Status report on the hadronic light-by-light contribution to the muon g-2 using twisted-mass fermions

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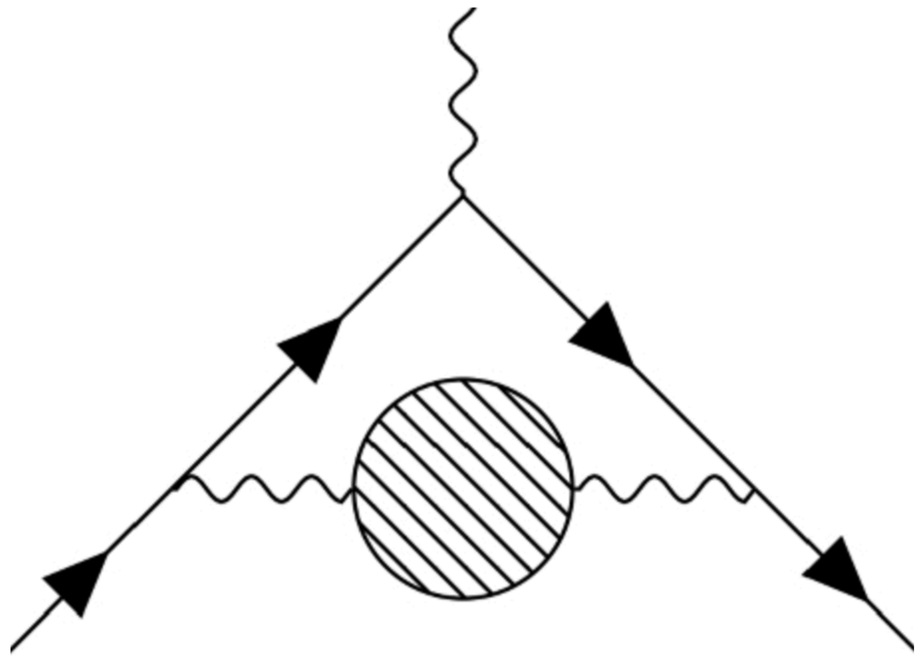
on behalf of the ETM Collaboration

Lattice 2024

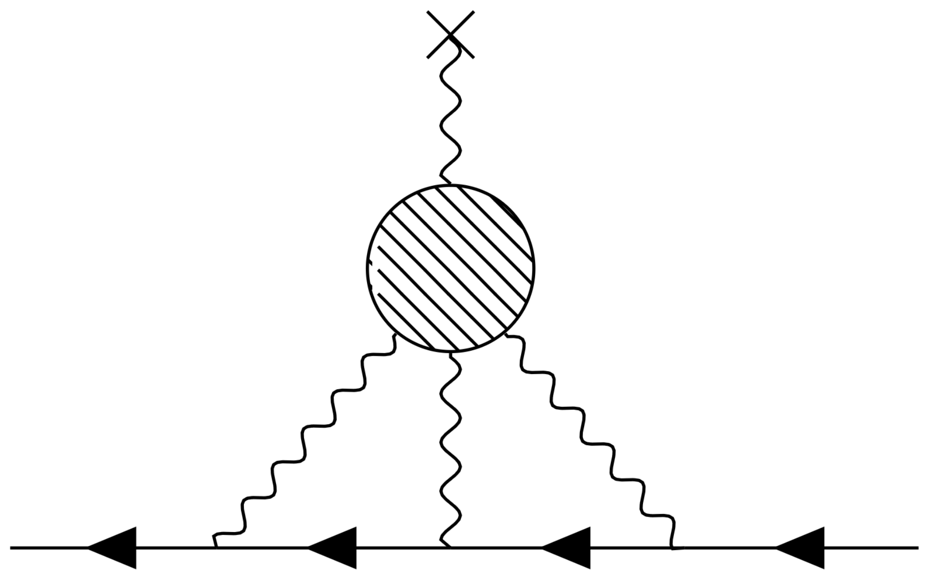
Liverpool, July 29th, 2024

Introduction

- g-2 puzzle: Theoretical uncertainty has to be reduced.
- Two hadronic diagrams contribute the most to the theoretical uncertainty.



- HVP @ $\mathcal{O}(\alpha_{QED}^2)$.



- HLbL @ $\mathcal{O}(\alpha_{QED}^3)$.

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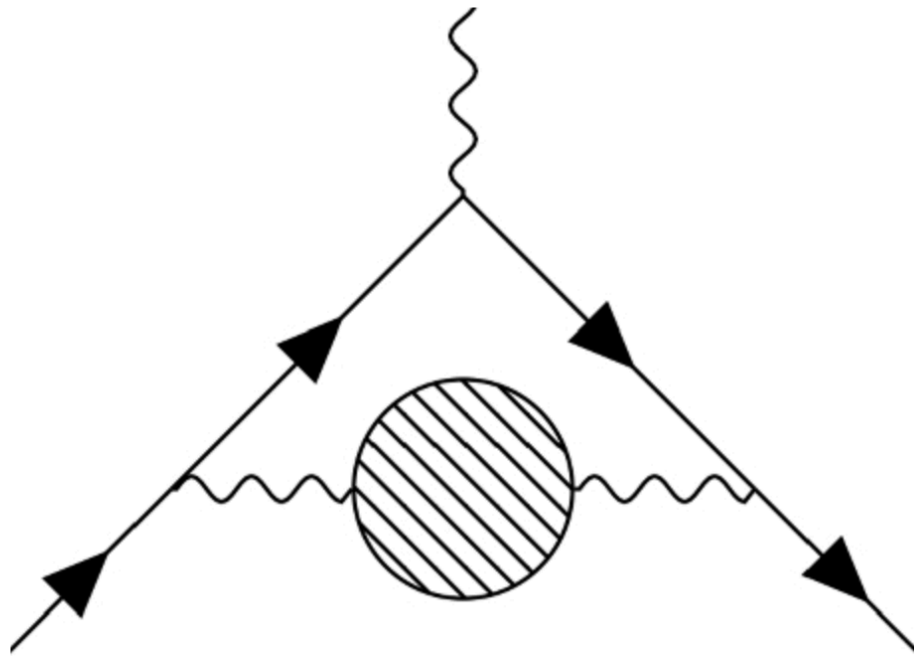
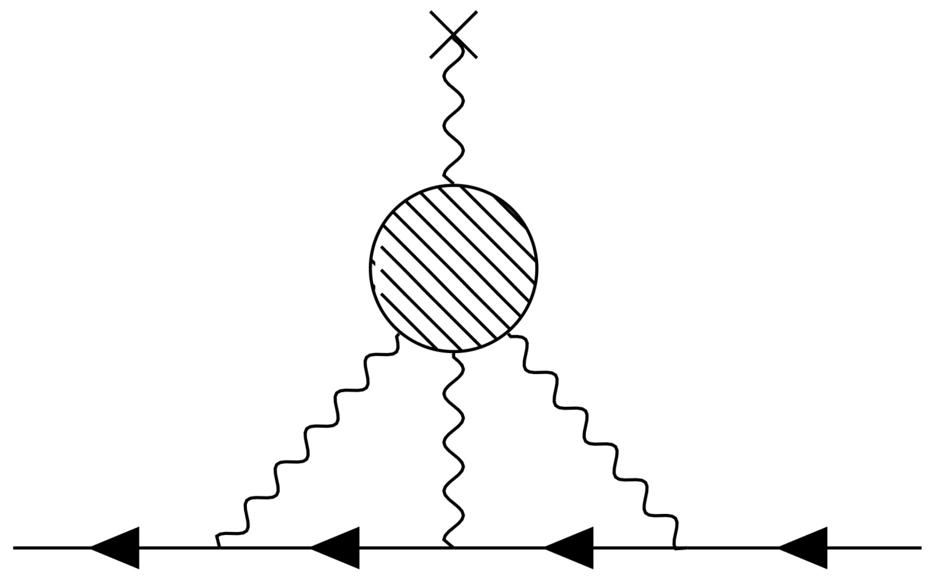




- HVP @ $\mathcal{O}(\alpha_{QED}^2)$.
- HLbL @ $\mathcal{O}(\alpha_{QED}^3)$.

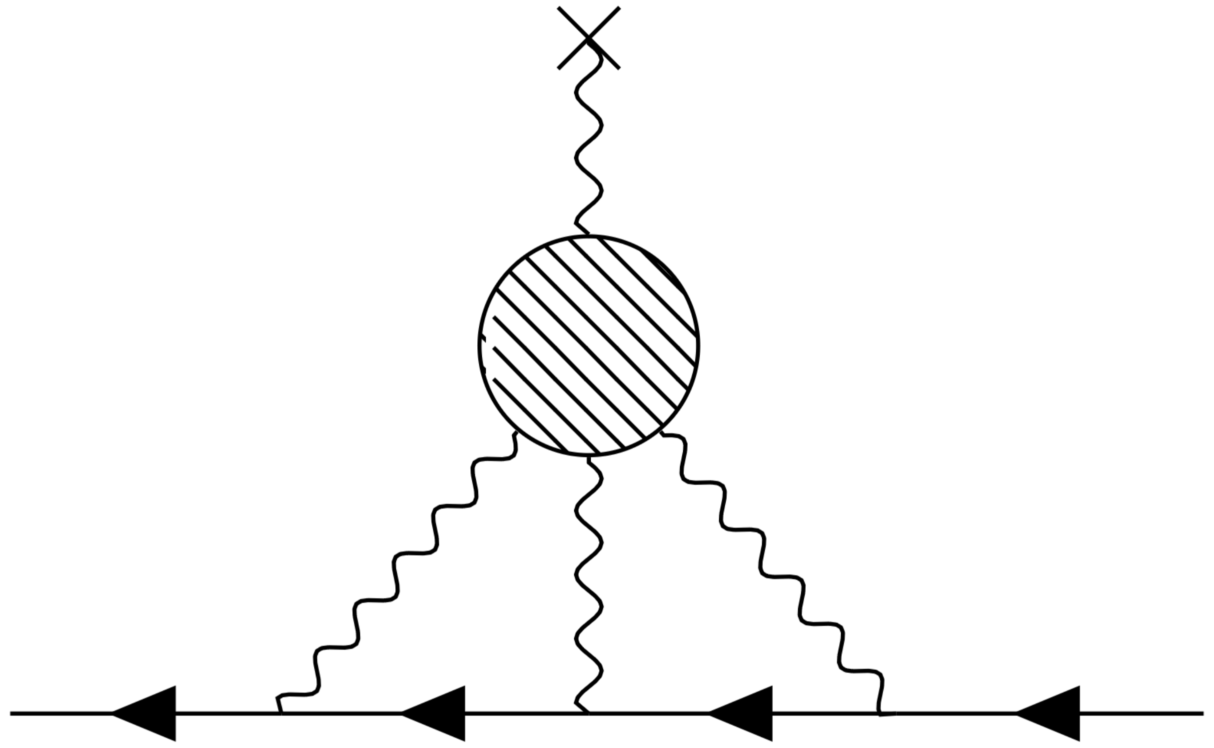
- Difficult to calculate (non-perturbative).
- Two main approaches:

  (1) Dispersion relations: Data-driven approach.

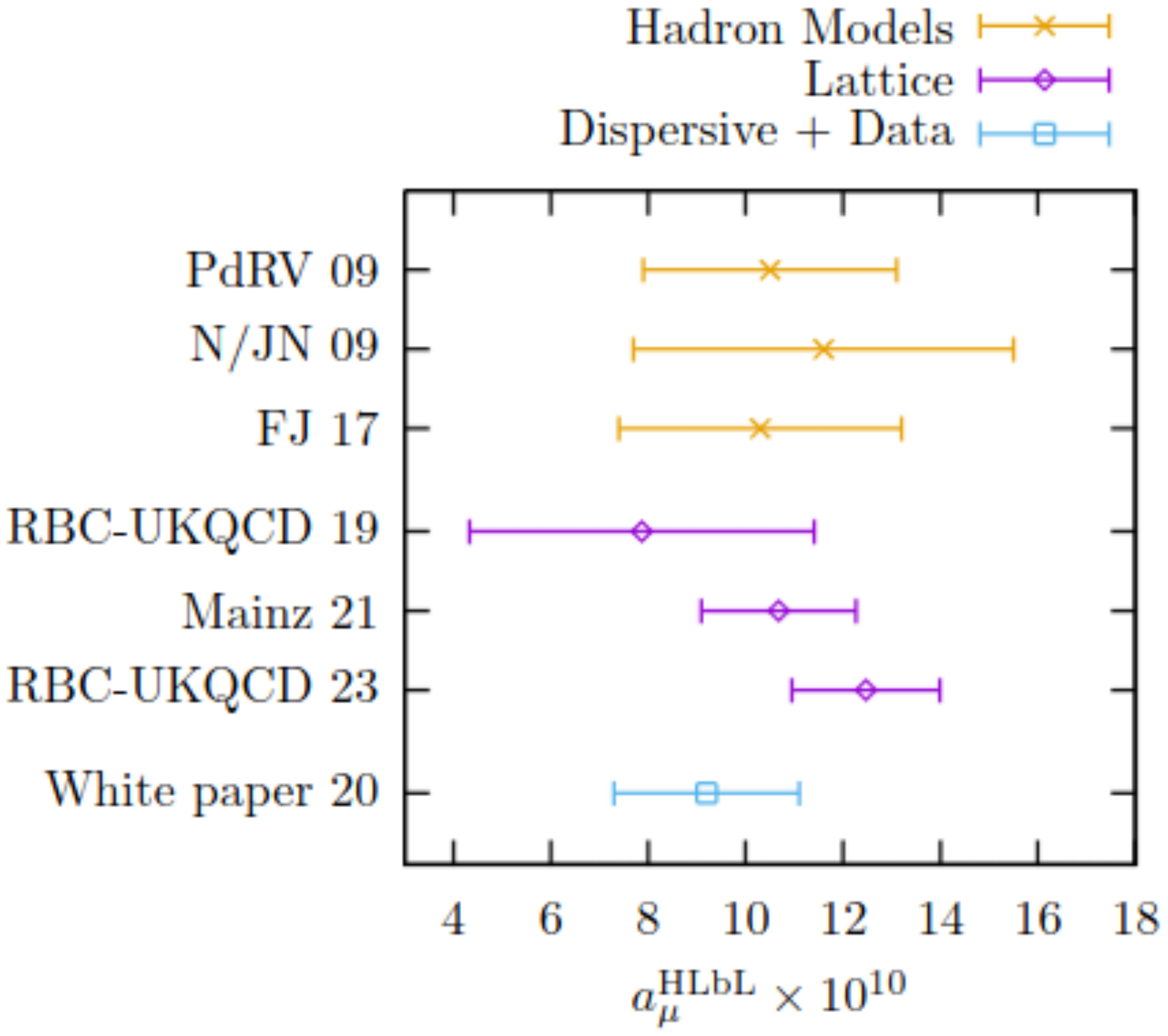
  (2) Lattice QCD: Calculations on a finite lattice.

Hadronic Light-by-Light (HLbL)

- We focus on the **full** lattice QCD calculation of HLbL (not only PS pole contributions).



- Difficult calculation: 4-pt function
- Sub-dominant contribution to the theoretical error @ $\mathcal{O}(\alpha_{QED}^3)$.
- **Good agreement** between lattice and dispersive.
- Uncertainty has to be significantly reduced.

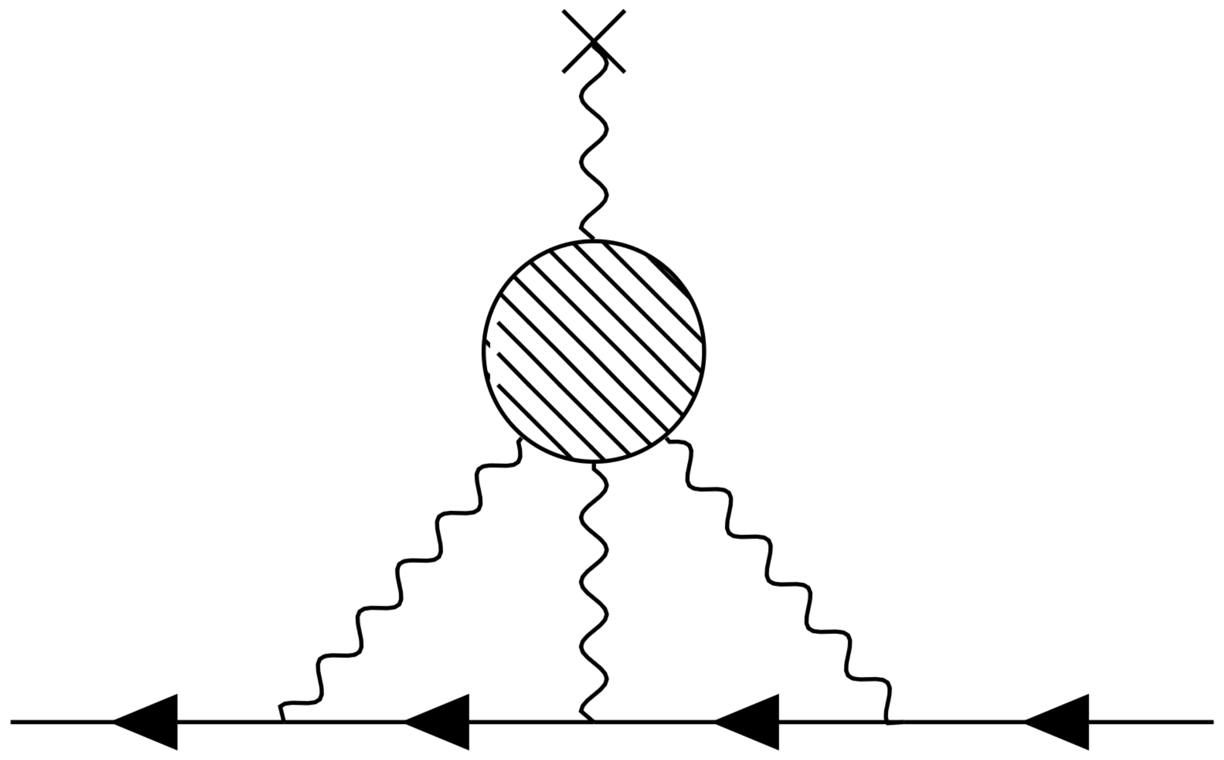


- RBC-UKQCD (2023)

Master formula for a_μ^{HLbL}

- We follow the Mainz approach

$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} \int_{x,y} \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)\, i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y).$$



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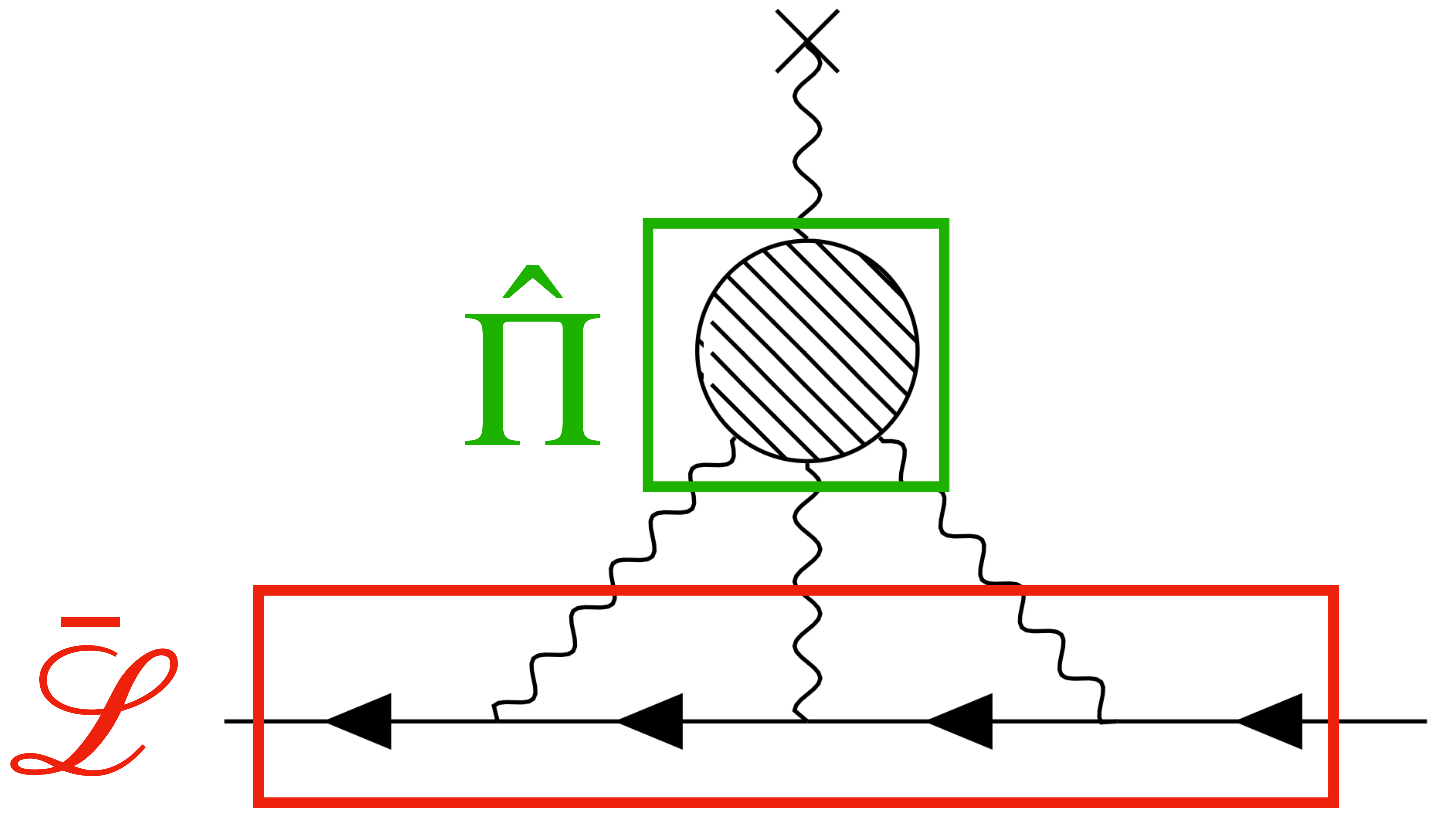

- QED kernel $\bar{\mathscr{L}}$: continuum and infinite volume.

- 4-pt function $\hat{\Pi}$: discrete lattice and finite volume.

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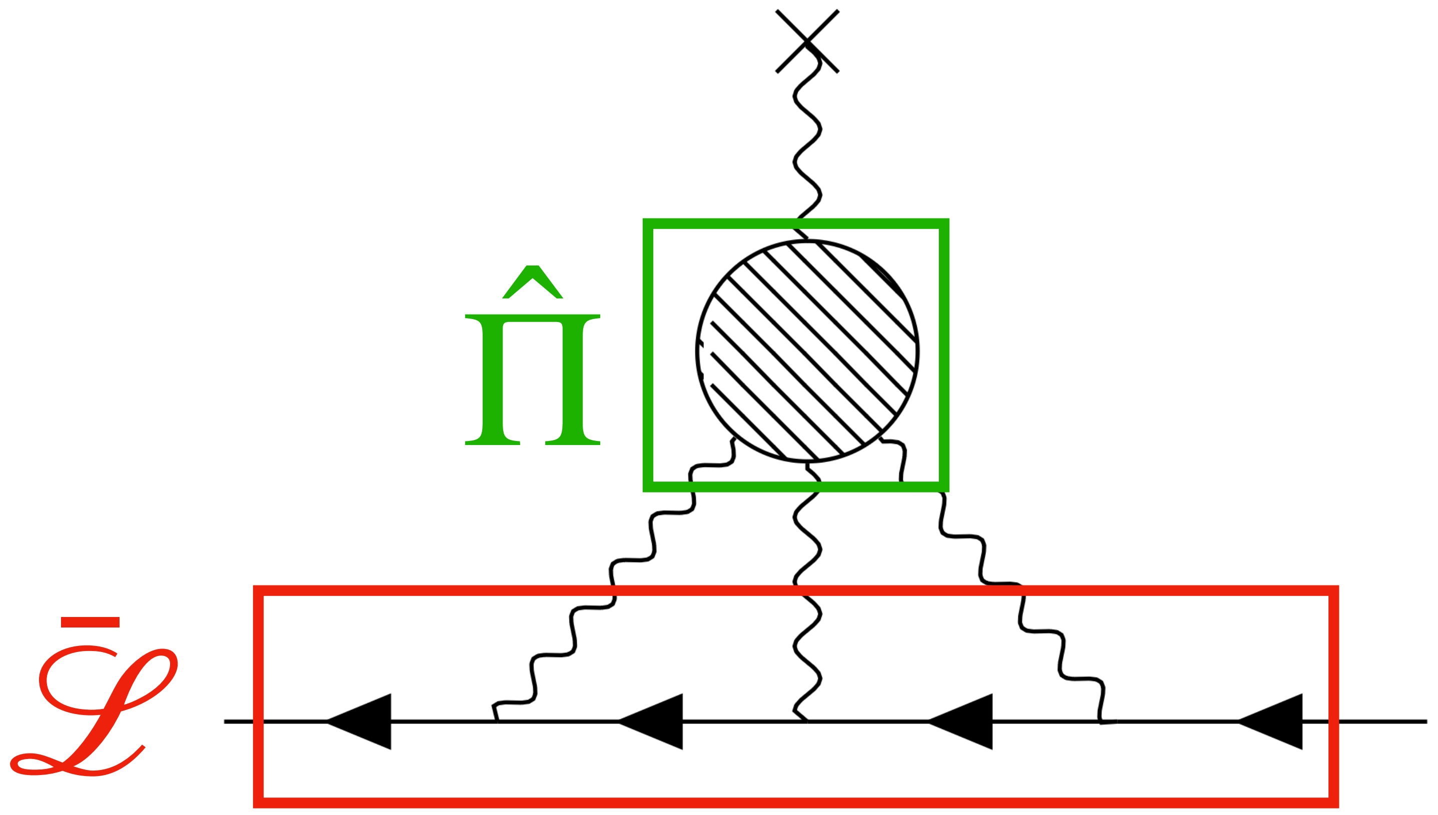

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$$a_\mu^{HLbL}(|y|_{max}) = \int_0^{y_{max}} d|y| f(|y|), \quad f(|y|) = \frac{m_\mu e^6}{3} 2\pi^2 |y|^3 \int_x \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)\; i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y).$$

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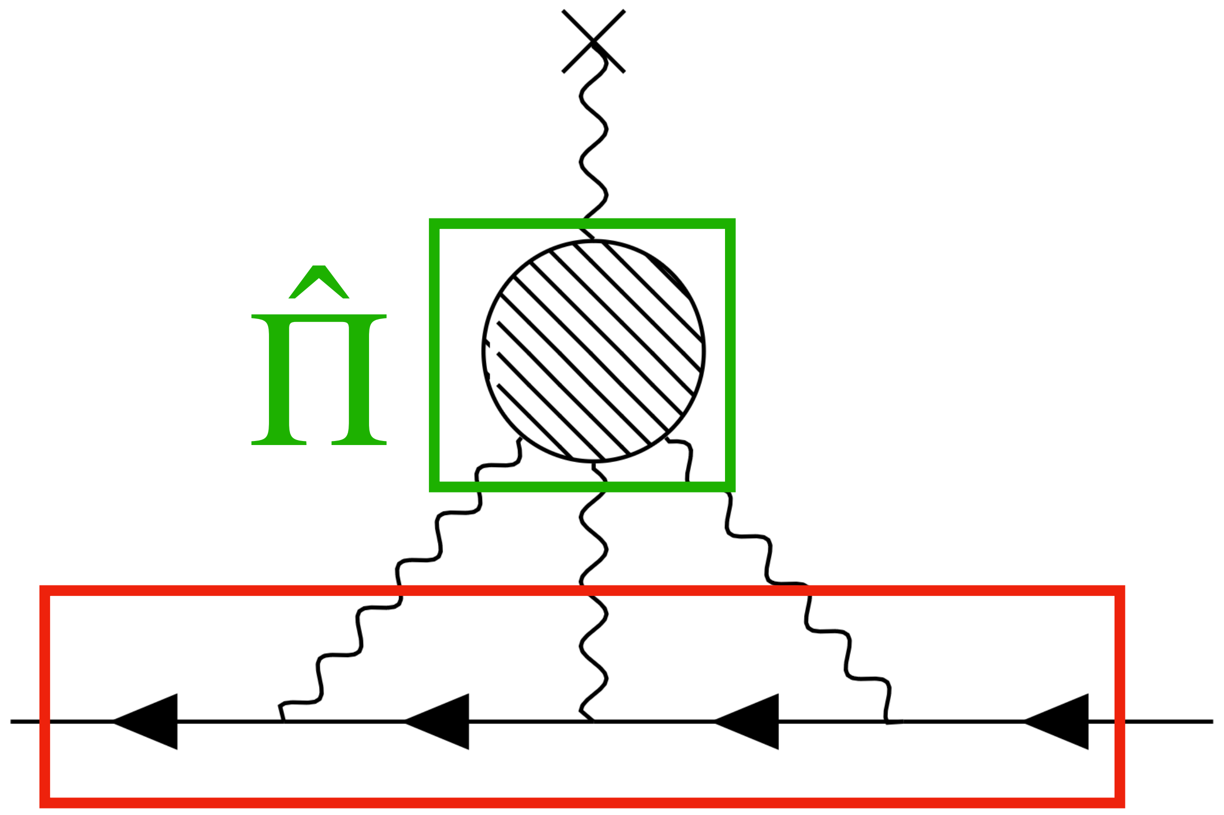

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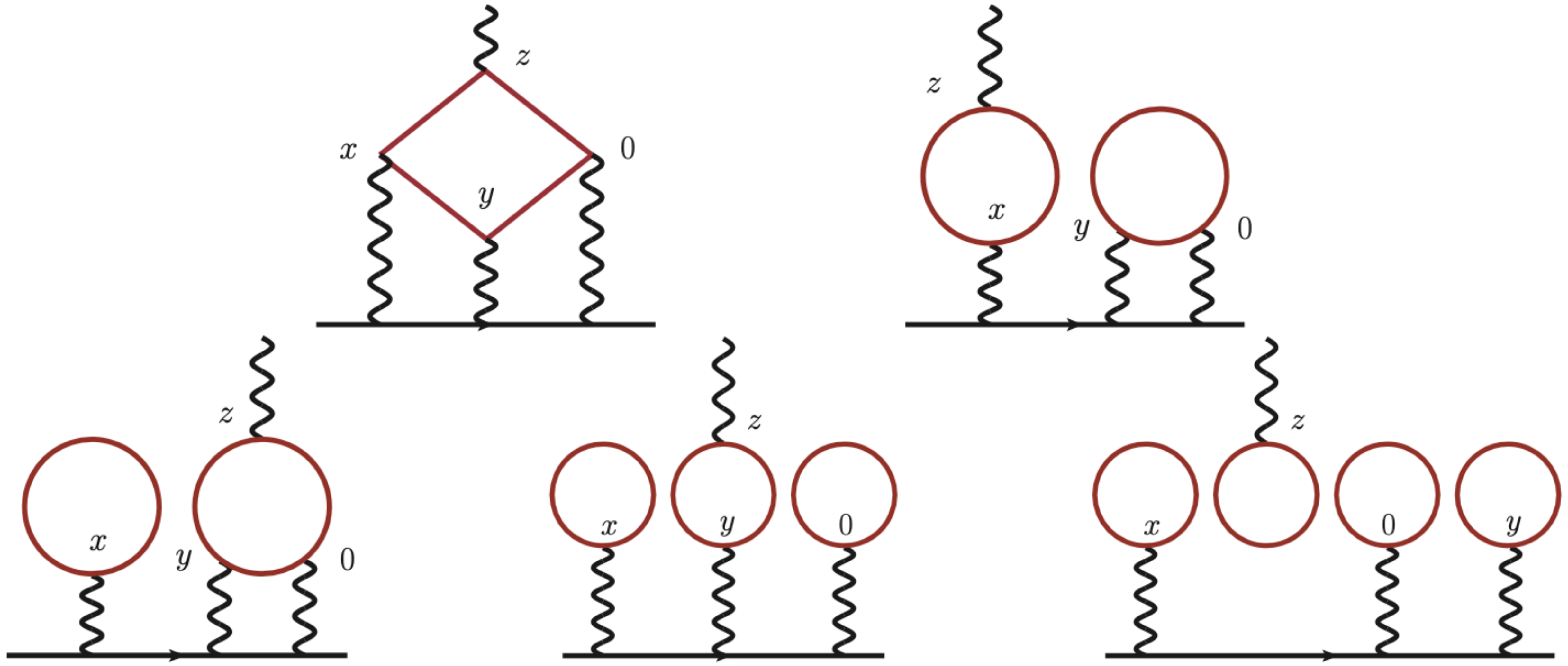
- Kernel freedom: $\bar{\mathscr{L}} \to \bar{\mathscr{L}} +$ anything that vanishes upon integration (in the continuum and infinite volume limit).

- We work with two kernels: Kernel 3 and Kernel $\Lambda = 0.4$ (as defined by Mainz).

Topologies

- For the HLbL contribution there are 5 distinct topologies (5 classes of Wick contractions).



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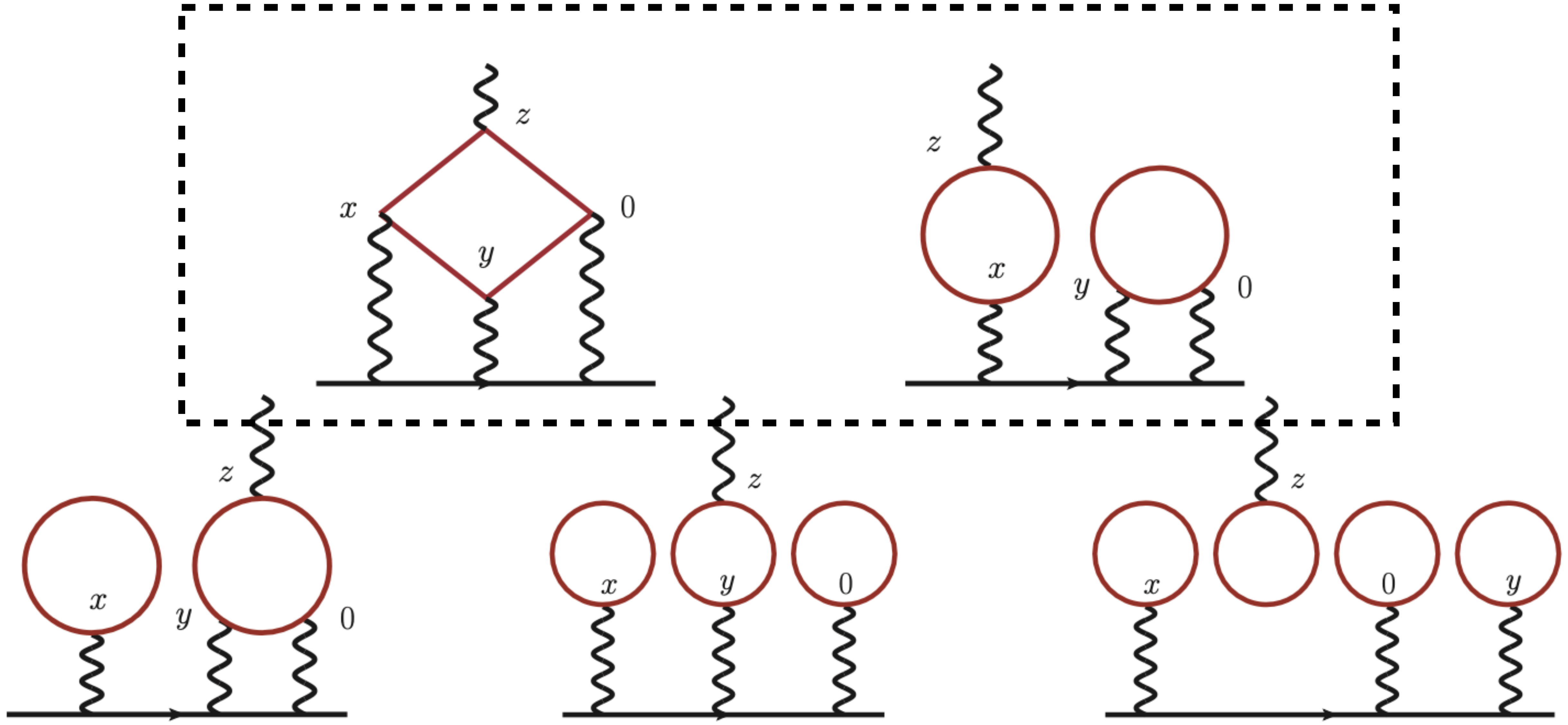

- Mainz (2021)

- We focus on the **fully-connected** and **2+2** (dominant ones).

Ensemble details and runs

- We use **twisted-mass fermions** on **2+1+1 gauge ensembles** at the **physical point**, generated by the Extended Twisted Mass Collaboration (ETMC).

Ensemble	$L^3 \cdot T/a^4$	M_π $[MeV]$	a $[fm]$	L $[fm]$	$M_\pi \cdot L$	Z_V	Z_A
cB211.072.64 (cB64)	$64^3 \cdot 128$	140.2(2)	0.07961(13)	5.09	3.62	0.706379(24)	0.74294(24)
cC211.060.80 (cC80)	$80^3 \cdot 160$	136.7(2)	0.06821(12)	5.46	3.78	0.725404(19)	0.75830(16)
cD211.054.96 (cD96)	$96^3 \cdot 192$	140.8(2)	0.05692(10)	5.46	3.90	0.744108(12)	0.77395(12)

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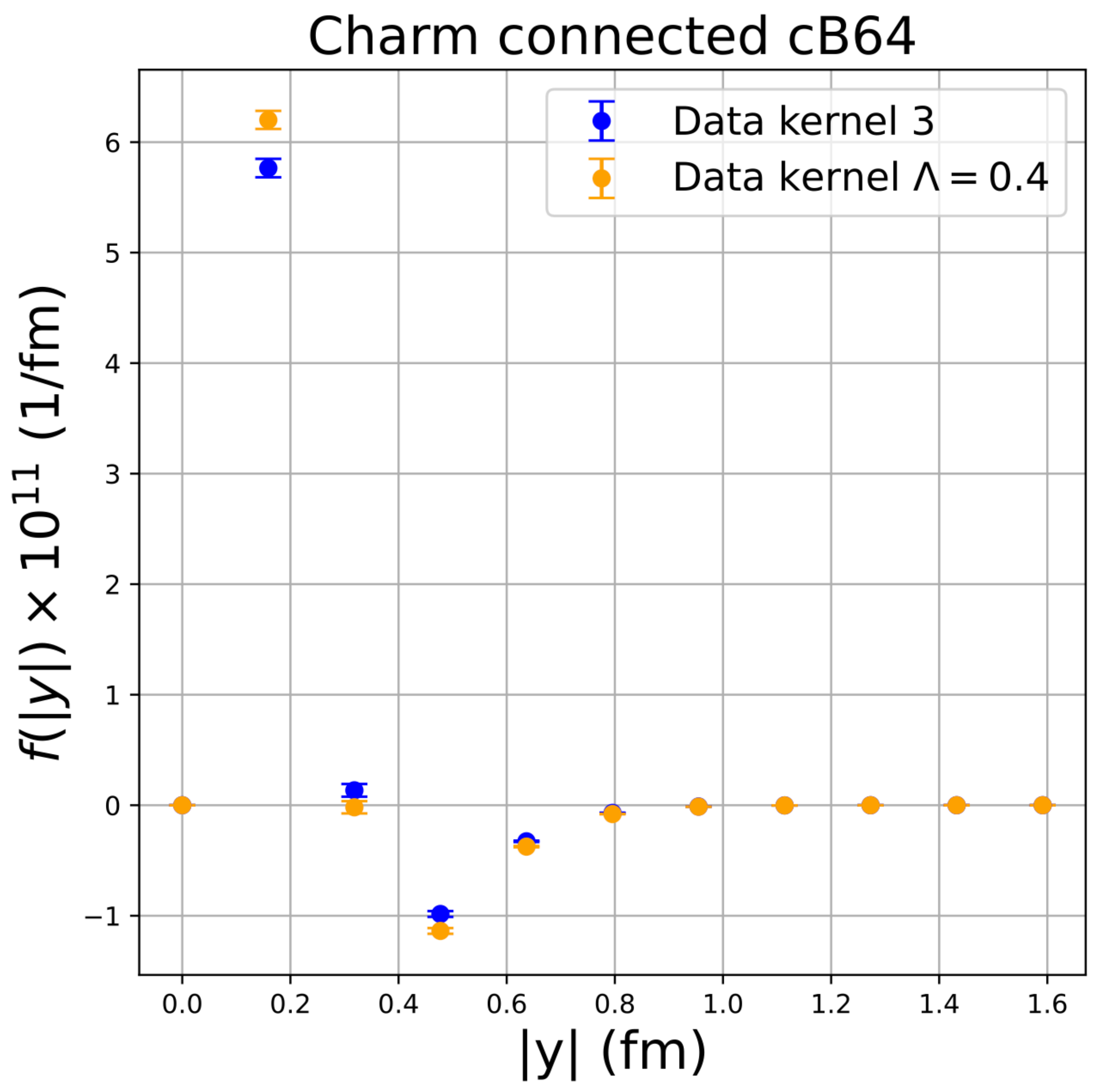
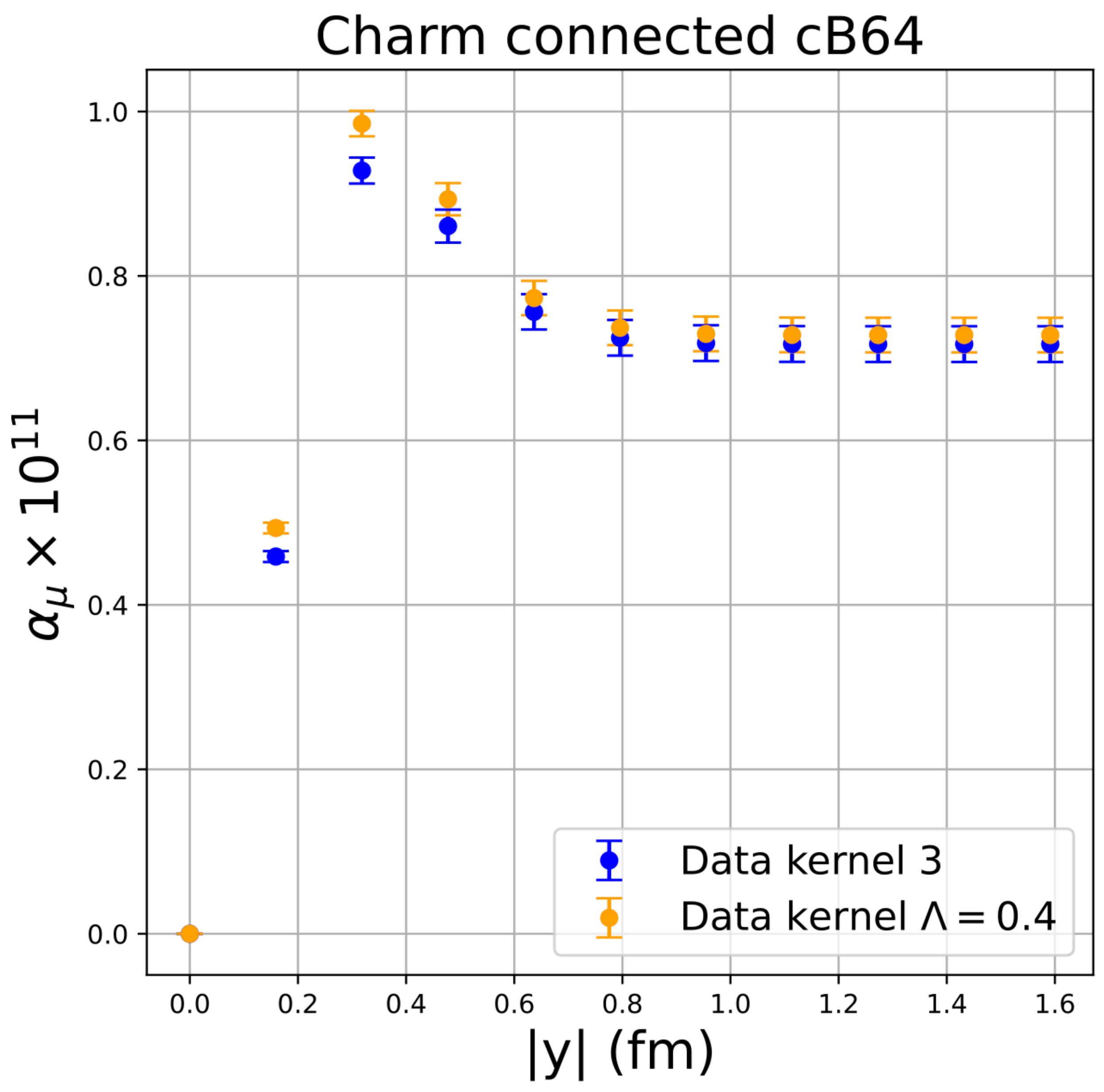
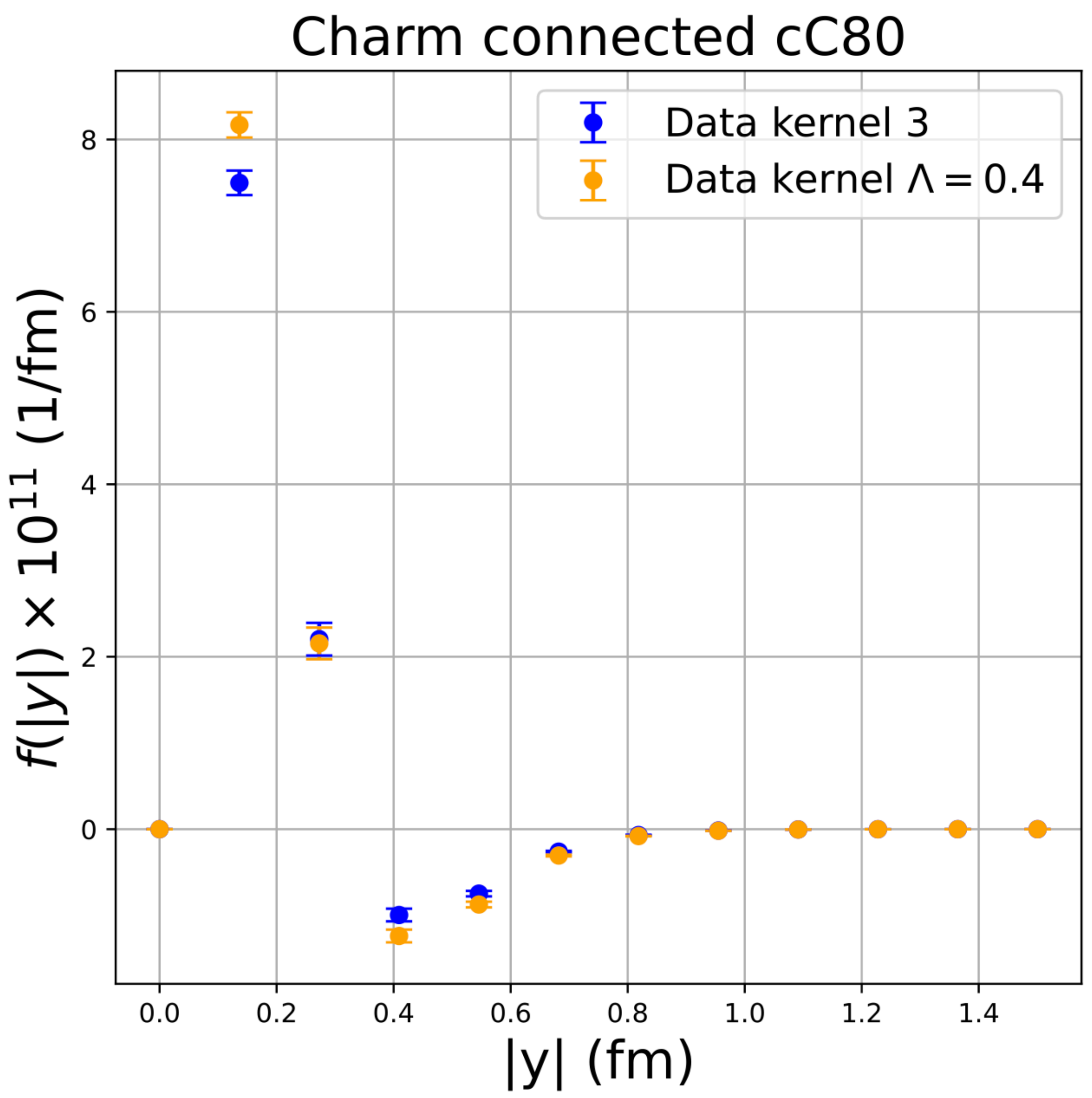
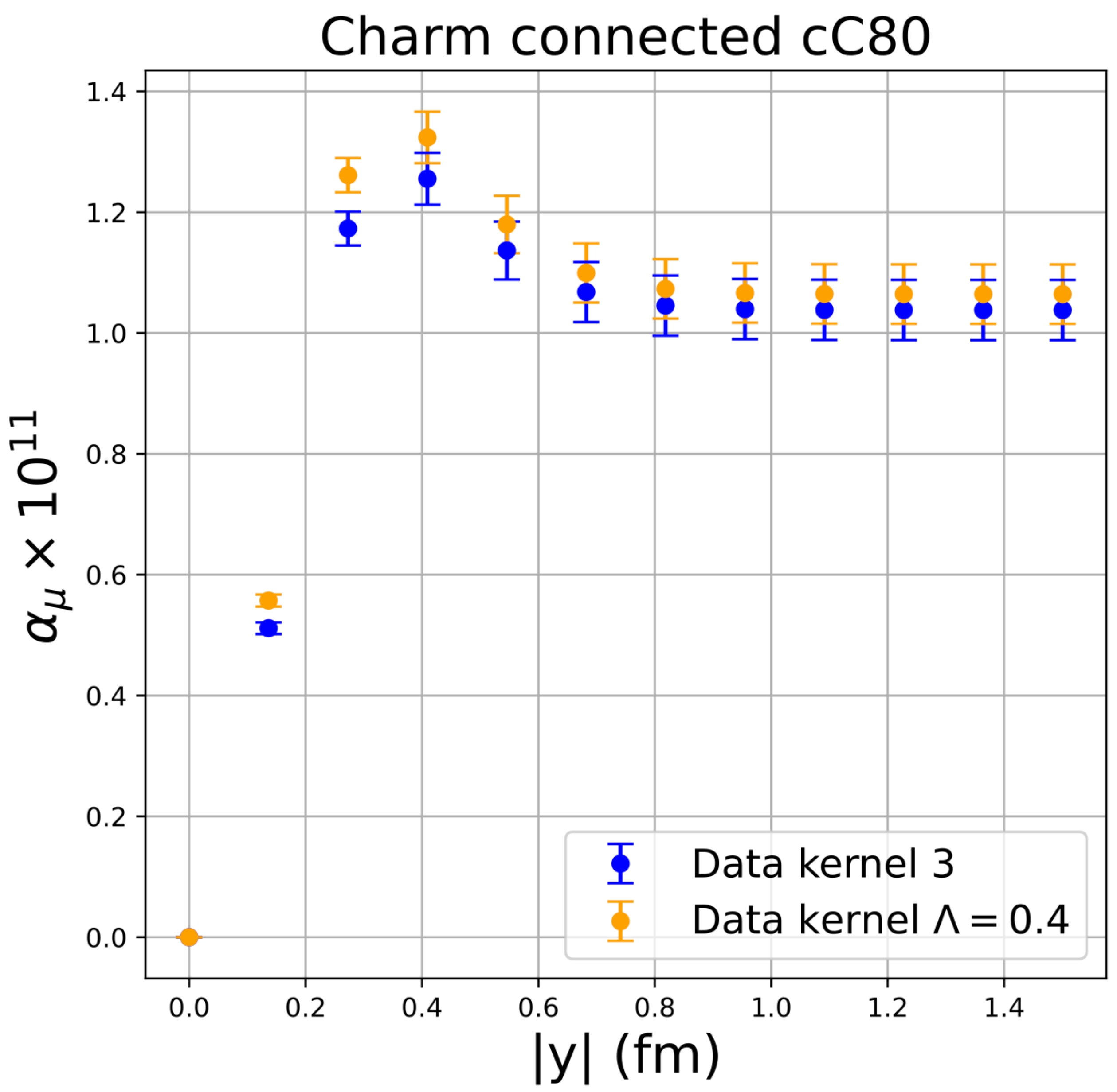
- Charm, Strange quarks: All three ensembles.
- Light quarks: cB64 so far.
- We work with two kernels (estimate of FVE and lattice artefacts): Kernel 3 and Kernel $\Lambda = 0.4$.
- Note: The results presented are preliminary.

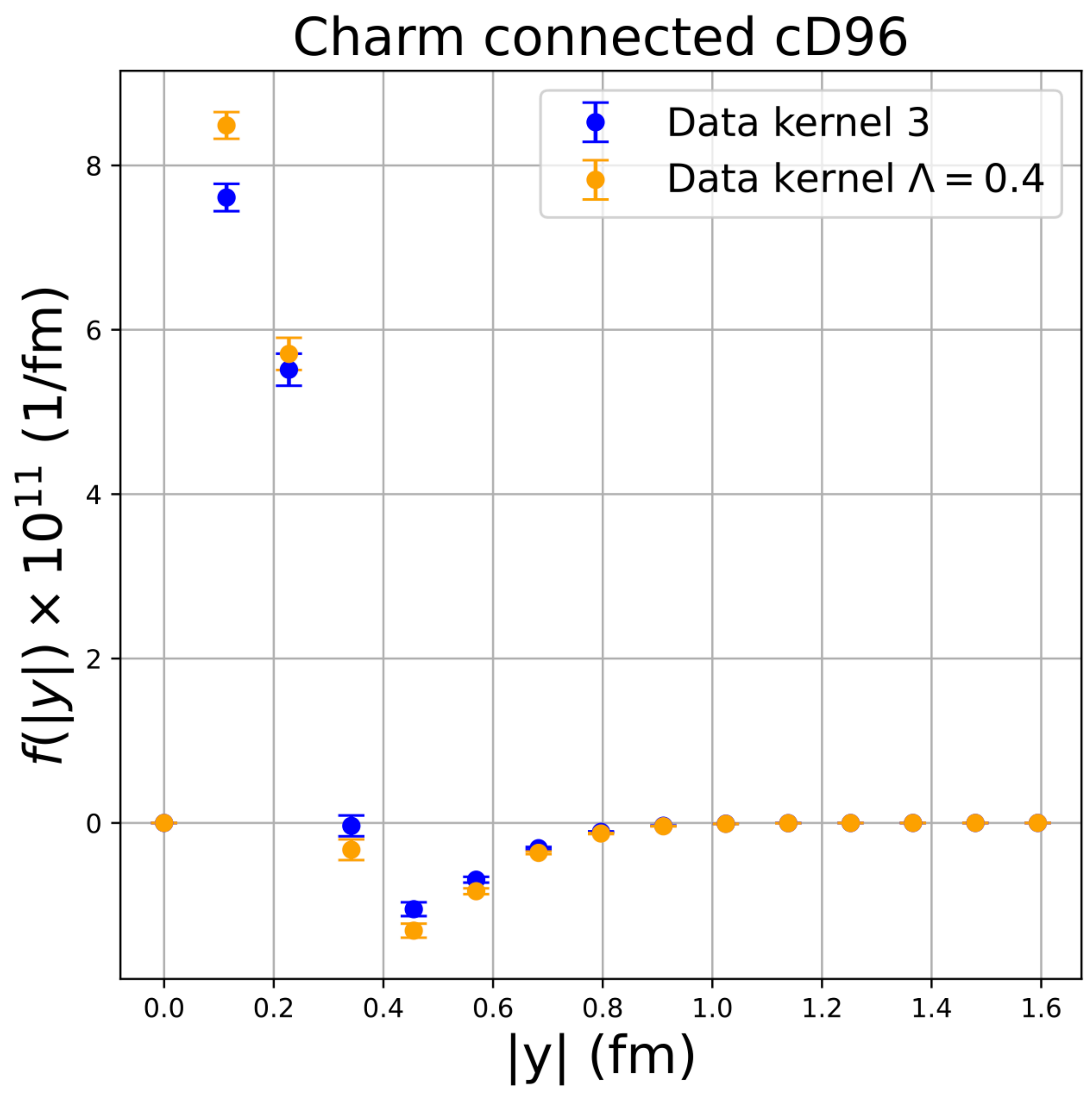
Charm connected: preliminary results

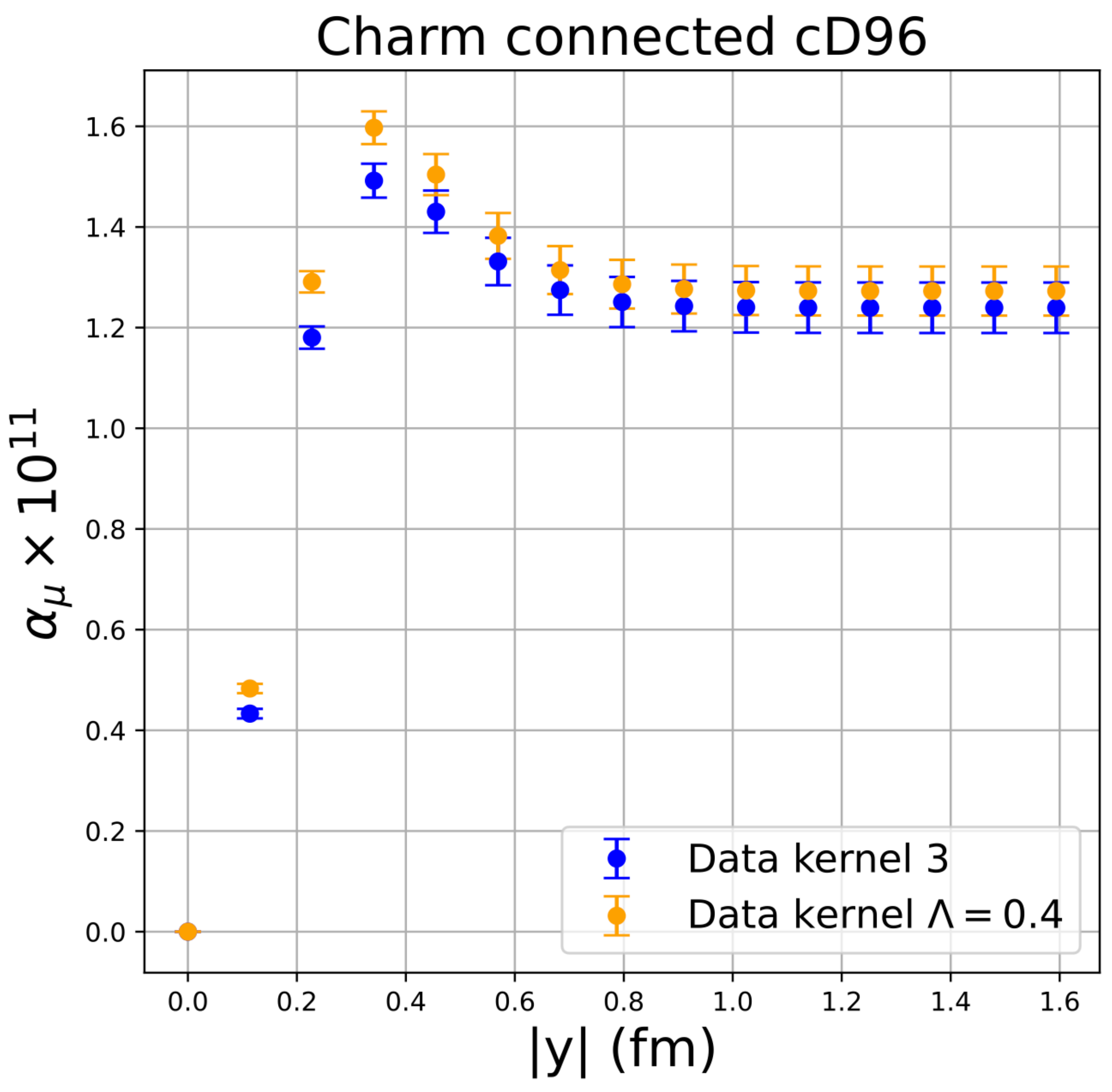






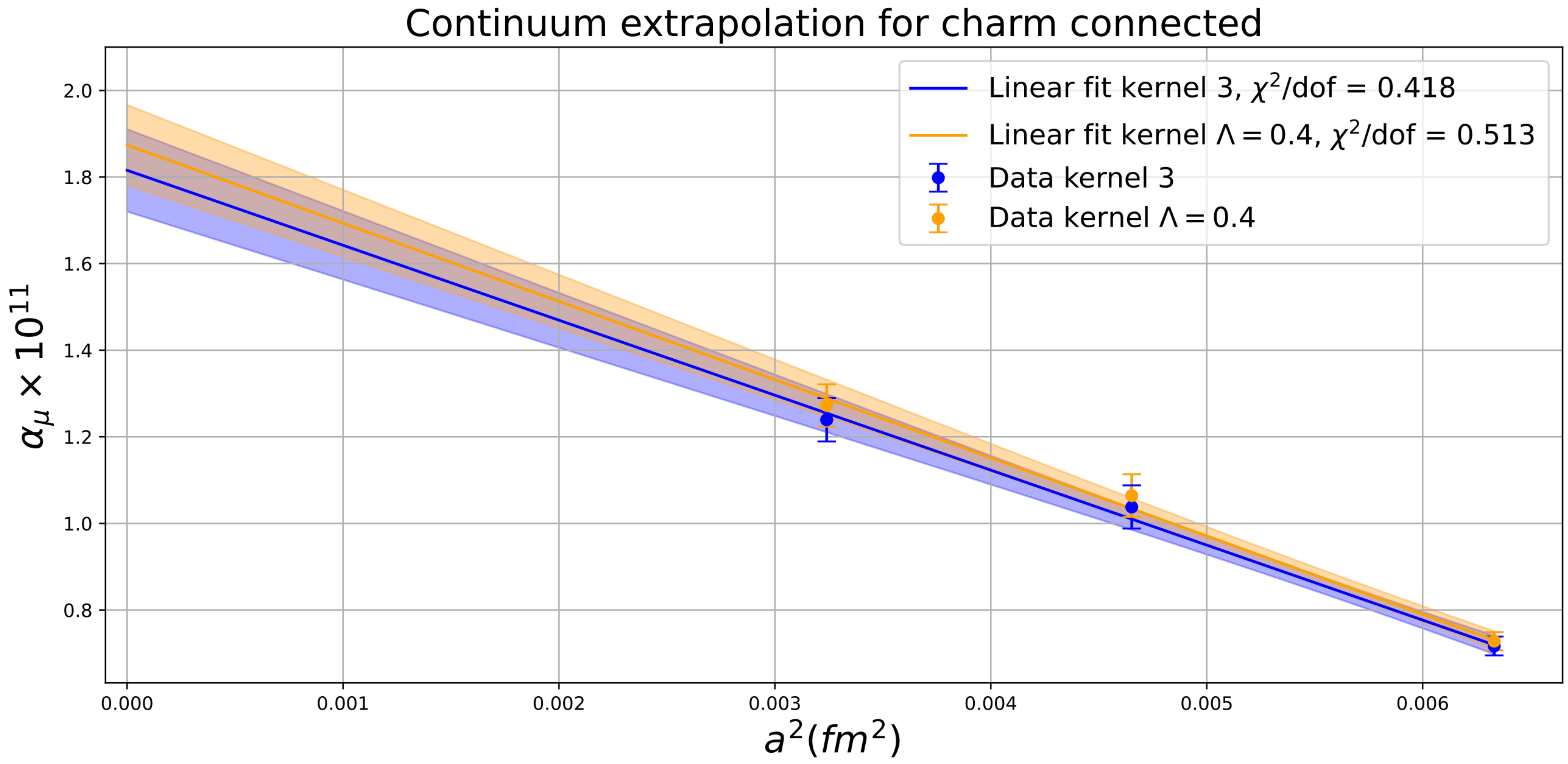






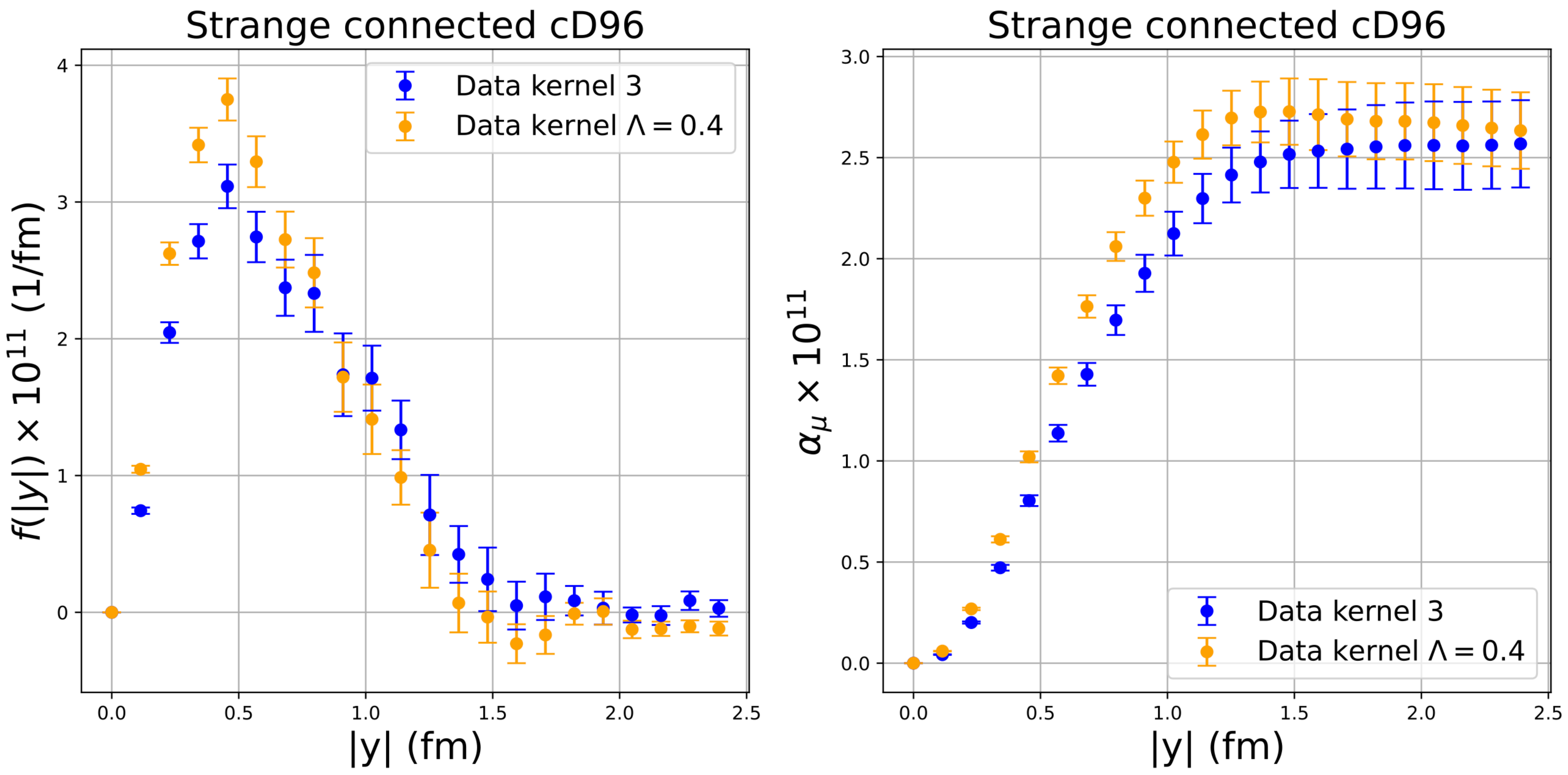
- Very good quality of data for both kernels.
- Only a few configurations needed.
- Small contribution to HLbL.

Charm connected: preliminary results



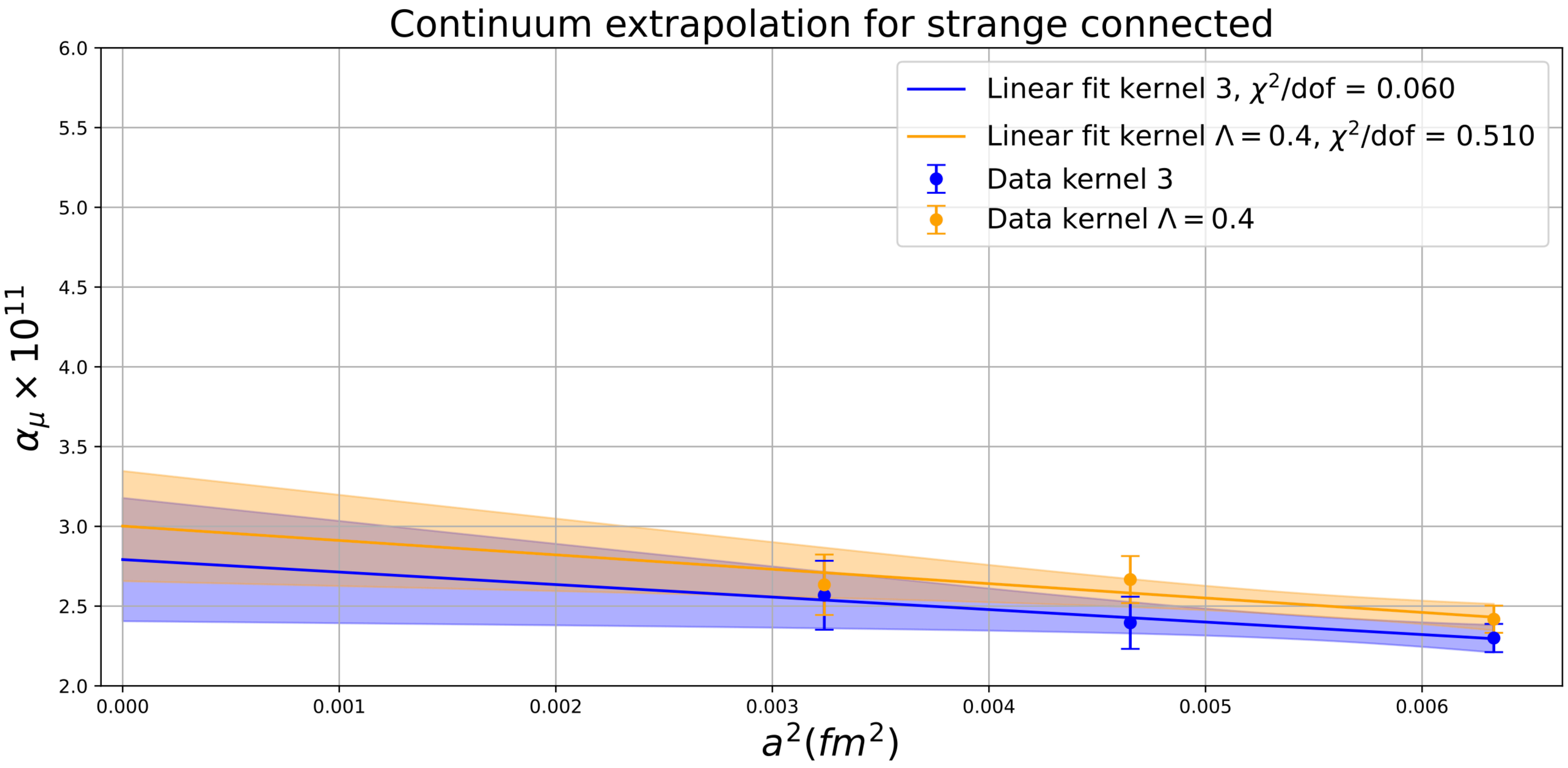
- Linear fit in a^2 describes the extrapolation accurately (expected for twisted-mass).
- Statistical error is under control.
- Systematic error to be included (in progress).

Strange connected: preliminary results

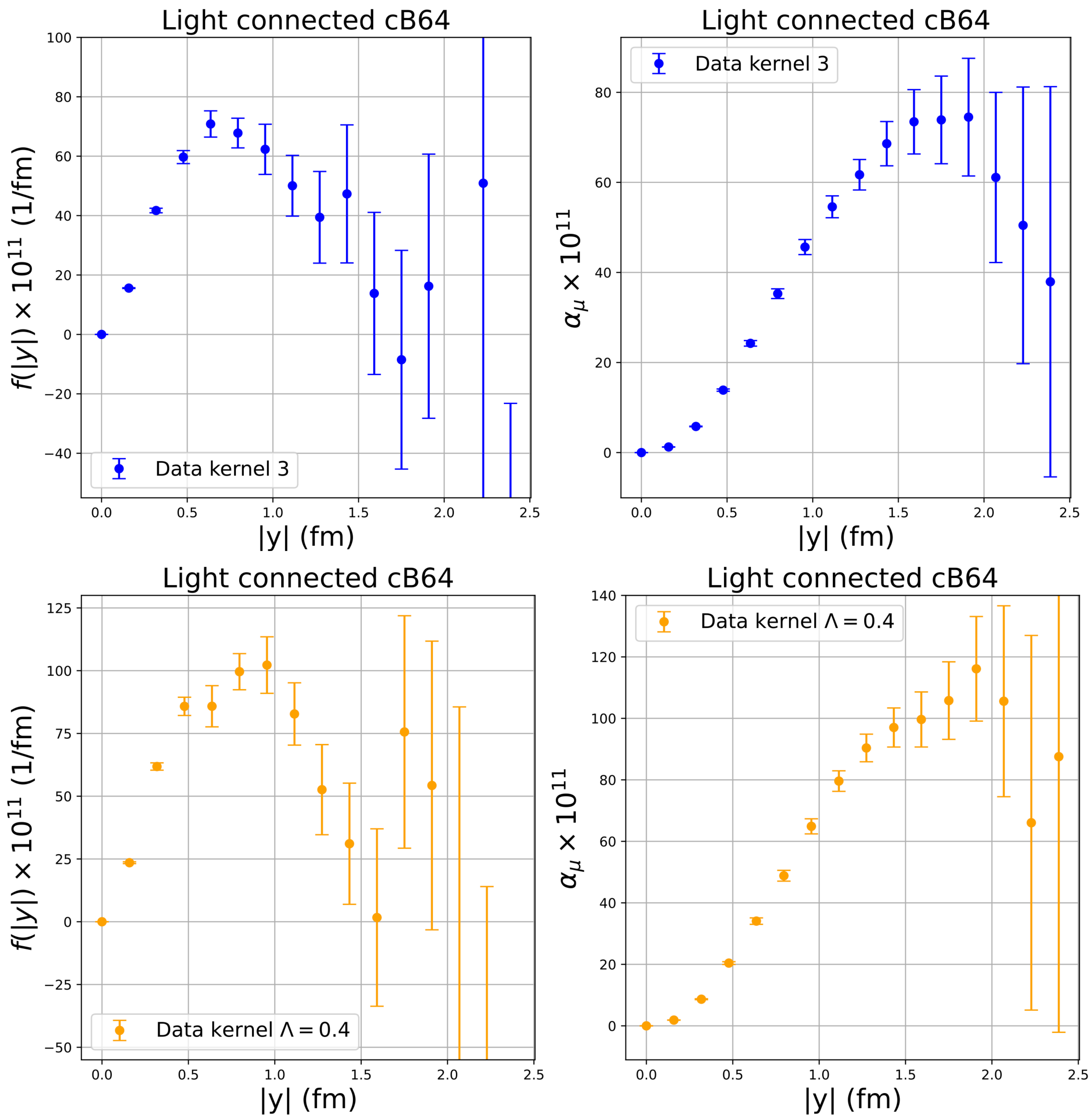


- Good quality of data for both kernels.
- Only a few configurations needed.
- Also, small contribution to HLbL.

Strange connected: preliminary results

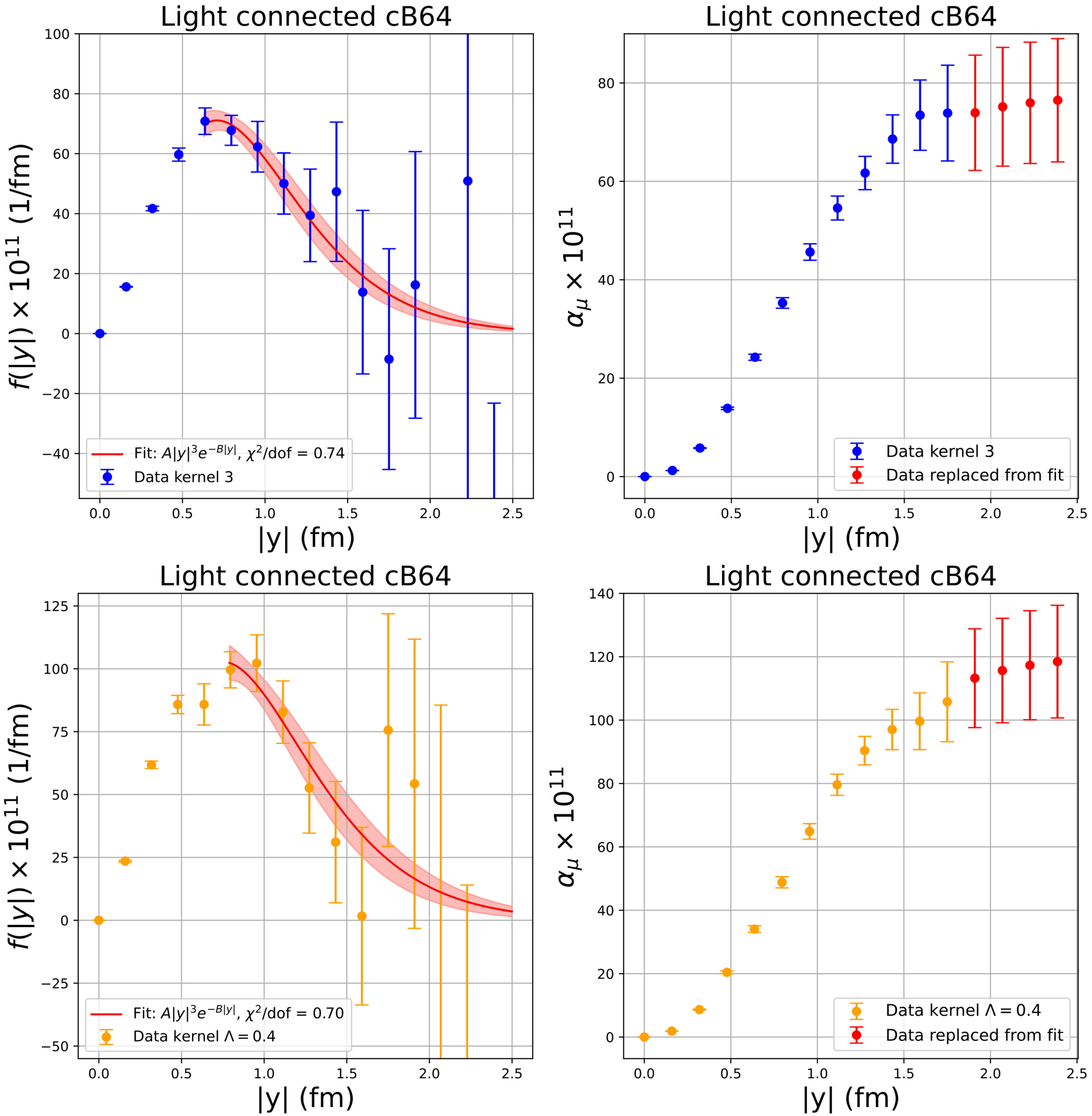


- Similar order of magnitude to charm-connected.
- Linear fit in a^2 seems to describe the extrapolation accurately.
- Discretisation effects smaller compared to charm-connected.
- Plan to run more configurations for cC80 and cD96.
- Statistical error is under control.
- Systematic error to be included (in progress).



Light connected: preliminary results

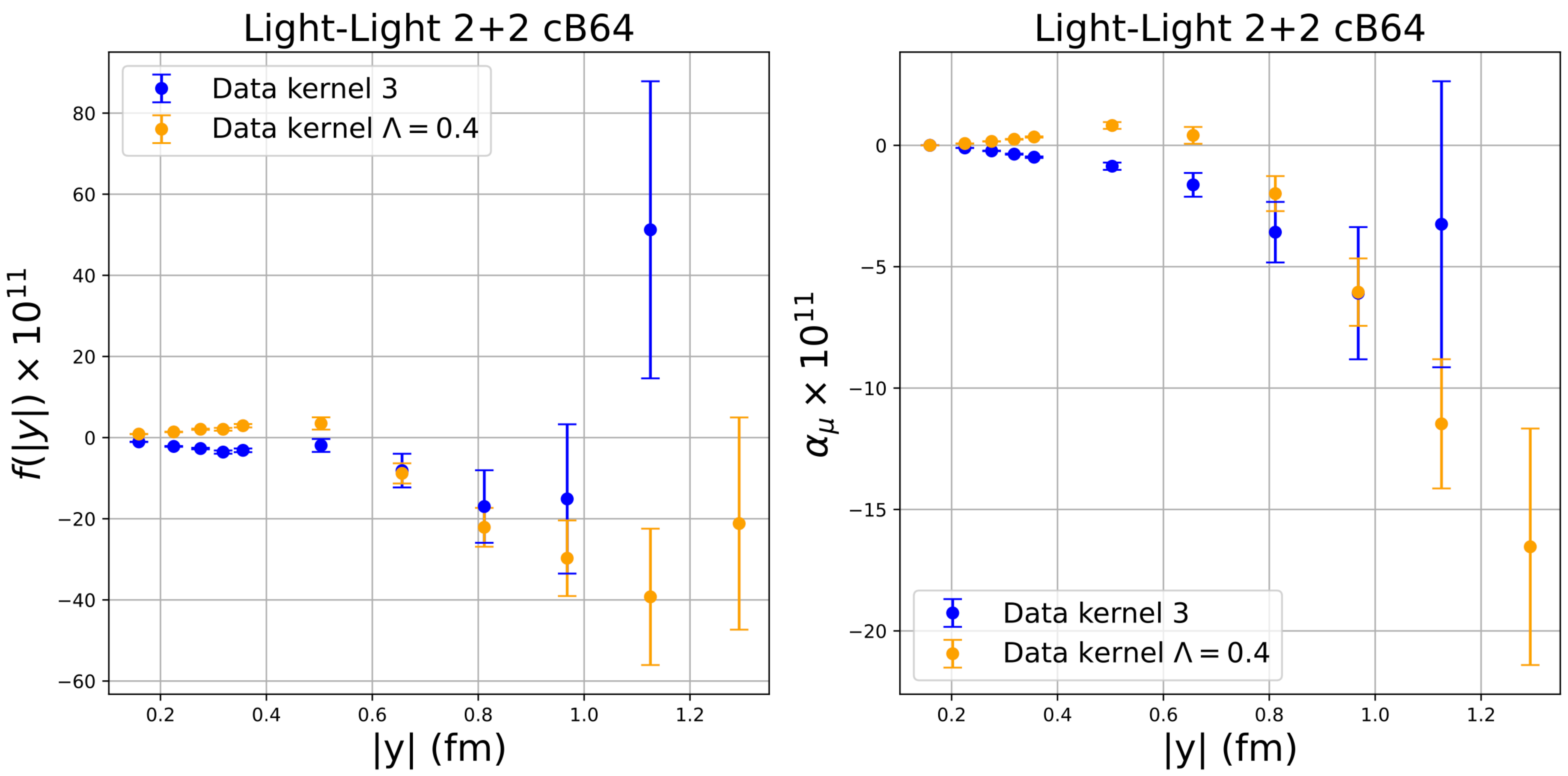
- Dominant contribution to HLbL.
- Good quality for the signal up to ~1.5 fm.
- After ~1.5 fm it becomes noisy: How can we deal with it?
    - Model the tail with the PS pole contributions (in progress); also better estimation of FVE.
    - Use a more general model to replace the data of the tail.



Light connected: preliminary results

- One such model: $A\,|\,y\,|^3 e^{-B|y|}$ (Mainz 2021).
- Good description of $f(\,|\,y\,|\,)$ after the peak.
- Replace the data after 1.9 fm with the model.
- Adds a systematic error that needs to be calculated (in progress).
- For both kernels ~95% of the signal comes from the pure data.

Light-Light 2+2: preliminary results



- Light-Light: Dominant contribution to α_μ^{disc}.
- Difficult to calculate (very noisy).
- Kernel 3: low statistics (to be increased).
- Kernel $\Lambda = 0.4$: good signal up to ~1.2 fm.
- Next steps:
    - Increase statistics after ~ 1 fm and extend data to larger distances;
    - Extend analysis to light-strange and strange-strange (in progress).

Conclusions and future directions

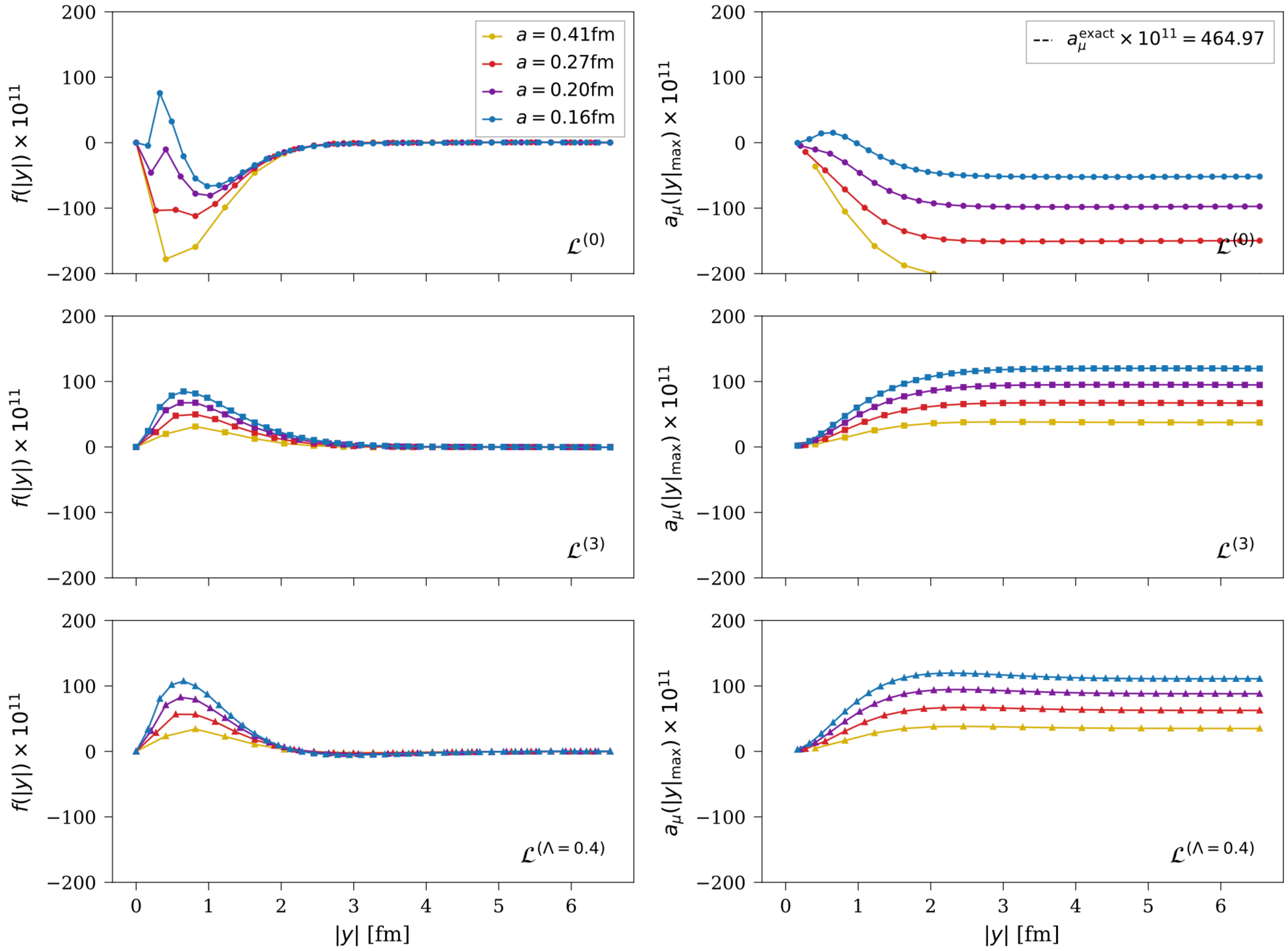
Charm and Strange connected	Light connected	Light-Light 2+2	Light-Strange and Strange-Strange 2+2

• Good signal for both kernels.
• Statistical error under control.
• Continuum extrapolation done.
• Systematic error to be added.
	• Good signal for both kernels up to ~1.5 fm.
	• Large distances: Replacement of tail with PS pole data (in progress) or a more general model, like $A\|y\|^3 e^{-B\|y\|}$.
	• Systematic error to be added.
	• Extend to cC80 and cD96.
		• Good signal for Kernel $\Lambda = 0.4$ up to ~1.2fm.
		• Kernel 3 needs more statistics.
		• Plans for both kernels:
		    • Increase statistics after ~1fm.
		    • Produce data for larger distances.
		• Extend to cC80 and cD96.
			• To be included in future runs.

Thank you for your attention!

Back-up slides

Lepton-loop cross-check



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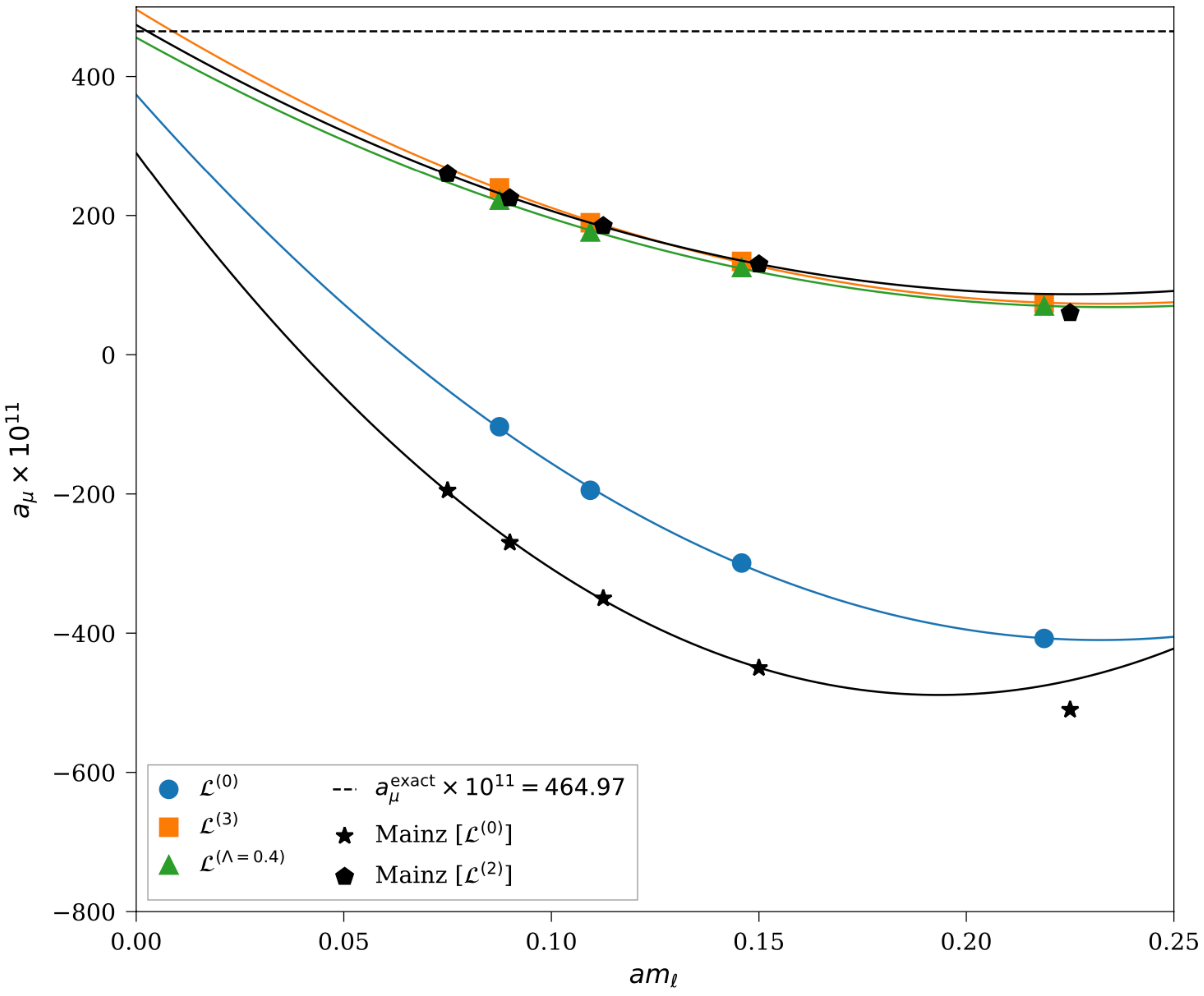
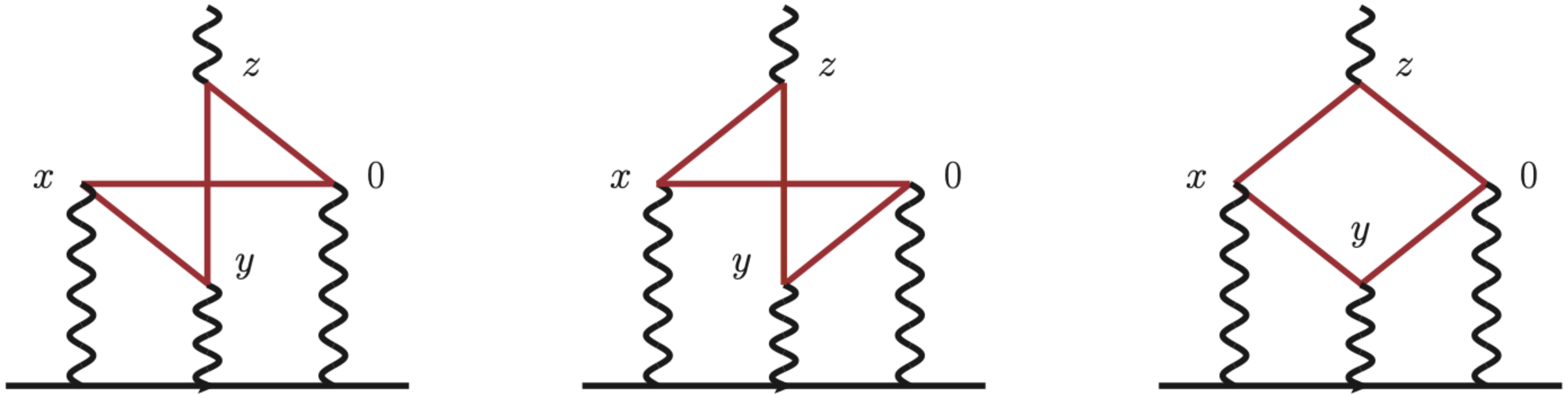


Fully-connected contribution



- Each diagram: 2 contractions with quark flow in opposite directions.
- To compute this contribution: Pick a reference diagram that is easier to compute - the 1st one in our case.
- Then, use a change of variables in the integrals to relate the other diagrams to the first.

Fully-connected contribution

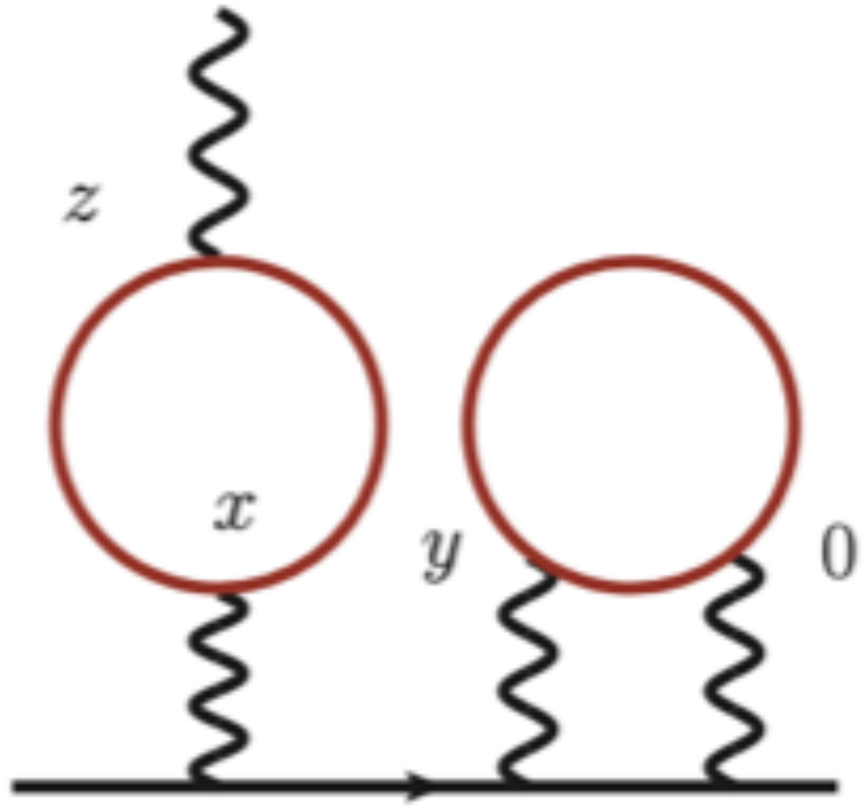
$$f^{(\mathrm{Conn.})}(|y|) = -\sum_{j\in u,d,s,c} \hat{Z}_{\mathrm{V}}^4 Q_j^4 \frac{m_\mu e^6}{3} 2\pi^2 |y|^3 \times$$
$$\int_x \left(\mathcal{L}'_{[\rho,\sigma]\mu\nu\lambda}(x,y) \int_z z_\rho \widetilde{\Pi}^{(1),j}_{\mu\nu\sigma\lambda}(x,y,z) + \bar{\mathcal{L}}^{(\Lambda)}_{[\rho,\sigma];\lambda\nu\mu}(x,x-y) x_\rho \int_z \widetilde{\Pi}^{(1),j}_{\mu\nu\sigma\lambda}(x,y,z) \right).$$

$$\widetilde{\Pi}^{(1),j}_{\mu\nu\sigma\lambda}(x,y,z) = -2\mathrm{Re}\left\langle \mathrm{Tr}\left[S^j(0,x)\gamma_\mu S^j(x,y)\gamma_\nu S^j(y,z)\gamma_\sigma S^j(z,0)\gamma_\lambda \right] \right\rangle_U.$$

← Calculate on the lattice

- $S^j(x,y)$: Propagators from inverting the Dirac equation on the lattice (not an easy job).
- Then, contract with the kernel.

2+2 disconnected contribution



- 2+2 Wick contractions written in terms of

$$\hat{\Pi}_{\mu\nu}(x,y) = \Pi_{\mu\nu}(x,y) - \langle \Pi_{\mu\nu}(x,y) \rangle_U \,,$$
$$\Pi_{\mu\nu}(x,y) \equiv -\,\mathrm{Re}\,\mathrm{Tr}\{S(y,x)\gamma_\mu S(x,y)\gamma_\nu\}.$$

- Important: One has to subtract the VEV of $\hat{\Pi}_{\mu\nu}$ to ensure that the two "disconnected" quark loops are still connected by gluons.

2+2 disconnected contribution

$$f^{(2+2)}(|y|) = -\sum_{i,j\in u,d,s,c} Q_i^2 Q_j^2 \hat{Z}_V^4 \frac{m_\mu e^6}{3} 2\pi^2 |y|^3 \times$$
$$\left\langle \int_x \left((\bar{\mathcal{L}}^{(\Lambda)}_{[\rho,\sigma];\mu\nu\lambda}(x,y) + \bar{\mathcal{L}}^{(\Lambda)}_{[\rho,\sigma];\nu\mu\lambda}(y,x)) \hat{\Pi}^i_{\mu\lambda}(x,0) \int_z z_\rho \hat{\Pi}^j_{\sigma\nu}(z,y) \right.\right.$$
$$\left.\left. + \bar{\mathcal{L}}^{(\Lambda)}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \hat{\Pi}^i_{\mu\nu}(x,y) \int_z z_\rho \hat{\Pi}^j_{\sigma\lambda}(z,0) \right) \right\rangle_U .$$

$$\hat{\Pi}_{\mu\nu}(x,y) = \Pi_{\mu\nu}(x,y) - \langle \Pi_{\mu\nu}(x,y) \rangle_U ,$$
$$\Pi_{\mu\nu}(x,y) \equiv -\,\mathrm{Re}\,\mathrm{Tr}\{S(y,x)\gamma_\mu S(x,y)\gamma_\nu\}.$$

Calculate on the lattice

- Many sub-contributions: light-light is the dominant one.
- Computationally (a lot) more challenging and noisy than the connected.

2+2 Disconnected Framework

- We define the objects $\hat{P}$ in analogy to the objects $\hat{\Pi}$ as following

$$\hat{P}^i_{\rho\sigma\nu} \equiv P^i_{\rho\sigma\nu} - \langle P^i_{\rho\sigma\nu} \rangle_U \,.$$

- Therefore, we can compute the final ensemble average as

$$\langle \ldots \rangle = \Big\langle \hat{P}^2_{\rho\sigma\nu}(w, w+y) \int d\zeta \, (\zeta + y_\rho) \, \hat{P}^1_{\rho\sigma\nu}(w+y, \zeta) + \hat{P}^3_{\rho\sigma\nu}(w+y, w) \int d\zeta \, \zeta \, \hat{P}^1_{\rho\sigma\nu}(w, \zeta) \Big\rangle_U .$$

- The quantity above can be averaged over all (w, w') sources, for each y, to increase statistics.

- $< \ldots >$ must be computed after finishing the inversions and contractions across the whole ensemble.