RTG 2575:
Rethinking
Quantum Field Theory

Error Scaling of Sea Quark Isospin-Breaking Effects

Alessandro Cotellucci

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NATIONALES HOCHLEISTUNGS RECHNEN

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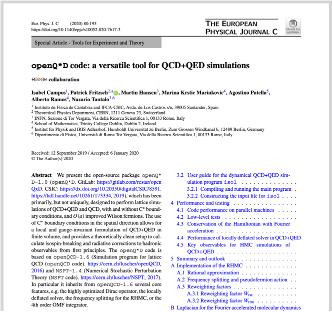

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Special Article - Tools for Experiment and Theory

openQ*D code: a versatile tool for QCD+QED simulations

RC*OR collaboration

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Abstract We present the open-source package openQ*D-1.0 (openQ*D. GitLab: https://gitlab.com/rcstar/openQxD. CSIC: https://dx.doi.org/10.20350/digitalCSIC/8591. https://hdl.handle.net/10261/173334, 2019), which has been primarily, but not uniquely, designed to perform lattice simulations of QCD+QED and QCD, with and without C* boundary conditions, and $O(a)$ improved Wilson fermions. The use of C* boundary conditions in the spatial direction allows for a local and gauge-invariant formulation of QCD+QED in finite volume, and provides a theoretically clean setup to calculate isospin-breaking and radiative corrections to hadronic observables from first principles. The openQ*D code is based on openQCD-1.6 (Simulation program for lattice QCD (openQCD code). https://cern.ch/luscher/openQCD, 2016) and NSPT-1.4 (Numerical Stochastic Perturbation Theory (NSPT code). https://cern.ch/luscher/NSPT, 2017). In particular it inherits from openQCD-1.6 several core features, e.g. the highly optimized Dirac operator, the locally deflated solver, the frequency splitting for the RHMC, or the 4th order OMF integrator.

Motivations

Isospin symmetry is an approximation as long as the target precision is above 1%:

$$\underbrace{\left(\frac{m_d - m_u}{\Lambda_{\text{QCD}}}\right) \sim 0.01}_{\text{Strong-IBE}} \qquad \underbrace{\frac{e^2}{4\pi} \sim 0.01}_{\text{Electromagnetic-IBE}} \quad .$$

- Many lattice computations of hadronic observables have reached 1% precision;
- Sea-quark effects are small, but a bound needs to be included in the error;

RM123 Method [PhysRevD.87.114505]

Monte Carlo simulation of $\mathrm{QCD}_{\mathrm{Iso}}$ + perturbative expansion in Isospin breaking term $S_{\mathrm{QCD+QED}} = S_{\mathrm{Iso}+\gamma} + S_{\mathrm{IB}}\left(\delta m_{ud}, e^2\right)$:

$$\langle \mathcal{O} \rangle = \frac{\int dU d\chi dA \; e^{-S_{\mathrm{Iso}}[U,\chi]} e^{-S_{\mathrm{IB}}[U,A,\chi]} e^{-S_{\gamma}[A]} \mathcal{O}[U,A,\chi]}{\int dU d\chi dA \; e^{-S_{\mathrm{Iso}}[U,\chi]} e^{-S_{\mathrm{IB}}[A,\chi,U]} e^{-S_{\gamma}[A]}}$$

$$= \langle \mathcal{O} \rangle_{\mathrm{Iso}} - \underbrace{\langle S_{\mathrm{IB}} \mathcal{O} \rangle_{\mathrm{Iso}\gamma,c} + \frac{1}{2} \langle S_{\mathrm{IB}} S_{\mathrm{IB}} \mathcal{O} \rangle_{\mathrm{Iso}\gamma,c}}_{valence-valence} + \underbrace{\langle S_{\mathrm{IB}} S_{\mathrm{IB}} \mathcal{O} \rangle_{\mathrm{Iso}\gamma,c}}_{sea-valence}$$

$$- \underbrace{\langle S_{\mathrm{IB}} \mathcal{O} \rangle_{\mathrm{Iso}\gamma,c} + \frac{1}{2} \langle S_{\mathrm{IB}} S_{\mathrm{IB}} \mathcal{O} \rangle_{\mathrm{Iso}\gamma,c} + \frac{1}{2} \langle S_{\mathrm{IB}} S_{\mathrm{IB}} \mathcal{O} \rangle_{\mathrm{Iso}\gamma,c}}_{sea-sea}$$

$$+ O\left(\delta m_{ud}^2, e^4, e^2 \delta m_{ud}\right).$$

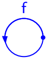
Sea-sea Diagrams

From the expansion:

$$\langle \mathcal{O} \rangle = \langle \mathcal{O} \rangle_{\text{Iso}} - \langle {\color{blue}S_{\text{IB}}} \mathcal{O} \rangle_{\text{Iso}+\gamma,c} + \frac{1}{2} \langle {\color{red}S_{\text{IB}} S_{\text{IB}}} \mathcal{O} \rangle_{\text{Iso}+\gamma,c}$$

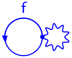
The sea-sea diagrams are:

$$\langle \mathcal{O} \rangle_{\text{sea}} = \sum_f \delta m_f \langle$$



mass

$$\mathcal{O} \rangle_{\text{Iso},c} + e^2 \sum_f q_f^2 \langle$$



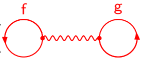
tadpole

$$\mathcal{O} \rangle_{\text{Iso},c}$$

$$+ e^2 \Bigg[\sum_f q_f^2 \langle$$

lightbulb

$$\mathcal{O} \rangle_{\text{Iso},c} + \sum_{fg} q_f q_g \langle$$



lanterns

$$\mathcal{O} \rangle_{\text{Iso},c} \Bigg] .$$

Scaling of the Error

Looking at the error of the insertion

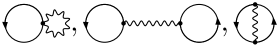
$$\sigma\left[\langle\mathcal{D}\mathcal{O}\rangle_{\mathsf{Iso},c}\right]$$

In the asymptotic limit, the gauge error factorizes [Harris et al., LATTICE2022 013]:

$$\sigma\left[\langle\mathcal{D}\mathcal{O}\rangle_{\mathsf{Iso},c}\right] \underset{a\to 0}{\sim} \sqrt{\langle\mathcal{D}^2\rangle_{\mathsf{Iso},c}\langle\mathcal{O}^2\rangle_{\mathsf{Iso},c}}$$

The leading result is:

$\sigma \Big[\langle$  $\mathcal{O}\rangle_{\mathsf{Iso},c} \Big] \underset{a\to 0}{\sim} \sigma_{\mathcal{O}} a^{-1}\sqrt{V}$

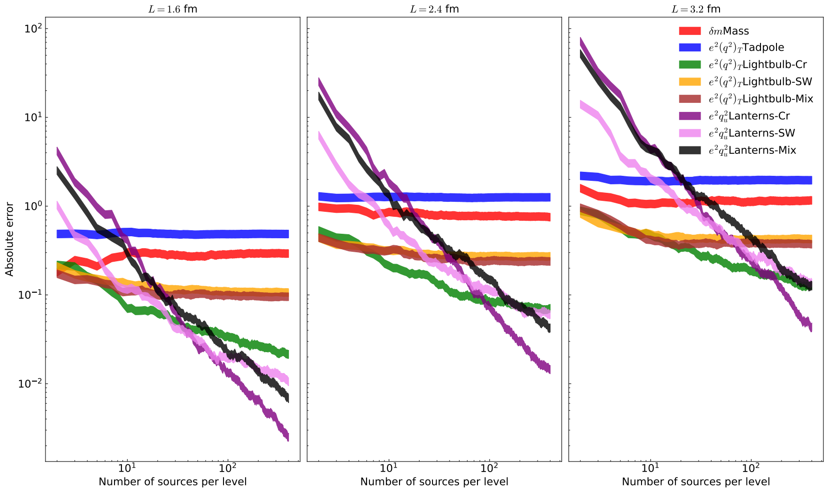
$\sigma\left[\langle\mathcal{D}\mathcal{O}\rangle_{\mathsf{Iso},c}\right] \underset{a\to 0}{\sim} \sigma_{\mathcal{O}} a^{-2}\sqrt{V}$ for $\mathcal{D} =$ 

Volume Scaling

Scaling of the diagrams' error with the number of sources

ensemble	a [fm]	M_π [MeV]	$M_\pi L$	n.cnfg
A420a00b334	0.0990(7)	413(8)	3.31	50
B420a00b334	0.0991(3)	415(3)	4.98	50
C420a00b334	0.0990(1)	415(1)	6.63	50

Parameters from [Bali et al., JHEP05(2023)035]



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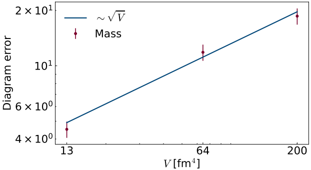
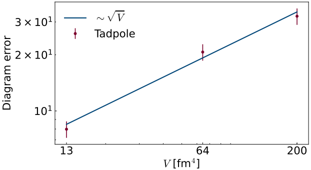
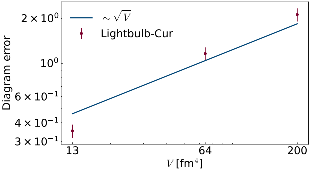
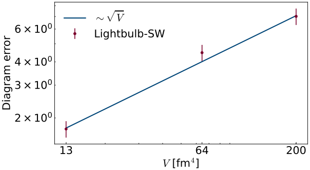
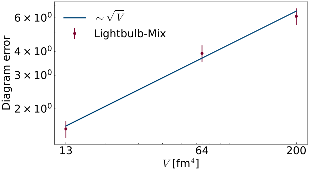








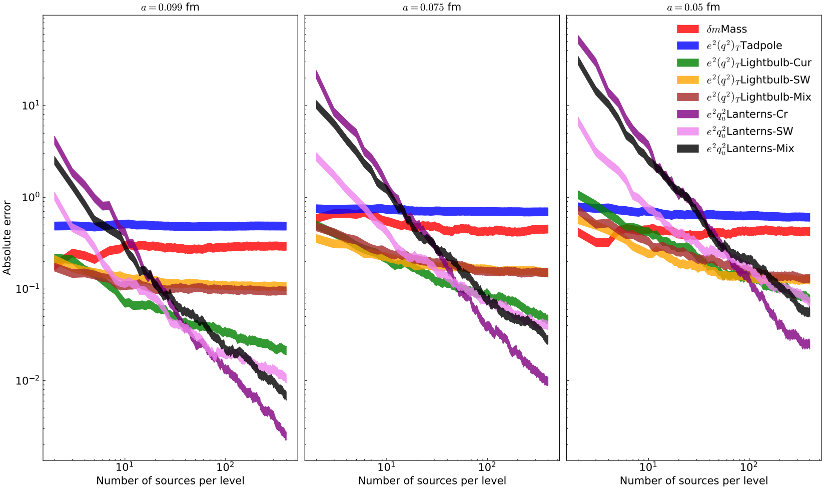


Continuum Scaling

Scaling of the diagrams' error with the number of sources

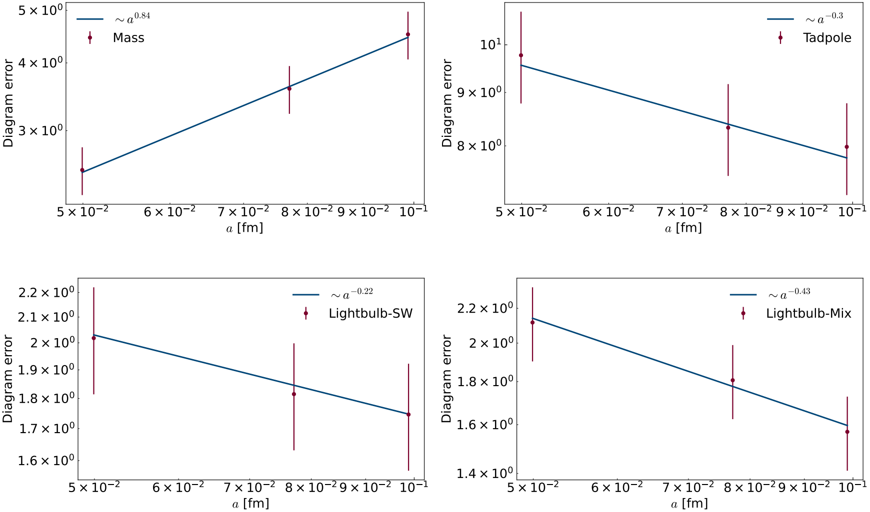
ensemble	a [fm]	M_π [MeV]	L [fm]	n.cnfg
C420a00b370	0.0499(2)	416(3)	1.596(6)	50
B420a00b346	0.0769(3)	414(2)	1.857(6)	50
A420a00b334	0.0990(7)	413(8)	1.58(1)	50

Parameters from [Bali et al., JHEP05(2023)035]



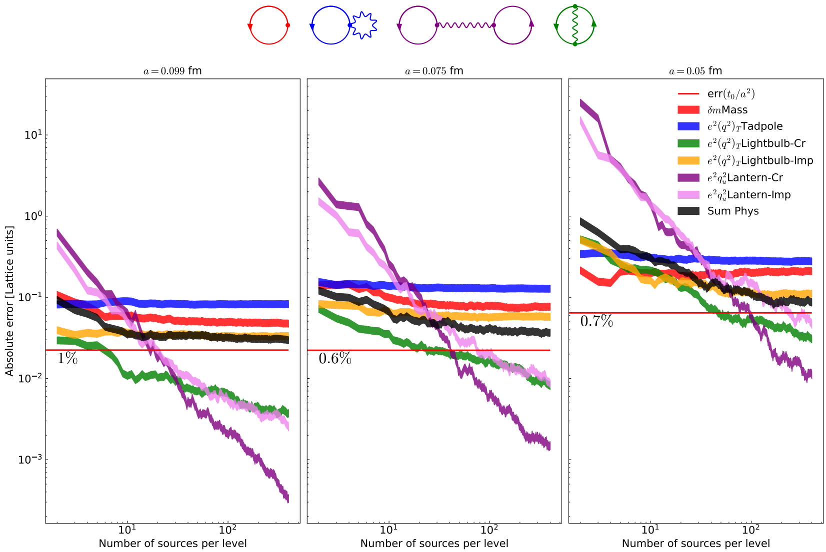
Continuum Scaling

Scaling of the diagrams' error



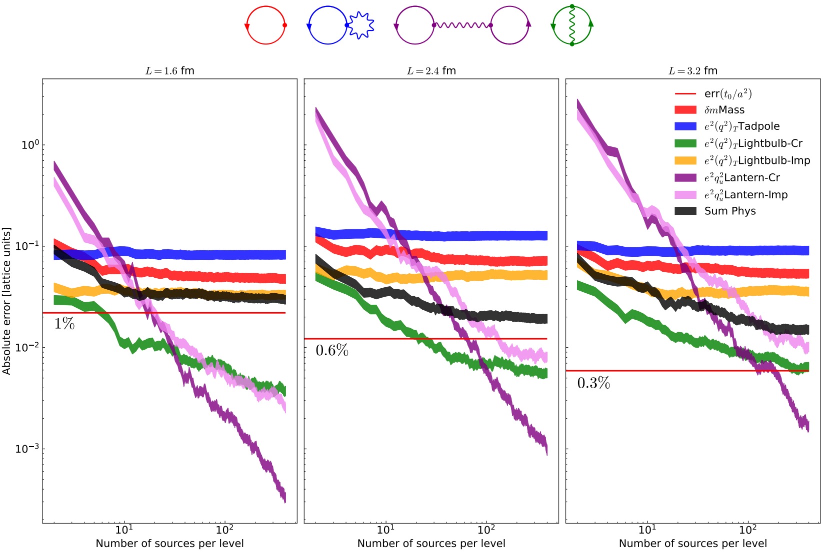
Continuum Scaling of the error of t_0/a^2

Scaling with the number of sources

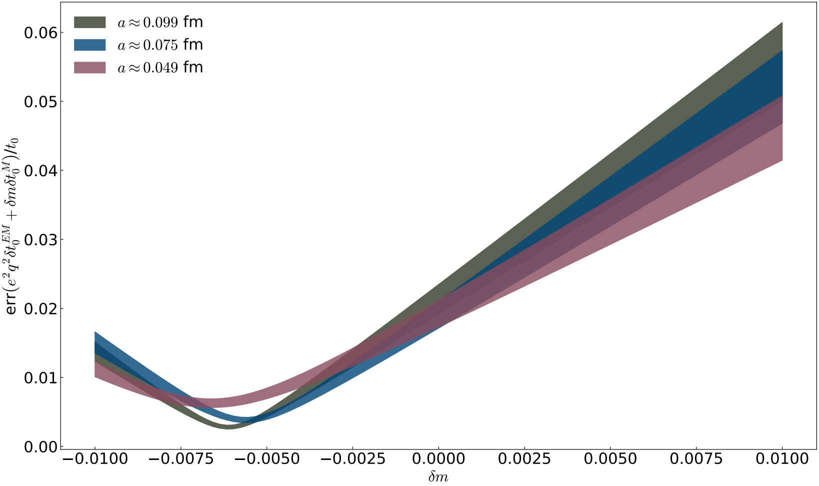


Infinite Volume Scaling of the error of t_0/a^2

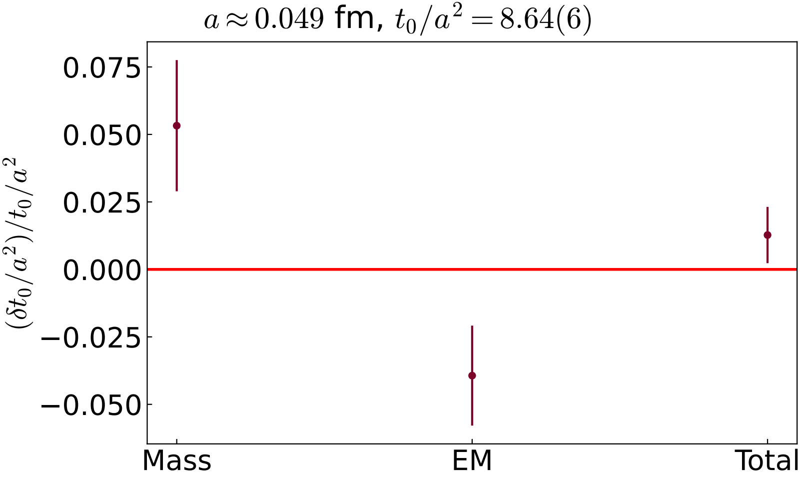
Scaling with the number of sources



Error Correlation for t_0/a^2



Correction to t_0/a^2



Conclusions and Outlook

- ✓ Sea quark isospin-breaking effects can be computed for $O(a)$-improved Wilson fermions, reaching the gauge noise for almost all the contributions;
- ✓ The gauge error of the diagrams does not diverge in the continuum limit;
- ✓ The gauge error of the diagrams diverges in the infinite volume limit;
- ✓ The correlation between the mass term and the tadpole term can be exploited with new discretization of the mass operator;
- Extend the statistics and $m_\pi \to m_{\pi^{\text{phys.}}}$;
- Study the effects on the HVP [for the latest update, see L. Parato's talk today @ 12:10];

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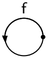
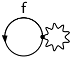
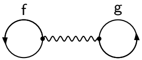
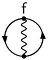
Thank you for your attention!

Backup: The Diagrams

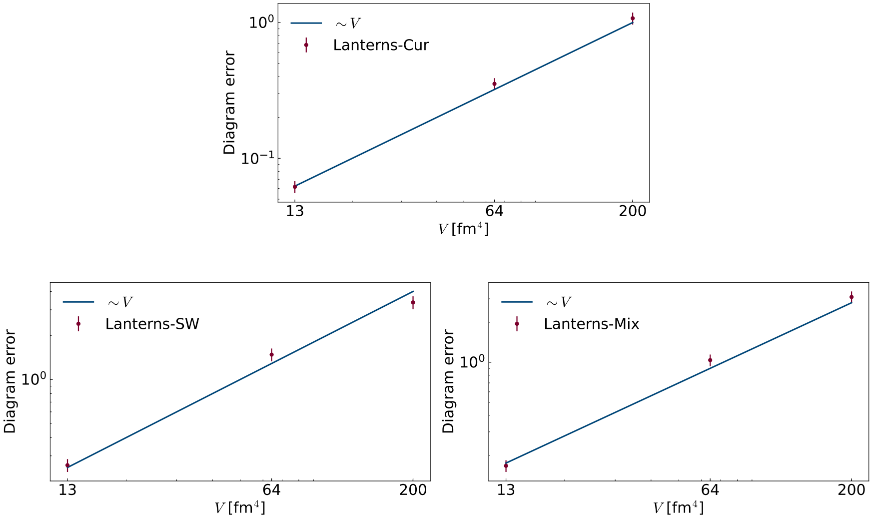
$$\frac{1}{2}\sum_{x} \text{Re tr}\left[D_f^{-1}(x,x)\right] =$$ 

$$\frac{1}{4}\sum_{xy} \langle \text{Re tr}\left[D_f^{-1}(x,y)T(y,x)\right]\rangle_\gamma =$$ 

$$\left.\begin{array}{l} \frac{1}{8}\sum_{xyzw,\mu} \langle \text{Im tr}\left[J(x,y)D_f^{-1}(y,x)\right] \text{Im tr}\left[J(z,w)D_g^{-1}(w,z)\right]\rangle_\gamma \\ \frac{c_f c_s}{128}\sum_{xy,\mu\nu\rho\sigma} \langle \text{Re tr}\left[\sigma_{\mu\nu}\hat{A}_{\mu\nu}(x)D_f^{-1}(x,x)\right] \text{Re tr}\left[\sigma_{\rho\sigma}\hat{A}_{\rho\sigma}(y)D_g^{-1}(y,y)\right]\rangle_\gamma \\ \frac{c_f}{16}\sum_{xyz,\mu\nu} \langle \text{Re tr}\left[\sigma_{\mu\nu}\hat{A}_{\mu\nu}(x)D_f^{-1}(x,x)\right] \text{Im tr}\left[J(y,z)D_g^{-1}(z,y)\right]\rangle_\gamma \end{array}\right\} =$$ 

$$\left.\begin{array}{l} \frac{1}{4}\sum_{xyzw,\mu} \langle \text{Re tr}\left[J(x,y)D_f^{-1}(y,z)J(z,w)D_f^{-1}(w,x)\right]\rangle_\gamma \\ -\frac{c_f^2}{64}\sum_{xy,\mu\nu\rho\sigma} \langle \text{Re tr}\left[\sigma_{\mu\nu}\hat{A}_{\mu\nu}(x)D_f^{-1}(x,y)\sigma_{\rho\sigma}\hat{A}_{\rho\sigma}(y)D_f^{-1}(y,x)\right]\rangle_\gamma \\ -\frac{c_f}{8}\sum_{xyz,\mu\nu} \langle \text{Im tr}\left[\sigma_{\mu\nu}\hat{A}_{\mu\nu}(x)D_f^{-1}(x,y)J(y,z)D_f^{-1}(z,x)\right]\rangle_\gamma \end{array}\right\} =$$ 

Backup: Volume Scaling for the Lanterns



Backup: Continuum Scaling for the other Diagrams