University of Colorado
Boulder

DOE Award No DE-SC0024053
lgt4hep.github.io

DOE Award No.
DE-SC0010005

Strong isospin breaking correction to HVP for the muon g-2

Jake Sitison
Fermilab Lattice, HPQCD, MILC Collaboration
2024 Lattice Conference

Acknowledgements

Fermilab Lattice and MILC

- Alexei Bazavov
- Pietro Butti
- David Clarke
- Carleton DeTar
- Aida El-Khadra
- Elvira Gámiz
- Steven Gottlieb
- Anthony Grebe
- Leon Hostetler
- William Jay
- Hwancheol Jeong
- Andreas Kronfeld
- Shaun Lahert
- Michael Lynch
- Andrew Lytle
- Ethan Neil
- Curtis Peterson
- James Simone
- Jacob Sitison
- Ruth Van de Water
- Alejandro Vaquero
- Shuhei Yamamoto

HPQCD

- Christine Davies
- Peter Lepage
- Craig McNeile
- Gaurav Ray

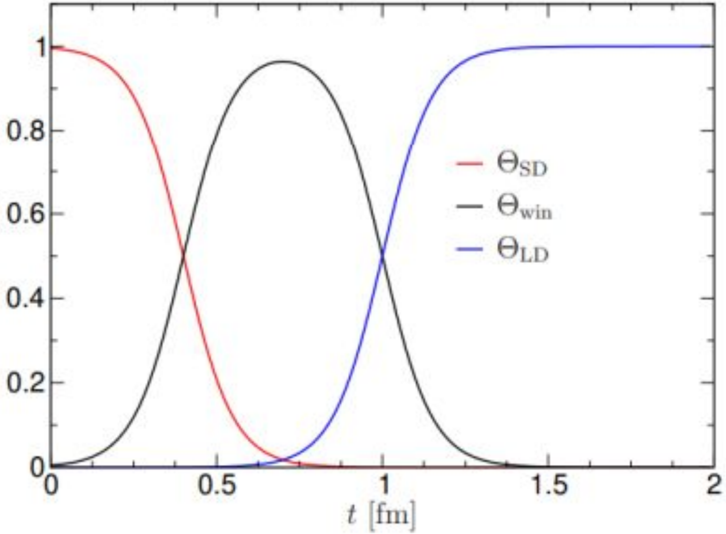
Computing Resources

- ACCESS
- ALCC
- Dirac
- ERCAP
- INCITE
- Indiana U
- LRAC
- USQCD
- XSEDE

Strong-Isospin Breaking

$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{ll}(\mathrm{conn.}) + a_\mu^{ss}(\mathrm{conn.}) + a_\mu^{cc}(\mathrm{conn.}) + \dots$$
$$+ a_\mu^{lsc\dots}(\mathrm{disc.}) + \boxed{\Delta a_\mu^{ud}(\mathrm{SIB})} + \Delta a_\mu^{ud}(\mathrm{QED})$$

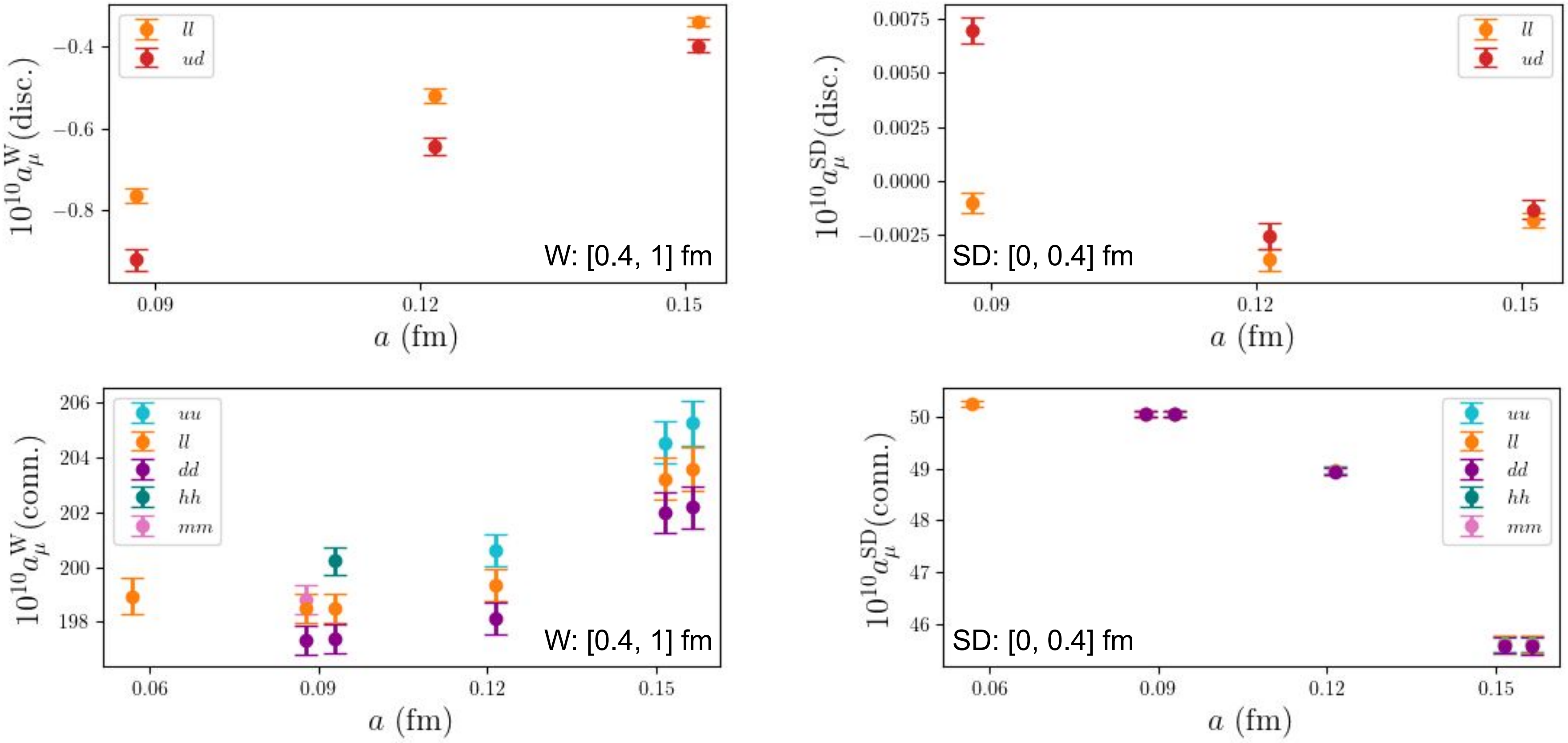
$$\Delta a_\mu^{ud}(\mathrm{SIB}) = a_\mu^{ud} - a_\mu^{ll} = \frac{4a_\mu^{uu} + a_\mu^{dd}}{5} - a_\mu^{ll}$$
$$= \Delta a_\mu^{ud}(\mathrm{SIB, conn.}) + \Delta a_\mu^{ud}(\mathrm{SIB, disc.})$$



	$10^{10} a_\mu^{\mathrm{HVP,LO}}$	$\frac{\Delta a_\mu^{ud}(\mathrm{SIB,conn.})}{a_\mu^{\mathrm{HVP,LO}}}$
Full	~ 700	$\sim 1\%$
W conn.	~ 200	$\lesssim 1\%$
SD conn.	~ 50	$\sim 0\%$
$\Delta a_\mu^{ud,\mathrm{W/SD}}(\mathrm{SIB, disc.}) \sim -10^{-1} \Delta a_\mu^{ud,\mathrm{W/SD}}(\mathrm{SIB, conn.})$		

(Blinded) data

$$a_\mu^{(\mathrm{W/SD}),\mathrm{blind}} = \begin{cases} \alpha^{\mathrm{W/SD,disc.}} a_\mu^{\mathrm{W/SD}}, & \text{disc.} \\ \alpha^{\mathrm{W/SD,conn.}} a_\mu^{\mathrm{W/SD}} + \beta, & \text{conn.} \end{cases}$$



Disconnected continuum fit

W: [0.4, 1] fm

$$\Delta a_\mu^{ud}(a, \{M_A\}) = \Delta a_\mu^{ud,\text{cont.}}(1 + F^{\text{disc}}(a) + F^M(\{M_A\}))$$

$$F^{\text{disc}}(a) = \sum_{j=1}^{n} c_j (a\Lambda)^{2j} \alpha_s^{\delta_{ij}\nu}$$

$$F^M(\{M_A\}) = C_{\text{sea}} \sum_{A=\pi,K} \delta M_A^2$$

$$\delta M_A^2 = \frac{M_{A,\text{ phys.}}^2 - M_{A,\text{ latt.}}^2}{M_{A,\text{ phys.}}^2}$$

$$\Lambda = 900 \text{ MeV}$$

Dependence	Variations
a^2	$n = 0, 1, 2^*$
α_s	$a^2\alpha_s^\nu,\ \nu = 0, 1, 2$
F^M	With and without
Data subsets	$a \in [0.09, 0.15], [0.09, 0.12], [0.012, 0.15]$ fm
*Only fits with sufficient DoF included	

- Model-based corrections (FV, M_π, TB) cancel exactly
- Coarse data subset to inflate systematic error

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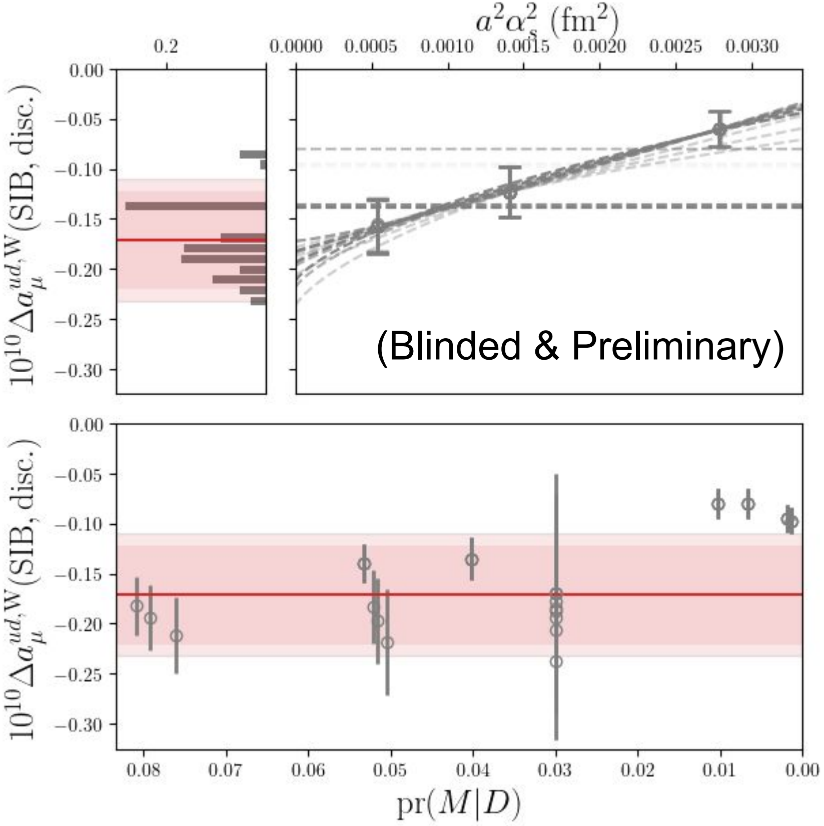
- Coarse data subset to inflate systematic error

Differences for SD: [0, 0.4] fm

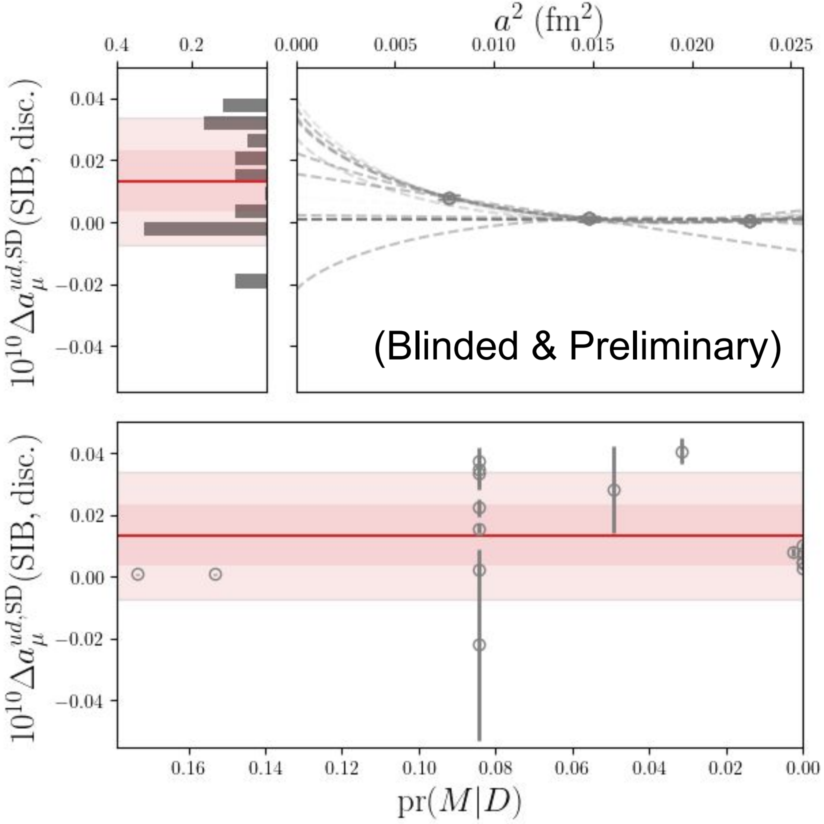
- α_s : LO discretization effect is a^2 in SD
- a^2 : $a^2 \log a$ variations included

Disconnected BMA analysis

W: 33 fits



SD: 17 fits



Connected chiral continuum fit

W: [0.4, 1] fm

$$a_\mu(a, M_\pi, \{M_A\}) = a_\mu(a, M_\pi)(1 + F^M(\{M_A\}))$$

$$a_\mu(a, M_\pi) = \sum_{i=-1}^{1} c_i(a)(M_\pi/\Lambda)^{2i}$$

$$c_i(a) = \sum_{j=0}^{n_i} c_{ij}(a\Lambda)^{2j}\alpha_s^{\delta_{1j}\nu}$$

$$\Lambda = 650 \text{ MeV}$$

Dependence	Variations
M_π^2	$\{M_\pi^{-2}, \log M_\pi^2, 0, M_\pi^4\} + M_\pi^0 + M_\pi^2$
a^2	$n = [0,0,0], \ldots, [2,3,2]^*$
α_s	$a^2\alpha_s^\nu$, $\nu = 0, 1, 2$
F^M	With and without
FV	CM, χPT (NLO,NNLO), HP
Data subsets	$a \in [0.06, 0.15], [0.06, 0.12], [0.09, 0.15]$ fm
*Only fits with sufficient DoF included	

- TB corrections have negligible effect on continuum results

Connected chiral continuum fit

W: [0.4, 1] fm

$$a_\mu(a, M_\pi, \{M_A\}) = a_\mu(a, M_\pi)(1 + F^M(\{M_A\}))$$

$$a_\mu(a, M_\pi) = \sum_{i=-1}^{1} c_i(a)(M_\pi/\Lambda)^{2i}$$

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α_s	$a^2\alpha_s^\nu,\ \nu = 0,1,2$
F^M	With and without
FV	CM, χPT (NLO,NNLO), HP
Data subsets	$a \in [0.06, 0.15], [0.06, 0.12], [0.09, 0.15]$ fm

*Only fits with sufficient DoF included

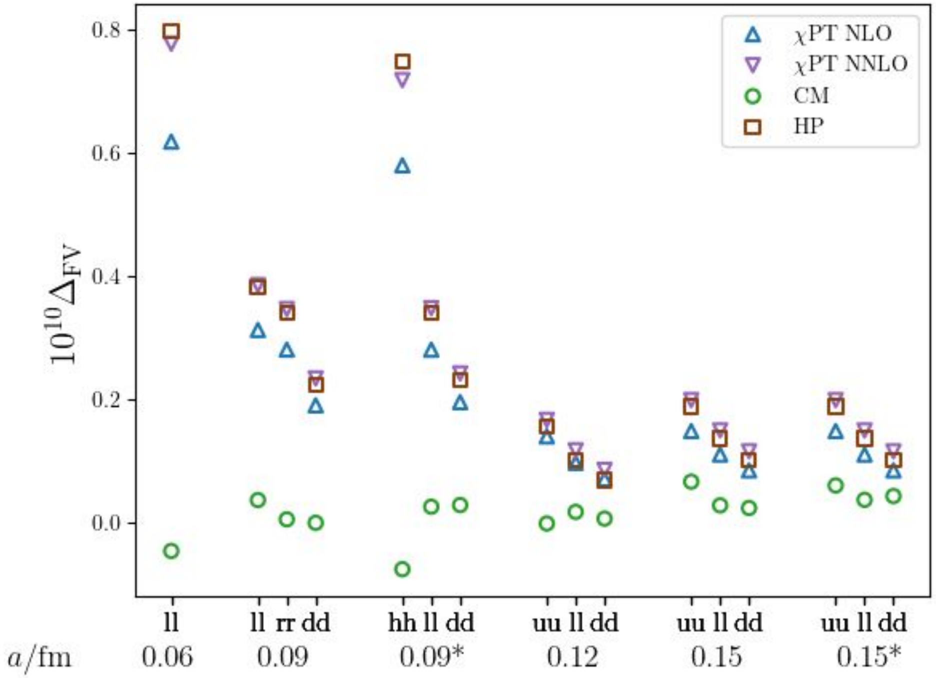
- TB corrections have negligible effect on continuum results

Differences for SD: [0, 0.4] fm

- M_π^2: M_π^{-2} destabilizes fits
- α_s : LO discretization effect is a^2 in SD
- FV : EFT model error expected to be larger than FV correction in SD

FV corrections

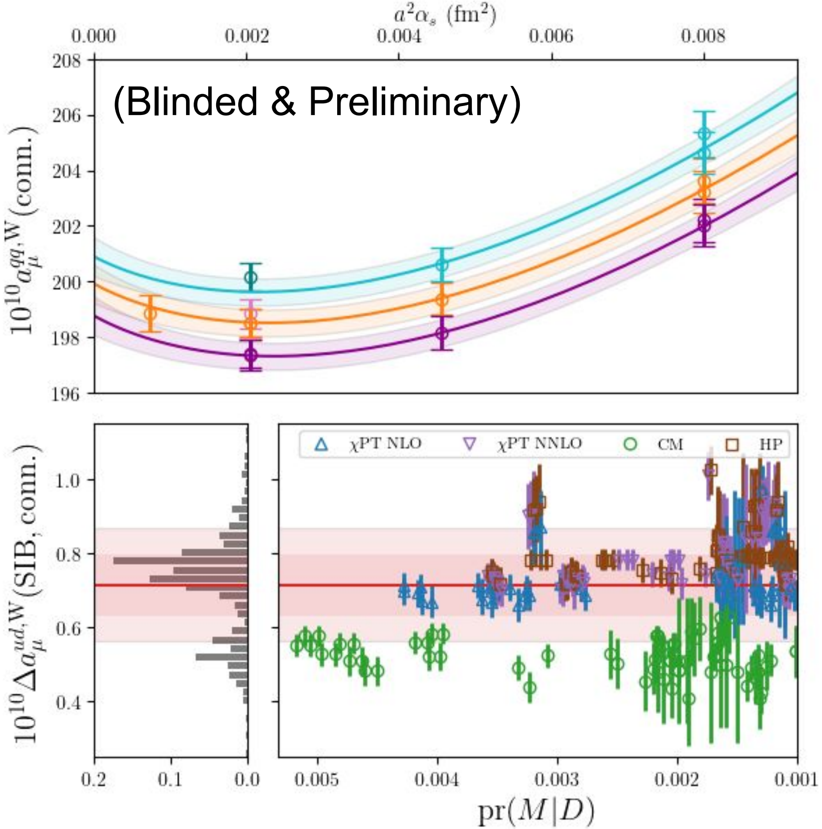
W: [0.4, 1] fm



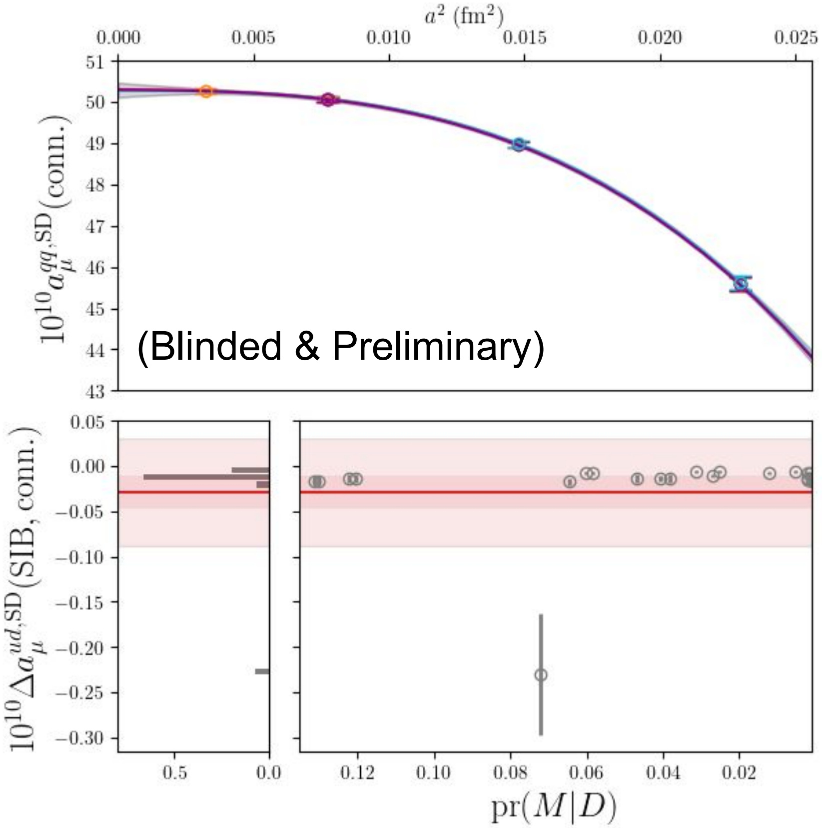
- Expected behavior shown
  - Decreasing with V
  - Decreasing with M_π
- χPT (NLO, NNLO) and HP agree
- CM generally less aggressive
  - Still sub-percent difference in correction to a_μ
  - Contributes to systematic error

Connected BMA analysis

W: 3768 fits



SD: 221 fits



(Blinded) results and error budget

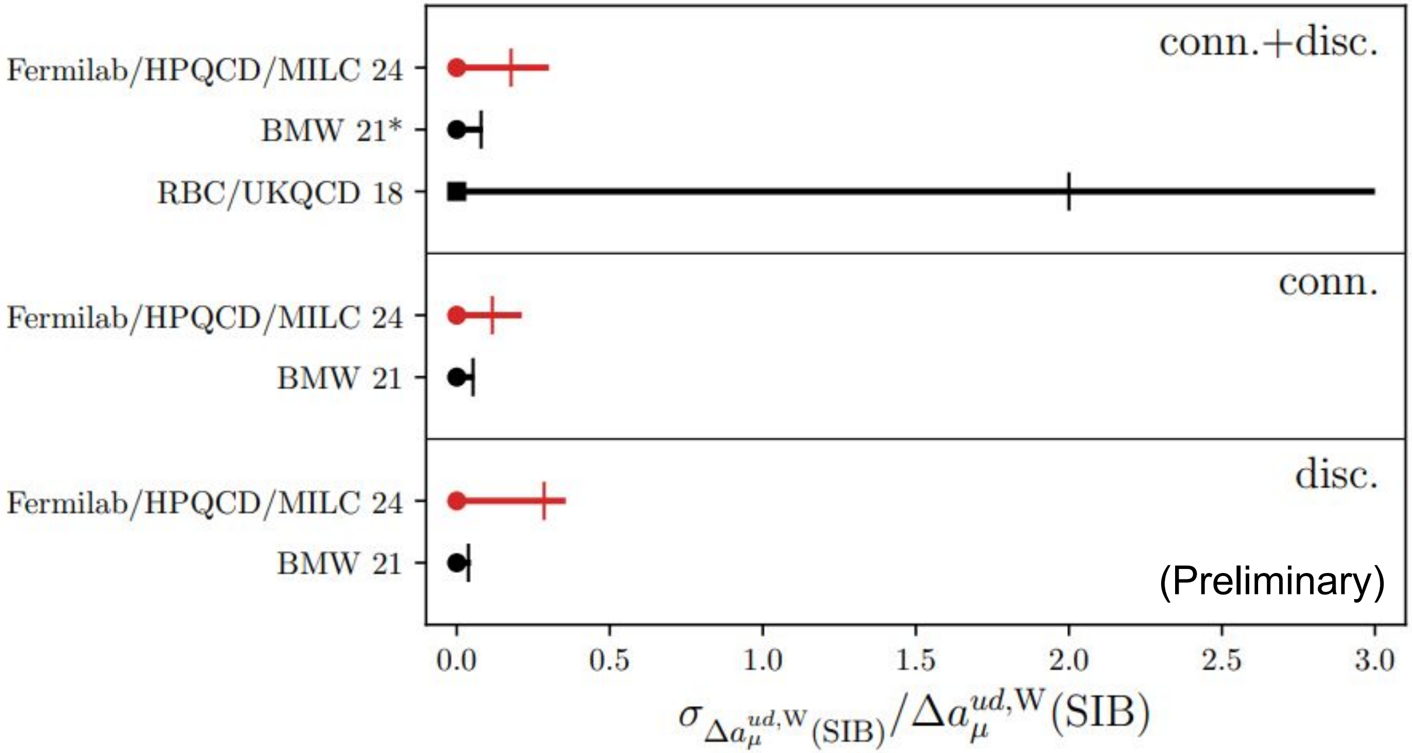
$10^{10}\Delta a_\mu^{ud}$(SIB) (Blinded & Preliminary)	W		SD	
	conn.	disc.	conn.	disc.
	0.71(8)(13)[15]	−0.17(5)(4)[6]	−0.029(19)(56)[59]	0.013(10)(18)[21]

Stat. Syst. Total

$\sigma_{\Delta a_\mu^{ud}}/a_\mu^{ll}$(conn.) (%) (Blinded & Preliminary)		W		SD	
		conn.	disc.	conn.	disc.
Statistics		0.037	0.014	0.010	0.011
Scale setting	w_0 (fm)	0.001	0.002	0.004	0.002
	w_0/a	0.001	~ 0	0.001	~ 0
Current renormalization (Z_V)		0.013	~ 0	0.035	~ 0
Finite-volume correction (Δ_{FV})		0.054	—	—	—
Remainder (cont. extrap.)		0.036	0.027	0.113	0.040
Total		0.076%	0.031%	0.119%	0.041%

	$10^{10}a_\mu^{ll}$(conn.)
W conn.	~ 200
SD conn.	~ 50

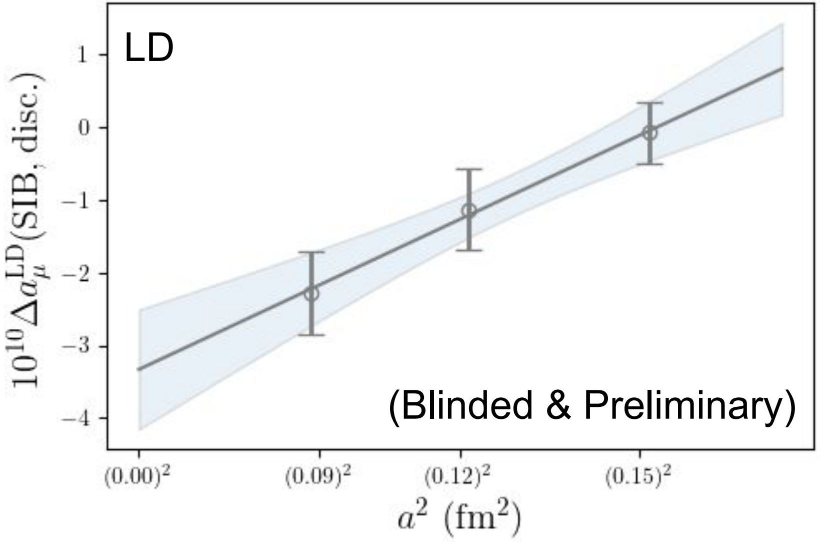
Comparison to previous results on W: [0.4, 1] fm



- No previous SD specific results

Conclusions

$10^{10}\Delta a_\mu^{ud}$(SIB) (Blinded & Preliminary)	W		SD	
	conn.	disc.	conn.	disc.
	0.71(8)(13)[15]	−0.17(5)(4)[6]	−0.029(19)(56)[59]	0.013(10)(18)[21]



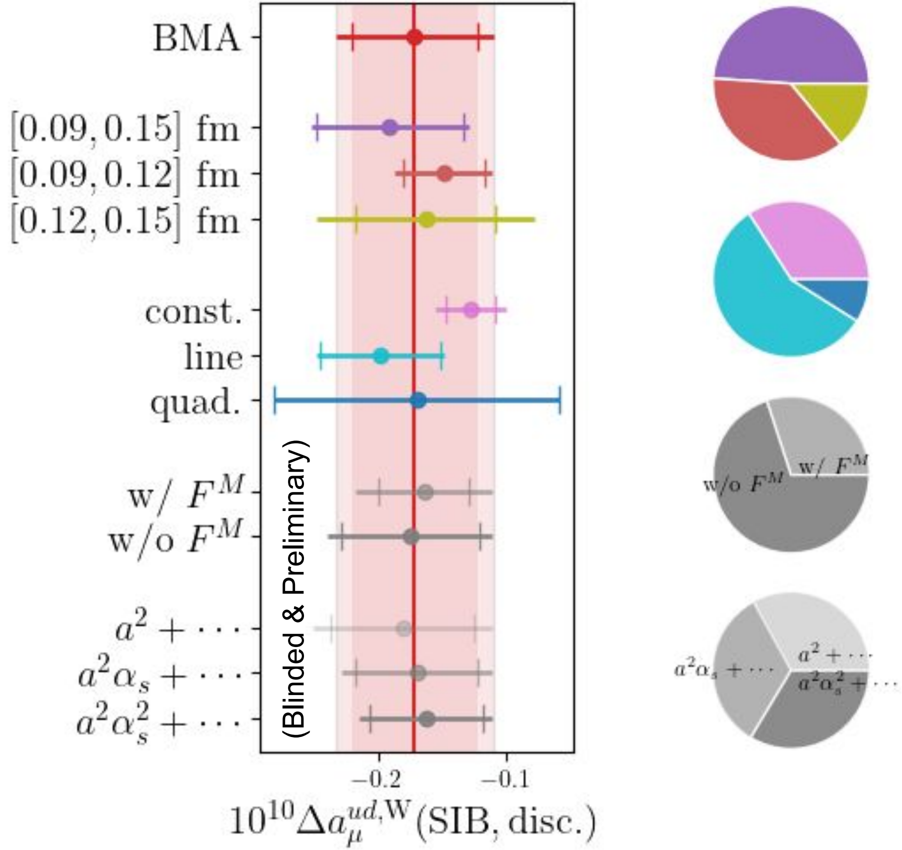
Connected LD in progress

BMW 21 (Full): disconnected -4.67(54)(69), connected 6.60(63)(53)

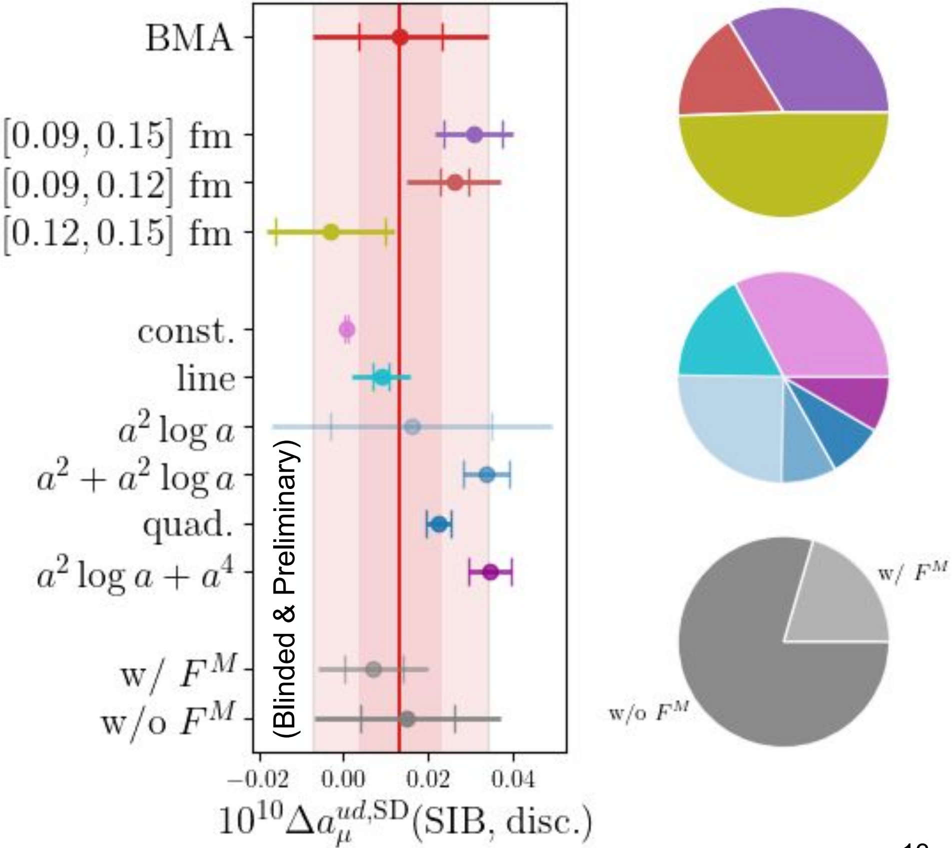
Backup slides

Disconnected BMA breakdown

W: 33 fits



SD: 17 fits



Connected BMA breakdown

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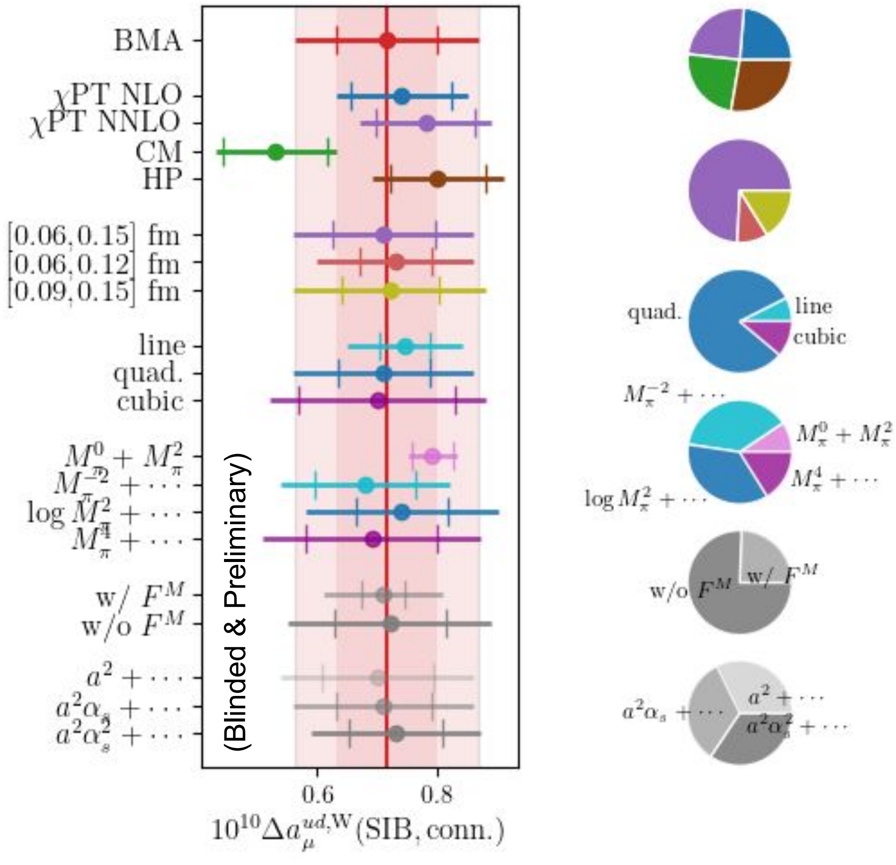

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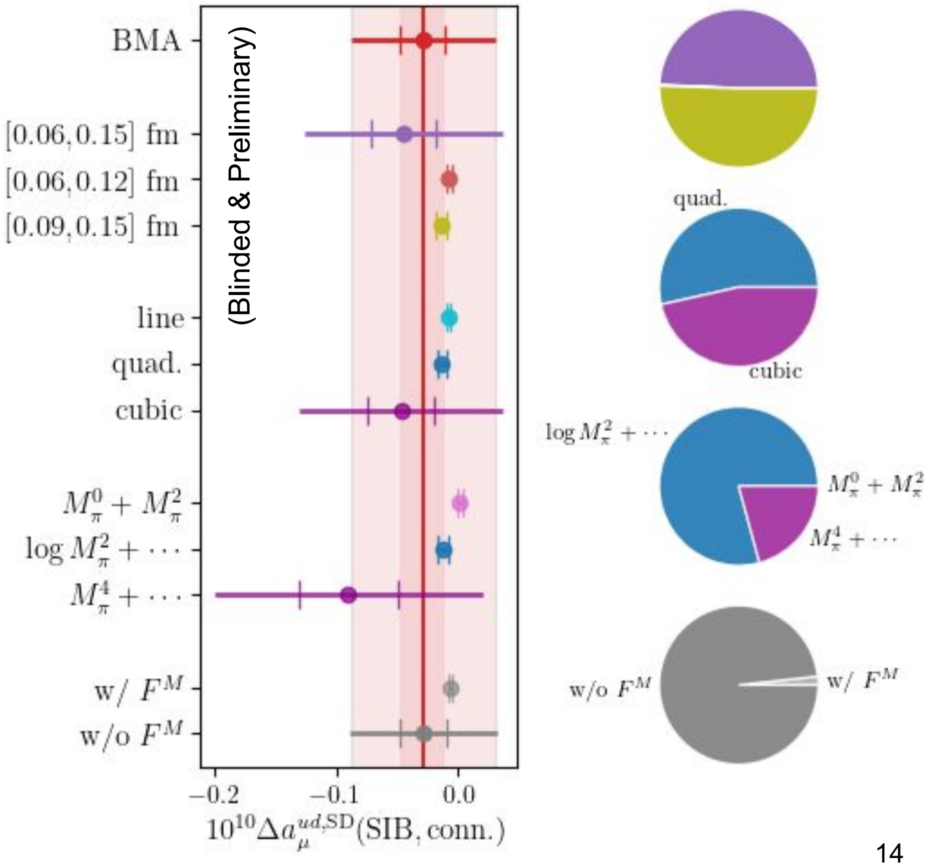

Error budget procedure

- Statistical, scale setting (w0, w0/a), and current renormalization (Zv) as in windows paper, e.g.,

$$\sigma^2_{a_\mu}(\text{stat}) = \sum_{i=1}^{N_M} \sigma^2_{a_\mu,i}(\text{stat})\text{pr}\left(M_i \middle| \{y\}\right)$$

- Finite volume is variance of center values (approximate, assumes each model has similar weight):

$$\sigma^2_{a_\mu}(\text{FV}) = \frac{1}{N_{FV}} \sum_{j=1}^{N_{\text{FV}}} \left(\langle a_\mu \rangle_j - \langle a_\mu \rangle_{\text{BMA}}\right)^2$$

- The remainder of the error comes from (chiral) continuum fit (up to coupling with the aforementioned sources). It is the difference of the errors above from the total in quadrature.

Physical masses

Pure QCD mesons (Antonin Portelli):

$$\begin{array}{r} \text{(isospin-broken} \\ \text{pure-QCD} \Rightarrow \\ \text{world)} \end{array} \begin{cases} f_{\pi^+} & = 130.50 \text{ MeV}, \\ M_{\pi^+} & = 134.977 \text{ MeV}, \\ M_{K^+} & = 491.405 \text{ MeV}, \\ M_{K^0} & = 497.567 \text{ MeV}, \\ M_{D_s} & = 1967.02 \text{ MeV}. \end{cases} \Rightarrow \begin{array}{l} M_{\pi^{ll}} = M_{\pi^+} = 134.977 \text{ MeV} \\ M_{\pi^{ls}} = \sqrt{(M_{K^+}^2 + M_{K^0}^2)/2} = 494.496 \text{ MeV} \end{array}$$

Pion masses (GMOR):

$$m_u/m_d = 0.4529(48)_{\text{stat}} \left({}^{+150}_{-67} \right)_{\text{syst}}$$

arXiv:1807.05556 (MILC)

$$M_\pi^{uu} = M_{\pi^+} \sqrt{\frac{2}{1 + m_d/m_u}} = 106.7(1.1) \text{ MeV}$$

$$M_\pi^{dd} = M_{\pi^+} \sqrt{\frac{2}{1 + m_u/m_d}} = 158.3(0.8) \text{ MeV}$$