Flavor mixing in charmonium and light mesons with optimal distillation profiles

J. A. Urrea-Niño, R. Höllwieser, J. Finkenrath, F. Knechtli, T. Korzec and M. Peardon

Lattice 2024, Liverpool, England, July 28 to August 3, 2024

FOR 5269

BERGISCHE UNIVERSITÄT WUPPERTAL

Motivation

Goal: Calculate the low-lying meson spectrum in $N_f = 3 + 1$ QCD. This includes different states related to $SU(3)_F$:

- Flavor-singlet: f_0, η_c, glueballs, 2-pion states, etc...
- Flavor-octet: π, a_0, octet 2-pion states, etc...

$\rightarrow$ Experimental studies of glueball candidates and other exotics in the light and charmonium regions can benefit from insights on the composition of these states.

$\rightarrow$ Tuning of masses in simulations is useful to study convenient setups, e.g heavy π's to raise the threshold of scalar glueball decay into $\pi\pi$.

This talk: Mixing between different kinds of flavor-singlet scalar operators.

- Do light meson, charmonium, gluonic and 2-pion operators talk to each other?

Strategy

Common approach:

- Study light meson states with $\bar{l}(t)\Gamma l(t)$ operators including disconnected correlations.
- Study charmonium states with $\bar{c}(t)\Gamma c(t)$ operators **disregarding** charm disconnected correlations.

!!! Mixing is artificially **eliminated**.

Since $\mathcal{O}_c = \bar{c}\Gamma c$ and $\mathcal{O}_l = \bar{u}\Gamma u + \bar{d}\Gamma d + \bar{s}\Gamma s$ are in the same symmetry channel on the lattice they create states with **non-zero overlap** onto the same set of energy eigenstates $|n\rangle$.
E.g The ground state of the flavor-singlet 0^{++} channel is f_0, not χ_{c0}.

$$C_l(t) \overset{t\to\infty}{\approx} |\langle f_0| \mathcal{O}_l |\Omega\rangle|^2 e^{-E_{f_0} t}$$

$$C_c(t) \overset{t\to\infty}{\approx} |\langle f_0| \mathcal{O}_c |\Omega\rangle|^2 e^{-E_{f_0} t} \neq |\langle \chi_{c0}| \mathcal{O}_c |\Omega\rangle|^2 e^{-E_{\chi_{c0}} t}$$

If we account for the mixing we do not need to rely on possible fake plateaus, e.g $|\langle \chi_{c0}| \mathcal{O}_c |\Omega\rangle| >> |\langle f_0| \mathcal{O}_c |\Omega\rangle| > 0$

We need a correlation matrix which accounts for all types of operators and their possible mixing:

$$\begin{pmatrix} \left\langle \mathcal{O}_l(t)\bar{\mathcal{O}}_l(0)\right\rangle & \left\langle \mathcal{O}_l(t)\bar{\mathcal{O}}_c(0)\right\rangle & \left\langle \mathcal{O}_l(t)\bar{\mathcal{O}}_{2\pi}(0)\right\rangle & \left\langle \mathcal{O}_l(t)\bar{\mathcal{O}}_g(0)\right\rangle \\ * & \left\langle \mathcal{O}_c(t)\bar{\mathcal{O}}_c(0)\right\rangle & \left\langle \mathcal{O}_c(t)\bar{\mathcal{O}}_{2\pi}(0)\right\rangle & \left\langle \mathcal{O}_c(t)\bar{\mathcal{O}}_g(0)\right\rangle \\ * & * & \left\langle \mathcal{O}_{2\pi}(t)\bar{\mathcal{O}}_{2\pi}(0)\right\rangle & \left\langle \mathcal{O}_{2\pi}(t)\bar{\mathcal{O}}_g(0)\right\rangle \\ * & * & * & \left\langle \mathcal{O}_g(t)\bar{\mathcal{O}}_g(0)\right\rangle \end{pmatrix}$$

- $O_g(t)$: gluonic operators, e.g built from Wilson loops.
- $O_{2\pi}(t)$: flavor-singlet 2-pion operators at zero total momentum.

$\rightarrow \left\langle \mathcal{O}_i(t)\bar{\mathcal{O}}_j(0)\right\rangle \neq 0$ for $i \neq j$ means **no decoupling** between operators.

We solve a GEVP M. Lüscher & U. Wolff, Nuclear Physics B 339, 222–252, B. Blossier et al. Journal of High Energy Physics 2009, 094–094:

$$C(t)v_n(t,t_0) = \rho_n(t,t_0)C(t_0)v_n(t,t_0)$$

Overlaps:

$$\langle n| \mathcal{O}_i |\Omega\rangle \propto [C(t_0)v_n(t,t_0)]_i$$

Noise in vectors and choice of t_0 play a large role here. J.J. Dudek et al. Phys. Rev. D 77, 034501 (2008)

Building the correlation matrix

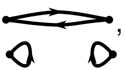
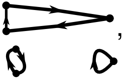
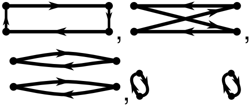
Improved Distillation: Control the contribution from each 3D Laplacian eigenvector. M. Peardon et al. Phys. Rev. D 80, 054506 (2009), J. A. Urrea-Niño, F. Knechtli, T. Korzec & M. Peardon. Phys. Rev. D 106, 034501 (2022)

Flavor-singlet channel:

- 1-particle: connected + disconnected 2-point function.
- 2-particle: $8 \otimes 8 = \mathbf{1} \oplus 8 \oplus 8' \oplus 10 \oplus \bar{10} \oplus 27$
  - **SU(3) CG-coefficients:** How to combine products of $|8, Y, I, I_z\rangle$ to obtain $|1, 0, 0, 0\rangle$? P. McNamee & F. Chillton, Rev. Mod. Phys 36 (1964)
  - **Wick contractions:** 2-point function involves 4 perambulators connecting sink and source times in few different diagrams.
  - **Spatial momentum:** Non-zero relative momentum of mesons requires further projection considerations in corresponding momentum little group.

Advantages:

- ✓ Same perambulators/elementals are used for all correlations.
- ✓ Singlet 2-pion diagrams are also useful for other flavor configurations, e.g octet.
- ✓ 1- and 2-particle loops are reusable.
- ✓ Gluonic operators are easy to include.

	$\mathcal{O}_l$	$\mathcal{O}_c$	$\mathcal{O}_{2\pi}$	$\mathcal{O}_g$
$\mathcal{O}_l$				
$\mathcal{O}_c$	-			
$\mathcal{O}_{2\pi}$	-	-		
$\mathcal{O}_g$	-	-	-	

Disconnected correlations are very **necessary** for flavor-singlets but have **large** statistical errors.

- We use a basis of different distillation profiles in the 1-particle blocks to saturate the spectrum $(\Gamma = \mathbb{I})$.
- 2-pion operator with standard distillation *for now*...

$N_f = 3 + 1$ ensembles

Wilson fermion action with non-perturbatively determined clover improvement + Lüscher-Weisz gauge action. R. Höllwieser et al. Eur. Phys. J. C 80, 349. P. Fritzsch et al. J. High Energ. Phys. 2018, 25 (2018)

A1	A1h
96×32^3	96×32^3
$a \approx 0.054$ fm	$a \approx 0.069$ fm
$m_\pi \approx 420$ MeV	$m_\pi \approx 800$ MeV
$N_v^{light} = 100$	$N_v^{light} = 200$
$N_v^{charm} = 200$	$N_v^{charm} = 200$

Control over decay thresholds:

- Quenched 0^{++} glueball $\approx$ 1800 MeV
- A1: Glueball $\to \pi\pi$, $\pi\pi\pi\pi$
- A1h: Glueball $\to \pi\pi$

0^{++} flavor-singlet correlation matrix at $t = a$

$C_{ij}(t) \to C_{ij}(t)/\sqrt{C_{ii}(a)C_{jj}(a)}$.

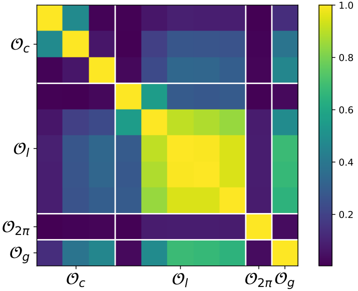


Figure: A1 ensemble

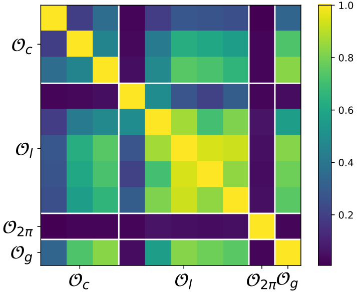
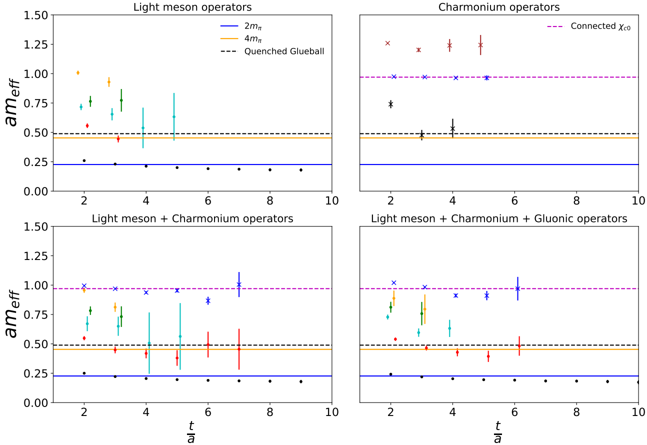


Figure: A1h ensemble

All operators "talk" to each other to some degree.

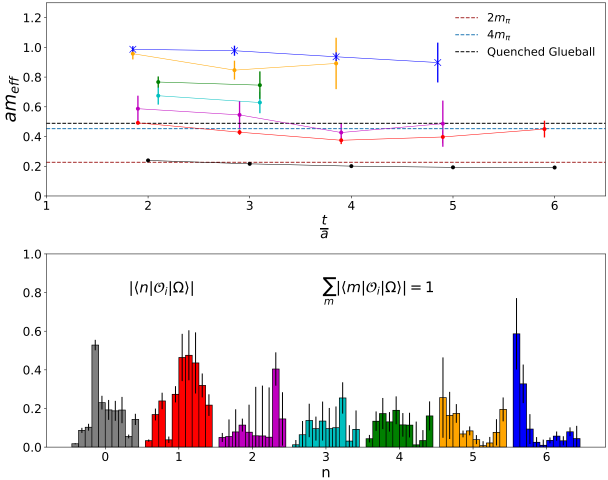
- $\mathcal{O}_g$: Sum of Laplacian eigenvalues. C. Morningstar et al. Phys. Rev. D 88, 014511
- $\mathcal{O}_{l/c}$: From SVD pruning of sub-blocks. J. Balog et al., Phys. Rev. D 60, 094508, F. Niedermayer et al., Nuclear Physics B 597, 413–450

0^{++} single-particle operator mixing in A1



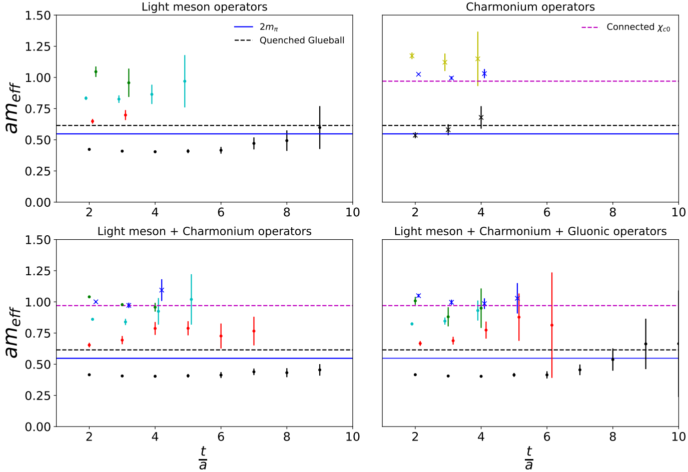
- Charmonium operators alone see a light state.
- Including $\mathcal{O}_g$ does not change the low-lying spectrum. Similar to R. Brett et al. 1909.07306.

0^{++} full operator mixing in A1



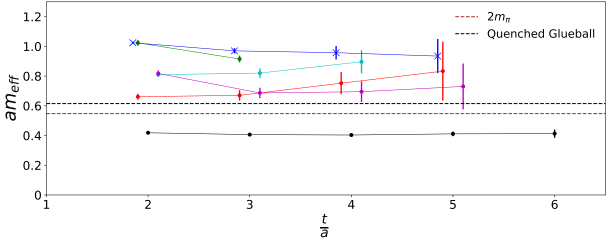
- 10 operators: $3 \times \mathcal{O}_c$, $5 \times \mathcal{O}_l$, $\mathcal{O}_{2\pi}$ and $\mathcal{O}_g$.
- 2-pion operator introduces an additional state.

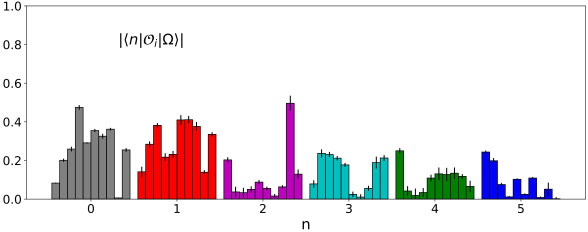
0^{++} single-particle operator mixing in A1h



- Charmonium operators alone see a light state.
- Including $\mathcal{O}_g$ does not change the low-lying spectrum.

0^{++} full operator mixing in A1h





- 10 operators: $3 \times \mathcal{O}_c$, $5 \times \mathcal{O}_l$, $\mathcal{O}_{2\pi}$ and $\mathcal{O}_g$.
- 2-pion operator introduces an additional state.

Conclusions and Outlook

- **All** flavor-singlet operators "talk" to each other and flavor-mixing off-diagonals are **non-negligible**.
- Mixing of 2-pion operator with one-particle states is the **smallest** and yields additional state very close to quenched 0^{++} glueball mass at both pion masses.
- ○○ Disconnected correlation in charmonium **introduces** a state in the GEVP in the region of light mesons for both pion masses.
- ○○ Statistical noise from disconnected correlations is a major problem.

Future work:

- ○ Different choices of Γ to better sample radial excitations.
- ○ More 2-pion operators: bact-to-back momentum + profiles in a larger volume.
- ○○ Better sampling methods for disconnected pieces, e.g multi-level sampling 2406.12656. See talk by L. Barca on Thursday!
- ○ ○ ○ Lüscher's method at different volumes to study scattering.

Thank you for your attention!

Pruning of correlation matrix

We start from a $N \times N$ correlation matrix $C(t)$ and perform an SVD at a reference time t_s

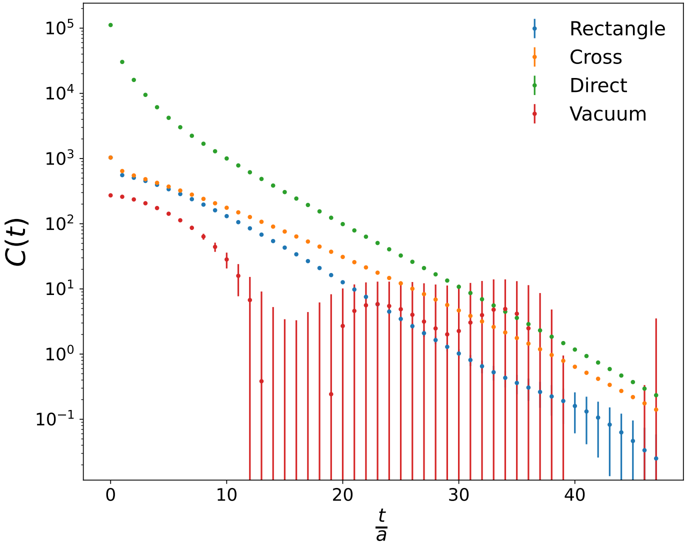
$$C(t_s) = UDU^\dagger,$$

which is symmetric thanks to hermiticity of $C(t)$. We then define a pruned $M \times M$, $M < N$, correlation matrix $\tilde{C}(t)$ as

$$\tilde{C}_{ij}(t) = u_i^\dagger C(t) u_j$$

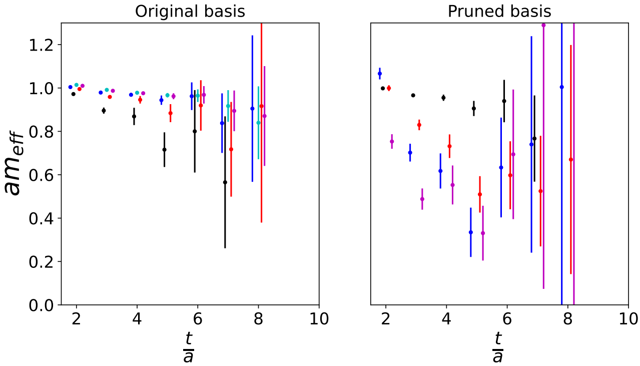
by projecting onto the singular vectors corresponding to the M largest singular vectors. $\tilde{C}(t)$ is not only smaller but better conditioned and less affected by statistical noise than $C(t)$. J. Balog et al., Phys. Rev. D 60, 094508, F. Niedermayer et al., Nuclear Physics B 597, 413–450

$\pi\pi$ Diagrams and their error in A1



Light state with charmonium operators in A1

Masses extracted from **diagonal** entries of the charmonium-only correlation matrix.



Both choices of basis see a lighter state but we need light meson operators to clearly resolve it and other surrounding ones.