Predicting the spectrum and decay constants of positive parity heavy-strange mesons using domain wall fermions

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Heavy-Strange Mesons

- D_s mesons in the quark model:

$c\bar{s}$ and $\bar{c}s$ particles from the quark model - (more are known from experiment compared to the bottom sector)

$n^{2s+1}\ell_J$	J^P	state
1^1S_0	0^-	$D_s^\pm$
1^3S_1	1^-	$D_s^{*\pm}$
1^3P_0	0^+	$D_{s0}^*(2317)^\pm$
1^3P_1	1^+	$D_{s1}(2460)^\pm$
1^1P_1	1^+	$D_{s1}(2536)^\pm$

Table: from *S. Navas et al. (Particle Data Group), Phys. Rev. D 110, 030001 (2024)*

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Below threshold narrow D_{s0}/D_{s1} states not expected in conventional quark model (e.g. [Godfrey, Kokoski; Phys. Rev. D 43, 1679 (1991)] predicts resonance above $D^{(*)}K$ threshold)

- The heavy quark sector is home to many exotic states to explore, with valance content beyond the conventional $q\bar{q}$ quark model - here are some candidates for molecular states

- Many D_s Mesons observed

CHARMED, STRANGE MESONS ($C = \pm1$, $S = \pm1$) (including possibly non-$q\bar{q}$ states)

$D_s^+ = c\bar{s}$, $D_s^- = \bar{c}s$, similarly for D_s^*'s

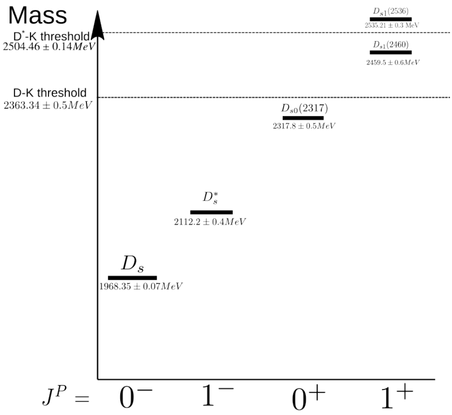
D_s^+ Branching Fractions [PDF]

Leptonic Decays of Charged Pseudoscalar Mesons [PDF]

• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(1^-)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}^*(2573)$	$0(2^+)$
$D_{s0}(2590)^+$	$0(0^-)$
• $D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{s1}^*(2860)^\pm$ was $D_{sJ}^*(2860)$	$0(1^-)$
• $D_{s3}^*(2860)^\pm$	$0(3^-)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

• Indicates established particles.

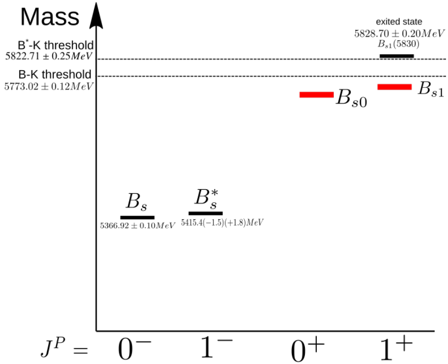
- Previous lattice and light cone sum rule calculations of decay constants



Masses From: R.L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022,

Less of the B_s Spectrum is Known From Experiment

- Red bars are states that have not been experimentally observed
- Heavy quark flavor symmetry: expect hyperfine splitting of D_s to be larger than B_s by factor of $\sim \frac{m_b}{m_c} \approx \frac{4.18\text{GeV}}{1.27\text{GeV}} \sim 3.3$
- Hints of these below threshold states [C. B. Lang, Daniel Mohler, Sasa Prelovsek, R. M. Woloshyn; arXiv:1501.01646]
- Less literature on decay constants and no prior lattice calculations for decay constants of positive parity states



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Molecular Mesons

- Weinstein+Isgur (1990): Level repulsion between S-wave 2-pseudoscalar scattering states and nearby strongly coupled scalar $q\bar{q}$ mesons $\rightarrow$ drives scattering state to below threshold bound state [Barnes, Close, Lipkin; arXiv:hep-ph/0305025], [Weinstein, Isgur; Phys. Rev. D41, 2236 (1990)]
- Look for same quantum numbers as meson pair with small binding energy

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- A compositeness criterion e.g. [Weinberg; Phys. Rev. 137, B672(1965)]

  ansantz $|\Psi\rangle = \begin{pmatrix} \lambda\,|\psi_0\rangle \\ \chi(\vec{k})\,|h_1 h_2\rangle \end{pmatrix}$

  Find χ, λ, and self energy in terms of same form factor $\Rightarrow$

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- Other related procedures [Morgan; Nucl. Phys. A543, 632(1992)], [Baru; arXiv:nucl-th/0410099]
- On the lattice: study ERE (good for near-threshold poles) from Lüscher's formula for $p\cot\delta(p)$ from finite volume energy levels [Guo *et al.*; arXiv:1705.00141]

Positive Parity Mesons and Semileptonic Decays

- Current for $b \to s$ transition has overlap with meson pairs (t_+ denotes kinematic threshold) $\to$ meson-meson scattering poles show up in form factors
- Factor out these poles for better convergence. e.g. in z expansion meson pole is mapped into the interior of the unit circle

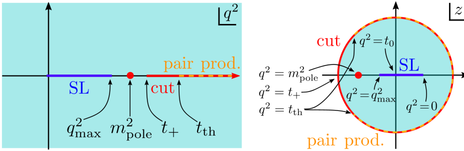

Figure: Figure from: T. Blake, S. Meinel, M. Rahimi, D. van Dyk "Dispersive bounds for local form factors in $\Lambda_b \to \Lambda$ transitions" arXiv:2205.06041 [hep-ph]

$$f(q^2) = \frac{1}{1 - q^2/m^{f\,2}_{\rm pole}} \left[a_0^f + a_1^f z(q^2) + a_2^f (z(q^2))^2 + \mathcal{O}(z^3) \right]$$

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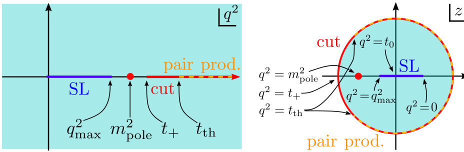

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- Consider derivatives of current twopoint functions χ in both a hadronic representation and with an OPE $\to$ get dispersive bounds on form factors - need decay constants

$$\chi^J_\Gamma|_{OPE} \geq \chi^J_\Gamma|_{1pt} + \chi^J_\Gamma|_{2pt}, \quad \text{e.g.} \quad \chi_V^{J=0}(Q^2)|_{1pt} = \frac{m^2_{B^*_{s,0}} f^2_{B^*_{s,0}}}{(m^2_{B^*_{s,0}} - Q^2)^{n+1}}$$

Previous Lattice Results

- [G. S. Bali, Phys. Rev. D68, 071501 (2003), arXiv:hep-ph/0305209] Early unquenched calculations for D_{s0}^* with quark-antiquark interpolators extract only above threshold states
- [D. Mohler, C. Lang, L. Leskovec, S. Prelovsek, and R. Woloshyn, Phys. Rev. Lett. 111, 222001 (2013); arXiv:1308.3175] Explores the $D_{s0}^*(2317)$ and DK scattering on the lattice
  - Includes meson-meson interpolating fields and distallation method allows for these at both source and sink + variational method to extract energy levels
  - Lüscher's formula gives $a_0 < 0$, a bound state for DK scattering. $J^P = 0^+$ state found 37(17) MeV below threshold.

Previous Lattice Results

- [G.S. Bali, S. Collins, A. Cox, A. Schäfer, Phys.Rev.D 96 (2017) 7, 074501; arXiv1706.01247] Studies $D_{s0}^*(2317)$ and $D_{s1}(2460)$
  - Has an almost physical pion mass ensemble, using two quark and meson-meson operators with the stochastic sources and the variational method to extract energy levels
  - $f_{D_{s0}^*} \sim 114$ MeV, $f_{D_{s1}} \sim 194$ MeV and get $a_0 < 0$ in both channels
- [C.B. Lang, D. Mohler, S. Prelovsek, R.M. Woloshyn, Phys.Lett.B 750 (2015) 17-21; arXiv:1501.01646] Explores the positive parity B_s states - predicts near threshold $J^P = 0^+, 1^+$ states
  - Uses stochastic distilation and variational method
  - Bound states at $m_{B_{s0}} = 5.711(13)(19)$ GeV and $m_{B_{s1}} = 5.750(17)(19)$ GeV

Our Operator Basis

$$\left.\begin{aligned}
\Phi^{(1)} &= \bar{b}s \\
\Phi^{(2)} &= \bar{b}\gamma^i \nabla_i s \\
J_{V0} &= \sqrt{Z_V^{ss} Z_V^{bb}}\rho_{V_0}\left[\bar{b}\gamma_0 s + \mathcal{O}(a)\text{-terms}\right] \\
\Phi^{(3)}(\vec{x},t) &= \sum_{\vec{y}} \Phi_K(\vec{x},t)\Phi_B(\vec{y},t) \;\; \text{where } \Phi_K = \bar{u}\gamma_5 s,\;\; \Phi_B = \bar{b}\gamma_5 u
\end{aligned}\right\} J^P = 0^+$$

$$\left.\begin{aligned}
\Phi^{(1)i} &= \bar{b}\gamma^i\gamma_5 s \\
\Phi^{(2)i} &= \bar{b}\gamma_5 \nabla^i s \\
J_{Ai} &= \sqrt{Z_V^{ss} Z_V^{bb}}\rho_{A_i}\left[\bar{b}\gamma_i\gamma_5 s + \mathcal{O}(a)\text{-terms}\right] \\
\Phi^{(3)i}(\vec{x},t) &= \sum_{\vec{y}} \Phi_K(\vec{x},t)\Phi_{B^*}(\vec{y},t) \;\; \text{with} \;\; \Phi_K = \bar{u}\gamma_5 s,\;\; \Phi_{B^*} = \bar{b}\gamma^i u
\end{aligned}\right\} J^P = 1^+$$

Meson-Meson operator only at the source

Lattice Setup

- Iwasaki gauge configurations from RBC/UKQCD Collaboration with 2+1 dynamical domain-wall fermions [Blum *et al.* (RBC/UKQCD Collaboration); arXiv:1411.7017], [Boyle *et al.* (RBC/UKQCD Collaboration); arXiv:1812.08791]
- We use domain wall fermions for light and strange quarks - our light/strange propagators are pre-computed
- Anisotropic clover action for heavy quarks tuned with $B_s^{(*)}/D_s^{(*)}$ dispersion/hyperfine splitting [Aoki *et al.* (RBC/UKQCD Collaboration); arXiv:1206.2554], [Meinel; arXiv:2309.01821]

Lattice Setup

- 7 ensembles we use:

Label	$N_s^3 \times N_t$	a [fm]	a^{-1} [GeV]	$am_{u,d}$	m_π [GeV]	$am_s^{(\mathrm{sea})}$	$am_s^{(\mathrm{val})}$	N_{ex}	N_{sl}
C00078	$48^3 \times 96$	0.114	1.7295(38)	0.00078	0.13917(35)	0.0362	0.0362	80	2560
C005LV	$32^3 \times 64$	0.111	1.7848(50)	0.005	0.3398(12)	0.04	0.0323	186	5022
C005	$24^3 \times 64$	0.111	1.7848(50)	0.005	0.3398(12)	0.04	0.0323	311	4976
C01	$24^3 \times 64$	0.111	1.7848(50)	0.01	0.4312(13)	0.04	0.0323	283	9056
F004	$32^3 \times 64$	0.083	2.3833(86)	0.004	0.3036(14)	0.03	0.0248	251	4016
F006	$32^3 \times 64$	0.083	2.3833(86)	0.006	0.3607(16)	0.03	0.0248	223	3568
F1M	$48^3 \times 96$	0.073	2.708(10)	0.002144	0.2320(10)	0.02144	0.02217	113	3616

- $3.86 \lesssim m_\pi L \lesssim 6.09$ (only C00078 is < 4) keep finite volume errors under control. Note also the near-physical pion mass ensemble
- We use all mode averaging. N_{ex} and N_{sl} denote number of exact and sloppy samples available [Shintani, Arthur, Blum, Izubuchi, Jung, Lehner; arXiv:1402.0244]

Lattice Setup

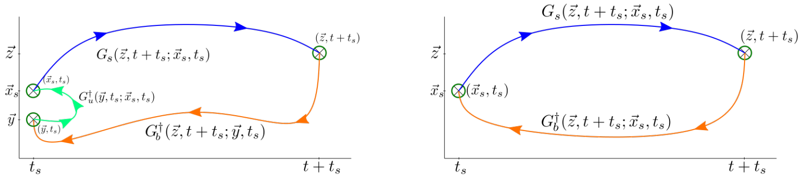
- Positive parity particles projected to zero momentum
- Quarks in Φ-fields smeared with Gaussian smearing
- Currents renormalized via the mostly non-perturbative method of [El-Khadra, Kronfeld, Mackenzie, Ryan, Simone; arXiv:hep-ph/0101023] , [Detmold, Lehner, Meinel; arXiv:1503.01421] where Z_V^{qq} computed nonperturbatively, residual matching factor ρ and $\mathcal{O}(a)$ improvement terms are calculated to 1-loop in lattice perturbation theory

$$J_{V0} = \sqrt{Z_V^{ss} Z_V^{bb}} \rho_{V_0} \left[\bar{s}\gamma_0 b + 2a \left(c_{V_0}^R \bar{s}\gamma_0\gamma_j \overrightarrow{\nabla}_j b + c_{V_0}^L \bar{s} \overleftarrow{\nabla}_j \gamma_0\gamma_j b \right) \right]$$

$$J_{Vi} = \sqrt{Z_V^{ss} Z_V^{bb}} \rho_{V_i} \left[\bar{s}\gamma_i b + 2a \left(c_{V_i}^R \bar{s}\gamma_i\gamma_j \overrightarrow{\nabla}_j b + c_{V_i}^L \bar{s} \overleftarrow{\nabla}_j \gamma_i\gamma_j b + d_{V_i}^R \bar{s} \overrightarrow{\nabla}_i b + d_{V_i}^L \bar{s} \overleftarrow{\nabla}_i b \right) \right]$$

$$J_{A0} = \sqrt{Z_V^{ss} Z_V^{bb}} \rho_{A_0} \left[\bar{s}\gamma_0\gamma_5 b + 2a \left(c_{A_0}^R \bar{s}\gamma_0\gamma_5\gamma_j \overrightarrow{\nabla}_j b + c_{A_0}^L \bar{s} \overleftarrow{\nabla}_j \gamma_0\gamma_5\gamma_j b \right) \right]$$

$$J_{Ai} = \sqrt{Z_V^{ss} Z_V^{bb}} \rho_{A_i} \left[\bar{s}\gamma_i\gamma_5 b + 2a \left(c_{A_i}^R \bar{s}\gamma_i\gamma_5\gamma_j \overrightarrow{\nabla}_j b + c_{A_i}^L \bar{s} \overleftarrow{\nabla}_j \gamma_i\gamma_5\gamma_j b + d_{A_i}^R \bar{s}\gamma_5 \overrightarrow{\nabla}_i b + d_{A_i}^L \bar{s} \overleftarrow{\nabla}_i \gamma_5 b \right) \right]$$



$C^{13}(t) = -\sum_{\vec{z},\vec{y}} \mathrm{Tr}[(G_s(\vec{z}, t+t_s; \vec{y}, t_s)\gamma_5 G_u(\vec{y}, t_s; \vec{x}_s, t_s))^\dagger \gamma_5 G_b(\vec{z}, t+t_s; \vec{x}_s, t_s)]$ $\quad C^{11}(t) = -\sum_{\vec{z}} \mathrm{Tr}[\gamma_5 G_s^\dagger(\vec{z}, t+t_s; \vec{x}_s, t_s)\gamma_5 G_b(\vec{z}, t+t_s; \vec{x}_s, t_s)]$

Figure: Constructing the sequential propagator $\Sigma(\vec{z}, t+t_s; \vec{x}_s, t_s) = \sum_{\vec{y}, y_0 = t+t_s} G_b(z, y)\gamma_5 G_u(y, x_s)$. C^{11} on the right for comparison

With $\Phi^{(3)}$ at source:

$$T(z, x) \equiv \sum_{\vec{y}} G_b(z; \vec{y}, t_s)\gamma_5 G_u(\vec{y}, t_s; x)$$

Solve for this propagator with a Dirac equation using light propagator as source $DT = \gamma_5 G_u$.

Derivative Source Propagators

Similarly with derivative operators at the source we need the combination of propagators and gauge links:

$$\tilde{G}^{ab}_{bi}(\vec{z}, t+t_s; \vec{x}_s, t_s) = \langle b(\vec{z}, t+t_s)^a \bar{\psi}'_{bi}(\vec{x}_s, t_s)^b \rangle \quad \text{where}$$

$$\bar{\psi}'_{bi}(\vec{x}_s, t_s)^b = \sum_y \bar{b}(\vec{y}, t_s)^c S_i^{(\vec{x}_s)}(\vec{y})^{cb} \quad \text{with} \quad S_i^{(\vec{x}_s)}(\vec{y})^{cb} = (\nabla_i^y)^{cf} \delta^{fb} \delta_{y^0 t_s} \delta_{\vec{y}, \vec{x}_s}$$

$$\tilde{G}^{ab}_{bi}(\vec{z}, t+t_s; \vec{x}_s, t_s) = \sum_y \langle b(z)^a \bar{b}(\vec{y}, t_s)^c S_i^{(\vec{x}_s)}(\vec{y})^{cb} \rangle =$$

$$= \frac{1}{2a}\left(G^{ac}_b(\vec{z}, t+t_s; x_s - \hat{i}a, t_s) U_i(\vec{x}_s - \hat{i}a, t_s)^{cb} - G^{ac}_b(\vec{z}, t+t_s; x_s + \hat{i}a, t_s) U_i^\dagger(\vec{x}_s, t_s)^{cb} \right)$$

$\tilde{G}_i$ is just the propagator calculated with a derivative on the source.

$$D\tilde{G}_i(z,x) = \sum_y DG_b(z,y) S_i^{(x)}(y) = \sum_y \delta_{z,y} S_i^{(x)}(y) = \nabla_i^z \delta_{z^0, t_s} \delta_{\vec{z}, \vec{x}_s}$$

Fitting

- All mode averaging is used
- Employ a 3-exponential simultaneous matrix fit for the matrix of correlators $\langle \phi_k \phi_l^\dagger \rangle$ ordered as in The slide exhibiting the operator basis

$$C_{ij} = A_i A_j \left[e^{-\exp(\tilde{E})t} + B_{1i} B_{1j} e^{-(\exp(\tilde{E}) + \exp(\tilde{\Delta E_1}))t} + B_{2i} B_{2j} e^{-(\exp(\tilde{E}) + \exp(\tilde{\Delta E_1}) + \exp(\tilde{\Delta E_2}))t} \right]$$

- 3 fit types: Unimproved current at both source and sink (4×3), improved current at sink (3×3), $\mathcal{O}(a)$ improvement terms at sink (3×3)
- Extract decay constant at $\vec{p} = 0$: $f = A_J \sqrt{\frac{2}{\exp(\tilde{E})}}$

Example Fits - bottom $J^P = 1^+$

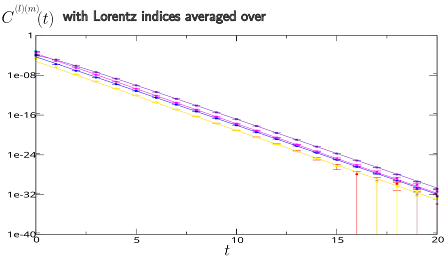


Figure: B_{s1} correlator matrix fit for C005

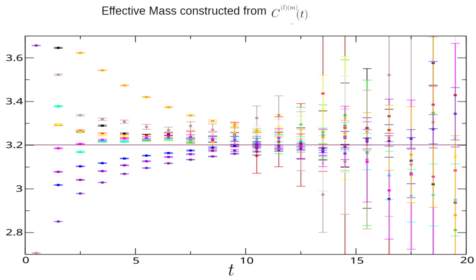


Figure: B_{s1} C005 effective mass plot

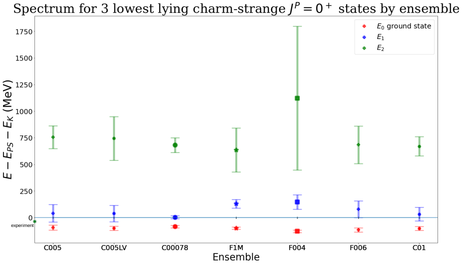


Figure: D_{s0}^* spectrum

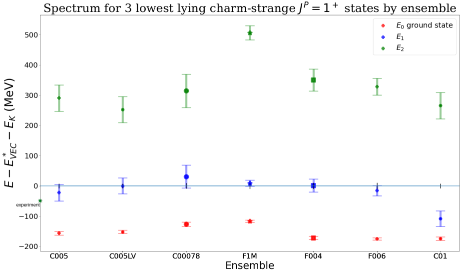
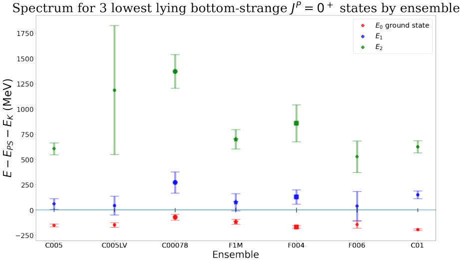


Figure: D_{s1} spectrum

- Ground state from $\sim 84 \pm 12$ MeV to 128 ± 11 MeV below threshold
- Compare with experiment ~ 40 MeV below threshold

- Ground state from 117 ± 4 MeV to 175 ± 4 MeV below threshold
- Near-physical pion mass ensemble: 126 ± 9 MeV below threshold
- Compare with experiment which predicts ~ 45 MeV

Preliminary Results - Finite Volume Spectrum: Bottom



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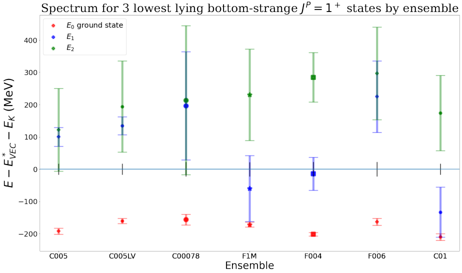

Figure: B_{s1} spectrum

- Ground state ranges from 71 ± 29 MeV to 192 ± 8 MeV below threshold (in some tension with Lang *et al.*)
- Compare with Lang *et al.* at 64(13)(19) MeV

- Ground state ranges from 156 ± 17 MeV to 210 ± 10 MeV below threshold
- Both substantially more deeply bound than Lang *et al.* which predicts 71(17)(19) MeV

Preliminary Results - Decay Constants: Charm

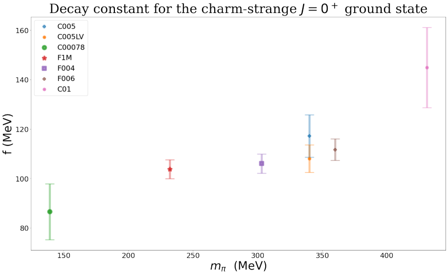

Figure: Averaged D_{s0}^* decay constant ranges from 87 $\pm$ 12 MeV to 145 $\pm$ 16 MeV
Compare to Bali *et al.* estimate of $\sim$ 114 MeV

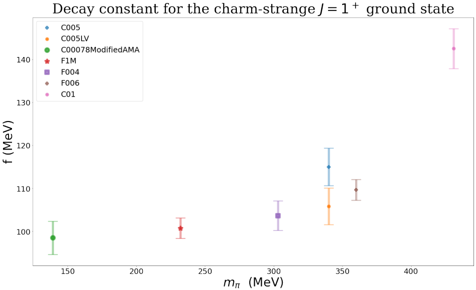

Figure: D_{s1} decay constant range from 99 $\pm$ 3 MeV to 142 $\pm$ 5 MeV.
Compare to Bali *et al.* estimate of $\sim$ 194 MeV

Preliminary Results - Decay Constants: Bottom

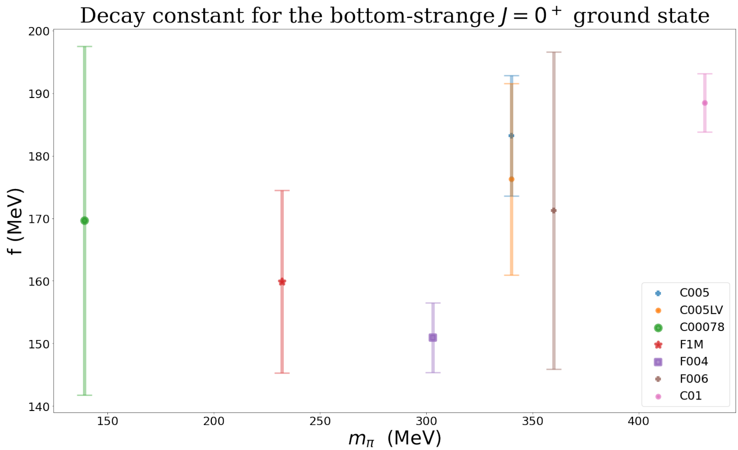

Figure: B_{s0}^* decay constants range from 151 $\pm$ 6 MeV to 188 $\pm$ 5 MeV

Reliable B_{s1} fits are still a work in progress

Conclusion and Future Prospects

- Preliminary finite volume results indicate below-threshold B_s and D_s bound states in the $J^P = 0^+, 1^+$ channels, these are surprisingly $>$ 100 MeV below threshold!
- Lattice extraction of positive parity D_s/B_s decay constants
- The multi-exponential fits are very unstable (especially C00078 w/ 80 AMA samples), very few fit ranges will converge
- While 4 quark operator at both source+sink would be helpful, it would require computing expensive light quark propagators

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- Akaike information criterion: average fits of different $(t_{\text{min}}, t_{\text{max}})$ weighted by $\chi^2/d.o.f.$ and number of data points to estimate systematics associated with fit range choice
- Lüscher's method to extract phase shifts in $D^{(*)}K$ / $B^{(*)}K$ scattering and infinite volume bound state mass
- Chiral and continuum extrapolation + further estimate all sources of systematic uncertainty