Quark Mass dependence of the $\Delta(1232)$ resonance parameters.

Srijit Paul

University of Maryland, College Park

In Collaboration:

Ferenc Pittler, Stefan Krieg, Luka Leskovec, Stefan Meinel, John Negele, Marcus Petschlies, Andrew Pochinsky, Sergey Syritsyn

LATTICE 2024, Liverpool

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- *Lowest lying* baryon resonance $\Delta(1232)$
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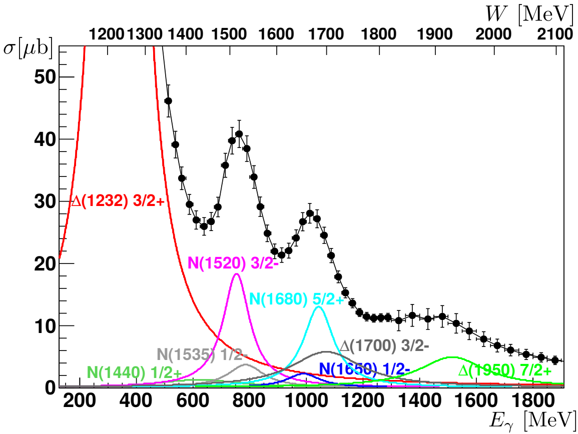

[CBELSA/TAPS(ELSA), 1604.02922]

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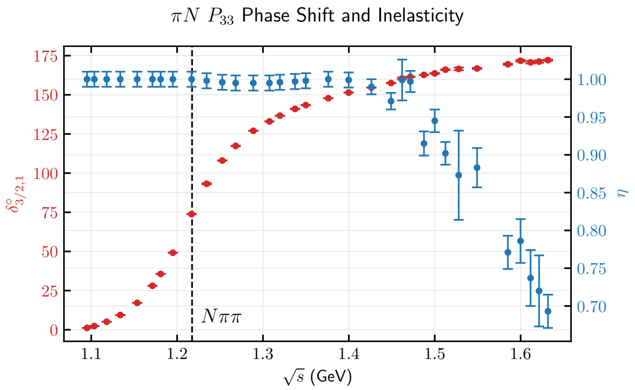

[gwdac.phys.gwu.edu]

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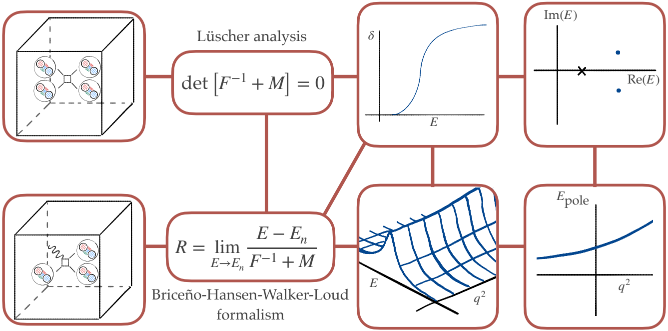

[Briceno.et.al., 1406.5965]

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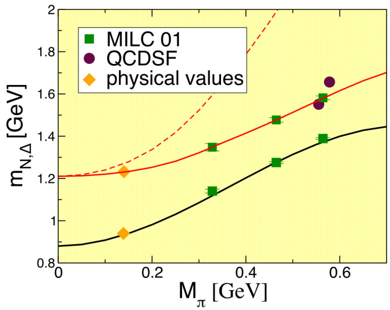

[*LATTICE 2005, Meiβner, hep-lat/0509029*]

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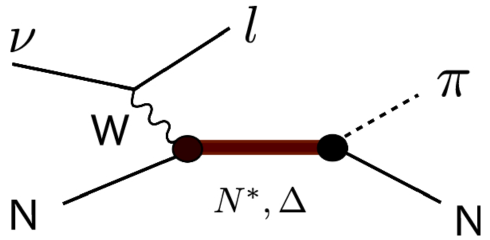

[*Snowmass 2021, 2203.09030*]

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[*Hoferichter.et.al(2024)*]

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  [*Hoferichter.et.al(2024)*]
- Lattice community $m \to m_{\text{phys}}$, significant contribution of *multihadron states* in computation of *hadronic observables*.

Lattice Setup

- Budapest-Marseille-Wuppertal simulations $N_f = 2+1$
  tree-level *clover-improved Wilson fermions* coupled to *doubly HEX smeared links.*,

[*Durr.et.al.,1011.2711*]

Label	$N_s^3 \times N_t$	L (fm)	a (fm)	m_π (MeV)	$m_\pi L$
A7	$24^3 \times 48$	2.8	≈ 0.116	≈ 247	3.6
A8	$32^3 \times 48$	3.7	≈ 0.116	≈ 249	4.7
A11	$24^3 \times 48$	2.8	≈ 0.116	≈ 199	3.6
A12	$32^3 \times 48$	3.7	≈ 0.116	≈ 199	4.7
A15	$48^3 \times 48$	5.6	≈ 0.116	≈ 137	*4.0*

Table: Parameters of the lattice gauge-field ensembles

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pion interpolator:

$$\pi(\vec{p}) = \sum_{\vec{x}} \bar{d}(\vec{x})\gamma_5 u(\vec{x}) e^{i\vec{p}\cdot\vec{x}}$$

Nucleon interpolators

$$N_\mu^{(1)}(\vec{p}) = \sum_{\vec{x}} \epsilon_{abc} \left(u_a(\vec{x})\right)_\mu \left(u_b^T(\vec{x}) C\gamma_5 d_c(\vec{x})\right) e^{i\vec{p}\cdot\vec{x}}$$

$$N_\mu^{(2)}(\vec{p}) = \sum_{\vec{x}} \epsilon_{abc} \left(u_a(\vec{x})\right)_\mu \left(u_b^T(\vec{x}) C\gamma_0\gamma_5 d_c(\vec{x})\right) e^{i\vec{p}\cdot\vec{x}}$$

Δ interpolators

$$\Delta_{\mu i}^{(1)}(\vec{p}) = \sum_{\vec{x}} \epsilon_{abc} \left(u_a(\vec{x})\right)_\mu \left(u_b^T(\vec{x}) C\gamma_i u_c(\vec{x})\right) e^{i\vec{p}\cdot\vec{x}}$$

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2-hadron interpolators: $N_\mu^{(1,2)}(\vec{p}_N)\pi(\vec{p}_\pi)$

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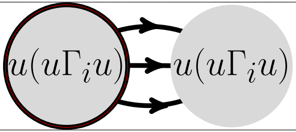
	Δ	$N\pi$
Δ		
$N\pi$		

[Silvi.et.al., 2101.00689]

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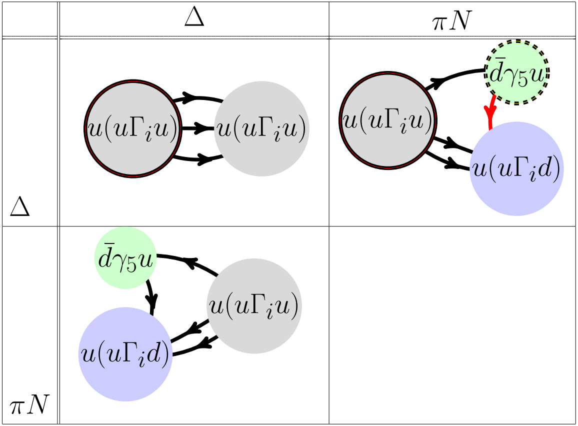

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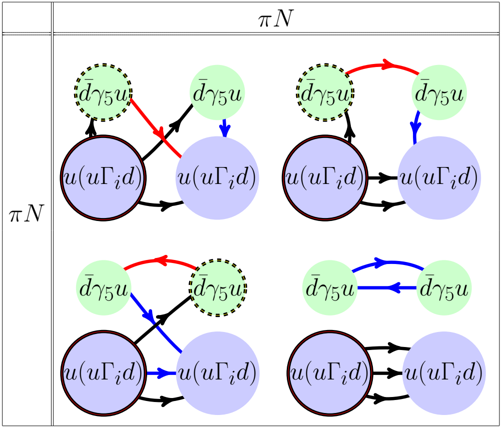

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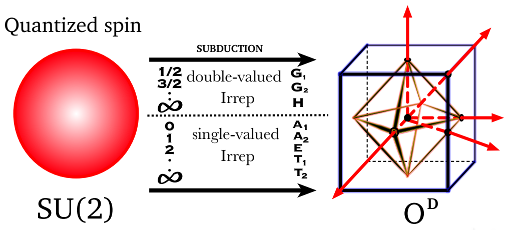

$\frac{L}{2\pi}\vec{P}_{\text{ref}}$ $[N_{\text{dir}}]$	$LG(\vec{P})$	$\Lambda(J) : \pi(0^-)$	$\Lambda(J) : N(\frac{1}{2}+)$	$\Lambda(J) : \Delta(\frac{3}{2}+)$
$(0,0,0)[1]$	O_h^D	$A_{1u}(0,4,\ldots)$	$G_{1g}\left(\frac{1}{2},\frac{7}{2},\ldots\right) \oplus G_{1u}\left(\frac{1}{2},\frac{7}{2},\ldots\right)$	$H_g\left(\frac{3}{2},\frac{5}{2},\ldots\right) \oplus H_u\left(\frac{3}{2},\frac{5}{2},\ldots\right)$
$(0,0,1)[6]$	C_{4v}^D	$A_2(0,1,\ldots)$	$G_1\left(\frac{1}{2},\frac{3}{2},\ldots\right)$	$G_1\left(\frac{1}{2},\frac{3}{2},\ldots\right) \oplus G_2\left(\frac{3}{2},\frac{5}{2},\ldots\right)$
$(0,1,1)[12]$	C_{2v}^D	$A_2(0,1,\ldots)$	$G\left(\frac{1}{2},\frac{3}{2},\ldots\right)$	$G\left(\frac{1}{2},\frac{3}{2},\ldots\right)$
$(1,1,1)[8]$	C_{3v}^D	$A_2(0,1,\ldots)$	$G\left(\frac{1}{2},\frac{3}{2},\ldots\right)$	$G\left(\frac{1}{2},\frac{3}{2},\ldots\right) \oplus F_1\left(\frac{3}{2},\frac{5}{2},\ldots\right) \oplus F_2\left(\frac{3}{2},\frac{5}{2},\ldots\right)$

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$\Delta(\frac{3}{2}+)$ Partial wave mixing with $J = \frac{1}{2}$ in $G_1, 2G$ Irreps.

[Göckler.et.al., *1206.4141*]

$$O^{\wedge,r,n}(\vec{P}) = \frac{\text{dim}_\Lambda}{\#_{LG(\vec{P})}} \sum_{R\in LG(\vec{P})} \Gamma^{\wedge}_{r,r}(R) U_R O(\vec{P}) U_R^\dagger, \quad r \in \{1, \dots, \text{dim}_\Lambda\}$$

[Morningstar.et.al.,*1303.6816*]

Total 1720 Operators.

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Variational analysis :

$$C(t)\, v(t,t_0) = \lambda(t,t_0) C(t_0)\, v(t,t_0)$$

Fit $[\lambda(t,t_0)]$ to single/multi exponential $[t_{min}, t_{max}]$, and
ratio fit $R(t,t_0)$ to single exponential for ΔE(Energy shift)

$$R(t,t_0) = \frac{\lambda(t,t_0)}{(C_\pi^{\vec{p}_1}(t) C_N^{\vec{p}_2}(t))}$$

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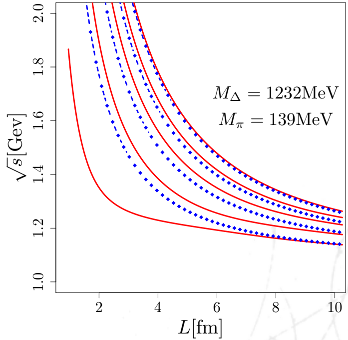
H_g Irrep Spectrum Volume dependence



Blue $N\pi$ states, Red Interacting levels.

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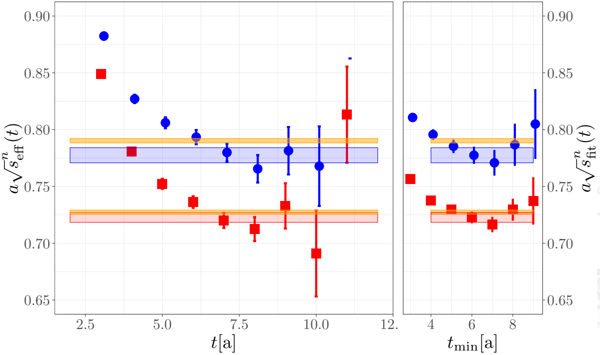
Effective masses in H_g irrep.



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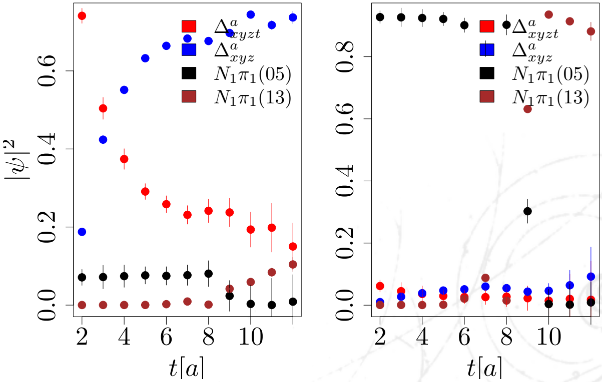
Eigenvector Overlaps in H_g irrep.



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For Baryons with spin J and partial wave l, $-J \leq \mu, \mu' \leq J$

$$\det(M_{Jl\mu,J'l'\mu'} - \delta_{JJ'}\delta_{ll'}\delta_{\mu\mu'} \cot \delta_{Jl}) = 0$$

δ_{Jl} is the scattering phase shift.

[Göckler.et.al., *1206.4141*]

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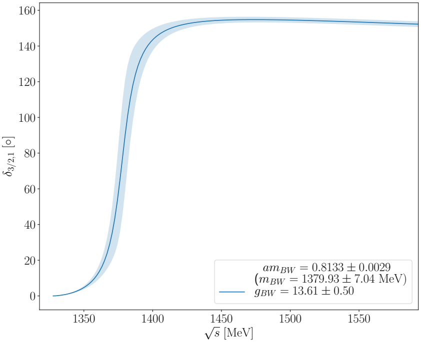

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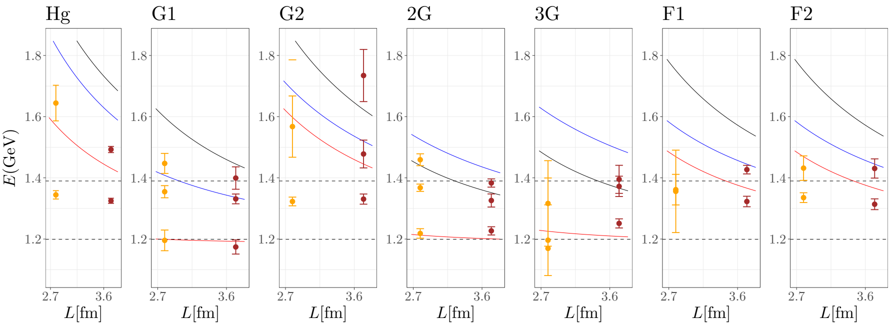
Hadron Spectrum

$N\pi$ spectrum $m_\pi \approx 199$ MeV.



Red, Blue, Black continuous lines: Non-Interacting $N\pi$ states.
Dashed lines are $N\pi$ and $N\pi\pi$ thresholds.

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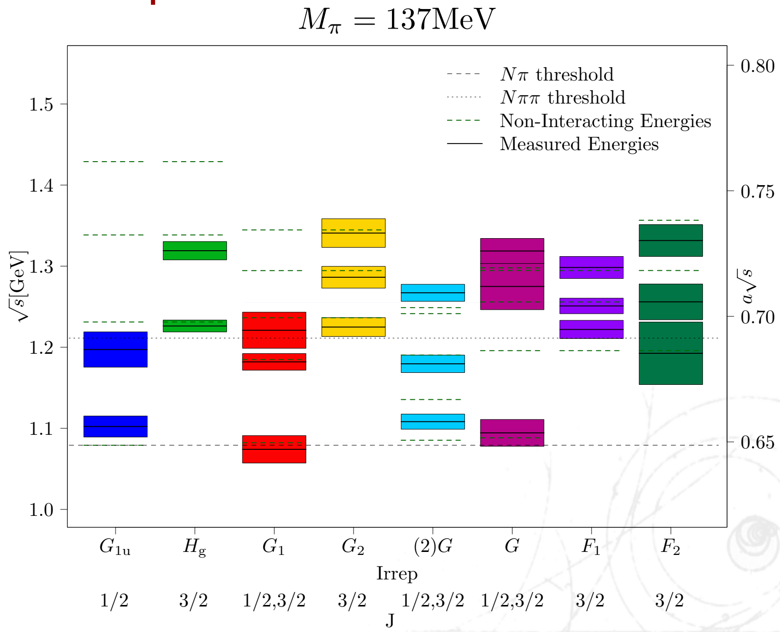
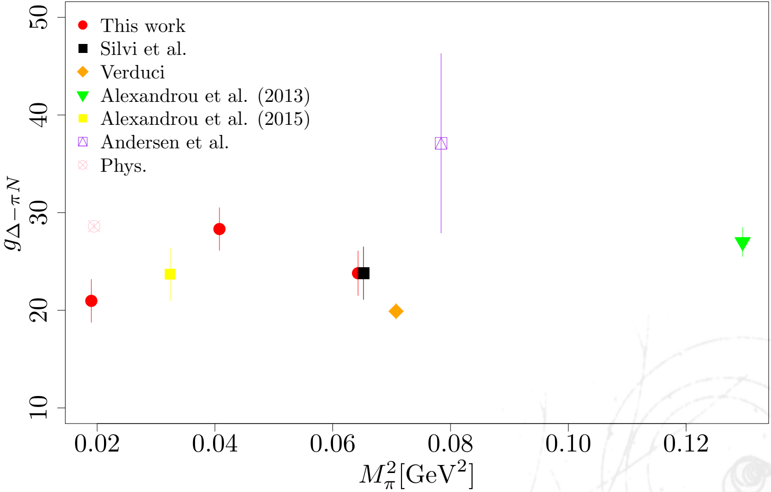
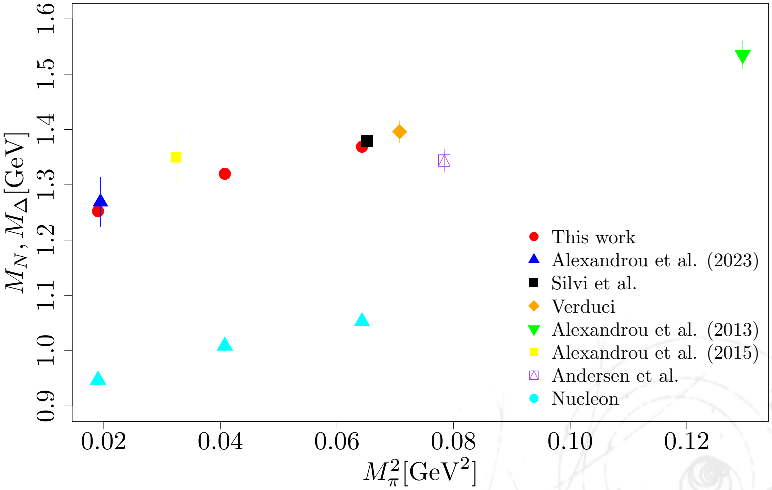


Pion Mass Dependence of $g_{\Delta-N\pi}$



Pion Mass Dependence of m_Δ



Summary

- Resonance parameters for $N\pi$ scattering in $I = \frac{3}{2}$ channel for $m_\pi = 250, 199, 137$ MeV on 5 ensembles.
- Both of single and multi-hadron interpolators become crucial in the extraction of the spectrum.
- Quark mass dependence studied: Consistent with m_Δ linear behaviour with m_π^2 near physical pion mass.
- Remarkable agreement with the experimental value of m_Δ.

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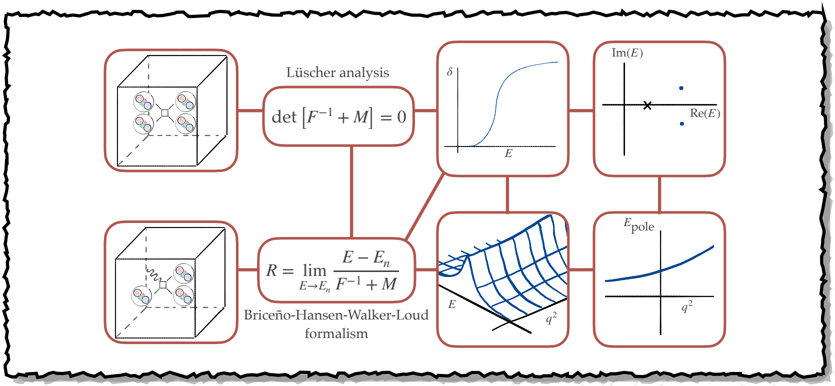

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