Update on the isospin breaking corrections to the HVP with C-periodic boundary conditions

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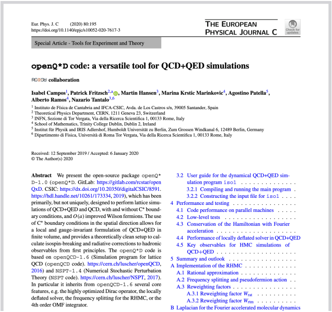

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Special Article - Tools for Experiment and Theory

openQ*D code: a versatile tool for QCD+QED simulations

RC* collaboration

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Abstract We present the open-source package openQ*D-1.0 (openQ*D. GitLab: https://gitlab.com/rcstar/openQxD. CSIC: https://dx.doi.org/10.20350/digitalCSIC/8591. https://hdl.handle.net/10261/173334, 2019), which has been primarily, but not uniquely, designed to perform lattice simulations of QCD+QED and QCD, with and without C* boundary conditions, and $O(a)$ improved Wilson fermions. The use of C* boundary conditions in the spatial direction allows for a local and gauge-invariant formulation of QCD+QED in finite volume, and provides a theoretically clean setup to calculate isospin-breaking and radiative corrections to hadronic observables from first principles. The openQ*D code is based on openQCD-1.6 (Simulation program for lattice QCD (openQCD code). https://cern.ch/luscher/openQCD, 2016) and NSPT-1.4 (Numerical Stochastic Perturbation Theory (NSPT code). https://cern.ch/luscher/NSPT, 2017). In particular it inherits from openQCD-1.6 several core features, e.g. the highly optimized Dirac operator, the locally deflated solver, the frequency splitting for the RHMC, or the 4th order OMF integrator.

Motivations

Isospin breaking in lattice calculations

Subpercent precision predictions hadronic quantities in lattice QCD must include

- $m_d \neq m_u$, as $\mathcal{O}((m_d - m_u)/\Lambda_{QCD}) \sim 1\%$
- QED effects, as $\mathcal{O}(\alpha_{EM}) \sim 1\%$
- dynamical charm quark, ...

RC* program: focus on the IB corrections (*masses of mesons, HVP, leptonic decays*)
(see arXiv:1509.01636, arXiv:1908.11673 arXiv:2209.13183v1)

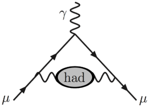
Goal of this (preliminary) analysis: Cross-check and compare two approaches to compute IB effects at fixed lattice spacing and volume:

1. Direct QCD+QED with dynamical U(1) and $m_u \neq m_d$
2. IsoQCD + RM123 (arXiv:1303.4896): perturbative expansion in δm_{ud} and α_{QED}, including all sea effects[1]

[1] This is the plan, all ingredients available. However, in this talk, no valence-disconnected or sea IB effect included.

Comparing two methods for calculating Isospin Breaking Effects

Target observable: HVP contribution to $(g-2)_\mu$.



Setup: QCD and QCD+QED gauge ensembles with Wilson fermions, $O(a)$ improved action with coeff. $c_{\text{sw}}^{SU(3)} = 2.18859$ and $c_{\text{sw}}^{U(1)} = 1$, *same volume and* β, but *different* κ_q *and* α

ensemble	lattice	β	α	κ_u	$\kappa_d = \kappa_s$	κ_c
A400a00b324	64×32^3	3.24	0	0.13440733	0.13440733	0.12784
A380a07b324	64×32^3	3.24	0.007299	0.13459164	0.13444333	0.12806355
		$\delta\beta$	$\delta\alpha$	δm_u	$\delta m_d = \delta m_s$	δm_c
$\delta\vec{\epsilon} = \epsilon^{\text{A380}} - \epsilon^{\text{A400}}$		0	0.007299	-0.00509422	-0.000996117	-0.00682735

(ensembles described in arXiv:2209.13183v1)

Steps:
- Compute all relevant observables at "LO"
- Compute derivatives: $\partial G/\partial m_f$, $\partial G/\partial e^2$, $\partial \phi_i/\partial m_f$, $\partial \phi_i/\partial e^2$ and derivatives to Z_V
- Combine $\delta\vec{\epsilon} \equiv (\delta\beta, \delta\alpha, \delta m_u, \delta m_{d/s}, \delta m_c)$ and derivatives to get IB effects to a_μ^{HVP}

Ensembles' setup

C-periodic (or C*) boundary conditions

- Local prescription for QED in a finite box w/ C-periodic BCs in spatial directions

$$A_\mu(x + L_i\hat{i}) = -A_\mu(x), \quad U_\mu(x + L_i\hat{i}) = U^*_\mu(x)$$

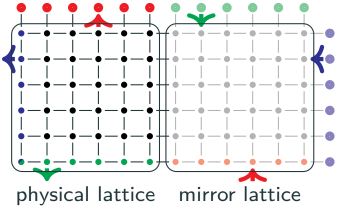
$$\psi(x + L_i\hat{i}) = C^{-1}\bar{\psi}(x)^T, \quad \overline{\psi}(x + L_i\hat{i}) = -\psi^T(x)C$$

- The lattice is doubled in the $\hat{1}$ direction: $L_1 = 2L$, while $L_k = L$ for $k = 2, 3$.
- C^* BCs in other directions: $\psi(x + L_k\hat{k}) = \psi(x + \frac{L_1}{2}\hat{1})$ for $k = 2, 3$
- Effective periodicity of $2L \to$ double sized Dirac operator w.r.t. PBCs

⊕ Only odd Matsubara modes $\to$ charged-states propagation allowed

⊕ Suppressed FV effects for meson masses and HVP w.r.t. to QED_L
arXiv:1509.01636 arXiv:2212.09565

⊖ Weak violation of flavor conservation (disappears as $V \to \infty$)



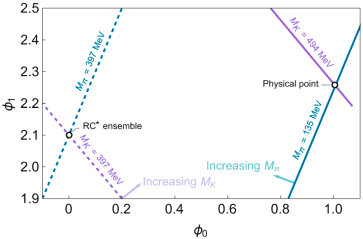
Lines of constant physics

Observable	Physical value	Target RC* value	Most sensitive bare parameter
$\sqrt{8t_0}$	0.415 fm*	0.415 fm	g_0^2
$\phi_0 = 8t_0(m^2_{K^\pm} - m^2_{\pi^\pm})$	0.992	0	$m_s - m_d$
$\phi_1 = 8t_0(m^2_{K^\pm} + m^2_{\pi^\pm} + m^2_{K^0})$	2.26	2.11	$m_u + m_d + m_s$
$\phi_2 = 8t_0(m^2_{K^0} - m^2_{K^\pm})/\alpha_R$	2.36	2.36, 0	$m_u - m_d$
$\phi_3 = \sqrt{8t_0}(m_{D_s^\pm} + m_{D^0} + m_{D^\pm})$	12.0	12.1	m_c
α_R	0.007297	(0, α^{phys})	e^2

* arXiv:1608.08900

Scheme described in arXiv:2209.13183v1

	A400 [MeV]	A380 [MeV]
$M_{K^\pm}$	398.5(4.7)	383.6(4.4)
$M_{\pi^\pm}$	398.5(4.7)	383.6(4.4)
M_{K^0}	398.5(4.7)	390.7(3.7)
$M^\pm_{D_s}$	1912.7(5.7)	1926.4(7.8)
$M_{D^\pm}$	1912.7(5.7)	1926.4(7.8)
M_{D^0}	1912.7(5.7)	1921.1(7.6)



Computing mass parameters shifts δm_f

Alternatively, mass shifts can be derived by matching IsoQCD+RM123 and QCD+QED schemes

$$\phi_i^{A400a00b324+RM123} = \phi_i^{A380a07b324}, \quad i = 0,1,2,3$$

LHS evaluated by expanding ϕ_i in isospin-breaking parameters $\delta m_f, e^2, \delta\beta = 0$, with target values as in previous slide:

$$\phi_0 \stackrel{!}{=} \phi_0^{(0)} = 0$$

$$\underbrace{\phi_1}_{2.11} \stackrel{!}{=} \underbrace{\phi_1^{(0)}}_{2.11} + 16 t_0^{(0)} m_{\pi^\pm}^{(0)} \left[\left(\sum_{f=d,s} \delta m_f \frac{\partial m_{K^0}}{\partial m_f} + e^2 \frac{\partial m_{K^0}}{\partial e} \right) + 2 \left(\sum_{f=u,d} \delta m_f \frac{\partial m_{\pi^\pm}}{\partial m_f} + e^2 \frac{\partial m_{\pi^\pm}}{\partial e} \right) \right]$$

$$\underbrace{\phi_2}_{2.36} \stackrel{!}{=} \underbrace{\phi_2^{(0)}}_{0} + 8 t_0^{(0)} \frac{2 m_{K^0}^{(0)}}{\alpha} \left[\left(\sum_{f=d,s} \delta m_f \frac{\partial m_{K^0}}{\partial m_f} + e^2 \frac{\partial m_{K^0}}{\partial e} \right) - \left(\sum_{f=u,s} \delta m_f \frac{\partial m_{K^\pm}}{\partial m_f} + e^2 \frac{\partial m_{K^\pm}}{\partial e} \right) \right]$$

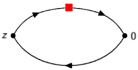
$$\underbrace{\phi_3}_{12.1} \stackrel{!}{=} \underbrace{\phi_3^{(0)}}_{12.1} + \sqrt{8 t_0^{(0)}} \left[\left(\sum_{f=u,c} \delta m_f \frac{\partial m_{D^0}}{\partial m_f} + e^2 \frac{\partial m_{D^0}}{\partial e} \right) + 2 \left(\sum_{f=d,c} \delta m_f \frac{\partial m_{D^\pm}}{\partial m_f} + e^2 \frac{\partial m_{D^\pm}}{\partial e} \right) \right]$$

$$\Rightarrow \begin{cases} \delta m_u & = -0.005100(59)_{stat}(10)_{syst}, \\ \delta m_d = \delta m_s & = -0.00114(58)_{stat}(10)_{syst}, \\ \delta m_c & = -0.00591(89)_{stat}(4)_{syst}. \end{cases}$$

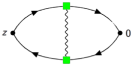
RM123: Feynman diagrams with our action and vector currents

Diagrams for leading IB effects (connected valence only here)

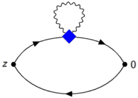
Derivatives from $O(a)$ improved Wilson action
$S^{\text{QCD+QED+SW}} = S_f(e, m_f) + S_{\text{SW}}(e) + \delta Sb$



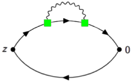
mass



exchange



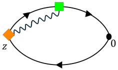
tadpole

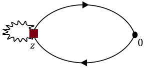


self-energy

■ (red)	$\propto \sum_x \bar{\psi}(x)\psi(x)$	from δ_m of mass term $(4 + m)\bar{\psi}\psi$ in S_f,
♦ (blue)	$\propto \sum_{x,\mu} T_\mu(x) A_\mu^2(x)$	from δ_e^2 of kinetic term $T(x)$ in S_f,
■ (green)	$\propto \sum_{x,\mu} V_\mu^c(x) A_\mu(x)$	from δ_e of kinetic term $T(x)$ in S_f, *or*
■ (green)	$\propto c_{\text{SW}}^{\text{U(1)}} \sum_{x,\mu} \delta_e D_{\text{SW}}(x)$	from $S_{\text{SW}} = S_{\text{SW}}\big\|_{e=0} + e \cdot \delta_e S_{SW} + O(e^3)$.

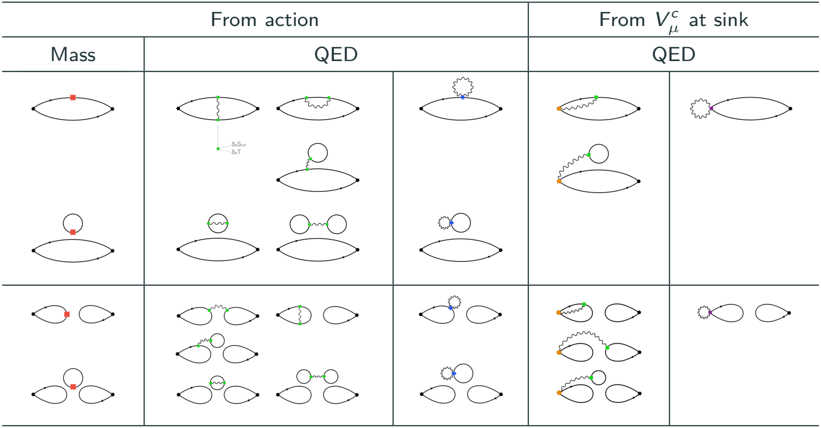
With $G^{c\ell}(t)$, if conserved current $V_\mu^c(x)$ defined at the sink, no additional propagators needed, but two additional diagrams appear:





■ (dark red)	$\propto \sum_{x,\mu} V_\mu^c(x) A_\mu^2(x)$	from $\delta_e^2 V_\mu^c$
♦ (orange)	$\propto \sum_{x,\mu} T_\mu(x) A_\mu(x)$	from $\delta_e V_\mu^c$

Diagrams for leading IB effects – complete list



■ $\propto \sum_x \bar{\psi}(x)\psi(x)$	■ $\propto \sum_{x,\mu} V^c_\mu(x) A_\mu(x)$	⬢ $\propto \sum_{x,\mu} V^c_\mu(x) A^2_\mu(x)$
◆ $\propto \sum_{x,\mu} T_\mu(x) A^2_\mu(x)$	■ $\propto c_{\mathrm{SW}}^{\mathrm{U(1)}} \sum_{x,\mu} \delta_e D_{\mathrm{SW}}(x)$	⬟ $\propto \sum_{x,\mu} T_\mu(x) A_\mu(x)$

Note: In our case ($q_d = q_s$ and $m_d = m_s$), the “isovector” current $\bar{\psi}_d \gamma_\mu \psi_d - \bar{\psi}_s \gamma_\mu \psi_s$ does not require valence-disconnected diagrams .

RM123 on HVP

IB corrections to the HVP

Using the local-local implementation for the correlator $G^{R,ll}(t) = Z_V G^{ll}(t) Z_V^T$

$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \sum_{f_1,f_2} \int_0^\infty dt\, G^{R,ll}_{f_1 f_2}(t) K(t; m_\mu)\,, \quad G_{f_1 f_2}(t) = \frac{1}{3} \sum_{\vec{x},k} \langle V_k^{f_1}(x) V_k^{f_2}(0) \rangle$$

in TMR arXiv:1107.4388v2 with $K(t; m_\mu)$ def as $\tilde{K}$ in arXiv:1705.01775.

a_μ^{HVP} receives two types of IB corrections:

1. Corrections to correlators

$$\delta_{(G)} a_\mu^{HVP} = \left(\tfrac{\alpha}{\pi}\right)^2 \int dt\, Z_V^{(0)} \delta G^{ll}(t) Z_V^{(0)^T} K(t; m_\mu)$$

$$G^{ll}(t) = G^{ll}(t)^{(0)} + \delta G^{ll}(t) = G^{ll}(t)^{(0)} + \sum_f \delta m_f \frac{\partial G^{ll}(t)}{\partial m_f}\bigg|_{(0)} + \frac{e^2}{2} \frac{\partial^2 G^{ll}(t)}{\partial e^2}\bigg|_{(0)}$$

2. Corrections to renormalization constants

$$\delta_{(Z)} a_\mu^{HVP} = \left(\tfrac{\alpha}{\pi}\right)^2 \int dt\, \left[Z_V^{(0)} G^{ll}(t)^{(0)} \delta Z_V^T + \delta Z_V G^{ll}(t)^{(0)} Z_V^{(0)^T} \right] K(t; m_\mu)$$

$$Z_V = Z_V^{(0)} + \delta Z_V = Z_V^{(0)} + \sum_f \delta m_f \frac{\partial Z_V}{\partial m_f}\bigg|_{(0)} + \frac{1}{2} e^2 \frac{\partial^2 Z_V}{\partial e^2}\bigg|_{(0)}$$

Alternatively, we also use conserved-local correlator $G^{R,cl}(t) = G^{ll}(t) Z_V^T$.

Renormalization constants Z_V : LO

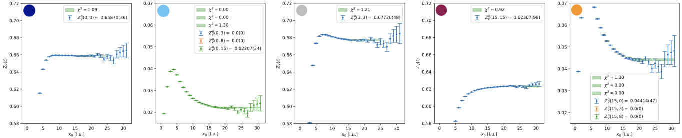
Renormalization conditions defined in *adjoint* basis of SU(4) generators $\lambda_3, \lambda_8, \lambda_{15}$ plus the identity $\lambda_0 = \mathbb{1}$ (see arXiv:hep-lat/0511014v3

$$V_\mu^{em} = \sum_{f=u,d,s,c} Q_f \bar{\psi}_f \gamma_\mu \psi_f \quad \rightarrow \quad \tilde{V}_\mu^{em} = \frac{1}{3} V_\mu^0 + V_\mu^3 + \frac{1}{\sqrt{3}} V_\mu^8 - \frac{1}{\sqrt{6}} V_\mu^{15}$$

with adjoint currents $V_\mu^{0,3,8} = \frac{1}{2}\mathrm{tr}(\lambda_{0,3,8}\mathcal{V})$, $V_\mu^{15} = \mathrm{tr}(\lambda_{15}\mathcal{V})$ and $[\mathcal{V}]_{f_1 f_2} = \bar{\psi}_{f_1}\gamma_\mu\psi_{f_2}$.

$$\left[\tilde{Z}_V\right]_{ab} \equiv \lim_{x_0 \to \infty} \tilde{G}^{cl}_{ad} \cdot (\tilde{G}^{ll})^{-1}_{db}\,, \quad \text{with } a, b = 0, 3, 8, 15$$

$$\tilde{Z}_V^{A400} = \begin{pmatrix} 0.6587(4) & 0.0000(0) & 0.0000(0) & 0.0221(2) \\ 0.0000(0) & 0.6772(5) & 0.0000(0) & 0.0000(0) \\ 0.0000(0) & 0.0000(0) & 0.6772(5) & 0.0000(0) \\ 0.0441(5) & 0.0000(0) & 0.0000(0) & 0.6231(10) \end{pmatrix}$$



Renormalization constants Z_V : IBE

Expand $\tilde{Z}_V = \lim_{x_0 \to \infty} \tilde{G}^{cl}(x_0)(\tilde{G}^{ll}(x_0))^{-1}$ at first order in δm_f and e^2

$$\delta \tilde{Z}_V = \sum_f \delta m_f \frac{\partial \tilde{Z}_V}{\partial m_f} + e^2 \frac{\partial \tilde{Z}_V}{\partial e^2} \,, \quad \text{w/}$$

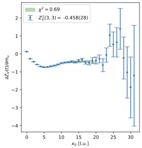
$$\frac{\partial \tilde{Z}_V}{\partial \varepsilon_i} = \lim_{x_0 \to \infty} \left[\frac{\partial \tilde{G}^{cl}}{\partial \varepsilon_i}(x_0) - \tilde{G}^{cl}(x_0) \, (\tilde{G}^{ll}(x_0))^{-1} \frac{\partial \tilde{G}^{ll}}{\partial \varepsilon_i}(x_0) \right] \cdot (\tilde{G}^{ll}(x_0))^{-1}$$

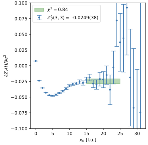
$\frac{\partial \tilde{Z}_V}{\partial e^2}$: requires 7 diagrams for G^{ll} + 10 diagrams for G^{cl}

$\frac{\partial \tilde{Z}_V}{\partial m_f}$: each ∂_{m_f} requires  for G^{ll} and G^{cl} and all f

$$\delta \tilde{Z}_V = \begin{pmatrix} -0.00002(19) & 0.000260(95) & 0.00027(11) & 0.000147(76) \\ 0.000230(93) & -0.00008(16) & 0.00027(11) & 0.000094(38) \\ 0.000133(54) & 0.00027(11) & -0.00005(19) & 0.000054(22) \\ 0.00030(15) & 0.00038(15) & 0.000217(87) & -0.000259(62) \end{pmatrix}$$

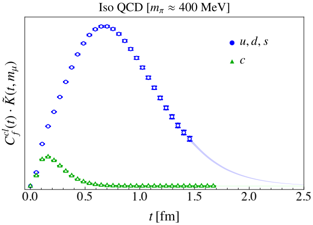
$$\frac{\delta \tilde{Z}}{\tilde{Z}} = \begin{pmatrix} -0.00003(29) & - & - & 0.0067(34) \\ - & -0.00012(24) & - & - \\ - & - & -0.00007(28) & - \\ 0.0068(34) & - & - & -0.00042(10) \end{pmatrix}$$

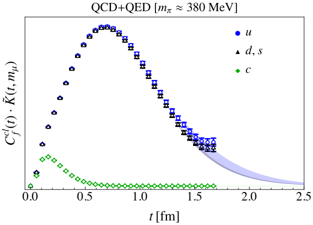




just two of many plots

Results for $a_\mu^{\rm HVP}$ from LO connected correlators





- Disconnected contributions currently ignored
- Tails are fitted to a single exponential at $t_{\rm cut} \in (1.2, 1.3)$ fm
- Local- correlators renormalized as $G^{R,cl} = G^{cl} \cdot Z_V^T$ and $G^{R,ll} = Z_V \cdot G^{ll} \cdot Z_V^T$

LO $a_\mu^{\rm HVP}$ on A400a00b324

type	$a_\mu^u \times 10^{10}$	$a_\mu^{d/s} \times 10^{10}$	$a_\mu^c \times 10^{10}$
ll	188.40(189)	47.11(47)	7.59(4)
cl	186.30(195)	46.58(49)	5.99(3)

type	$am_V^{u,d,s}$	am_V^c
ll	0.2896(35)	0.8548(3)
cl	0.2908(40)	0.8546(3)

LO $a_\mu^{\rm HVP}$ on A380a07b324

type	$a_\mu^u \times 10^{10}$	$a_\mu^{d/s} \times 10^{10}$	$a_\mu^c \times 10^{10}$
ll	194.0(2.3)	47.2(6)	7.55(4)(4)
cl	192.2(2.2)	46.8(6)	5.95(4)(3)

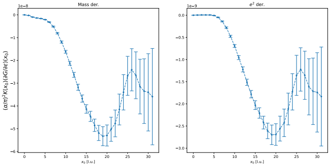
type	am_V^u	$am_V^{d,s}$	am_V^c
ll	0.2795(27)	0.2807(4)	0.8496(4)(3)
cl	0.2791(31)	0.281(4)	0.8496(4)(3)

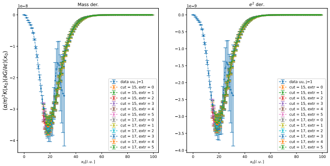
$$\delta G(x_0) = \sum_f \frac{\partial G(x_0)}{\partial m_f} \delta m_f + \frac{\partial G(x_0)}{\partial e^2} e^2$$

- Same derivatives already needed for Z_V, now for all x_0
- Tails' corrections $A = A^{(0)} + \delta A$ and $m_{\text{eff}} = m_{\text{eff}}^{(0)} + \delta m$ from 2-params linear fit:

$$\frac{G^{(1)}(x_0) - G^{(0)}(x_0)}{G^{(0)}(x_0)} = \delta A / A^{(0)} - \delta m \, x_0$$

- Procedure repeated for different fit ranges (light quarks) for systematic effects





Corrections to $\delta a_\mu^{u,s,d,c}$ from Z_V and $G(t)$: summary of results (1)

Corrections from renormalization constants $\times 10^{10}$			
loc-loc	$\delta_{Z_v} a_\mu^{HVP,uu}$	$=$	$-510(33)\,\delta m_u - 22(3)\,e^2$
	$\delta_{Z_v} a_\mu^{HVP,dd/ss}$	$=$	$0.016(23)\,\delta m_u - 128(8)\,\delta m_{d/s} - 1.4(2)\,e^2$
	$\delta_{Z_v} a_\mu^{HVP,cc}$	$=$	$0.003(4)\,\delta m_u - 6.37(2)\,\delta m_c - 0.578(2)\,e^2$
cons-loc	$\delta_{Z_v} a_\mu^{HVP,uu}$	$=$	$-252(16)\,\delta m_u - 11(2)\,e^2$
	$\delta_{Z_v} a_\mu^{HVP,dd/ss}$	$=$	$0.008(11)\,\delta m_u - 63(4)\,\delta m_{d/s} - 0.68(11)\,e^2$
	$\delta_{Z_v} a_\mu^{HVP,cc}$	$=$	$0.0012(16)\,\delta m_u - 2.516(9)\,\delta m_c - 0.2282(8)\,e^2$
Corrections from correlator $\times 10^{10}$			
loc-loc	$\delta_G a_\mu^{HVP,uu}$	$=$	$-4364(266)\,\delta m_u - 216(14)\,e^2$
	$\delta_G a_\mu^{HVP,dd/ss}$	$=$	$-1091(67)\,\delta m_{d/s} - 13.5(9)\,e^2$
	$\delta_G a_\mu^{HVP,cc}$	$=$	$-59.2(3)\,\delta m_c - 3.119(13)\,e^2$
cons-loc	$\delta_G a_\mu^{HVP,uu}$	$=$	$-4591(288)\,\delta m_u - 227(15)\,e^2$
	$\delta_G a_\mu^{HVP,dd/ss}$	$=$	$-1148(72)\,\delta m_{d/s} - 14.2(1.0)\,e^2$
	$\delta_G a_\mu^{HVP,cc}$	$=$	$-57.2(2)\,\delta m_c - 3.295(14)\,e^2$

IB effects to A400a00b324 from valence, connected diagrams.

Corrections to $\delta a_\mu^{u,s,d,c}$ from Z_V and $G(t)$: summary of results (2)

Corrections from renormalization constants $\times 10^{10}$					
loc-loc	$\delta_{Z_v} a_\mu^{HVP,uu}$	$=$	$2.60(17) - 2.02(28)$	$=$	0.56(27)
	$\delta_{Z_v} a_\mu^{HVP,dd/ss}$	$=$	$0.127(8) - 0.128(18)$	$=$	0.019(18)
	$\delta_{Z_v} a_\mu^{HVP,cc}$	$=$	$0.04347(14) - 0.05300(18)$	$=$	-0.015(6)
cons-loc	$\delta_{Z_v} a_\mu^{HVP,uu}$	$=$	$1.28(8) - 1.01(18)$	$=$	0.28(14)
	$\delta_{Z_v} a_\mu^{HVP,dd/ss}$	$=$	$0.063(4) - 0.062(10)$	$=$	0.0094(91)
	$\delta_{Z_v} a_\mu^{HVP,cc}$	$=$	$0.01717(16) - 0.02093(7)$	$=$	0.094(91)
Corrections from correlator $\times 10^{10}$					
loc-loc	$\delta_G a_\mu^{HVP,uu}$	$=$	$22.2(1.4) - 19.8(1.3)$	$=$	2.35(26)
	$\delta_G a_\mu^{HVP,dd/ss}$	$=$	$1.09(7) - 1.24(8)$	$=$	0.01(6)
	$\delta_G a_\mu^{HVP,cc}$	$=$	$0.4042(20) - 0.2860(12)$	$=$	0.064(52)
cons-loc	$\delta_G a_\mu^{HVP,uu}$	$=$	$23.4(1.5) - 20.8(1.4)$	$=$	2.49(27)
	$\delta_G a_\mu^{HVP,dd/ss}$	$=$	$1.14(7) - 1.30(9)$	$=$	0.01(7)
	$\delta_G a_\mu^{HVP,cc}$	$=$	$0.3905(14) - 0.3022(13)$	$=$	0.04(5)

by using the theoretical mass shifts of slide 2

Outlook

Outlook

- Goal of this ongoing work: compare two methods for computing IB effects.
- At the moment, the analysis includes all valence-connected terms,
- However, the plan is to obtain a full comparison by including all diagrams.
- Sea IB effects computed by A. Cotellucci (see talk), to be included in analysis.
- Done/Computed:
  - Mass derivatives of mesons in ϕ_i: $\pi^{\pm}, \pi^0, K^{\pm}, K^0, D^{\pm}, D^0, D_s^{\pm}$.
  - Bare parameters' shift to match IsoQCD+RM123 to QED+QCD computed.
  - Renormalization constants of local vector currents.
  - Derivatives of Z_V and $G(t)$.
- Next step: perform analysis using "isovector" current

$$\bar{d}\gamma_\mu d - \bar{s}\gamma_\mu s$$

  physically well defined, does not include any disconnected diagram.
- Ensembles with smaller pion masses, larger volumes, smaller a are being generated.
- Next update Muon g-2 Theory Initiative workshop at KEK.

Thank you for listening!