Static-light meson spectroscopy with optimal distillation profiles

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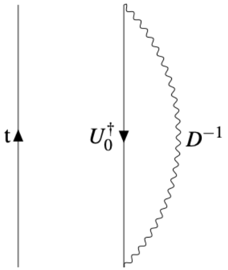
Motivation

- Static-light = static limit of B-mesons $\rightarrow$ leading term in HQET (Heavy quark effective theory)
- Spectroscopy of static-light system with different angular momentum $\rightarrow$ hybrid string-breaking
- Useful to investigate excited state contamination in B-system ($B\pi$-system)

Therefore, our **goal**:

- Spectroscopy of static-light and static-charm system with and without angular momentum
- Investigate improved distillation for this setup
- Set first step towards hybrid string-breaking

Static-light meson operators



Construction of the operators:

[C. Michael and J. Peisa, Phys. Rev. D 58, 034506 (1998)],

[J. Foley et al., Phys. Rev. D 75, 094503 (2007)]

Treat static quark as spinless color source $Q_a, \bar{Q}_a$

$\rightarrow$ light-quark ψ spin labels states

- Lattice symmetry group: double cover of full cubic group O_h^D
- Subduction to irreps of O_h^D

$$\frac{1}{2}^{\pm} \rightarrow G_1^{\pm}, \quad \frac{3}{2}^{\pm} \rightarrow H^{\pm}, \quad \frac{5}{2}^{\pm} \rightarrow G_2^{\pm} \oplus H^{\pm}$$

- Example operator (γ_5-diagonal basis):

$$\mathcal{O}^{(G_1^{\pm})} = \bar{Q}\left(\psi_1 \pm \psi_3\right)$$

Temporal gauge links are HYP2-smeared, spatial gauge links 3D APE-smeared.

Static-light correlators with improved distillation

Distillation [M. Peardon et al., Phys. Rev. D 80, 054506 (2009)]

- The columns of $V(t)$ are the N_v lowest modes of the 3D gauge covariant Laplacian at time $t \to \nabla^2(t)v_i(t) = \lambda_i(t)v_i(t)$
- Static and light quark propagation:

$$D^{-1}(t_1,t_2) \to \tau = V^\dagger(t_1)D^{-1}(t_1,t_2)V(t_2)$$
$$\mathcal{P}(\vec{x};t_1,t_2) \to \tau^{stat}(\vec{x};t_1,t_2) = V^\dagger(\vec{x};t_1)\mathcal{P}(\vec{x};t_1,t_2)V(\vec{x};t_2)$$

$\mathcal{P}(\vec{x};t_1,t_2)$: temporal Wilson line

Improved Distillation [J. A. Urrea-Niño, F. Knechtli, T. Korzec & M. Peardon, Phys. Rev. D 106, 034501 (2022)]

- Contribution of each v_i is modulated by a profile function $\rho_i(t) = \rho(\lambda_i(t))$

Local Static-light correlator

$$C(\vec{x}; t_2, t_1) = - \left\langle \sum_{i,j} \rho_i(t_2)\rho_j(t_1)\, \tau^{stat}_{ji}(\vec{x}; t_1, t_2)\ \left(\mathrm{Tr}_{spin}[\Gamma \tau_{ij}(t_2, t_1)]\right) \right\rangle_{gauge}$$

- Γ a 4×4 matrix that picks the needed spin components of τ

We can formulate a GEVP with 7×7 matrix using different Gaussian profiles.

Note: For derivative-based operators τ^{stat} involves derivatives of the v_i.

Results

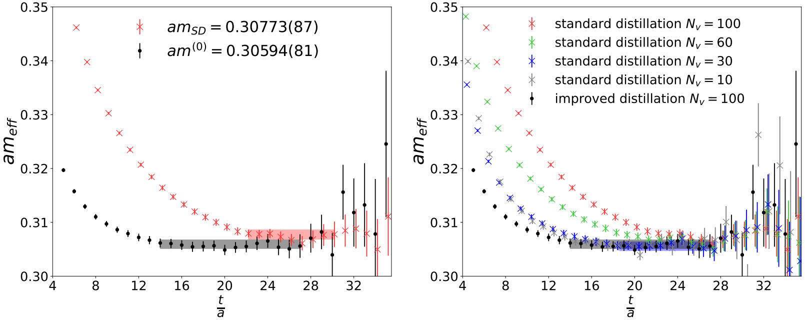
Measure static-light and static-charm spectrum in two different $N_f = 3 + 1$ QCD ensembles:

A1	A1h
$32^3 \times 96$	$32^3 \times 96$
$a \approx 0.05359$ fm	$a \approx 0.0690$ fm
$m_\pi \approx 420$ MeV	$m_\pi \approx 800$ MeV
$N_v = 100$	$N_v = 200$
$N_{config} = 4000$	$N_{config} = 2000$

$A1$ was generated at the $SU(3)$ flavor symmetric point.

Ensembles have been generated using the action of [P. Fritzsch et al., J. High Energ. Phys. 2018, 25 (2018)], see [R. Höllwieser et al., Eur. Phys. J. C 80, 349 (2020)].

Comparison standard and improved distillation



- Left: Same number of eigenvectors ($N_v = 100$)
  - $\rightarrow$ High supression of excited state contamination when using the optimal profiles
- Right: Different number of eigenvectors for standard distillation
  - $\rightarrow$ Improved distillation still shows less excited state contamination

Optimal meson profiles

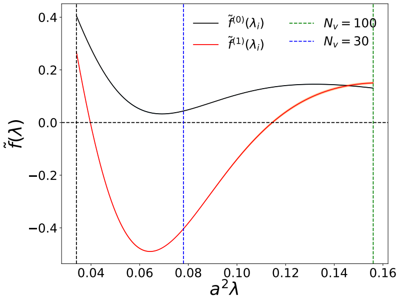


Figure: A1 ensemble.

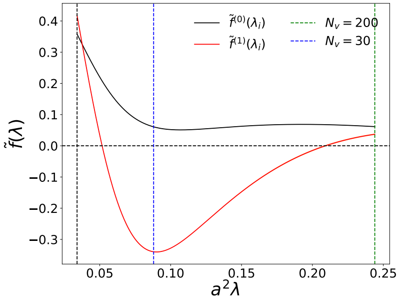


Figure: A1h ensemble.

→ The higher the state, the more structure in the optimal profile

→ Larger eigenvalues still have non-negligible contribution

→ Larger eigenvalues needed for excited states

Static-light meson spectrum with improved distillation

G_1^+ with local operators and $N_v = 100$

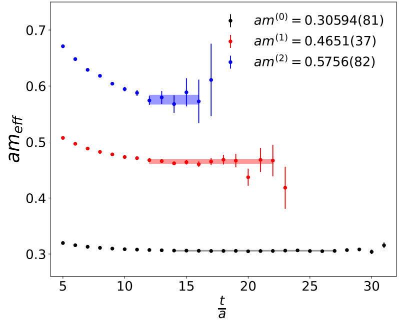


Figure: A1 ensemble.

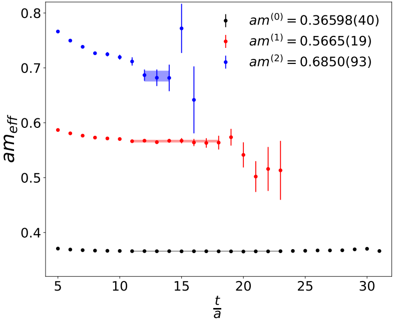


Figure: A1h ensemble.

Three states can be resolved using the basis of different profiles.

Static-light meson spectrum

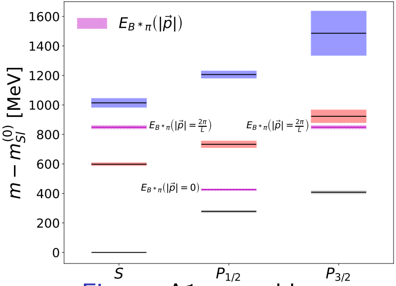


Figure: A1 ensemble.

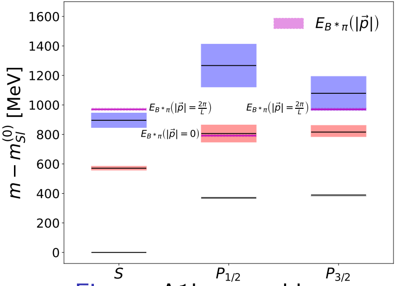


Figure: A1h ensemble.

$E_{B^*\pi}(|\vec{p}|)$: Energy of non-interacting B^* and π with momentum $\vec{p}$ ($m_{B^*} = m_B$ due to heavy quark spin symmetry)

→ Splittings show dependence on light quark masses

→ A1: Levels are most likely radial excitations of B

→ A1h: Level splittings are closer to non-interacting $B^*\pi$ states

[O. Bär et al., Eur. Phys. J. C 83 8, 757 (2023)], [J. Foley et al., Phys. Rev. D 75, 094503 (2007)]

Static-charm meson spectrum

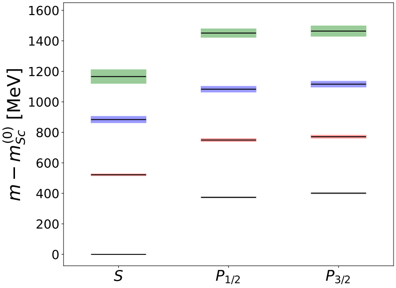


Figure: A1 ensemble.

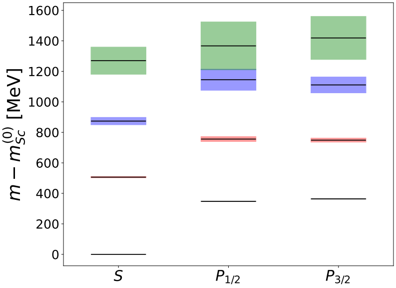


Figure: A1h ensemble.

→ Level splittings of both ensembles agree due to subtraction of ground state static-charm quark mass → dependence on charm mass cancels out

→ Splittings show only slight dependence on light quark masses

Mass-splittings

Splitting	$m_\pi \approx 420$ MeV	$m_\pi \approx 800$ MeV
$1P_{1/2} - 1S$	$277.9(6.9)$ MeV	$369.9(5.6)$ MeV
$1P_{3/2} - 1P_{1/2}$	$130(12)$ MeV	$14.8(7.6)$ MeV
$2S - 1S$	$598.2(9.7)$ MeV	$571(13)$ MeV

Table: Static-light meson.

Splitting	$m_\pi \approx 420$ MeV	$m_\pi \approx 800$ MeV	PDG
$1P_{1/2} - 1S$	$373.6(4.2)$ MeV	$347.5(2.2)$ MeV	
$1P_{3/2} - 1P_{1/2}$	$27.4(1.5)$ MeV	$16.2(1.4)$ MeV	
$2S - 1S$	$521.8(6.2)$ MeV	$506.3(4.3)$ MeV	$596.7(1.1)$ MeV

Table: Static-charm meson.

- → $2S - 1S$ static-charm splitting does not agree with PDG value
- → $2S - 1S$ larger than $1P_{1/2} - 1S$
- → $1P_{1/2} - 1S$ increases with m_π while $1P_{3/2} - 1P_{1/2}$ decreases
- → Static-light splittings show higher dependence on m_π

Conclusion and Outlook

- By using improved distillation we obtain enhanced plateaus of different radial and orbital excitations of the static-light and static-charm meson
- States on A1 $\rightarrow$ radial excitations; States on A1h $\rightarrow$ closer to $B^*\pi$ states
- Static-light results show higher dependence on light quark masses than static-charm results

Outlook:

- Identify $P_{3/2}$ states using two-derivative operators
- Include $B^*\pi$ interpolating operators
- Two static-light mesons which are relevant for hybrid string-breaking

Thank you for your attention!

Pruning

Start with 7×7 profile matrix. **Prune** the matrix to make it more numerically stable:
Perform *Singular Value Decomposition* (SVD)

$$C(t) = USV^\dagger$$

with S a diagonal matrix containing the singular values, U contains the left singular vectors, V the right singular vectors. Project onto the singular vectors u_i corresponding to the N_p largest singular values at a reference time t_S where the pruning vectors are stable:

$$C_S(t)_{ij} = u_i^\dagger(t_S) C(t) u_j(t_S)\,, \quad t \geq t_S \quad .$$

Use this $N_p \times N_p$ matrix for the GEVP.

[J. Balog et al., Phys. Rev. D 60, 094508], [F. Niedermayer et al., Nuclear Physics B 597, 413–450]

Multiparticle states

Subduction of pion representations for the smallest lattice momenta $|\vec{p}|$: [J. Foley et al., Phys. Rev. D 75, 094503 (2007)]

$\vec{p}$	Irreducible content
$(0,0,0)$	A_1^-
$(1,0,0)$	$A_1^- \oplus E^- \oplus T_1^+$
$(1,1,0)$	$A_1^- \oplus E^- \oplus T_1^+ \oplus T_2^+ \oplus T_2^-$

$\rightarrow$ Obtain two-particle states by taking direct product of pion representation with G_1^+
Compare with energy of non-interacting two-particle state

$$E_{B^*\pi} \approx E_{sl} + E_\pi \qquad \text{with}$$

$$\cosh\left(aE_\pi(p)\right) = \cosh\left(am_\pi\right) + \sum_{k=1}^{3}\left(1 - \cos\left(a\frac{2\pi|n_k|}{L}\right)\right)$$

Static-light meson spectrum

How does the static-light meson spectrum change when using different numbers of eigenvectors for the improved distillation?

Example: G_1^+ with local operators

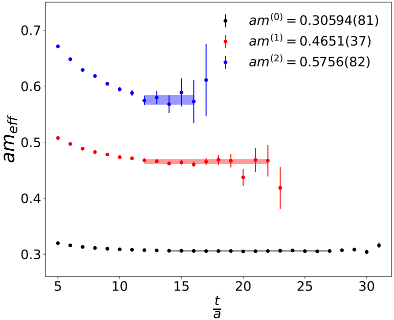


Figure: 100 eigenvectors.

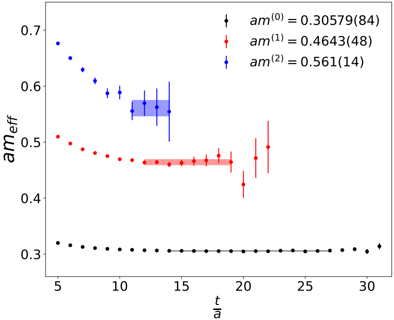


Figure: 50 eigenvectors.

→ All determined states agree

→ 50EV spectrum only slightly noisier

Optimal static-light meson profiles for both cases:

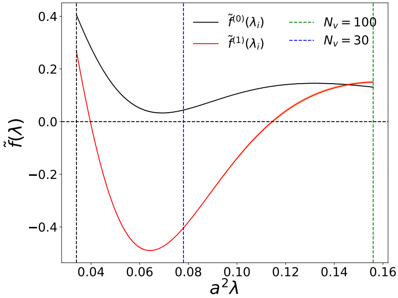


Figure: 100 eigenvectors.

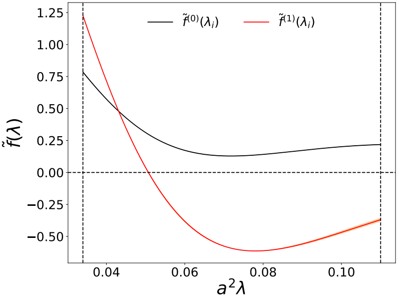


Figure: 50 eigenvectors.

$\rightarrow$ More eigenvalues necessary to obtain better resolution of higher excited states