Precision charmonium spectroscopy on CLS ensembles

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Lattice 2024, Liverpool

July 30, 2024

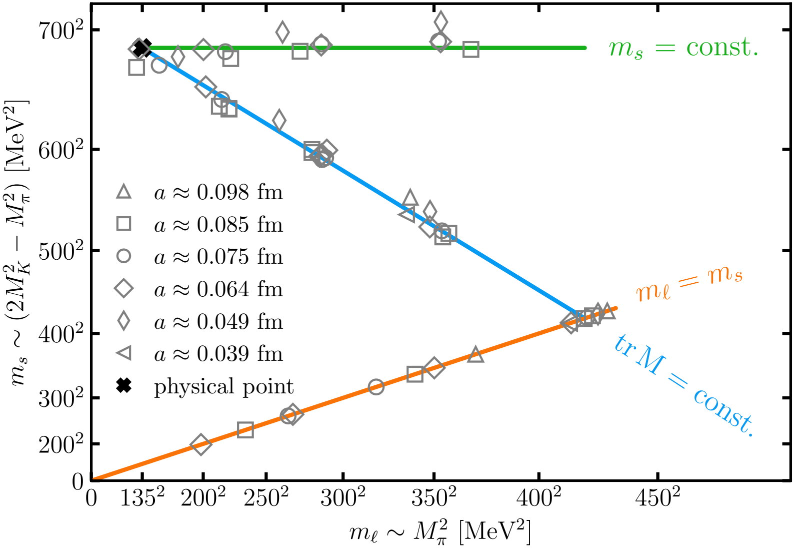
Motivation and scope

- Precision study of stable charmonia to set baseline for investigations of XYZ states.
- Compute low lying charmonium masses and decay constants from isoQCD quark-line connected correlation functions as a benchmark for future studies including electromagnetic and annihilation effects.
- $m_c \not\ll a^{-1}$: how big are cut-off effects? Are they bigger for the mass than for fine structure splittings $\sim m_c v$ or spin-independent splittings $\sim m_c v^2$?

For the moment being, we only study $J = 0$ and $J = 1$ charmonia
(smeared interpolating operators without derivatives).

I will only show **very preliminary** results for a reduced set of observables
(M_{η_c} and $M_{J/\psi} - M_{\eta_c}$) on a reduced set of gauge ensembles.

CLS ensembles in the quark mass plane



$N_f = 2 + 1$

NP $O(a)$ improved Wilson fermions

3 mass trajectories

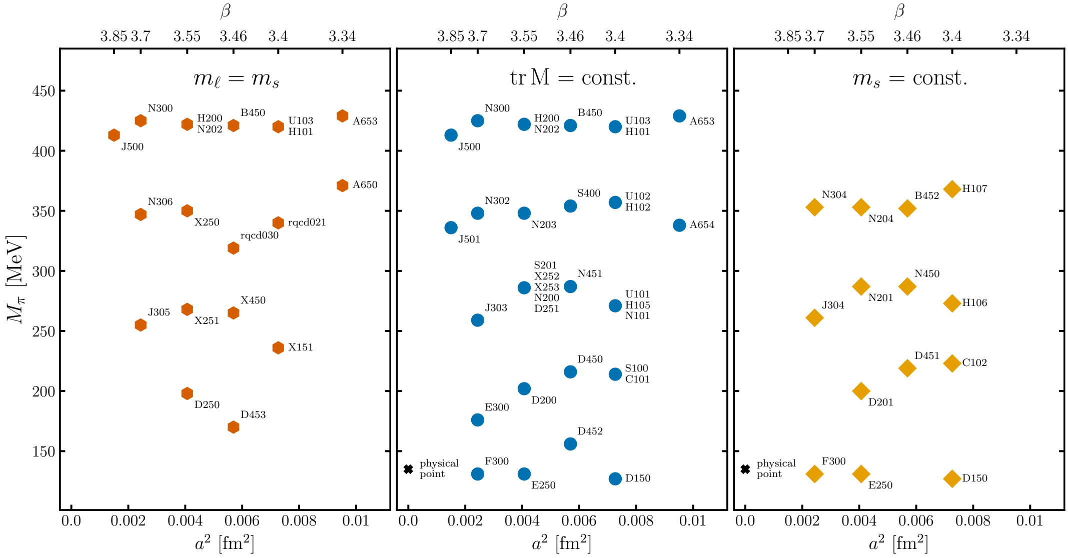
6 lattice spacings

Many volumes, including $M_\pi L > 4$

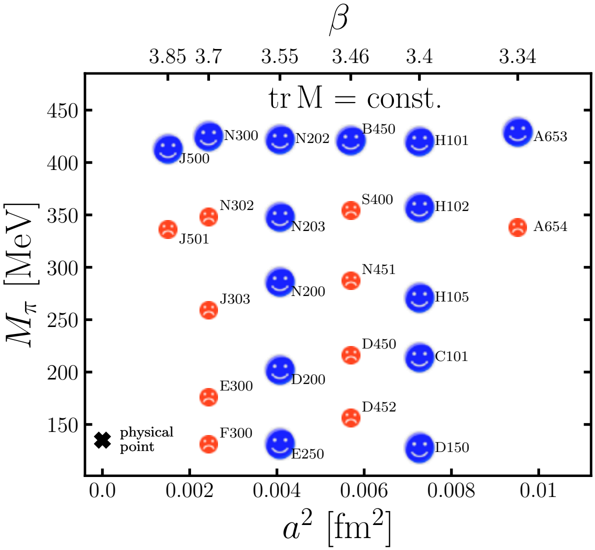
Usually a few 1000 MDUs

⇝ Georg von Hippel Friday data session

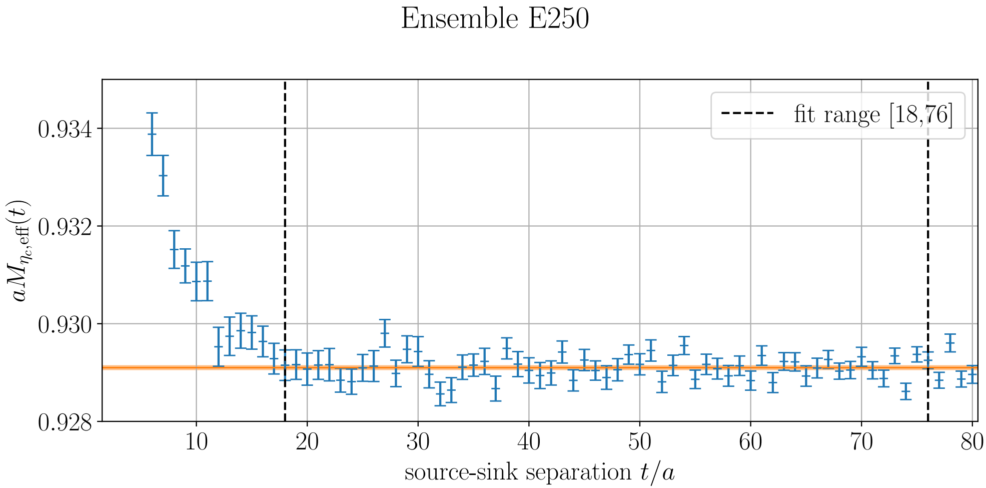
Ensembles available for this analysis



Results presented here



Example for a mass determination



$a \approx 0.064\,\text{fm}$, $M_\pi \approx 131\,\text{MeV}$, $V = 192 \cdot 96^3 a^4$, $\qquad aM(t) = \text{arccosh}\left[\frac{C(t+a)+C(t-a)}{2C(t)}\right]$

Definitions and input

Scale set via $t_{0,\mathrm{ph}}^{-1/2} = 1.362(8)\,\mathrm{GeV}$ [RQCD, 2211.03744], obtained from m_{Ξ}.

Using this scale, we define the isoQCD physical quark mass point ($\{m_q\} \to \{m_{q,\mathrm{ph}}\}$) as

$$M_\pi = 135\,\mathrm{MeV}, \quad M_K = 494\,\mathrm{MeV}, \quad M_{\overline{D}} = \frac{2M_D + M_{D_s}}{3} = 1899.4\,\mathrm{MeV}.$$

Define $\overline{M}^2 = \frac{2M_K^2 + M_\pi^2}{3} \approx 2B_0\overline{m}, \quad \delta M^2 = 2\left(M_K^2 - M_\pi^2\right) \approx 2B_0(m_s - m_\ell).$

To leading non-trivial order in ChPT neither charmonium masses nor $M_{\overline{D}}$ depend on δM^2.

Quantities in lattice units: $\mathbf{M}$, $\mathbf{t}_0$ etc. Then for $m_q \to m_{q,\mathrm{ph}}$, $L \to \infty$, $a \to 0$:

$$\mathbf{M}_\psi(\{m_q\}, L, a)\sqrt{\mathbf{t}_0(\{m_q\}, a)} \longrightarrow M_\psi\sqrt{t_{0,\mathrm{ph}}}, \quad \mathbf{t}_0(\{m_q\}, a)a^2 \longrightarrow t_{0,\mathrm{ph}} \quad \text{etc.}$$

Use t_0^*, defined at the point where $M_\pi = M_K$ and $12t_0^* M_\pi^2 = 1.11$, for the continuum limit extrapolation:

$$\frac{1}{\mathbf{t}_0^*} = \frac{a^2}{t_0^*}.$$

Extrapolation strategy

Ideally, we would simulate at fixed values of the $O(a)$ improved coupling constant $g^2 = g_0^2[1 + b_g(g_0^2)a\overline{m}]$.

Instead, we simulate at fixed $\beta = 6/g_0^2$ values, thereby changing the lattice spacing by an $O(a)$ effect between different points in the quark mass plane.

To cancel this, we must extrapolate dimensionless ratios of quantities, e.g., $\mathbf{M}_1/\mathbf{M}_2$ or $\mathbf{M}\sqrt{\mathbf{t}_0}$.

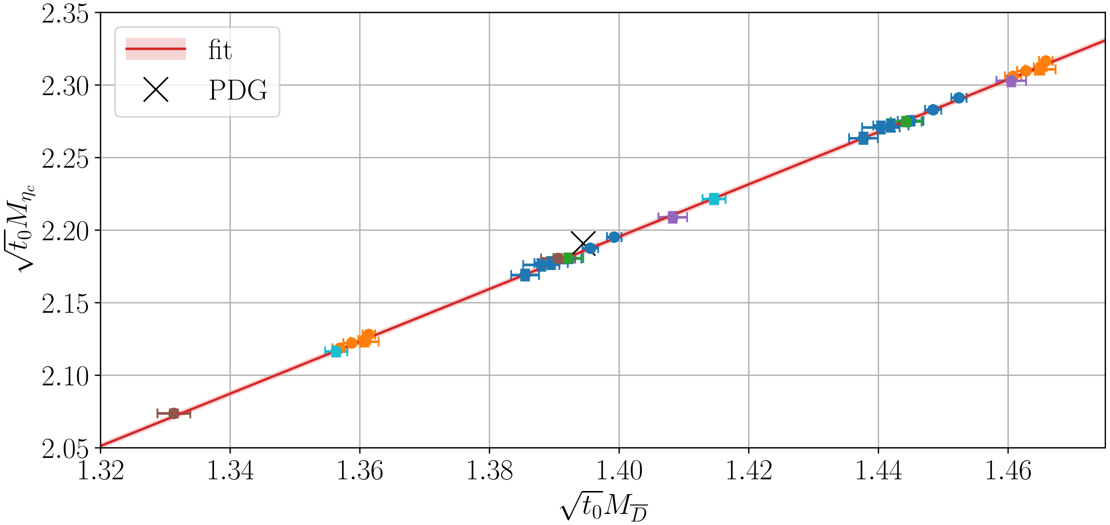
Accounting for this, we carry out a global fit to the charmonium mass data of the type:

$$\mathbf{M}_\psi = \frac{1}{\sqrt{\mathbf{t}_0}}\left\{M_\psi\sqrt{t_{0,\mathrm{ph}}} + c_c\left[\mathbf{M}_{\overline{D}}\sqrt{\mathbf{t}_0} - M_{\overline{D}}\sqrt{t_{0,\mathrm{ph}}}\right] + \bar{c}\left[\overline{\mathbf{M}}^2\mathbf{t}_0 - \overline{M}^2 t_{0,\mathrm{ph}}\right] + \frac{c_a}{\mathbf{t}_0^*} + \ldots\right\}.$$

The fit parameter M_ψ is then the charmonium mass in GeV at the physical point.

For each ensemble, two charm quark masses have been realized, bracketing the physical one.

Charm quark mass interpolation **(preliminary)**

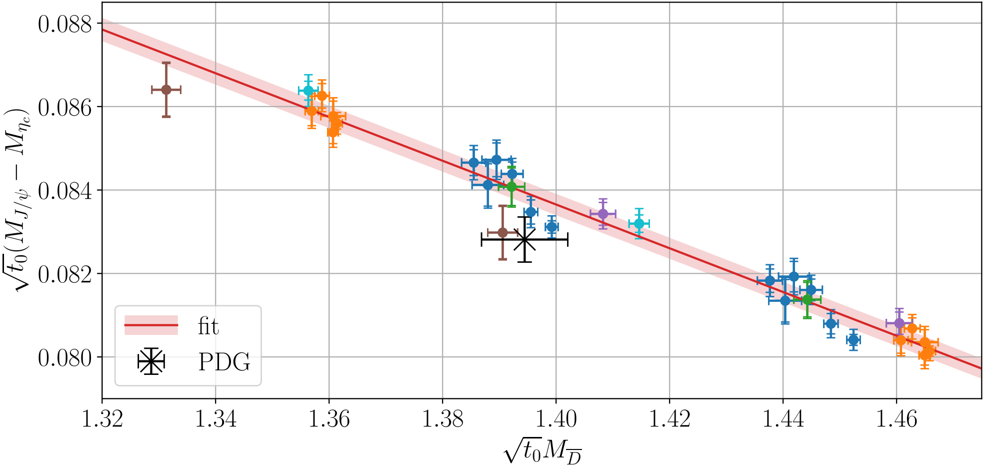


Fit details

$$\mathbf{M}_\psi = \frac{1}{\sqrt{\mathbf{t}_0}}\left\{M_\psi\sqrt{t_{0,\mathrm{ph}}} + c_c\left[\mathbf{M}_{\overline{D}}\sqrt{\mathbf{t}_0} - M_{\overline{D}}\sqrt{t_{0,\mathrm{ph}}}\right] + \bar{c}\left[\overline{\mathbf{M}}^2\mathbf{t}_0 - \overline{M}^2 t_{0,\mathrm{ph}}\right] + \frac{c_a}{\mathbf{t}_0^*} + \frac{c_{ac}}{\mathbf{t}_0^*}\mathbf{M}_{\overline{D}}\sqrt{\mathbf{t}_0}\right\}$$

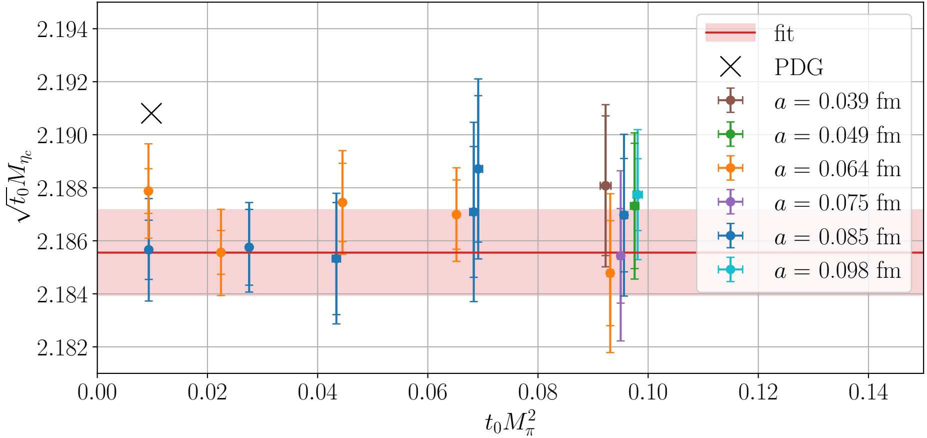
- On each ensemble, correlations between $\mathbf{M}_\pi$, $\mathbf{M}_K$ and the $\mathbf{M}_{\overline{D}}$ and $\mathbf{M}_\psi$ for the two charm quark masses are taken into account: 6×6 covariance matrices (5×5 for $m_s = m_\ell$).
- "x-errors" are included.
- $\mathbf{t}_0$ enters for each ensemble and $\mathbf{t}_0^*$ for each β-value (priors with errors).
- $t_{0,\mathrm{ph}}^{-1/2} = 1.362(8)\,\mathrm{GeV}$ added as a prior.
- 5 fit parameters for now. In the future, we will systematically incorporate further effects, including a^3, $\mathbf{M}_{\overline{D}}^2$, $\overline{\mathbf{M}}^2\delta\mathbf{M}^2$, $\delta\mathbf{M}^4$ and other corrections.

$1S$ fine structure splitting versus the D meson mass (preliminary)



Uncertainty of $t_{0,\text{ph}}$ was removed from the fit band and added to the experimental value.

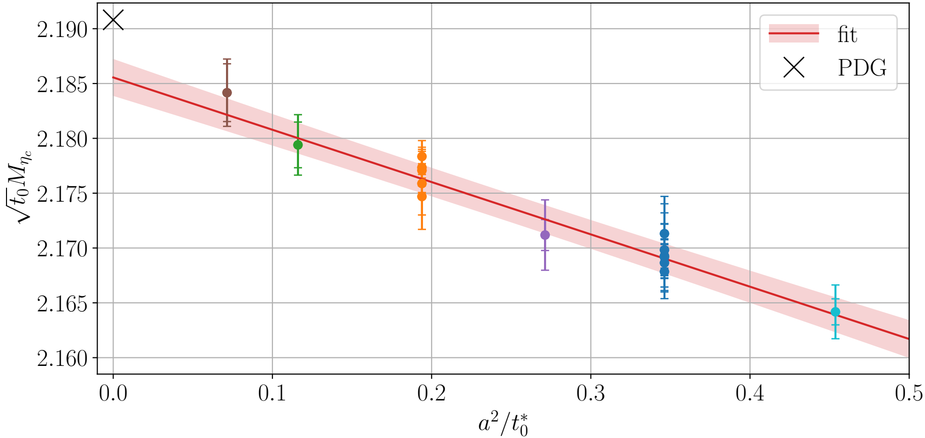
Pion mass dependence of the η_c mass (preliminary)



Error smaller than that of $\sqrt{t_{0,\text{ph}}}$ because charm quark mass set via $\mathbf{M}_{\overline{D}} = M_{\overline{D}}\sqrt{t_{0,\text{ph}}/\mathbf{t}_0}$.

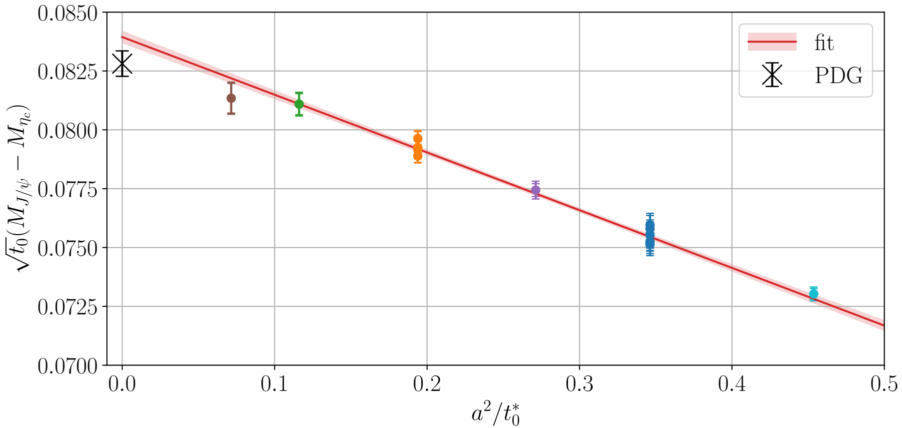
$\curvearrowright$ cancellations: only the error of $M_{\eta_c} - 2M_{\overline{D}}$ approximately scales with that of $\sqrt{t_{0,\text{ph}}}$.

The η_c mass: continuum limit **(preliminary)**



Fit band includes the uncertainty of $t_{0,\mathrm{ph}}$. Correlated $\chi^2/\mathrm{dof} = 23.3/25$.

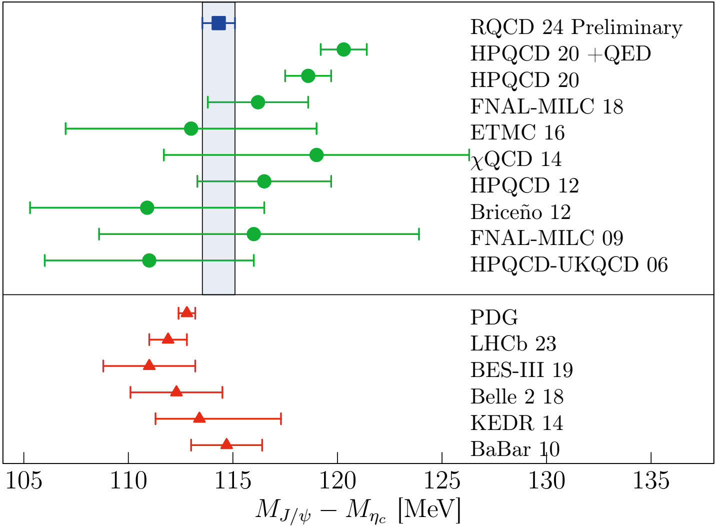
The $1S$ fine structure splitting: continuum limit **(preliminary)**



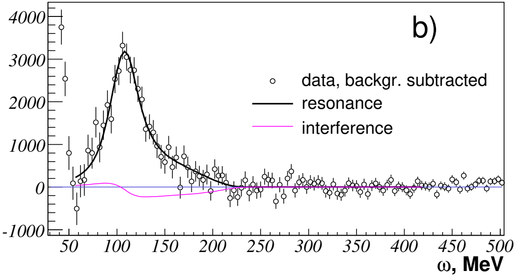
Uncertainty of $t_{0,\mathrm{ph}}$ only included in the "PDG" value. Correlated $\chi^2/\mathrm{dof} = 64.1/25$.

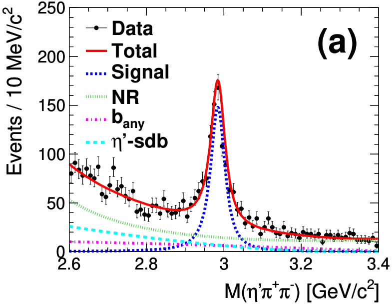
For the moment being, we inflate our error by $\sqrt{\chi^2/\mathrm{dof}}$. Extra fit parameters in the future.

The fine structure splitting in isoQCD. Connected two-point function only.



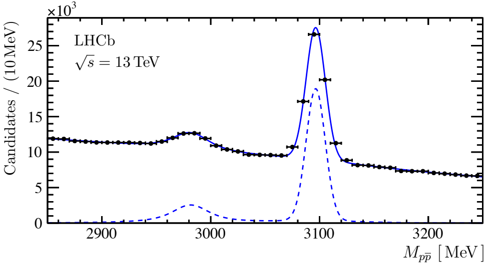
Experimental status

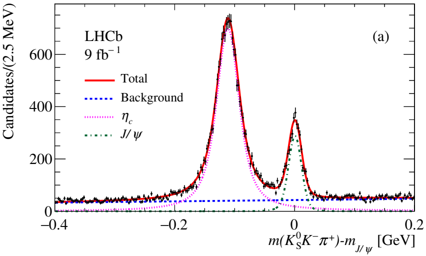




[KEDR, 1406.7644]

[Belle, 1805.03044]





[LHCb, 1911.03326]

[LHCb, 2304.14891]

Summary and outlook

- From potential models and [HPQCD, 2005.01845], one would expect an increase of the $1S$ fine structure splitting by up to 2 MeV, due to electromagnetism.
- Given the fact that the decay width of the η_c is 30 MeV, the agreement with experiment may be surprising. Also $M_{\eta_c} = 2977(4)$ MeV (**preliminary**) vs. 2984 MeV (PDG).
- It is not straightforward to include annihilation diagrams: the axial anomaly may decrease the splitting. In addition: many decay channels and mixing with other flavour-singlet 0^{-+} states.
- Sea quark effects are very significant (as expected). The quenched result of 77(2)(6) MeV [QCD-TARO, hep-lat/0307004] went up to 114.3(5)(6) MeV (**preliminary**).

Future plans

- Include more ensembles and carry out more sophisticated fits and analyses of systematics.
- Compute masses and decay constants of other charmonium states.