Status of the ETMC calculation of a_μ^{HVP} in isoQCD

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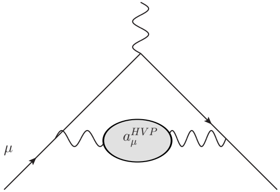
Hadron Vacuum Polarization from Lattice QCD

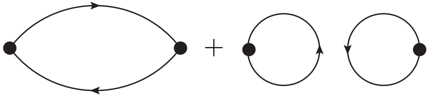


- Time-Momentum representation [D. Bernecker,H. B. Meyer (2011)]

$$a_\mu^{HVP} = 3\alpha_{em}^2 \int_0^\infty dt\, K(t)V(t) \qquad V(t) = \frac{1}{3}\sum_{i=1}^{3} \int d\boldsymbol{x}\, \langle J_i(\boldsymbol{x}, t), J_i(0)\rangle$$

$$J_\mu = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d - \tfrac{1}{3}\bar{s}\gamma_\mu s + \tfrac{2}{3}\bar{c}\gamma_\mu c$$

- In isoQCD $m_u = m_d$ two diagrams contribute



Twisted mass clover fermions

- Gluon Iwasaki action
- Automatic $\mathcal{O}(a)$-improvement
- Ensembles $\approx$ physical point
- Twisted-mass (TM) and Osterwalder-Seiler (OS) currents differ by $\mathcal{O}(a^2)$

$$J_\mu^{TM} = \bar{\psi}^+ \gamma_\mu \psi^- , \qquad J_\mu^{OS} = \bar{\psi}^+ \gamma_\mu \psi^+$$

  $\pm$ the sign of the twisted Wilson parameter
- Different renormalization $Z_A J_\mu^{TM}$ and $Z_V J_\mu^{OS}$
  - Z_A and Z_V from Ward-identity $< 0.1\%$ uncertainties
- Disconnected term can be evaluated only with the OS current

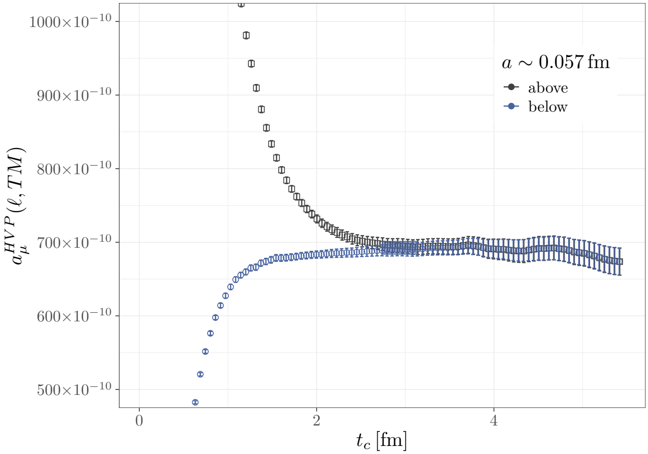
- Four lattice spacings $a \in 0.08 - 0.05$ fm
  - Finest (E112) $a \sim 0.05$ fm measurements in progress
- Leading QED and strong-isospin breaking (SIB) contributions.
  - "Valence leading isospin breaking contributions to a_μ^{HVP}", A. Evangelista
  - Sea effects computation in progress
- Lattice $L \in 5.1 - 5.5$ fm plus dedicated ensembles to check finite volume extrapolations $L \sim 7.6$ fm (B96)
- isoQCD prescription, $M_\pi^{iso} = 135.0$ MeV, $M_K^{iso} = 494.2$ MeV, $M_{D_s}^{iso} = 1969.0$ MeV, $F_\pi = 130.4$ MeV
- blinded analysis, 4 analysis groups $V_\ell^{blinded}(t) = V_\ell(t) + \sum_n c_n e^{-M_n t}$

- Statistical relative uncertainties increase exponentially at large Euclidean times.
- [RBC/UKQCD, (2018)] windows
- [RBC/UKQCD, (2018)] [BMW-DMZ, (2024)] combine lattice determination in a very long window with data-driven calculation from $e^{+}e^{-}$ annihilation and τ-decay experiments at very low energy
- Bounding methods [C. Lehner (2016)] [S. Borsanyi et al (2017)]
  - Improved bounding method [M. Bruno, T. Izubuchi, C. Lehner, A. Meyer (2023)]
- Low mode Averaging (LMA)
  [H. Neff, et al. (2001)] [L. Giusti, et al. (2004)] [T. A. DeGrand, S Schaefer, (2004)]
  poster "Smeared R-ratio in isoQCD with Low Mode Averaging" F. Margari

- Here we use

$$c(t_c)e^{-m_{eff}(t_c)(t-t_c)} \leq c(t > t_c) \leq c(t_c)e^{-E_{2\pi}(t-t_c)}$$

- $E_{2\pi}$ the energy of the lightest two charged pion state in free theory



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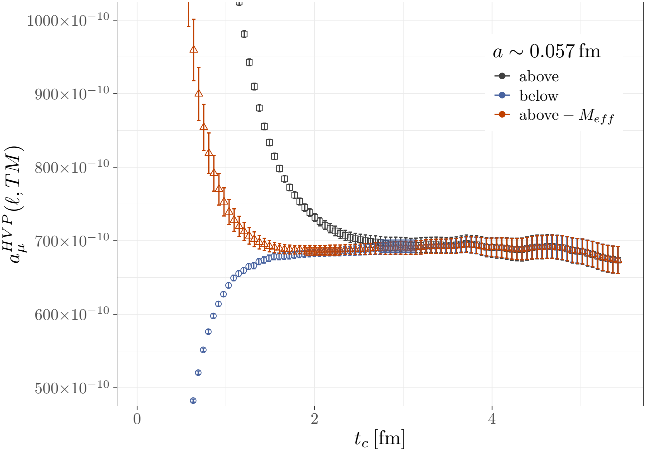

- A better bound using estimated ground state energy in progress ($\pi^+\pi^0$ in this mixed action setup)

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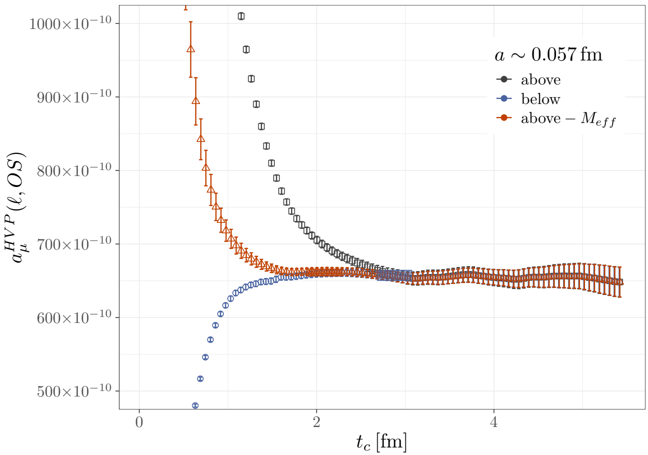
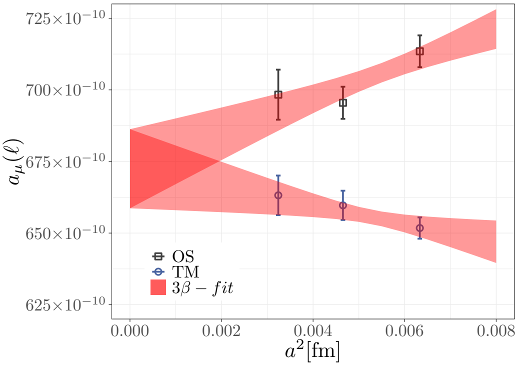


- A better bound using estimated ground state energy $(\pi^+\pi^-)$ in progress

ℓ-quark connected [Preliminary]

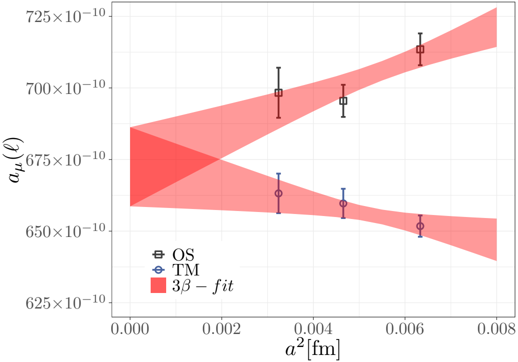


- Data extrapolated to $L = 5.46$ fm using Meyer-Lellouch-Lüscher-Gounaris-Sakurai (MLLGS) approach
- Correction to isoQCD pion point with MLLGS, lattice calculation in progress
- Significant reduction of uncertainty possible

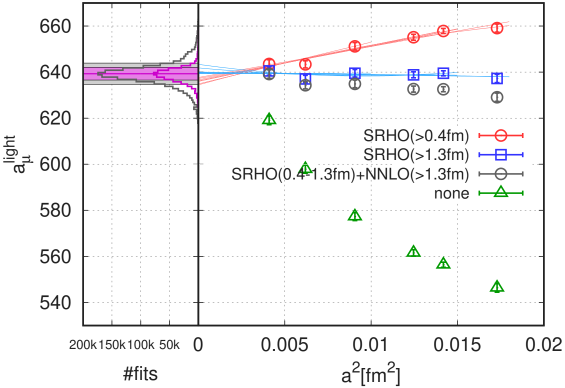
* Statistic at the two finest lattice spacing not final, better bounding and a finer lattice spacing
* Considering models to reduce lattice artefacts and improve continuum extrapolation

ℓ-quark connected

- Considering models to reduce lattice artefacts and improve continuum extrapolation

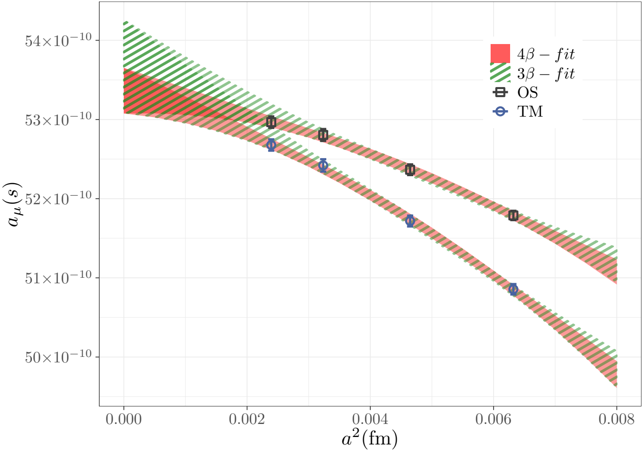


Blinded analysis, mean values comparison meaningless



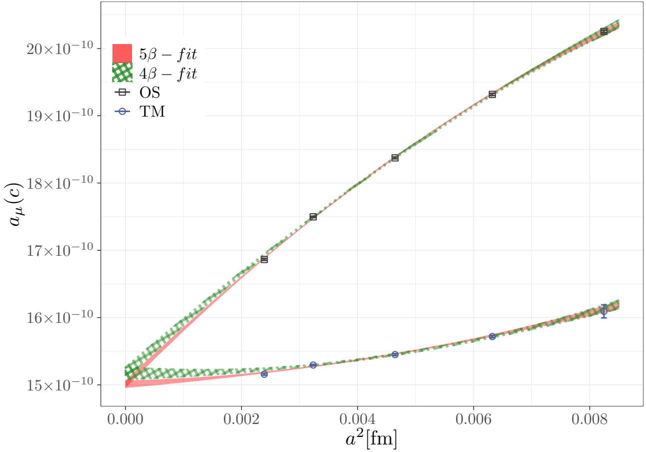
[Figure from BMW (2021)]

s-quark [Preliminary]



- No loss of signal at large t
- correlator computed without LMA
- Finer lattice spacing E112 reduces uncertainties by a factor of 2

c-quark [Preliminary]



- finer lattice spacing E112 reduces uncertainties by a factor of 2
- Coarsest lattice spacing A extrapolated to the physical point

Work in progress

- Increase statistic for two lattice spacing D96 and C80
- Finer lattice spacing with LMA: E112
- Ensemble at larger volume $L \sim 7.6$ fm $\rightarrow$ data-driven FSE corrections B96
- Numerical corrections to isoQCD pion point
- Disconnected contribution: preliminary result at $a \sim 0.08$ fm $\rightarrow$ 10% statistical error
- Exploring models to reduce lattice artefacts

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Thank you for your attention

Backup

- LMA + HIERARCHICAL PROBING [A. Stathopoulos, (2013)]
- $a \sim 0.08$ fm B64

$a_\mu(\ell)$ up to t_c