Muon g-2:
I=1 FV effects on LO HVP

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with BMW + DMZ

hep-lat/2407.10913

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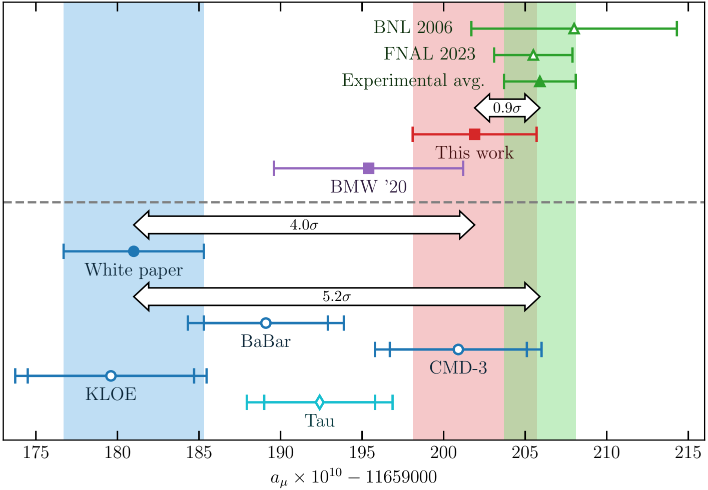
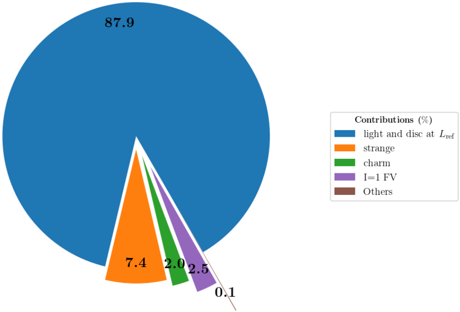
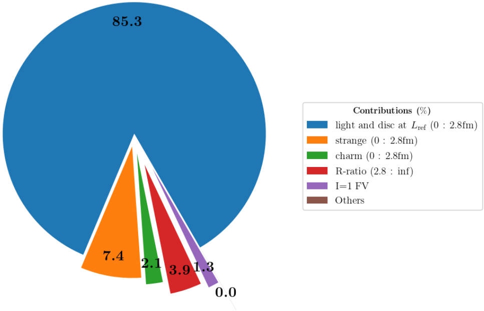
BMW - DMZ 2024


BNL 2006
FNAL 2023
Experimental avg.
0.9σ
This work
BMW '20
4.0σ
White paper
5.2σ
BaBar
CMD-3
KLOE
Tau
175
180
185
190
195
200
205
210
215
$a_\mu \times 10^{10} - 11659000$

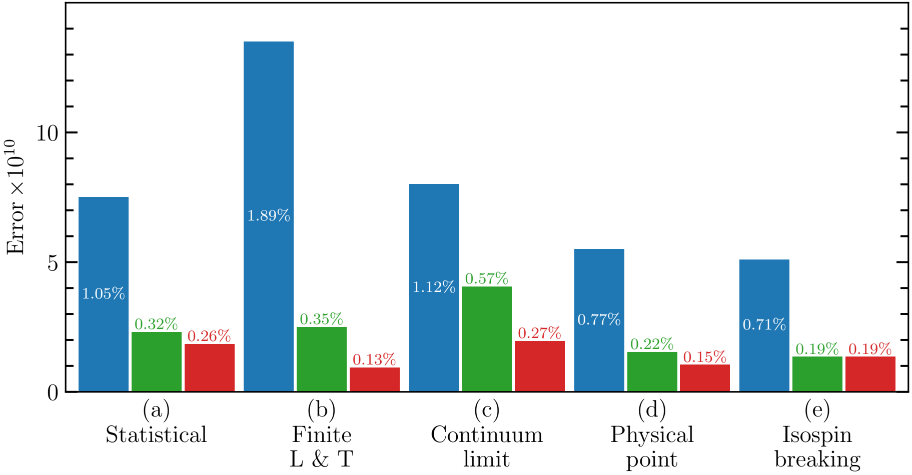
Contributions to $a_\mu^{\mathrm{LO-HVP}}$ from BMW20



Contributions to $a_\mu^{\mathrm{LO-HVP}}$ from BMW24



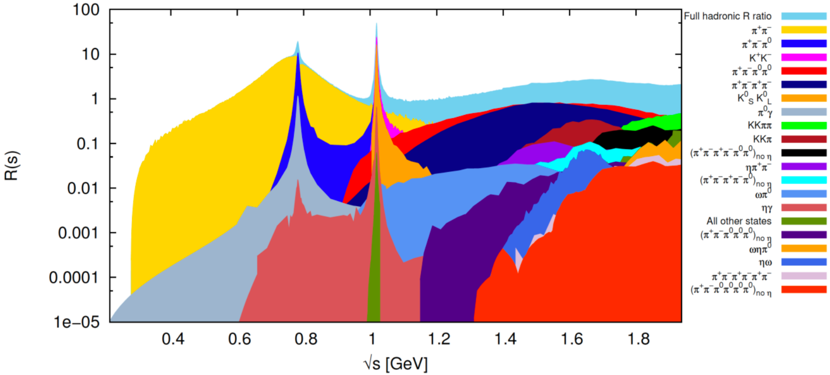
Errors



Error ×10^10
0
5
10
1.05%
0.32%
0.26%
1.89%
0.35%
0.13%
1.12%
0.57%
0.27%
0.77%
0.22%
0.15%
0.71%
0.19%
0.19%
(a) Statistical
(b) Finite L & T
(c) Continuum limit
(d) Physical point
(e) Isospin breaking

Pheno estimate of FV effect

- Models based on $\pi\pi$ states should reproduce FV effects very well



Plot from KNT [hep-ph/1802.02995]

Pheno estimate of FV effect

- We split the integration over Euclidean time in two:

$$\Delta_L a_{\mu,I=1}^{\mathrm{LO-HVP}} = \left(\int_0^{t^*} dt + \int_{t^*}^{\infty} dt\right) K(t)\,\Delta_L C(t)$$

- We adopt two methods for estimating $\Delta_L C(t)$: Hansen-Patella (HP) and Meyer-Lellouch-Lüsher (MLL)

- Series for $\Delta_L C(t)$ with different convergence properties at different t

- FV correlator from a spectral sum

$$C(t, L) = \sum_{n=0}^{n_{\max}} |A_n(L)|^2 e^{-|t| E_n(L)} + \mathrm{res}(n_{\max}, L)$$

- $E_n(L)$ from Lüsher [Nucl.Phys.B 354]

$$\phi\left(\frac{k_n L}{2\pi}\right) + \delta_1^1(k_n) = n \in \{1, 2 \ldots n_{\max}\} .$$

- $|A_n(L)|$ from Meyer [PRL 107.072002] Lellouch-Lüsher [Commun.Math.Phys. 219]

$$|A_n|^2 = \frac{2k_n^5}{\pi E_n^2} \frac{|F_\pi(k_n)|^2}{\mathbb{L}(k_n)} , \qquad \mathbb{L}(k_n) = \frac{kL}{2\pi} \frac{\partial \phi(\frac{kL}{2\pi})}{\partial(\frac{kL}{2\pi})}\bigg|_{k=k_n} + k \frac{\partial \delta_1^1(k)}{\partial k}\bigg|_{k=k_n} .$$

- We use $L_{\mathrm{ref}} = 6.272$ fm $\implies n_{\max} = 8$

Hansen Patella

- [10.1007/JHEP10(2020)029], [PRL 123.172001]
- Up to terms of order $\exp\left(-m_\pi L\sqrt{2+\sqrt{3}}\right)$:

$$\Delta_L C(x_0) = -\sum_{\vec{n}\neq 0}\int_{-\infty}^{\infty}\frac{dp_3}{2\pi}\frac{e^{-|\vec{n}|L\sqrt{m_\pi^2+p_3^2}}}{24|\vec{n}|\pi L}\int_{-\infty}^{\infty}\frac{dk_3}{2\pi}\cos(k_3 x_0)\Re\, T(-k_3^2, -p_3 k_3)$$

- Faster convergence at small time slices
- Dominated by the exchange of single pion: a larger pole part is expressed in terms of $F_\pi(-k^3)$, a smaller regular part from NLO ChPT.

Inputs

- Inputs required: $F_\pi(s)$ and $\delta_{11}(s)$
- Common choice: Gounaris Sakurai (timelike) Monopole model (spacelike)
- We adopt the parametrisation from DHMZ [hep-ph/1908.00921] in both space and timelike regions:

$$\cot\delta(s) = \frac{\sqrt{s}}{2k^3(s)}(m_\rho^2 - s)\left(\frac{2m_\pi^3}{m_\rho^2 s} + B_0 + B_1 w(s)\right)$$

$$F_\pi(s) = (1 + \alpha_V s)\Omega(s)$$

- Ω is the Omnès function

$$\Omega(s) = \exp\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds'\,\frac{\delta(s')}{s'(s'-s)}$$

- New fits to KLOE, BaBar, CMD3

Systematics I: Truncation

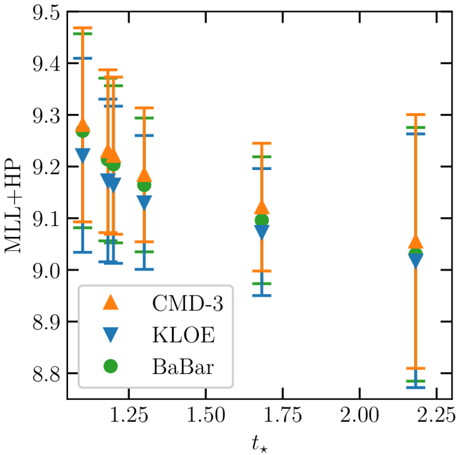
- For HP, we have three terms and we take the last one, $|\vec{n}|^2 = 3$, as an estimate of the systematic error
- MLL is truncated at $n = 8$ included: we take the term $n = 8$ as an estimate of the remainder.
- The sign of the remainder is known: we add it with 100% error

$$C(t, L) = \sum_{n=0}^{n_{\max}=8} |A_n(L)|^2 e^{-|t|E_n(L)} + |A_8(L)|^2 e^{-|t|E_8(L)} \pm |A_8(L)|^2 e^{-|t|E_8(L)}$$

- The time at which we switch from HP to MLL should not be relevant as long as both expansions are solid. We use $t_\star = (M_\pi L/4)^2/M_\pi$ suggested in [hep-lat/1306.2532] (1.682 fm)

Systematics I: Truncation

- Truncation errors from MLL and HP are assumed to be fully correlated



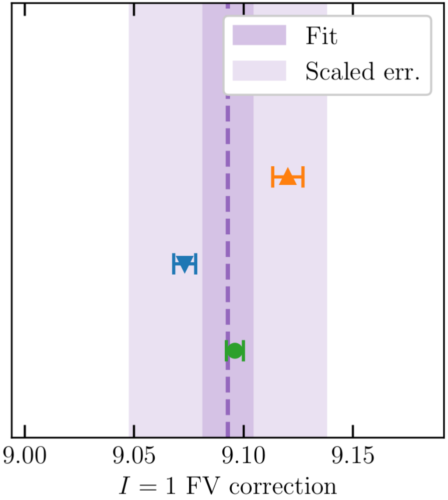
Systematics II: Regular part of the Compton amplitude

- The regular part of the Compton amplitude used in HP is much smaller than the 1-pion exchange
- It is however non-negligible for window quantities peaked at shorter distances
- Estimated in NLO CHPT: we give it 100% uncertainty

$$T_{\text{reg}}(k^2, kp) = \frac{7m^2 - 4k^2}{6\pi^2 f_\pi^2}\left[\sigma \coth^{-1}\sigma\right]\Big|_{\sigma=\sqrt{1-4m^2/k^2}} + P(k^2)$$

Systematics III: Parameter error

- Propagate errors from fit parameters accounting for correlations
- Generate parameters from multivariate gaussian distribution
- Without the inclusion of systematic errors, different experimental datasets lead to incompatible results



Systematics IV: Inelasticities

- Inelastic effects are part of the FV effect but are not straightforward to incorporate into our formalism where we use the elastic phase shift

- They are non-negligible for energies $\gtrsim M_\omega + M_\pi$: should not propagate significantly into FV effects in a_μ

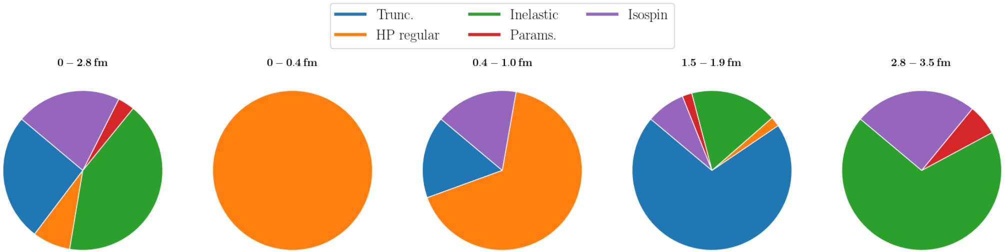
- From ChPT we expect these effects to be order s^3.

$$F_\pi(s) = \left[1 + \alpha_V s + O(s^3)\right] \, \Omega(s)$$

- We compute the FV effects with and without the known $O(s)$ term, to obtain an upper bound to $O(s^3)$ effects.

Systematics: final result

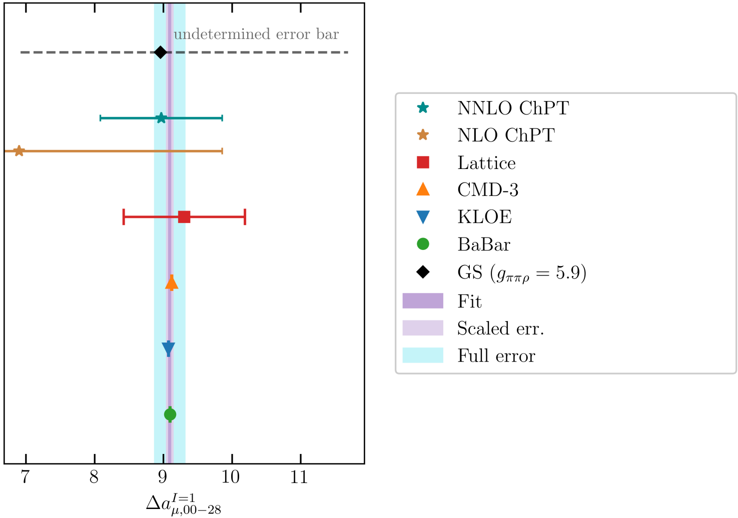
window	trunc. (MLL+HP)	HP(regular)	inel.	exp.	iso.	quad. sum
00-28	0.11	0.06	0.14	0.04	0.10	0.22
00-04	0.00	0.01	0.00	0.00	0.00	0.01
04-10	0.01	0.02	0.00	0.00	0.01	0.03
15-19	0.06	0.01	0.03	0.01	0.02	0.07
28-35	0.00	0.00	0.10	0.03	0.06	0.12



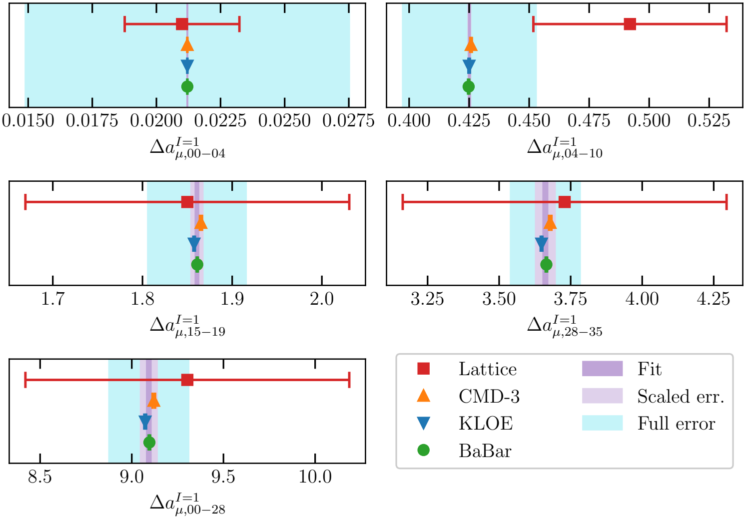
Results for the pheno estimate

window	$\Delta a_\mu^{I=1}(L_{\text{ref}} \to \infty)$
00-28	9.09(0.22)
00-04	0.02(0.01)
04-10	0.43(0.03)
15-19	1.86(0.07)
28-35	3.66(0.12)

Comparison with GS, 4HEX, ChPT



Comparison with 4HEX & ChPT



Thank you

Appendix

ChPT, I=1

window	$\Delta a_\mu^{I=1}(L_{\mathrm{ref}} \to \infty)$ NLO	NNLO	$\Delta a_\mu^{I=1}(L_{\mathrm{big}} \to \infty)$ NLO	NNLO
00-28	6.90	8.97	0.39	0.43
00-04	0.01	0.02	0.00	0.00
04-10	0.33	0.40	0.01	0.01
15-19	1.48	1.87	0.07	0.07
28-35	2.46	3.45	0.25	0.29

window	$\Delta a_\mu^{\mathrm{I}=1}(L_{\mathrm{ref}} \to L_{\mathrm{big}})$ from `4hex`	$\Delta a_\mu^{\mathrm{I}=1}(L_{\mathrm{big}} \to \infty)$ from NNLO	$\Delta a_\mu^{\mathrm{I}=1}(L_{\mathrm{ref}} \to \infty)$ from `4hex`+NNLO	$\Delta a_\mu^{\mathrm{I}=0}(L_{\mathrm{ref}} \to \infty)$ from N4LO est.
00-28	8.94(57)(67)	+0.43-0.06	9.31(88)	0.00(19)
00-04	0.02(00)(00)	+0.00-0.00	0.02(00)	0.00(01)
04-10	0.48(02)(04)	+0.01-0.00	0.49(04)	0.00(00)
15-19	1.78(12)(13)	+0.07-0.00	1.85(18)	0.00(03)
28-35	3.55(50)(27)	+0.29-0.11	3.73(57)	0.00(16)