# Precise decay rate for $\eta_b \to \gamma\gamma$ with Highly Improved Staggered Quarks

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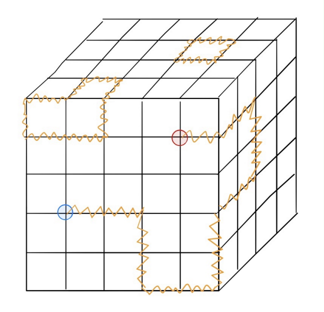
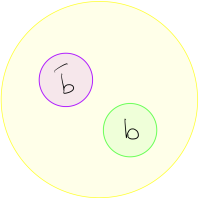
w/ Christine Davies, G. Peter Lepage
HPQCD Collaboration



1 August 2024

# Heavionium decays

- ⋆ Decays with photons can be used as tests of our understanding of internal structure of mesons from strong interaction physics
- ⋆ $\eta_b \to \gamma\gamma$ has not yet been seen; our result is a prediction for Belle II
- ⋆ Builds on $\eta_c \to \gamma\gamma$ where our result vastly improved picture from the lattice
  - ▸ Experimental results give no clear consensus for $\Gamma(\eta_c \to \gamma\gamma)$



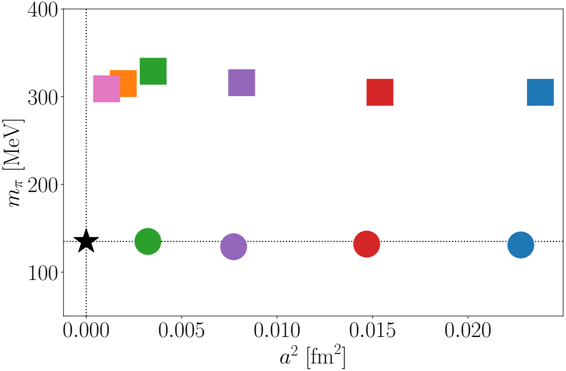

**This work**

- ⋆ Precise calculation by using Highly Improved Staggered Quark (HISQ) action
- ⋆ Calculate these decays with realistic sea
  - ▸ Effect of 2+1+1 quarks
- ⋆ $2-3\%$ uncertainties for $\Gamma(\eta_b \to \gamma\gamma)$
- ⋆ Combining our result with NRQCD calculation, can get new total $\eta_b$ decay width

> ⋆ Full details of previous calculation for $\eta_c \to \gamma\gamma$ process (& $J/\psi$ radiative decays) in Phys Rev **D** 108 (2023) [arXiv:2305.06231]

# Lattice details

- $2+1+1$ HISQ gauge ensembles provided by MILC Collaboration
- Lattice spacings from $\approx 0.15$ fm down to $\approx 0.03$ fm depending on process
- Combination of $m_s/m_l = 5$ and physical $m_l$
- Valence heavy quarks $m_c \leq m_h \leq m_b$ also use HISQ formalism
- Tuned $m_b$ to match $\eta_b$ mesons



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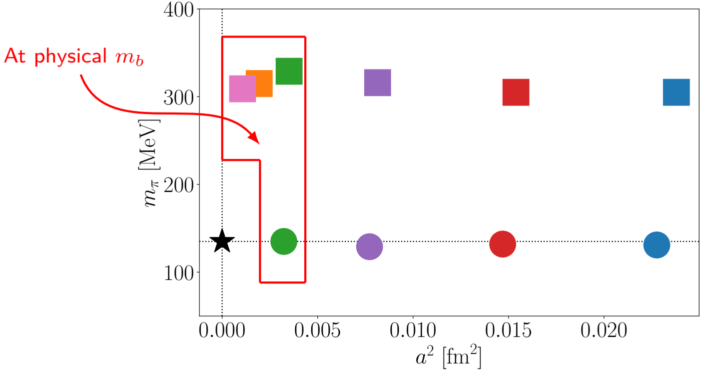
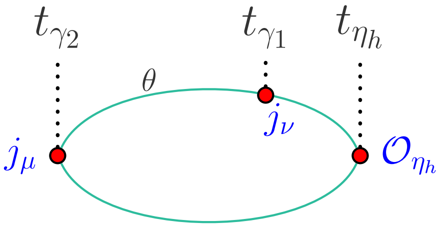
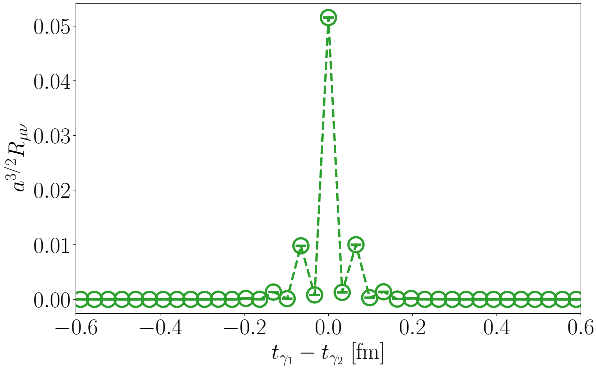





Ji & Jung [hep-lat/0101014] & [hep-lat/0103007]:

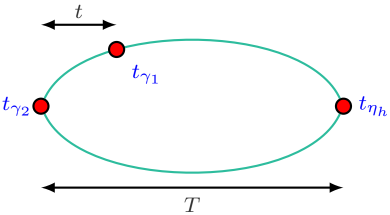
$$\tilde{C}_{\mu\nu}(t_{\gamma_2}, t_{\eta_h}) = a \sum_{t_{\gamma_1}} e^{-\omega_1 (t_{\gamma_1} - t_{\gamma_2})} C_{\mu\nu}(t_{\gamma_1}, t_{\gamma_2}, t_{\eta_h})$$

- For on-shell photons: $\omega_1 = |\vec{q}_1| = |\vec{q}_2| = \frac{M_{\eta_h}}{2}$
- Momentum twist, $\theta$, to tune $\omega_1$
- Require component of momentum orthogonal to photon polarisations (also mutually orthogonal)
  - We chose to put all momentum in $y$ direction (photons in $x$ and $z$)
- Vector currents require renormalisation; we use RI-SMOM scheme HPQCD '19 [1909.00756]

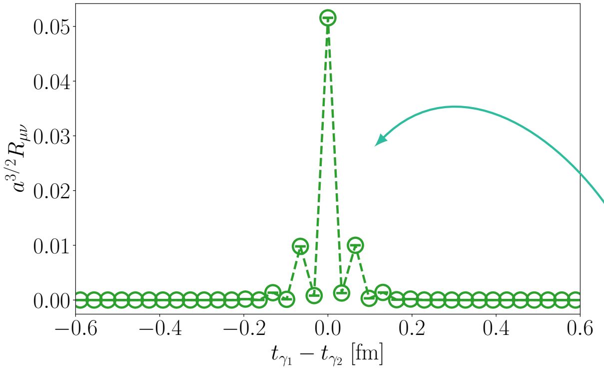



$$\tilde{C}_{\mu\nu}(t_{\gamma_2}, t_{\eta_h}) = \sum_{t_{\gamma_1}} a e^{-\omega_1 (t_{\gamma_1} - t_{\gamma_2})} C_{\mu\nu}(t_{\gamma_1}, t_{\gamma_2}, t_{\eta_h})$$

| $a$ [fm] | $m_s/m_l$ | $T$ |
|---|---|---|
| 0.059 | 5 | 14, 17, 22, 25, 30 |
| 0.057 | 27.5 | 12, 17, 22, 27, 32, 37 |
| 0.044 | 5 | 22, 29, 36, 43 |
| 0.033 | 5 | 21, 30, 39, 48 |



# A look at the integrand

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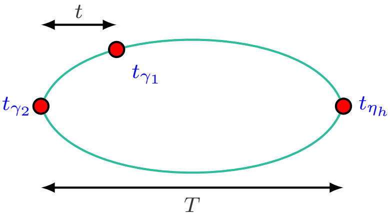
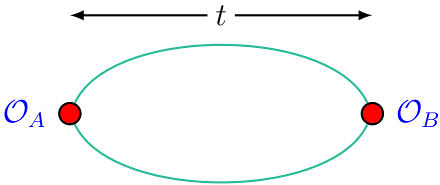


# Fitting correlators

Fit two sets of correlators:




$$C_{\eta_h}(t, t_{\eta_h}) = \sum_n^N a_n^2 \left(e^{-E_n t} + e^{-E_n (N_t - t)}\right)$$

and

$$\tilde{C}_{\mu\nu}(t_{\gamma_2}, t_{\eta_h}) = \sum_n^N a_n b_n \left(e^{-E_n (t_{\gamma_2} - t_{\eta_h})} + e^{-E_n (N_t - t_{\gamma_2} + t_{\eta_h})}\right)$$

Extract form factor $F_{\mathrm{latt}}(0,0)$ by:

Ground state

$$\frac{F_{\mathrm{latt}}(0,0)}{a} = b_0 Z_V^2 \sqrt{\frac{2}{aM_{\eta_h}}} \frac{L_s}{\theta\pi},$$

which relates to the width for two on-shell photons:

$$\Gamma(\eta_h \to \gamma\gamma) = \pi \alpha_{\mathrm{em}}^2 Q_h^4 M_{\eta_h}^3 F(0,0)^2.$$

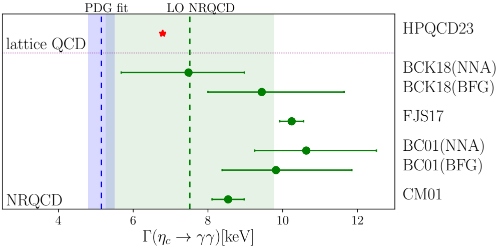
# Nonrelativistic relations: $\eta_c \to \gamma\gamma$

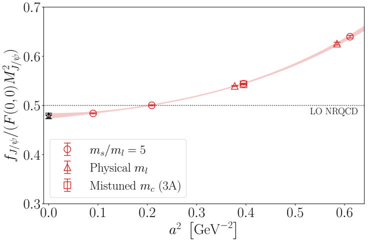
(Czarnecki & Melnikov '01 [hep-ph/0109054]):

Expectation in nonrelativistic limit:

$$\frac{\Gamma(J/\psi \to e^+e^-)}{\Gamma(\eta_c \to \gamma\gamma)} \approx \frac{3}{4}\left(1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(v^2/c^2\right)\right)$$

$$\frac{f_{J/\psi}}{F(0,0)M^2_{J/\psi}} = \frac{1}{2}\left(1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(v^2/c^2\right)\right)$$



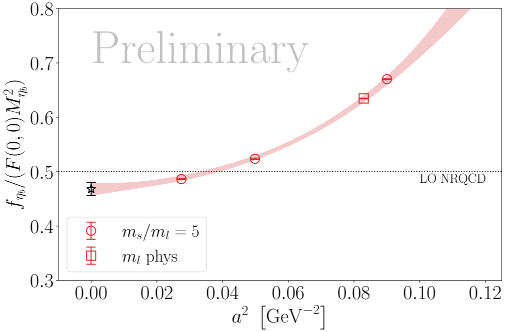




$$\text{Our result}: \frac{f_{J/\psi}}{F(0,0)M^2_{J/\psi}} = 0.4786(57)_{\text{fit}}(14)_{\text{syst}}$$

$M_{J/\psi}$, $f_{J/\psi}$ & $\Gamma(J/\psi \to e^+e^-)$ (for LO NRQCD central value) from HPQCD '20 [2005.01845]

# $\eta_b \to \gamma\gamma$ RESULTS

# $\eta_b \to \gamma\gamma$ from ratio with decay constant



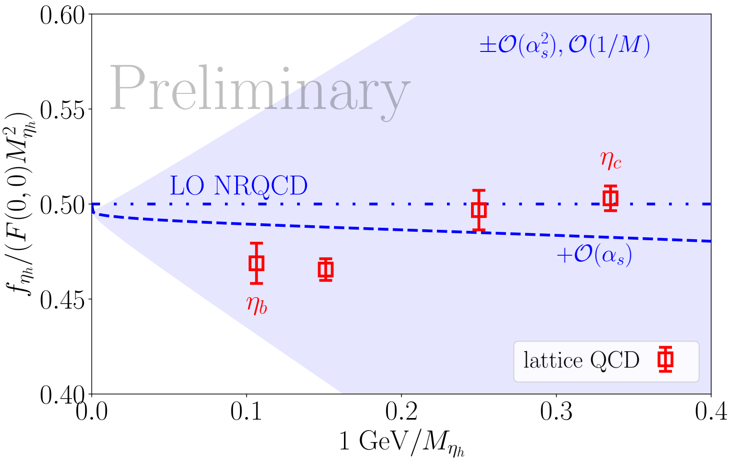

$$R^{\text{latt}}_{\eta_b} = R^{\text{phys}}_{\eta_b}\left[1 + \sum^{i_{\max}} \kappa^{(i)}_{a\Lambda}(a\Lambda)^{2i} + \kappa_{\text{val},b}\delta^{\text{val},b} + \kappa_{\text{sea},c}\delta^{\text{sea},c} + \kappa^{(0)}_{\text{sea},uds}\delta^{\text{sea},uds}\{1 + \kappa^{(1)}_{\text{sea},uds}(a\tilde{\Lambda})^2 + \kappa^{(2)}_{\text{sea},uds}(a\tilde{\Lambda})^4\}\right]$$

$$R^{\text{phys}}_{\eta_b} = 0.468(12); \quad F(0,0) = 0.01751(53)\ \text{GeV}^{-1}$$

$$\Gamma(\eta_b \to \gamma\gamma) = 0.526(32)\ \text{keV}$$

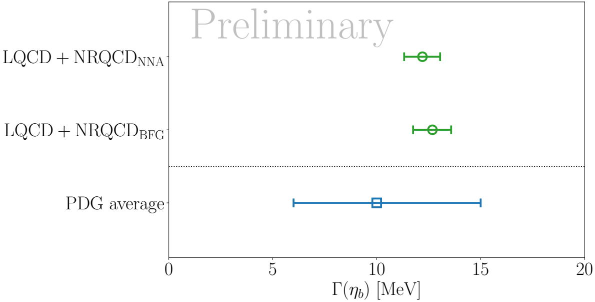
$f_{\eta_b}$ result for conversion from HPQCD '21 [2101.08103]

# Comparison with LO NRQCD




$$R_{\eta_h} \equiv \frac{f_{\eta_h}}{F_{\eta_h}(0,0)M_{\eta_h}^2} = \frac{1}{2}\left(1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(v^2/c^2\right)\right)$$

★ Can get full width $\Gamma(\eta_b)$ by combining our decay width with $\frac{\Gamma(\eta_b)}{\Gamma(\eta_b \to \gamma\gamma)}$ from NRQCD




$$\Gamma(\eta_b)_{\text{NNA}} = 12.20 \left({}^{+42}_{-47}\right)_{\text{NRQCD}} (74)_{\text{LQCD}} \text{ MeV}$$

$$\Gamma(\eta_b)_{\text{BFG}} = 12.68 \left({}^{+47}_{-53}\right)_{\text{NRQCD}} (77)_{\text{LQCD}} \text{ MeV}$$

# Inclusive width

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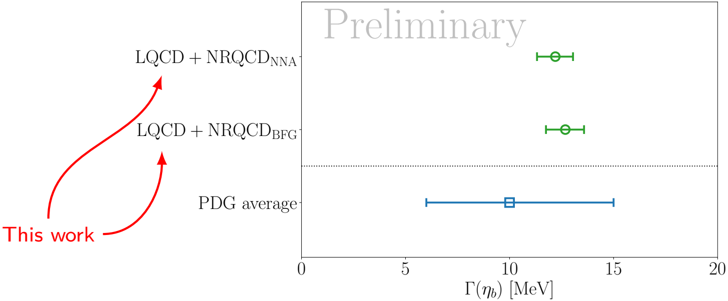


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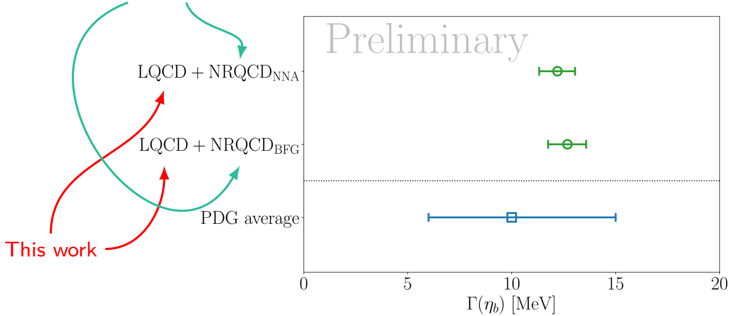
(Brambilla, Chung & Komijani [1810.02586])

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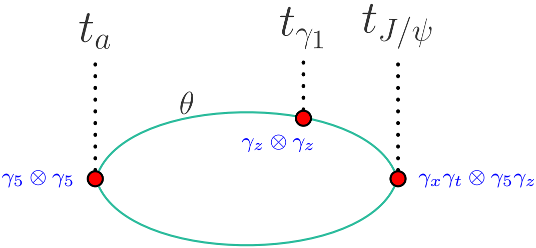
$$J/\psi \to \gamma a$$

# $J/\psi \to \gamma a$

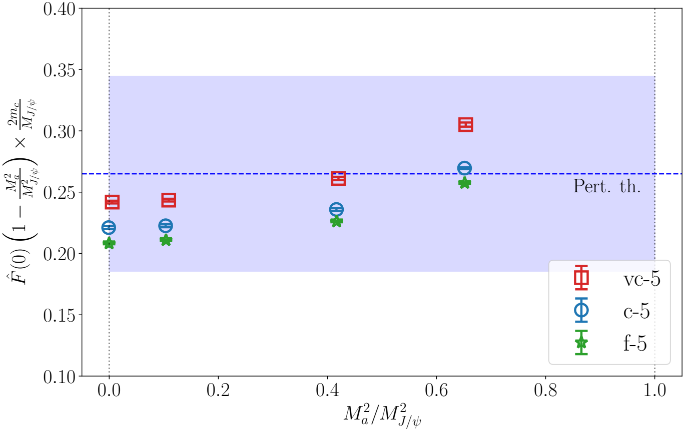
w/ Christine Davies, G. Peter Lepage & Sophie Renner

★ Analogously calculate process where $\gamma$ and an axion-like particle couple to charm



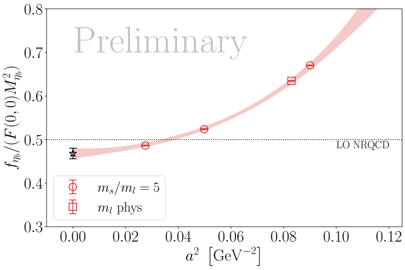

$$\hat{F}(0) = b_0 Z_V \sqrt{2M_{J/\psi}} \frac{L_s}{\theta\pi}$$

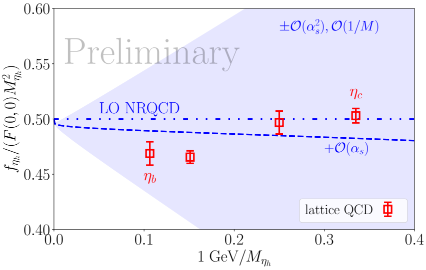
$$\Gamma(J/\psi \to \gamma a) = \frac{1}{24}\frac{C_{cc}^2}{f^2}\alpha_{\rm em} Q_c^2 M_{J/\psi}^3 \left(1 - \frac{M_a^2}{M_{J/\psi}^2}\right) \hat{F}(0)^2$$



0.40
0.35
0.30
0.25
0.20
0.15
0.10
$\hat{F}(0)\left(1-\frac{M_a^2}{M_{J/\psi}^2}\right)\times\frac{2m_c}{M_{J/\psi}}$
0.0
0.2
0.4
0.6
0.8
1.0
$M_a^2/M_{J/\psi}^2$
Pert. th.
vc-5
c-5
f-5

# Summary

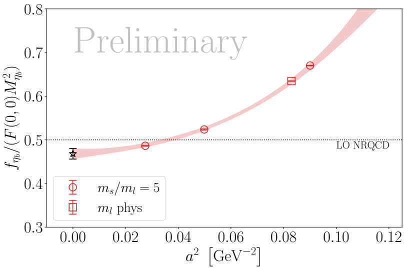


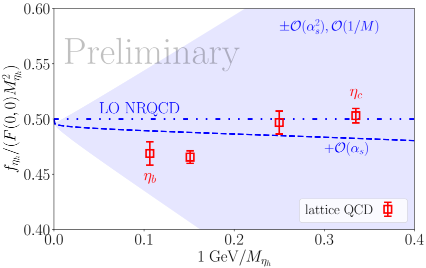




- ★ $\Gamma(\eta_b \to \gamma\gamma) = 0.526(32)$ keV Preliminary!
  - ▶ $\Gamma(\eta_b) = 12.20(88)$ MeV (NNA); $\Gamma(\eta_b) = 12.68(93)$ MeV (BFG) Preliminary!
- ★ We're hoping results from experiment for $\eta_b \to \gamma\gamma$ will be forthcoming

Belle II?

- ★ Similar analysis being performed for decay with axion-like particle: $J/\psi \to a\gamma$




Thank you!

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