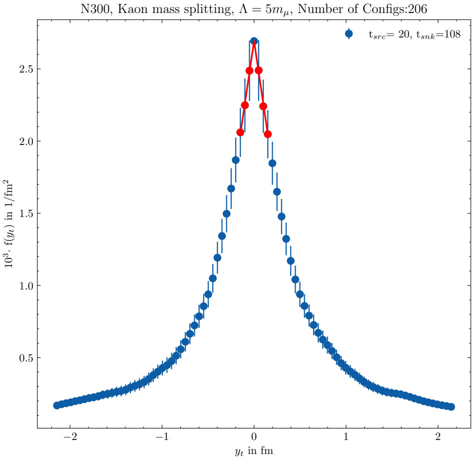
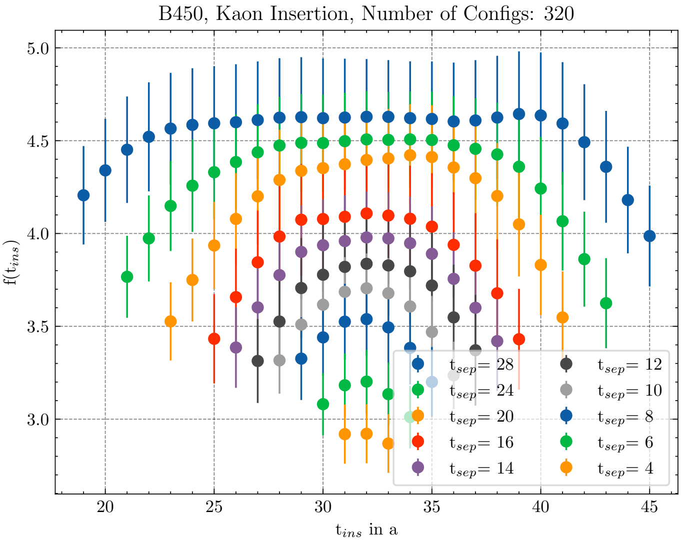


Fig. 14.b)
$y_t = 0$ from linear fit.

Kaon Mass Insertion



Variance composition of $a_\mu^{HVP,NLO,38}$