Hadronic τ data and Lattice QCD+QED simulations for the muon $(g-2)$

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for the RBC/UKQCD collaborations

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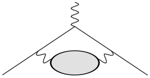
$(g-2)_\mu$

Lattice



Hadronic Vacuum Polarization (HVP) contribution to a_μ

Lattice

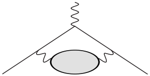
Time-momentum representation [Bernecker, Meyer, '11]

$$G^\gamma(t) = \frac{1}{3}\sum_k \int d\vec{x}\, \langle j_k^\gamma(x) j_k^\gamma(0)\rangle \quad \to \quad a_\mu = 4\alpha^2 \sum_t w_t G^\gamma(t)$$

Windows in Euclidean time [RBC/UKQCD '18]

$$a_\mu^W = 4\alpha^2 \textstyle\sum_t w_t\, G^\gamma(t)\,[\Theta(t,t_0,\Delta) - \Theta(t,t_1,\Delta)]$$

$$t_0 = 0.4 \text{ fm} \quad t_1 = 1.0 \text{ fm} \quad \Delta = 0.15 \text{ fm}$$

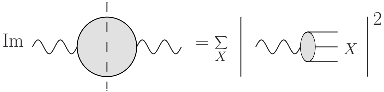


Hadronic Vacuum Polarization (HVP) contribution to a_μ

Dispersive

$$a_\mu = \frac{\alpha}{\pi} \int \frac{ds}{s} \, K(s, m_\mu) \, \frac{\mathrm{Im}\Pi(s)}{\pi}$$

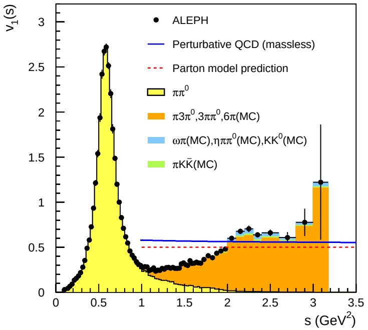
[Brodsky, de Rafael '68]

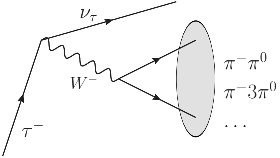


$$\frac{4\pi^2\alpha}{s} \frac{\mathrm{Im}\Pi(s)}{\pi} = \sigma_{e^+e^- \to \gamma^\star \to \mathrm{had}}$$

Motivations

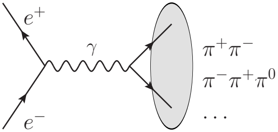
τ decays





$V - A$ current

Final states $I = 1$ charged



EM current

Final states $I = 0, 1$ neutral

τ data can improve $a_\mu[\pi\pi]$

$\to$ 72% of total Hadronic LO

$\to$ competitive precision on a_μ^W

Status

From the 2020 $(g-2)$ White Paper

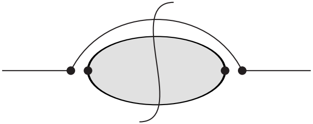
" ... it appears that, at the required precision to match the e^+e^- data, the present understanding of the IB corrections to τ data is unfortunately not yet at a level allowing their use for the HVP dispersion integrals. "

"The ratio $|F_0(s)/F_-(s)|^2$ is the most difficult to estimate reliably, since a number of different IB effects may contribute."

Recent reappraisal of τ data [Davier et al '23]

but no model-independent answer (yet)

$$\mathcal{M}_f(P,q,p_1\cdots p_{n_f}) = \frac{G_F V_{ud}}{\sqrt{2}}\,\bar{u}_\nu(-q)\gamma^L_\mu u_\tau(P)\,\langle \mathrm{out}, p_1\cdots p_{n_f}|\mathcal{J}^-_\mu(0)|0\rangle$$



$$d\Gamma = \frac{1}{4m} d\Phi_q \sum_f d\Phi_f \sum_{\mathrm{spin}} |\mathcal{M}_f|^2$$

$$= \frac{1}{4m} d\Phi_q \frac{G_F^2 |V_{ud}|^2}{2} \mathcal{L}_{\mu\nu}(P,q)\,\rho^{\mathrm{w}}_{\mu\nu}(p)$$

Charged spectral density isospin limit $= \rho^{\mathrm{w},0}$ $\quad\left[d\Phi_q = \frac{d^3q}{(2\pi)^3 2\omega_q}\right]$

$$\frac{d\Gamma(s)}{ds} = G_F^2|V_{ud}|^2 \frac{m^3}{16\pi^2}\left(1+\frac{2s}{m^2}\right)\left(1-\frac{s}{m^2}\right)^2 \rho^{\mathrm{w},0}(s)$$

$$= G_F^2|V_{ud}|^2 \frac{m^3}{16\pi^2}\,\kappa(s)\,\rho^{\mathrm{w},0}(s)$$

W REGULARIZATION

Short-distance effects

[Sirlin '82][Marciano, Sirlin '88][Braaten, Li '90]

Effective Hamiltonian $H_W \propto G_{\rm F} O_{\mu\nu}$

$G_{\rm F}$ low-energy constant; 4-fermion operator $O_{\mu\nu}$

At $O(\alpha)$ new divergences in EFT $\rightarrow$ need regulator, Z factors

$$\frac{1}{k^2} = \frac{1}{k^2 - m_W^2} - \frac{m_W^2}{k^2(k^2 - m_W^2)}$$

[Sirlin '78]

1. universal UV divergences re-absorbed in $G_{\rm F}$
2. process-specific corrections in S_{EW}, like a Z factor

Effective Hamiltonian at $O(\alpha)$: $H_W \propto G_{\rm F} S_{EW}^{1/2} O_{\mu\nu}$

matching required as noted by [Carrasco et al '15][Di Carlo et al '19]

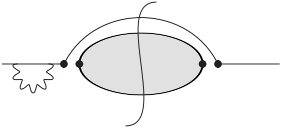
Wave-function renormalization

$$Z_\tau = 1 + \frac{\alpha}{2\pi}\left[\log\frac{m_\tau}{\mu} + 2\log\frac{m_\gamma}{m_\tau} + \cdots\right]$$

$$\frac{d\Gamma}{ds} \simeq 2 \times \frac{1}{2}[Z_\tau - 1]|\mathcal{M}|^2$$

$$\delta Z_\tau \equiv \frac{\alpha}{2\pi}\log(m_W/m_\tau)$$

[Sirlin '82]



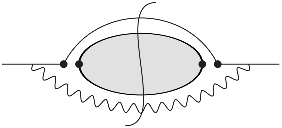
τ Bremsstrahlung

[Cirigliano et al '00, '01][MB et al, in prep]

$$\frac{d\Gamma}{ds}\ \frac{\alpha}{\pi}\left[G_{\log}(s,m_\gamma) + \ldots\right]$$

$$G_{\log}(s,m_\gamma) = \log\frac{m_\gamma}{m_\tau} + \cdots$$

$$\delta\kappa(s) \equiv G_{\log}(s,m_\tau) + \ldots$$



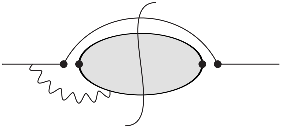
$$\frac{d\Gamma}{ds} \mathrel{+}= G_F^2|V_{ud}|^2\,\frac{m^3}{16\pi^2}\,\kappa(s)\,\rho^{\mathrm{w},0}(s)\left[\delta Z_\tau + \delta\kappa(s)\right]$$

Virtual photon loop

$\delta Z_{\kappa\rho} \propto \frac{\alpha}{\pi} \log(m_W/m_\tau)$ [Sirlin '82]

[Cirigliano et al '01]

Finite parts EFT and 2π

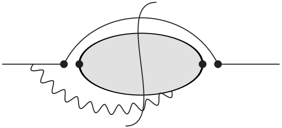


$\tau - \pi$ bremsstrahlung interfence

From EFT and 2π [Cirigliano et al' 00, '01]

Structure-independent captured by EFT
Structure-dependent meson dominance

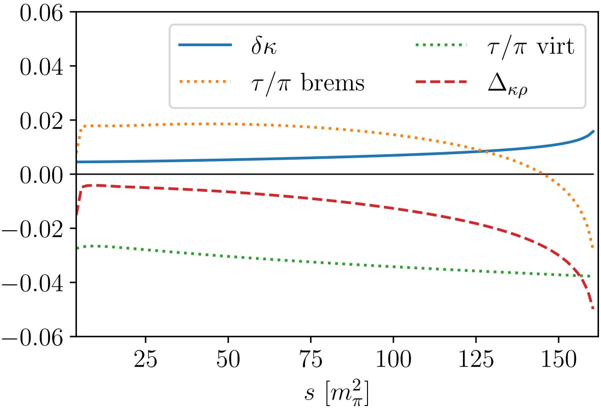
[Flores-Talpa et al. '06, '07]



$$\frac{d\Gamma}{ds} \mathrel{+}= G_F^2 |V_{ud}|^2 \frac{m^3}{16\pi^2} \kappa(s)\, \rho^{w,0}(s) \left[\delta Z_{\kappa\rho} + \Delta_{\kappa\rho}(s)\right]$$

Long-distance corrections

Let's take a look



$\delta\kappa$ is channel and m_γ independent [MB et al, in prep]
$\Delta_{\kappa\rho} \to 2\pi$, point-like, m_γ independent [Cirigliano et al '01, '02]

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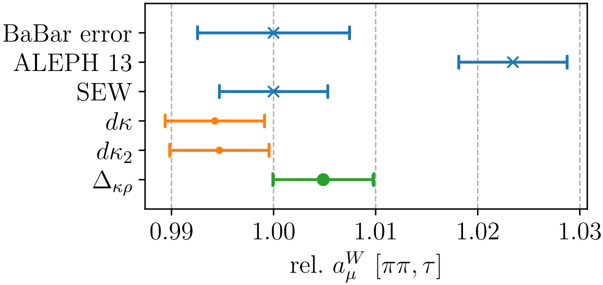
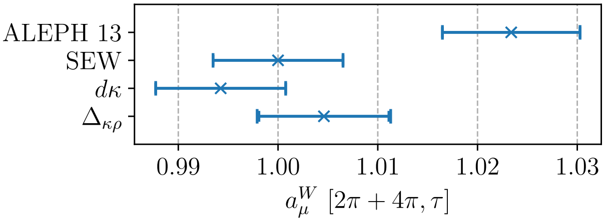
Towards a_μ^W

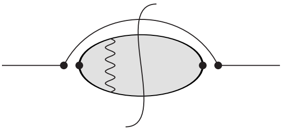
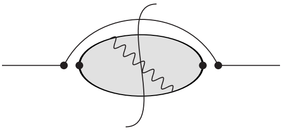
Preliminary



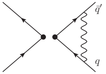


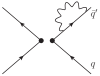
Isospin breaking

Final state

  $+ \ldots$

[Braaten,Li '90]

$-Q_u Q_d \times$ 

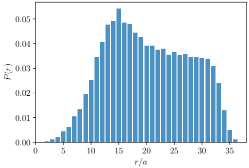
$+ \, (Q_u^2 + Q_d^2) \times \frac{1}{2}$ 

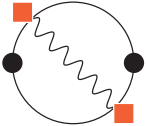
Naive scaling $\frac{\alpha}{\pi} \log(aM_w) \simeq 0.8 - 1\%$ so delicated matching required

Sampling strategy

Example

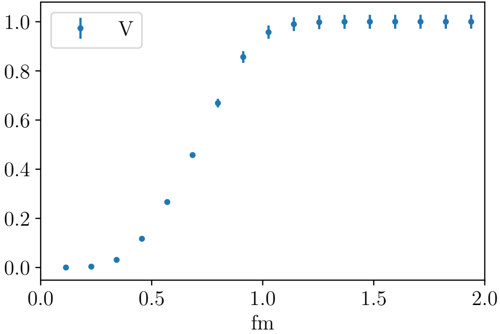
$O(10^3)$ points $\to$ $O(10^6)$ pairs





contract photon offline
$\to$ study QED_L vs QED_∞

[preliminary]



ongoing study 48I, 64I, 96I
3 lattice spacings, phys. mass

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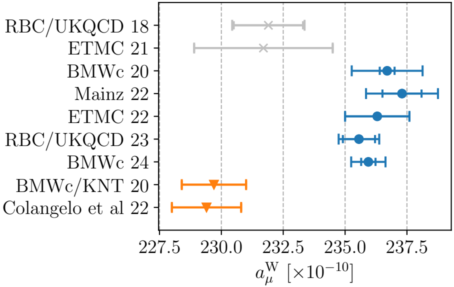
Conclusions

...and outlooks

hadronic τ-decays can shed light on tension lattice vs e^+e^-



τ data competitive on intermediate window

blinded analysis of Aleph

initial+mixed rad.cors. analytic

final radiative from LQCD+QED

Remaining work (in progress) to finalize full formalism [MB et al, in prep]

- W-regularization and short-distance corrections
- non-factorizable effects: beyond EFT?

Thanks for your attention

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Definitions

Hadronic currents

$$\mathcal{J}^{\gamma}_{\mu} = Q_{\mathrm{u}}\overline{u}\gamma_{\mu}u + Q_{\mathrm{d}}\overline{d}\gamma_{\mu}d$$

$$\mathcal{J}^{-}_{\mu} = \overline{u}\gamma_{\mu}d\,, \quad \mathcal{J}^{1}_{\mu} = \frac{Q_{\mathrm{u}}-Q_{\mathrm{d}}}{\sqrt{2}}\overline{u}\gamma_{\mu}d$$

Hadronic phase-space factor, i labels hadrons

$$d\Phi_f(p) \equiv (2\pi)^4\delta^4(p - \sum_i p_i)\, S_f \prod_i \frac{d^3p_i}{(2\pi)^3 2\omega_i}$$

Charged spectral densities

$$\begin{aligned}
\rho^{\mathsf{w}}_{\mu\nu}(p) &= \frac{1}{2\pi}\int d^4x\, e^{ipx}\,\langle 0|\mathcal{J}^{+}_{\mu}(x)\,\mathcal{J}^{-}_{\nu}(0)|0\rangle \\
&= \frac{1}{2\pi}\sum_f \int d\Phi_f\,\langle 0|\mathcal{J}^{+}_{\mu}(0)|p_1\cdots,\mathrm{out}\rangle\langle p_1\cdots,\mathrm{out}|\mathcal{J}^{-}_{\nu}(0)|0\rangle \\
&= (p^2 g_{\mu\nu} - p_{\mu}p_{\nu})\,\rho^{\mathsf{w}}(s) \qquad [s = p^2]
\end{aligned}$$

Electronic rate

$$\Gamma_e = \Gamma(\tau \to e\overline{\nu}\nu) = \frac{\mathcal{B}_e \Gamma}{\mathcal{B}} = \frac{G_F^2 m_\tau^5}{192\pi^3}$$

Used to normalize exp. data $\Gamma_e \times \frac{1}{\Gamma}\frac{d\Gamma}{ds}$

$O(\alpha)$ correction finite in Fermi theory [Kinoshita, Sirlin '59]

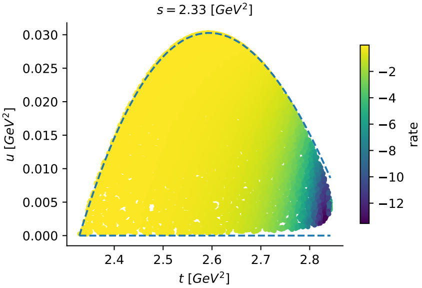
$\rightarrow$ 0.4% correction

Special care required to avoid double counting [Erler '02]

Capitano

A numerical n-particle phase-space integrator

- Grid/GPT backend, support for several parallelization schemes
- partial support for 1-loop Passarino-Veltman functions
- no support for MCMC yet (needed for $>=$6 particles)
- currently private, soon public `github.com/mbruno46`



Used to cross-check analytic formulae

Example: Dalitz plot τ Bremsstrahlung
$\rightarrow$ wrong boundary: finite m_γ effects