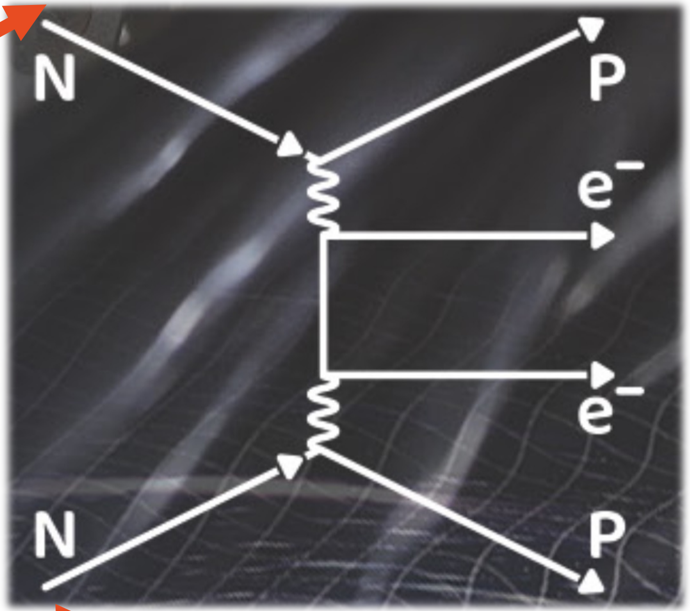
Lattice Calculation of Proton-Proton Fusion Matrix Element

Zi –Yu Wang (王子毓)

In collaboration: Xu Feng, Lu-chang Jin, Chuan Liu

Neutrinoless Double Beta Decay

2023 Long Range Plan
For Nuclear Science



If $0\nu2\beta$ is observed:

→ Direct evidence for the Majorana nature of neutrinos

→ LNV process

Challenges for LQCD

➢ Significant progress in the pion sector

- @physical pion mass
- Include both LD and SD matrix elements

A. Nicholson et al., PRL121 (2018) 172501

X. Feng, L. Jin, X. Tuo, S. Xia, PRL122 (2019) 022001

X. Tuo, X. Feng, L. Jin, PRD100 (2019) 094511

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- But only NPLQCD collaboration has report their pioneering calculations in nucleon sector @m_π=806 MeV

M. Savage et al., PRL 119 (2017) 6, 062002
P. E. Shanahan et al., PRL119 (2017)6, 062003
Z. Davoudi et al., arXiv: 2402.09362

Anthony Grebe's plenary talk on 8.3

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  - Pseudo plateau
  - Complicated contractions
  - FV corrections
  - Other systematic effects

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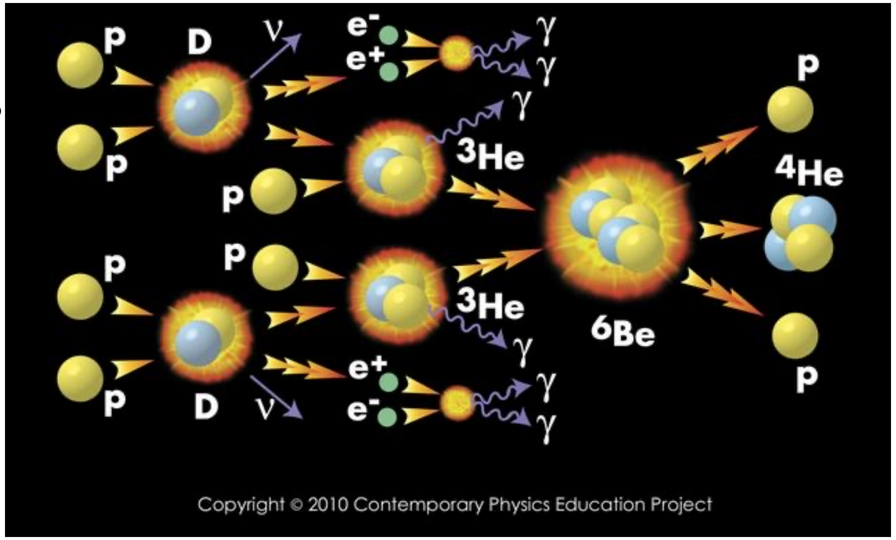
- What are the challenges?
  - **Signal-to-noise problem**
  - Pseudo plateau
  - Complicated contractions
  - FV corrections
  - Other systematic effects
- A simpler case of 1^{st} order weak transiton: Proton-Proton Fusion

Proton-Proton Fusion

➢ The Proton-Proton Fusion Process

- Initiating the **proton-proton fusion chain reaction** that provides the dominant energy production mechanism in stars.
- It is related to **neutrino-induced deuteron-breakup reaction**, which is relevant to the measurement of **neutrino oscillations**.
- A first step towards understanding g_A **quenching** in nuclei.



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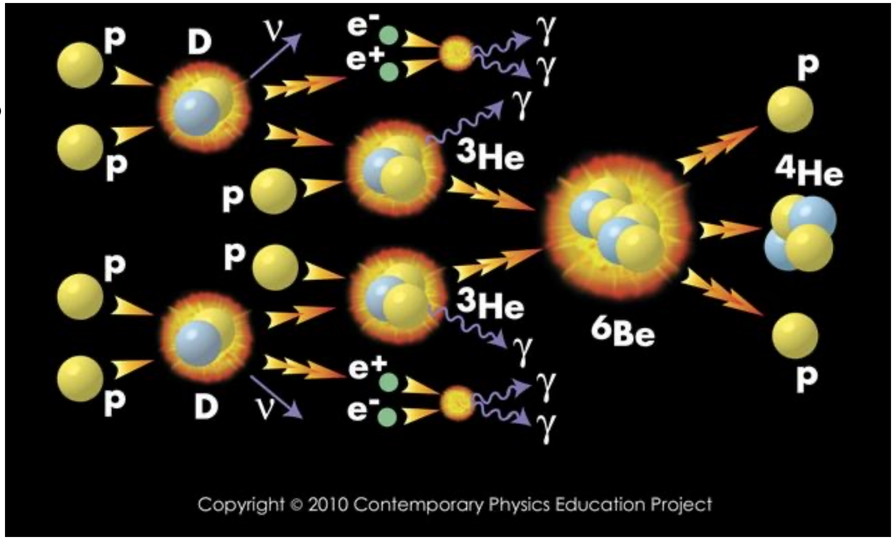

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➢ Lattice QCD Calculation of Proton-Proton Fusion

- P-P fusion process is hard to access in laboratory, but relatively easy to study by lattice calculation.
- All the techniques we developed here can be used for future double beta decay calculations.

Lattice Calculation

➢ Ensemble (RBC/UKQCD):

- 2+1 flavour **domain wall fermion + Iwasaki** gauge action,
- 162 cfgs, 128 meas./cfg.
- Field Sparsening: Computational cost reduced to 1/8
- Based on the Qlattice package by Luchang Jin.

Y. Li et al., PRD 103 (2021) 1, 014514
W. Detmold, PRD 104 (2021) 3, 034502

β	am_l	$L^3 \times T$	L(fm)	a^{-1}(GeV)	m_π(MeV)
2.13	0.01	$24^3 \times 64$	~ 2.65	1.7844(49)	432.2(1.4)

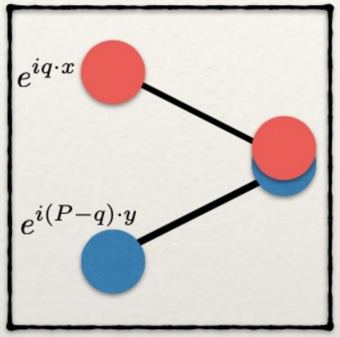
Two-nucleon Interpolators

To bind or not to bind: A question of various two-nucleon interpolators

Lattice 2023: FNAL
3rd August, 2023

André Walker-Loud, Lattice 2023

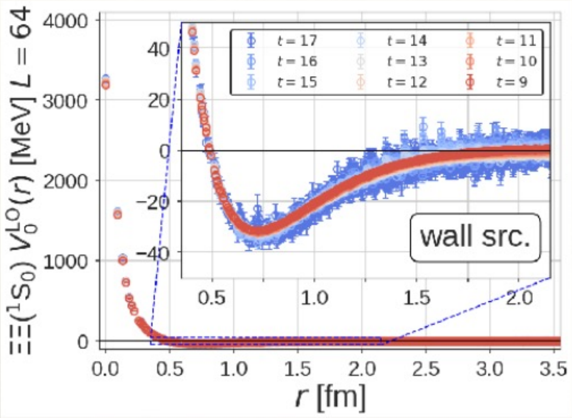
NPLQCD,
Yamazaki et al.,
CalLat (2015)



Compact, hexa-quark creation operator

Deep bound di-nucleons

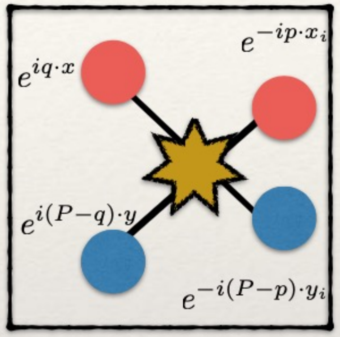
HAL QCD Potential



diffuse - wall source

no bound state

"Mainz" (Distillation)
CoSMoN (stochastic LapH
NPLQCD (sparsened momentum)



momentum-space
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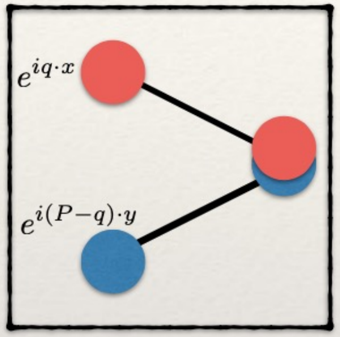

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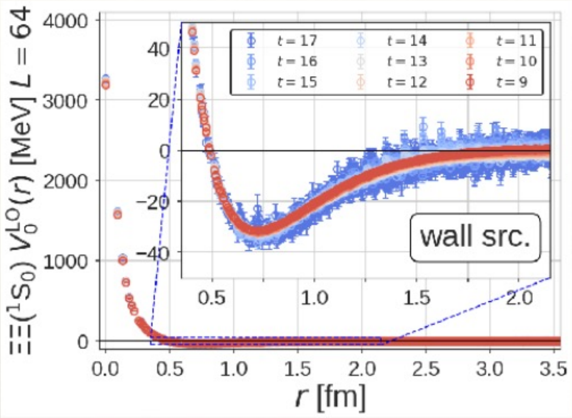

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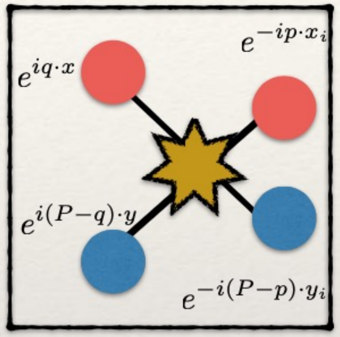

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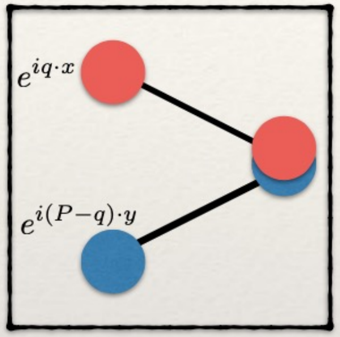

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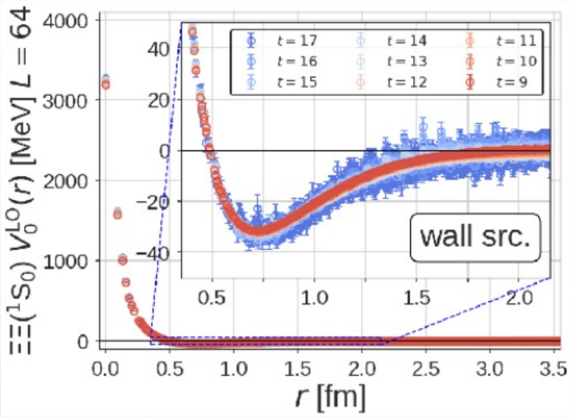

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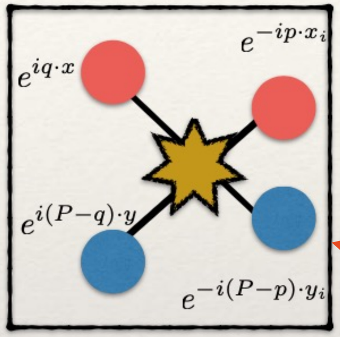

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"An open question until now, but the path forward seems clear"

This work: Utilizing in matrix element calculation

Interpolators Comparison

➢ Hexa-quark Interpolators

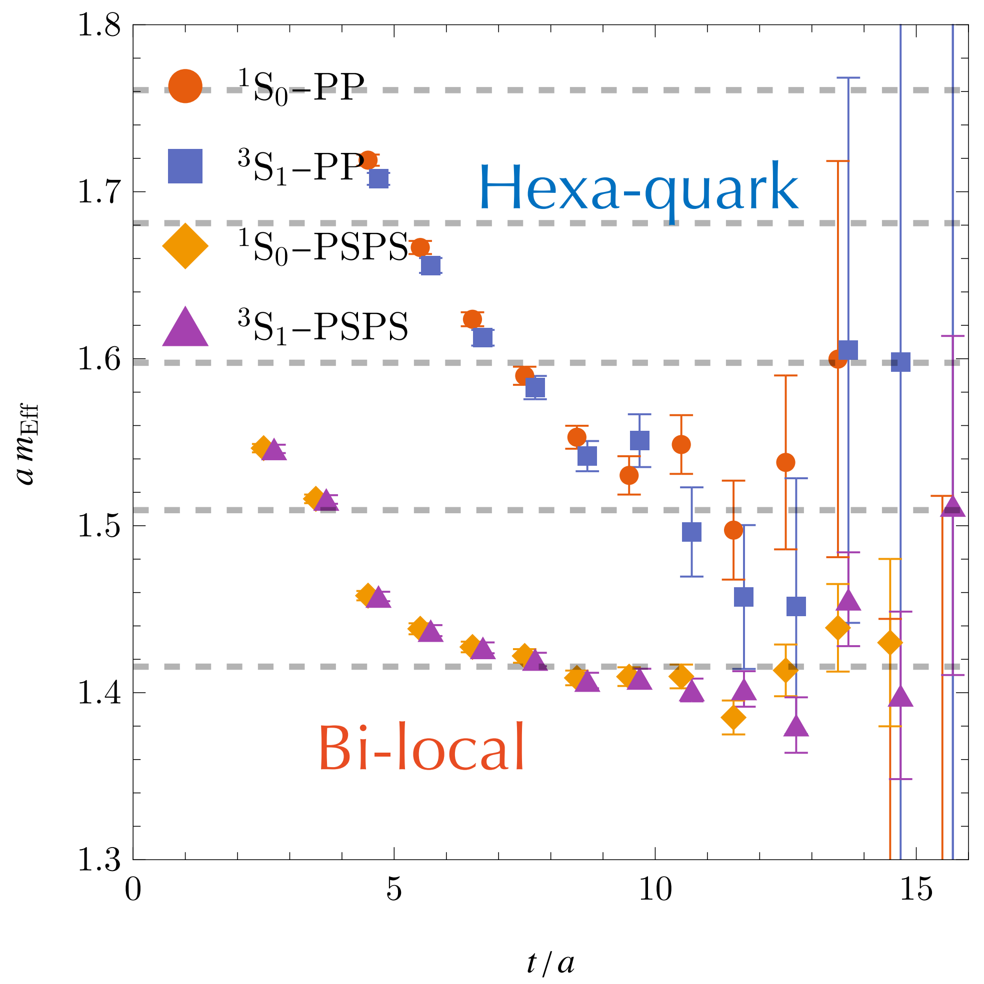
- All six quarks are located on one site
- Significant excited-state contamination:

$$\sum_x N(x)N(x) = \sum_p \tilde{N}(p)\tilde{N}(-p)$$

➢ Bi-local Interpolators

- Resemble the bi-local nature of NN system
- Suppressed excited-state contamination:

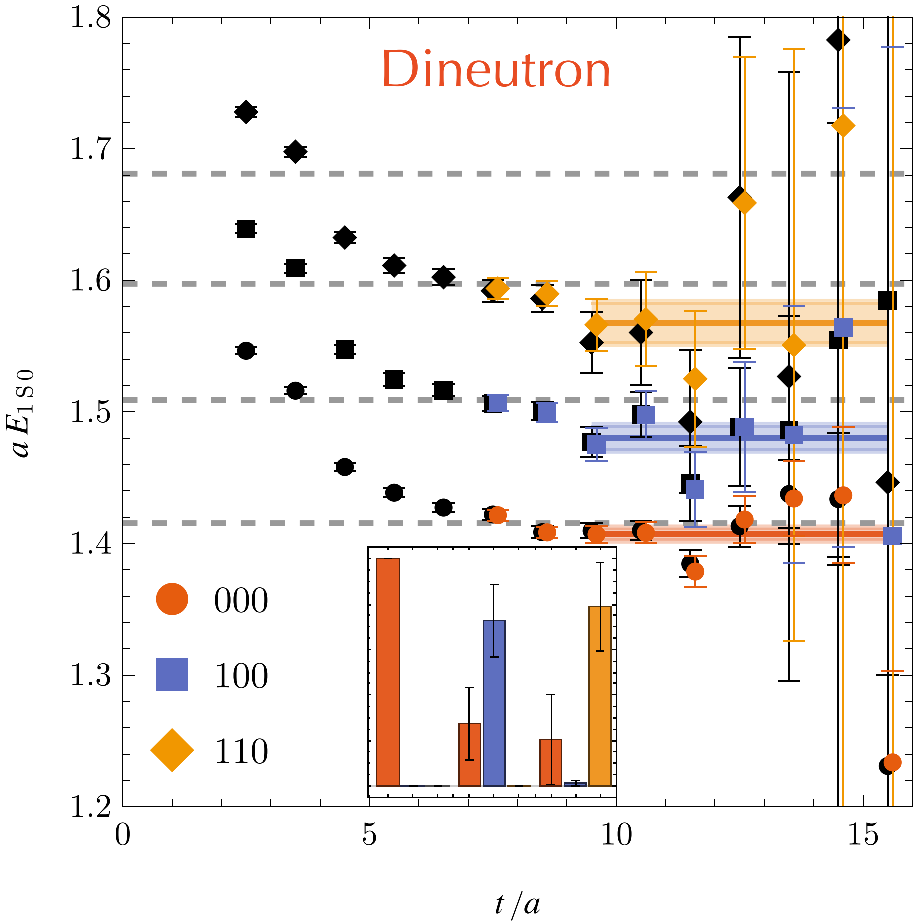
$$\sum_{x,y} N(x)N(y) = \tilde{N}(0)\tilde{N}(0)$$

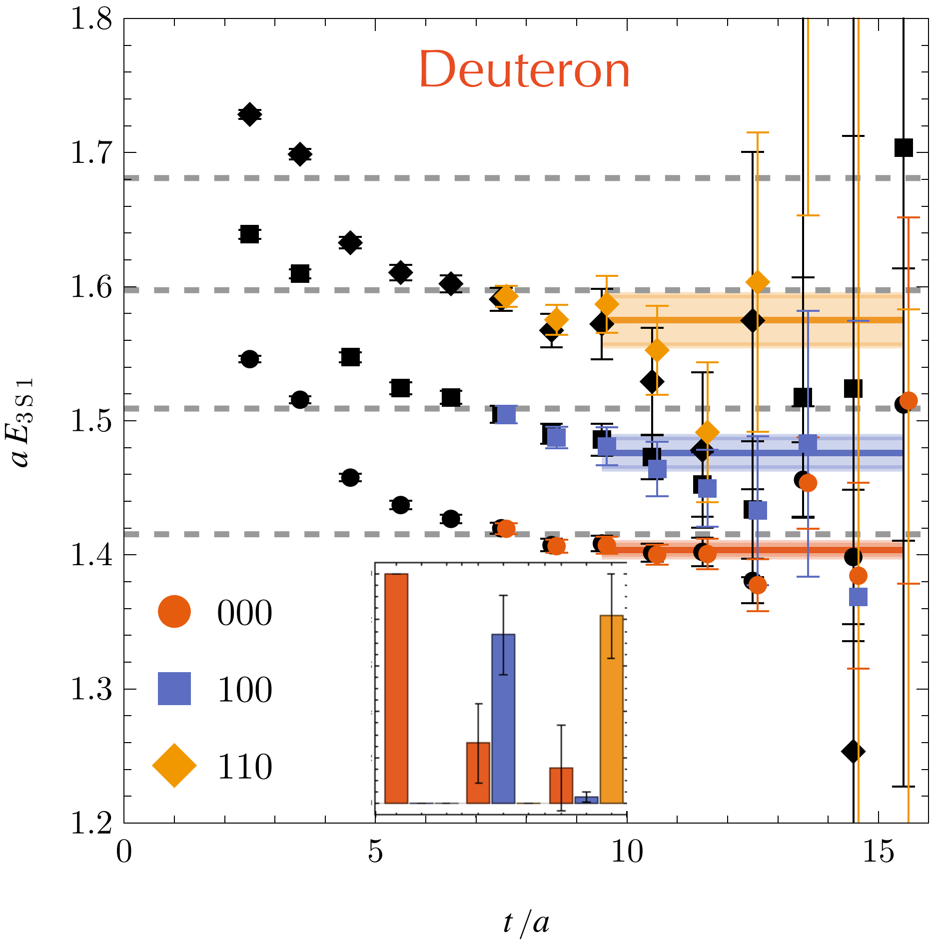


■ All the propagators are APE smeared.

Ground-State Determination I: Variational Analysis

- Momentum space op. set $\{O_q | q = 000, 100, 110\}$





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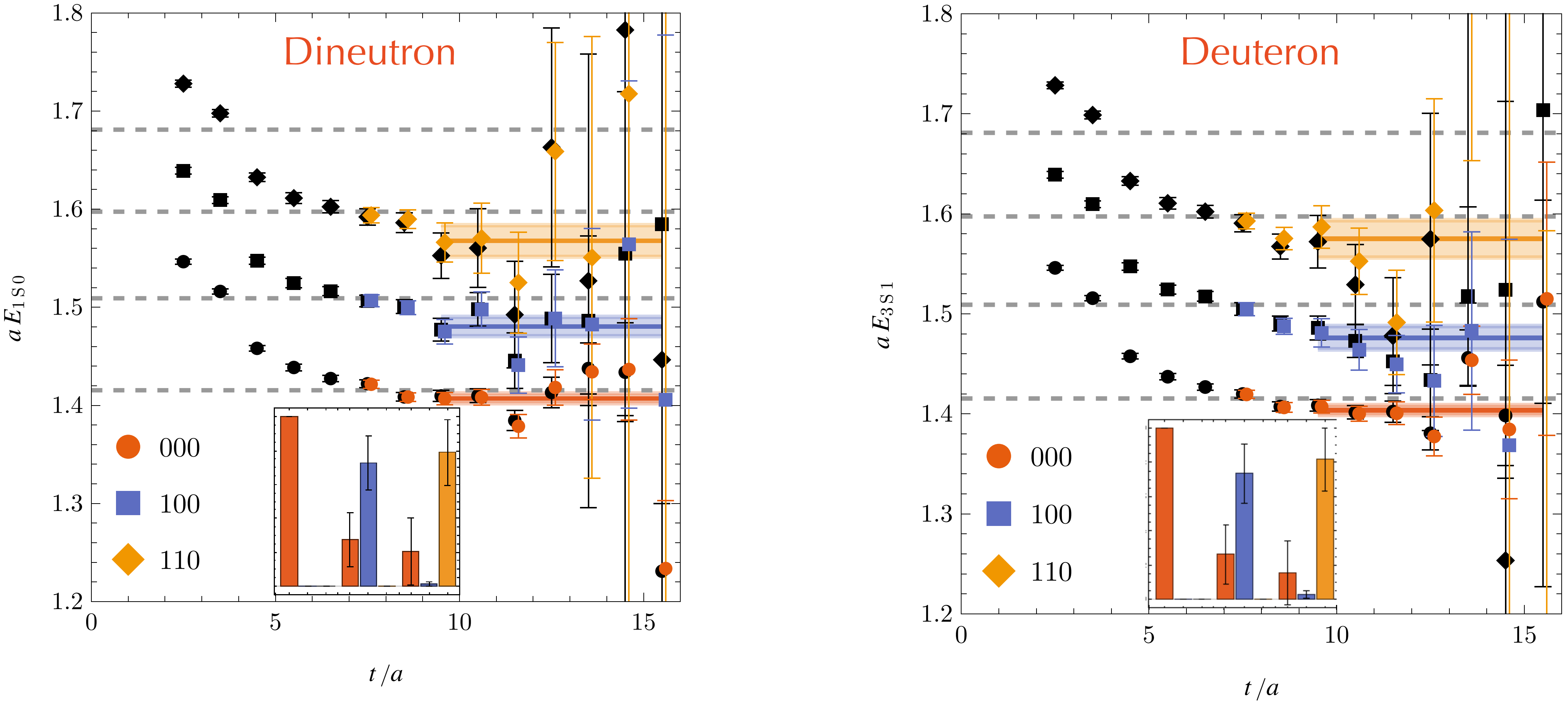
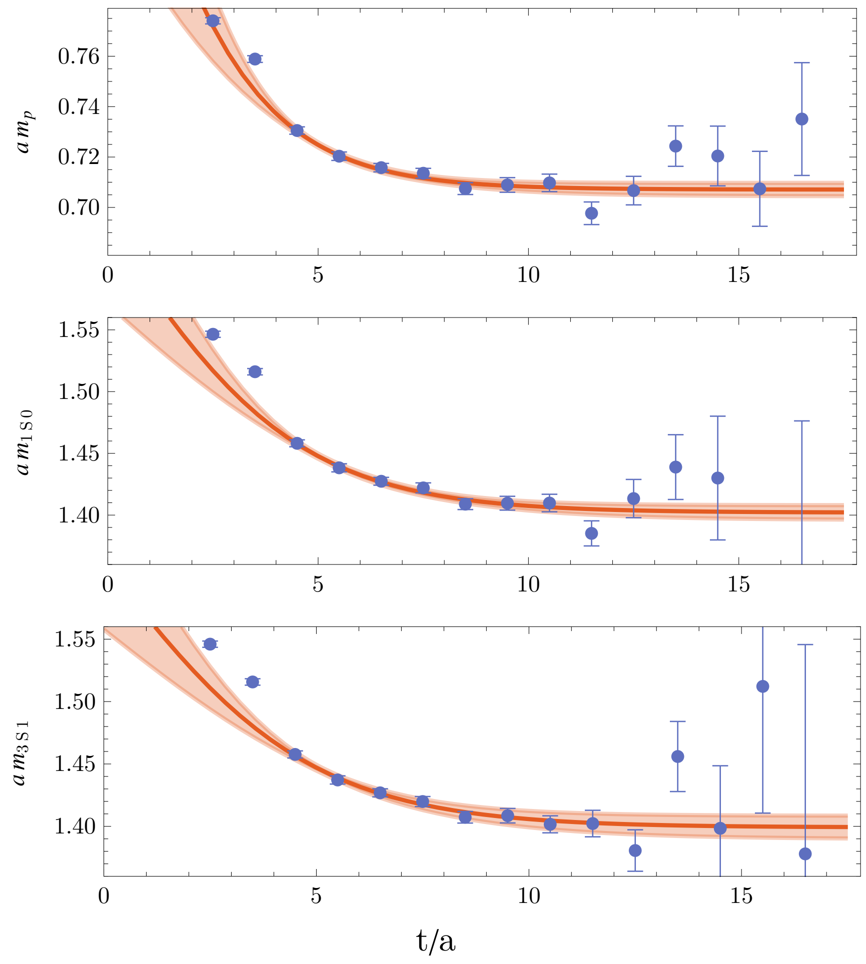


The overlap between GEVP ground state and non-zero momentum state is weak, but it is crucial to the energy difference.

Ground-State Determination II: Multi-state Fit

- Multi-state Fit ($t/a = 4 - 16$)

$$C(t) = A_0 e^{-E_0 t}(1 + r\, e^{-\delta\, t})$$



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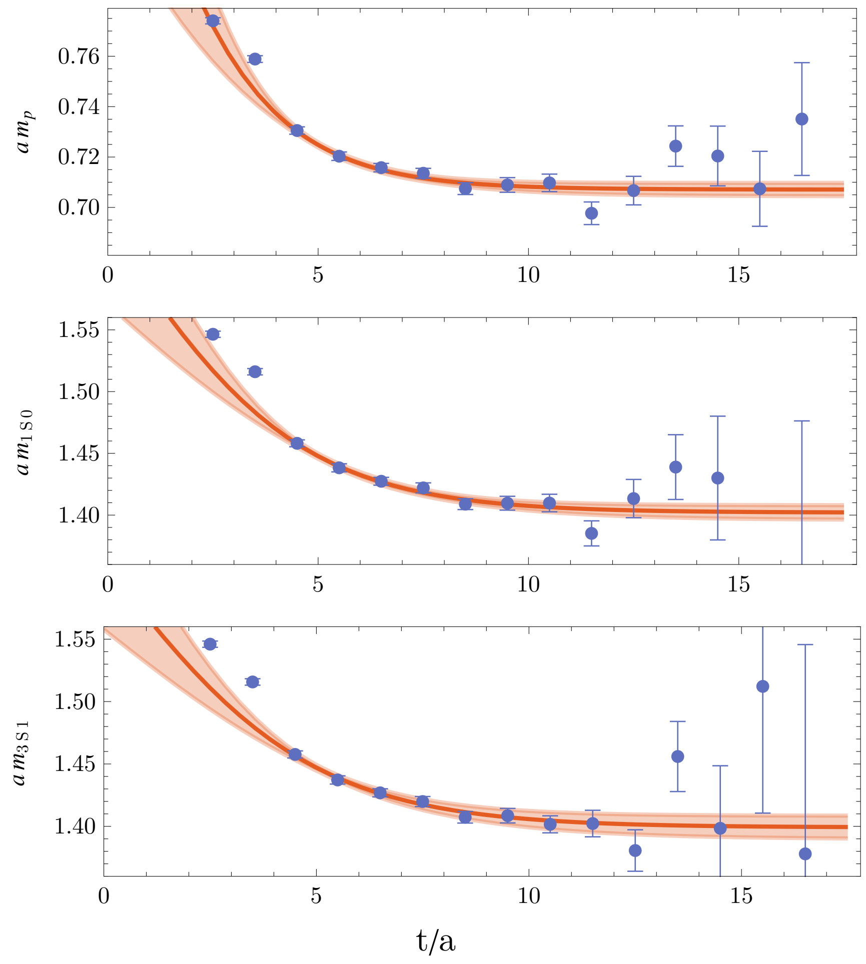

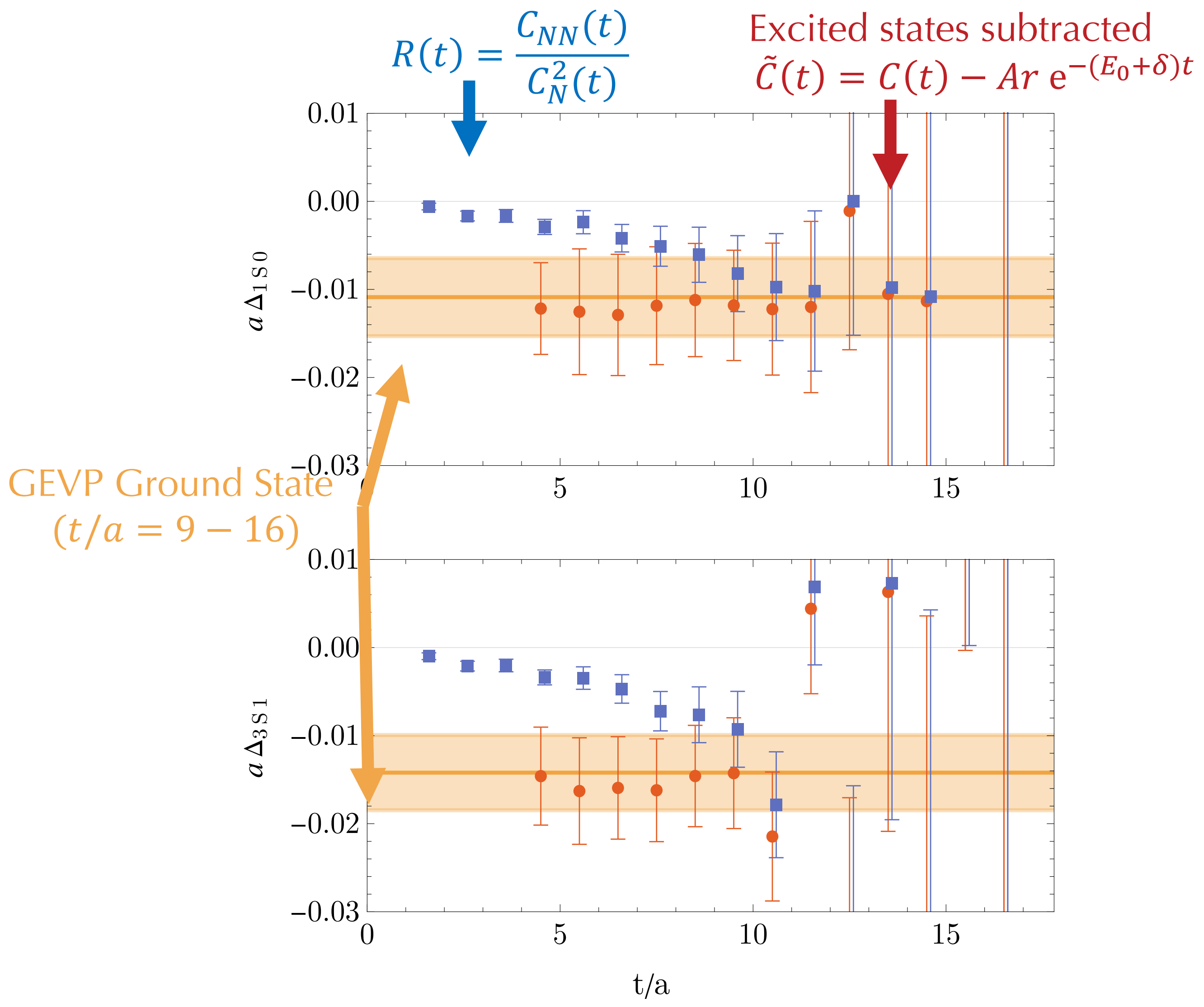

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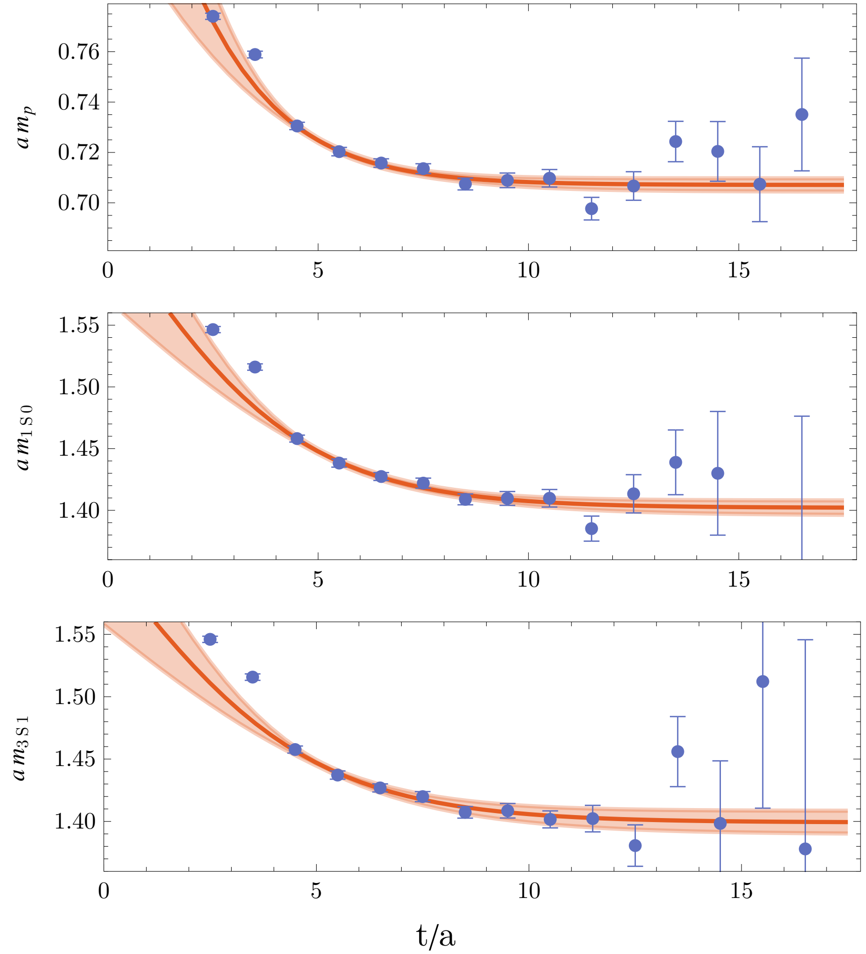

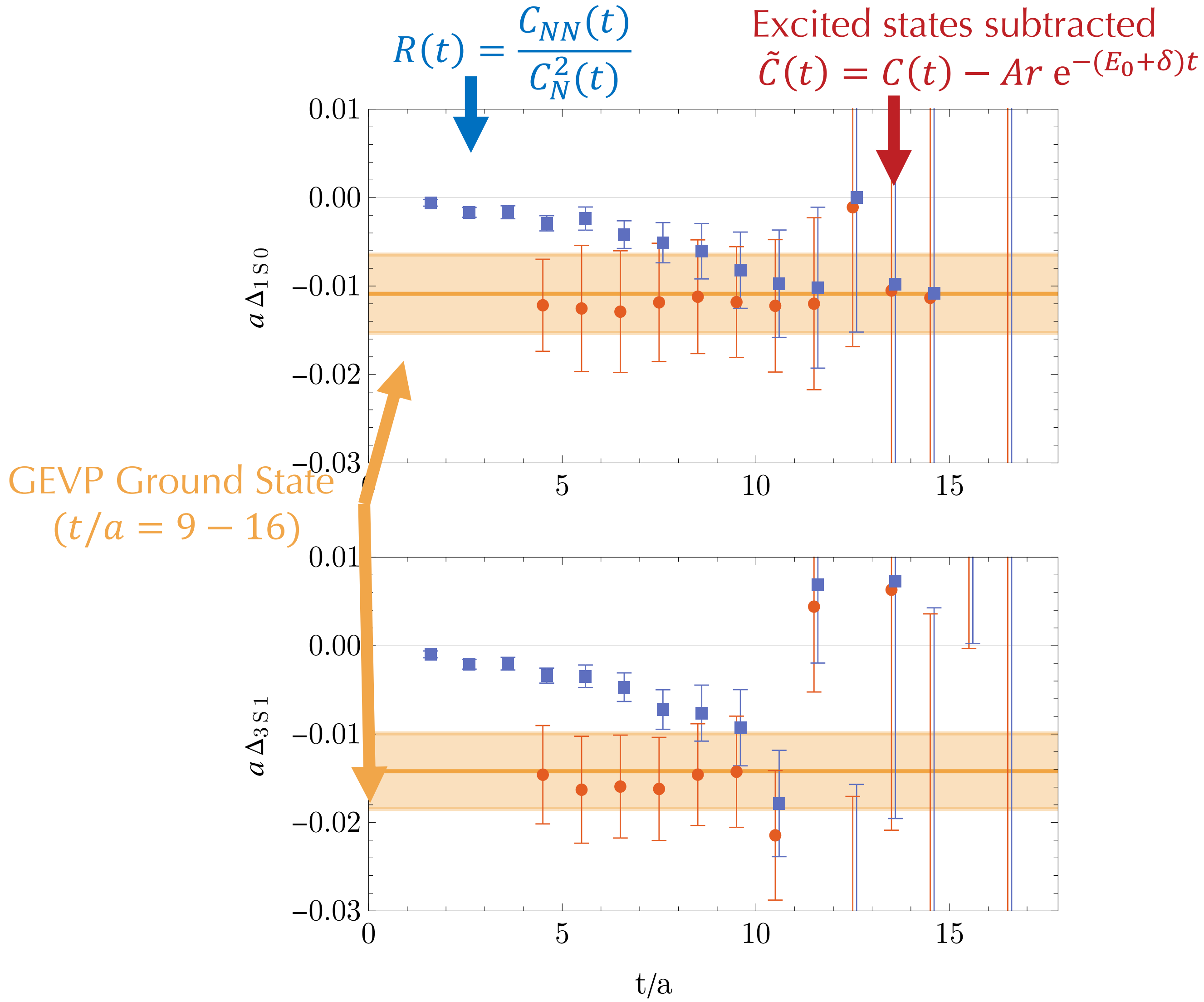

The (elastic) excited-state contamination is effectively removed by GEVP at large time slices.

Summary of the Ground State Energy

➢ Results

- We present the result of variational method as our final result. ($t_i/a = 9, t_f/a = 16$)

$$m_p = 1.2649(42)\,\mathrm{GeV},$$

$$m_{^1\mathrm{S}_0} = 2.5104(90)\,\mathrm{GeV},$$

$$m_{^3\mathrm{S}_1} = 2.5045(89)\,\mathrm{GeV},$$

$$\Delta_{^1\mathrm{S}_0} \equiv m_{^1\mathrm{S}_0} - 2m_p = -19.4(7.9)\,\mathrm{MeV},$$

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Yan Lyu, Phys.Rev.D 105 (2022) 7, 074512

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- The ES contamination from single nucleon interpolator still need to improve in the future calculation.
- Since we already have a GEVP spectrum, let us take a look at their scattering properties.

Scattering Phase Shift of Dineutron/diproton (1S0)

- Lüscher's Method: FV spectrum → scattering phase shift

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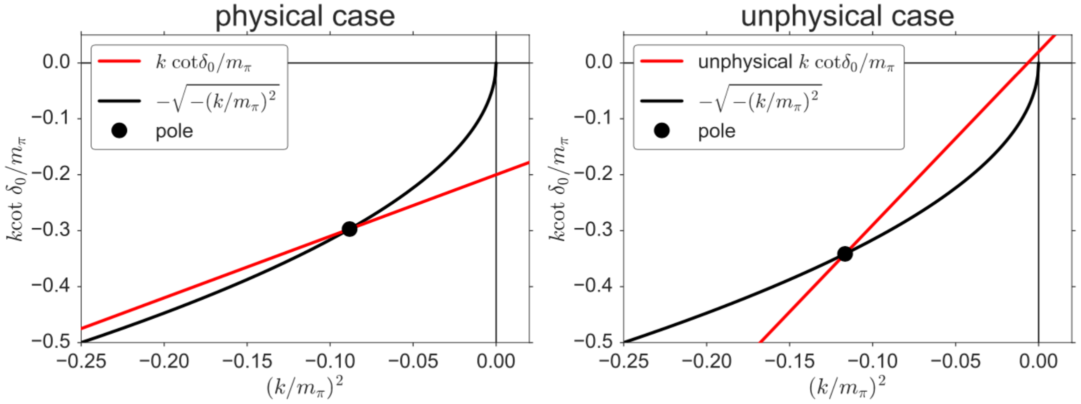

T. Iritani, PRD(2017), 96(3), 034521

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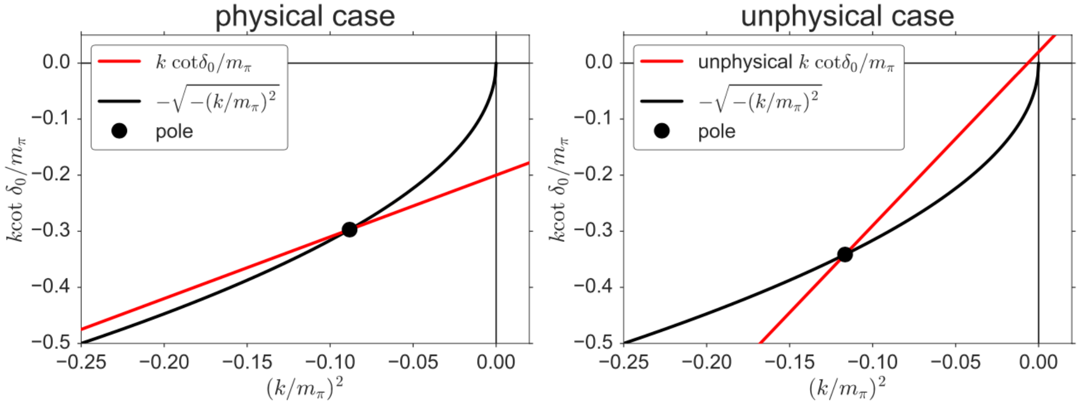

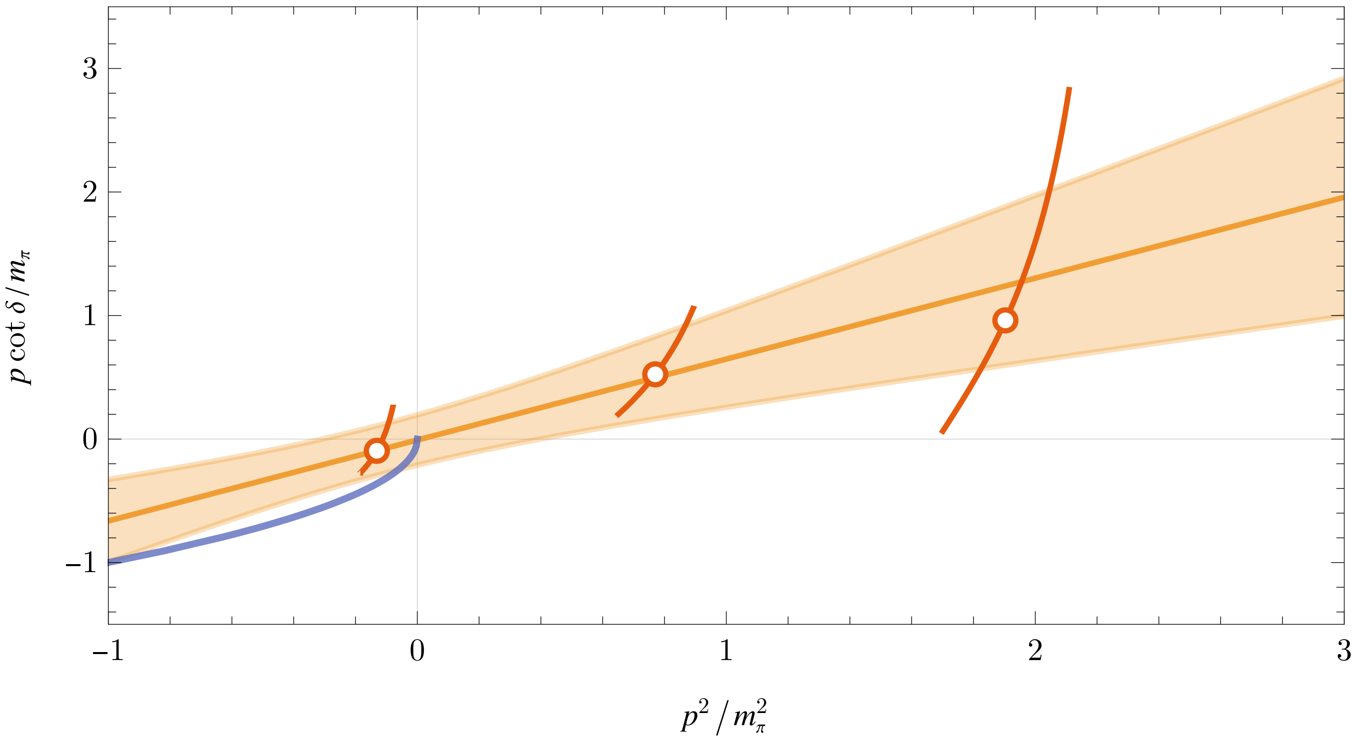

T. Iritani, PRD(2017), 96(3), 034521

- Current result of 1S0 channel:
  - No deep bound state
  - shallow bound state or scattering state

Scattering Phase Shift of Deuteron (3S1)

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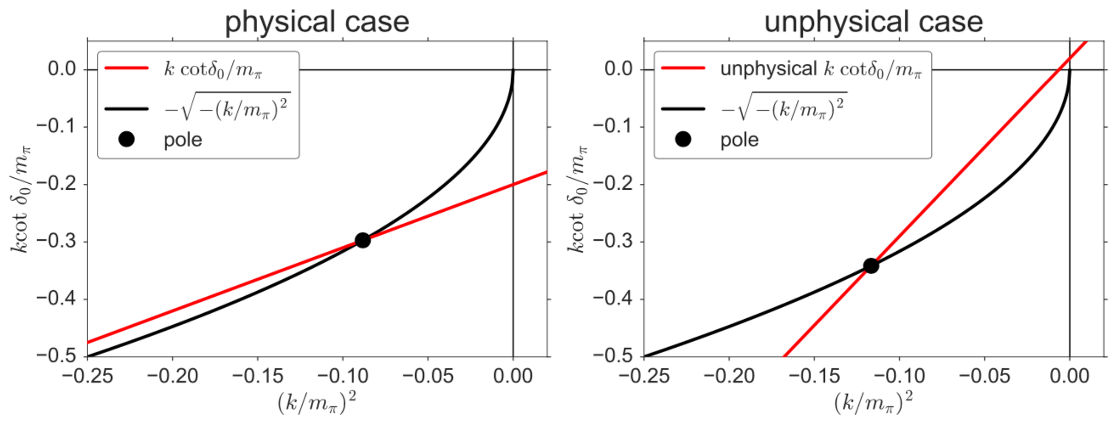
- Consistency check for a physical bound state:



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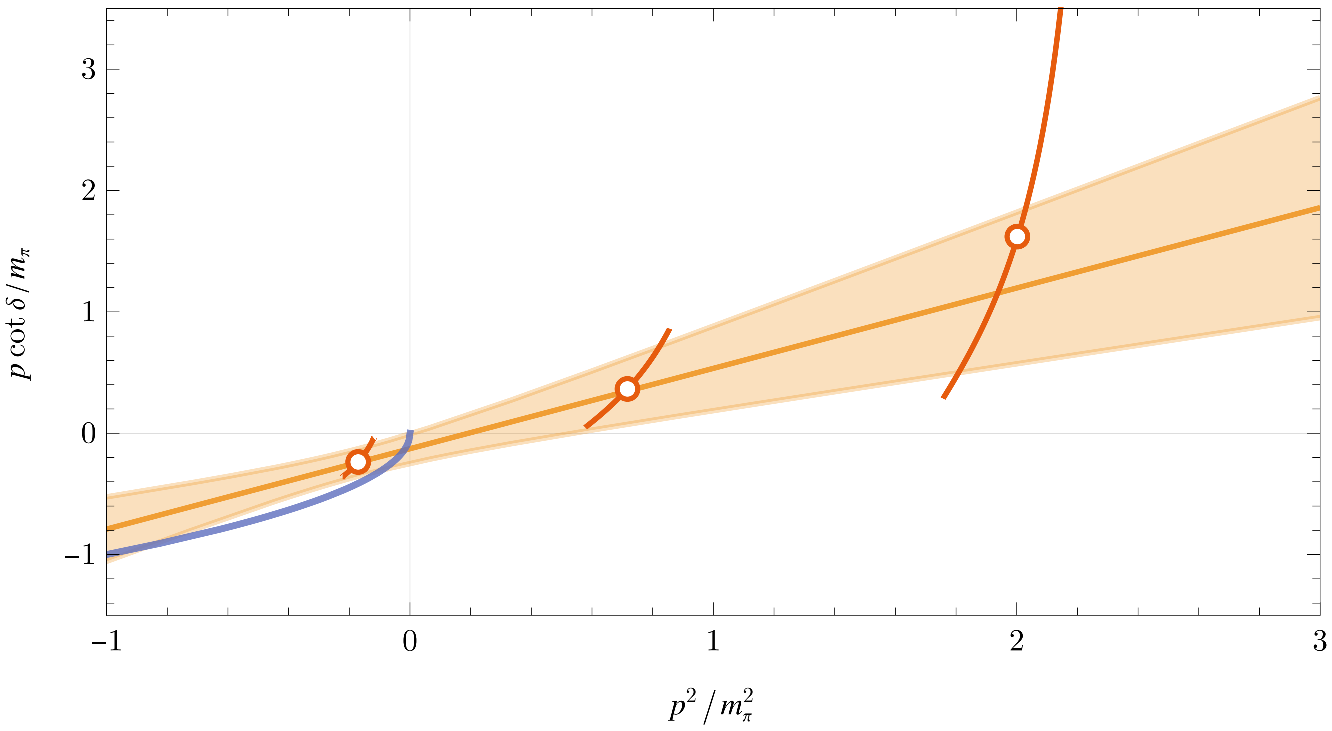

- Current result of 3S1 channel:
  - No deep bound state
  - Most likely to be a shallow bound state
  - Binding energy: $E = 3(6)$ MeV

Summary of Two-nucleon Spectroscopy

➢ Results

- The **deuteron** is likely to be a **shallow bound state** at $m_\pi \sim 432\ \mathrm{MeV}$.
- The **dineutron/diproton** is a **scattering state** or a **shallow bound state**.
- A conclusive result still needs further improvement in accuracy and systematics.

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- The excited state energy is above the inelastic threshold.

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- The excited state energy is above the inelastic threshold.
- As the pion mass continues to decrease, a larger spatial volume will be essential to ensure that the two-nucleon scattering is elastic. This is beyond the scope of this work.

Calculation of Matrix elements

➢ Methods

- Isospin rotation: Flavour changing process → Flavour conserving process

$$\langle p|\mathcal{J}^{+}|n\rangle = \sqrt{2}\,\langle p|\mathcal{J}_3|p\rangle\,,$$

$$\langle d|\mathcal{J}^{+}|pp(^1\mathrm{S}_0)\rangle = -\,\langle d|\mathcal{J}_3|pn(^1\mathrm{S}_0)\rangle\,.$$

- Sequential-source propagator technique: 3pt → 2pt
- Matrix elements extracted from the ratio: $R_3(t) = \frac{C_3(t)}{C_2(t)} = \left(\frac{\langle f\,|\,\mathcal{J}\,|\,i\rangle}{Z_A}\right) t + \cdots,$

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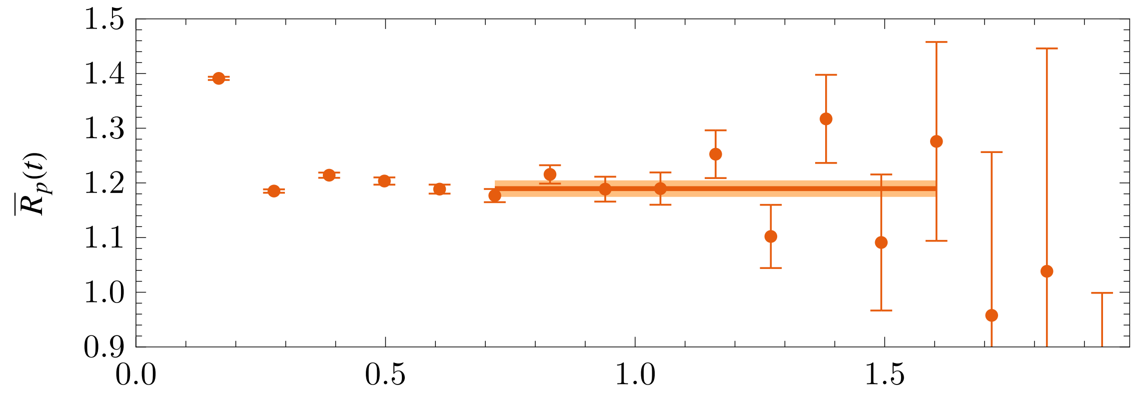
➢ Determination of nucleon axial charge

$$g_A = 1.190(10),$$



Consistent with RBC/UKQCD (same ensamble):
$g_A = 1.186(36)$

Yamazaki, et al., PRL, 100 (2008), 171602

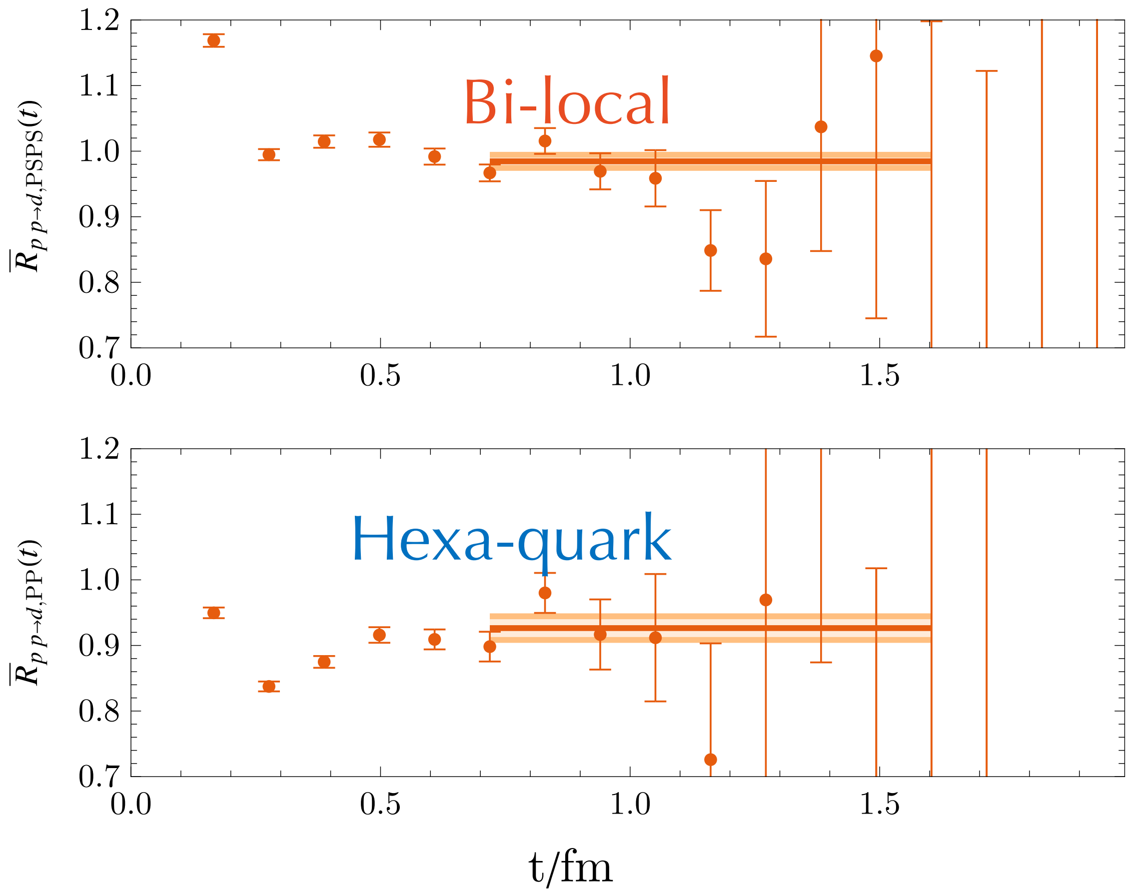
NME of Proton-Proton Fusion

- Applying the same techniques to two-nucleon interpolators: $R_3(t) = \frac{C_3(t)}{C_2(t)} = \left(\frac{\langle f \,|\, \mathcal{J} \,|\, i \rangle}{Z_A}\right) t + \cdots,$

→ Matrix elements of proton-proton fusion.

$$\frac{\langle d|\mathcal{J}|pp\rangle_{\rm PSPS}}{\sqrt{2}g_A} = 0.984(10),$$

$$\frac{\langle d|\mathcal{J}|pp\rangle_{\rm PP}}{\sqrt{2}g_A} = 0.926(18),$$



NME of Proton-Proton Fusion

- Applying the same techniques to two-nucleon interpolators: $R_3(t) = \frac{C_3(t)}{C_2(t)} = \left(\frac{\langle f\,|\,\mathcal{J}\,|\,i\rangle}{Z_A}\right) t + \cdots,$

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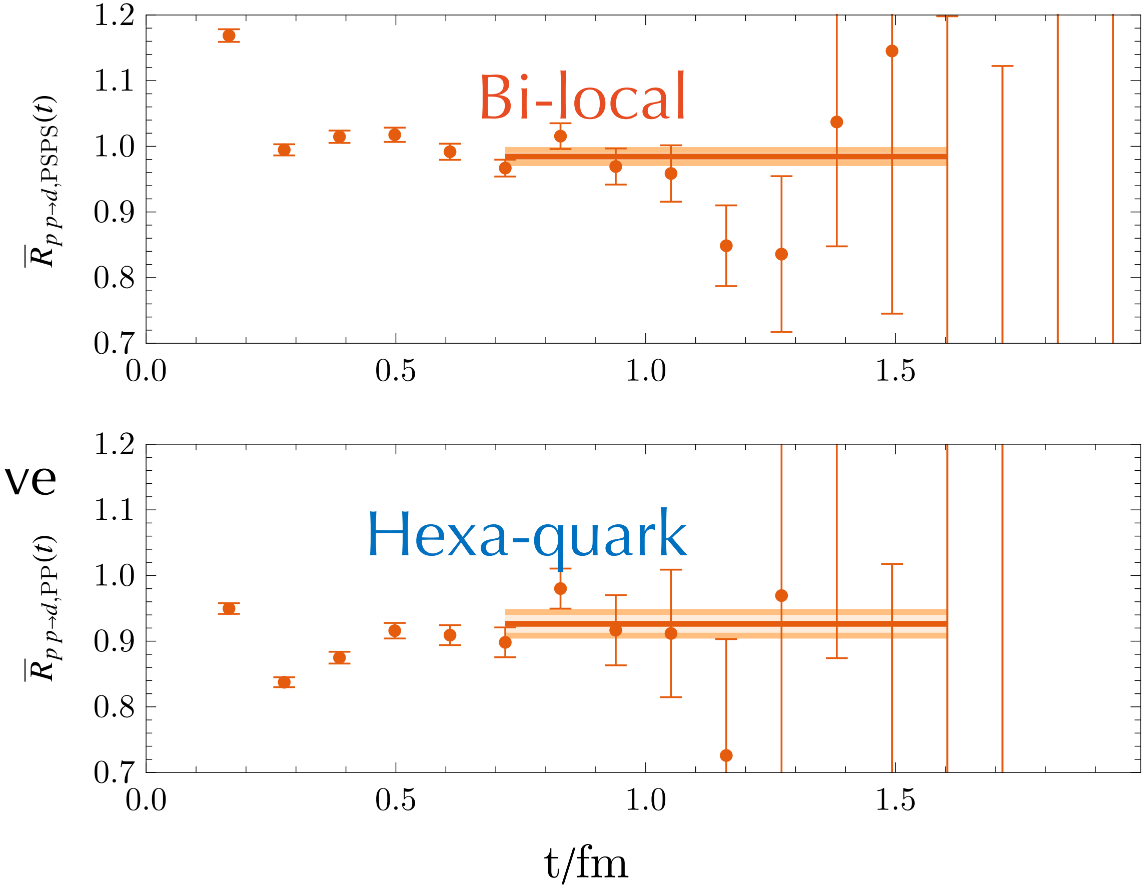

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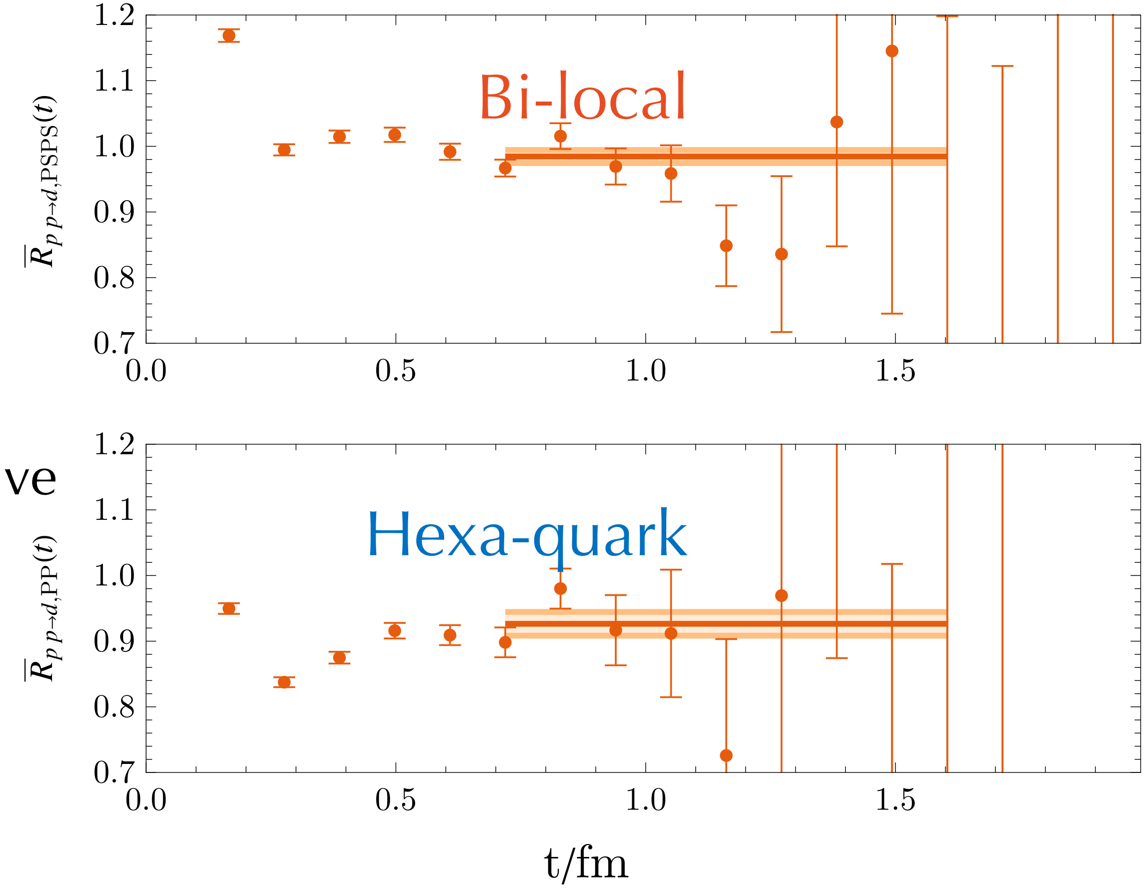

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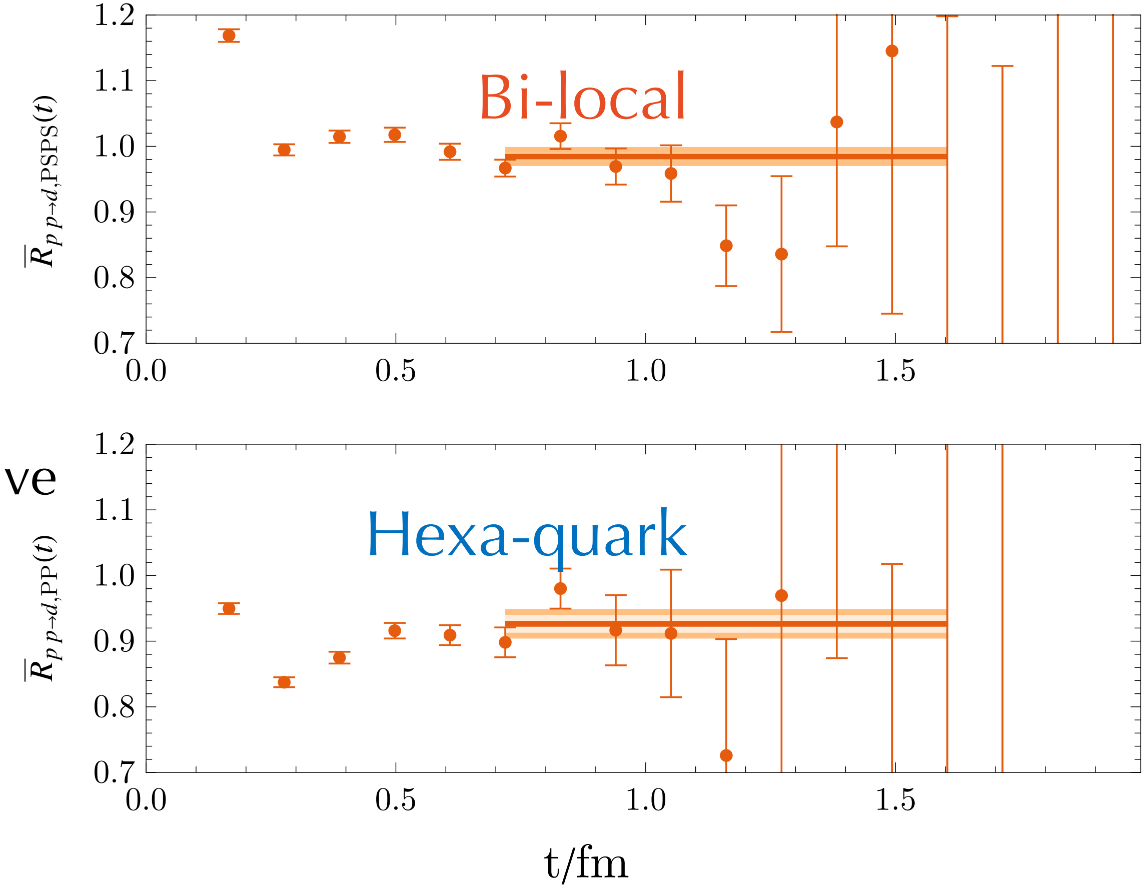

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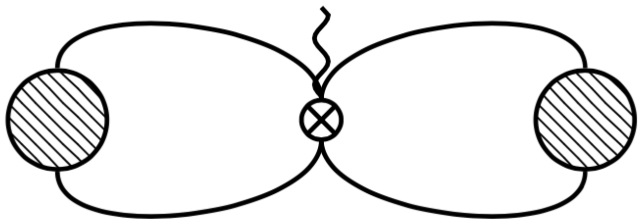
Finite Volume Analysis



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  - The FV correction for $2 \to 1 \to 2$ process:

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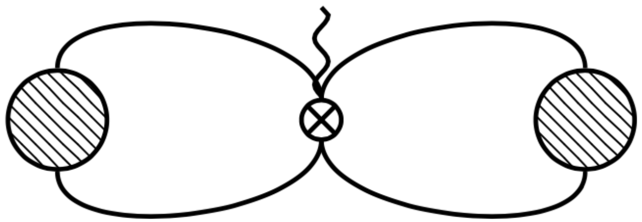

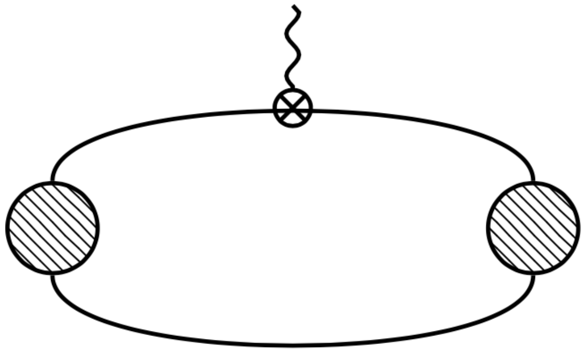

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- The one-body contribution "triangle diagram":
  - Related to nucleon axial charge g_A
  - FV formalism has present by:
    - Relativistic: R.A. Briceño & M. T. Hansen PRD 94 (2016) 1, 013008
    - Non-relativistic: Z. Davoudi & S. V. Kadam PRD 102 (2020) 11, 114521

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Joseph Moscoso's talk on 7.30.

➢ A nice example of charge conservation in FV analysis

R.A. Briceño et al., PRD 100 (2019) 11, 114505

- If applying the FV formalism of $2 + J \to 2$ process to charge operator: $\widehat{\mathrm{Q}} \equiv \int \mathrm{d}^3\mathbf{x}\, \mathcal{J}^0(x)\,,$

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Two key ingredient

- Relation between Lellouch-Lüscher factor and the kinematic function G(P,L)
- Relation from Ward-Takahashi identity:

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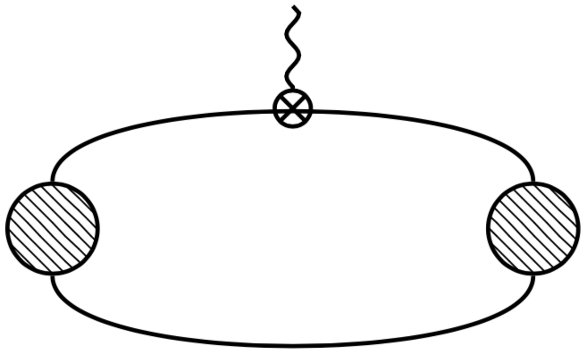

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Z. Davoudi, S. V. Kadam, PRD 102 (2020) 11, 114521

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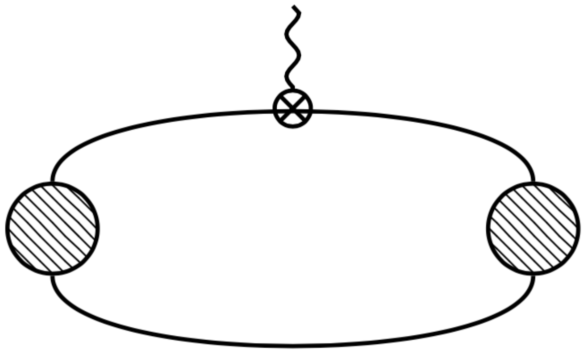

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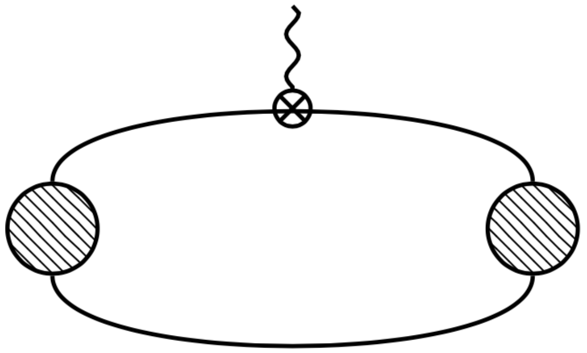

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$$F_0(E) = \left(\frac{1}{L^3}\sum_{\mathbf{p}} - \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\right)\frac{1}{E - \mathbf{p}^2/m + i\epsilon},$$
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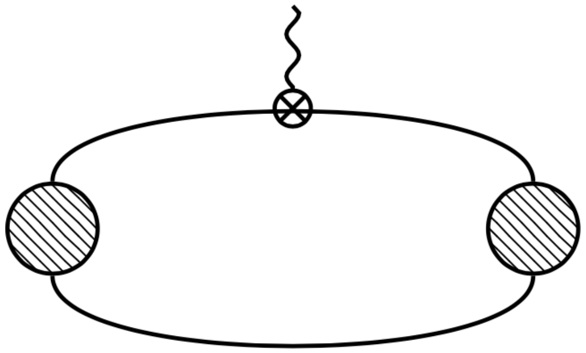

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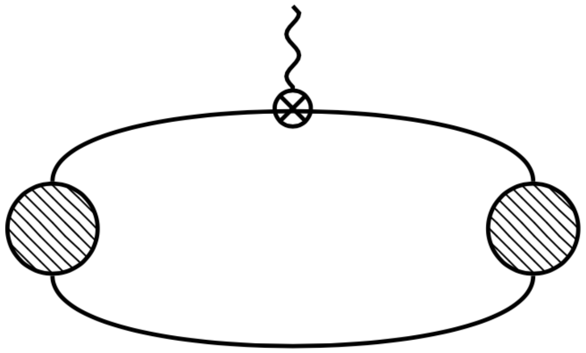

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- Now, we have all the key ingredients:

$$L^6 \left|\langle E_n, L | \mathcal{J}^{1B} | E_n, L\rangle\right|^2 = g_A^2 \left(\frac{-\mathcal{M}^2(E_n)\frac{\mathrm{d}}{\mathrm{d}E}F_0(E_n) - \mathcal{M}^2(E_n)\frac{\mathrm{d}}{\mathrm{d}E}\mathcal{M}^{-1}(E_n)}{-\mathcal{M}^2(E_n)\frac{\mathrm{d}}{\mathrm{d}E}\left[F_0(E_n) + \mathcal{M}^{-1}(E_n)\right]} \right)^2$$
$$= g_A^2.$$

ME of one-body contribution exactly reproduced g_A when $E_i = E_f$

Numerical Results of FV Correction

➢ FV Correction of one-body contribution

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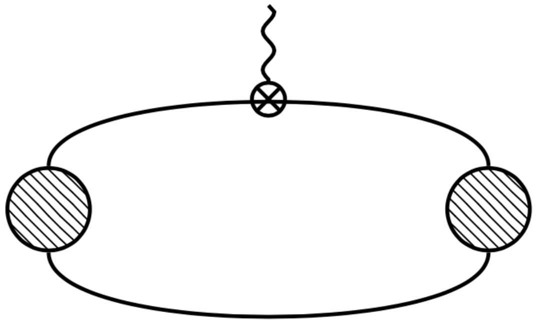
$$\equiv g_A(1 - \delta_{1\mathrm{B}}).$$



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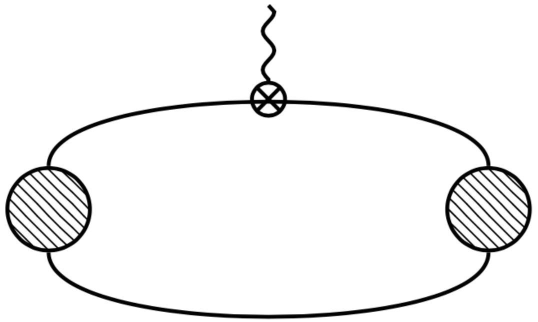

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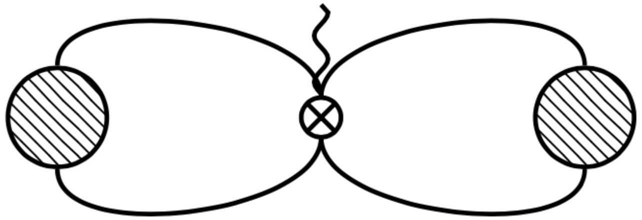
➢ FV Correction of two-body contribution

- LL-factors for both initial and final states.

$$L^6 \left|\langle E_f, L \,|\, \mathcal{J}^{2\mathrm{B}} \,|\, E_i, L\rangle\right|^2 = \mathcal{R}(E_f) W_{2\mathrm{B}}^2(E_i, E_f) \mathcal{R}(E_i)$$
$$L^3 \left|\langle E_f, L \,|\, \mathcal{J}^{2\mathrm{B}} \,|\, E_i, L\rangle\right| \equiv L_{1,A}^{sd-2b}(1 - \delta_{2\mathrm{B}})$$



short-distance two-body contribution

- The numerical result is $\delta_{2B} = 0.19(45)$.

Matching to Low Energy Constant in Pionless EFT

➢ Matching Directly at unphysical pion mass

PRD 105 (2022) 9, 094502

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- The LEC constrained by our work ($m_\pi \sim 432$ MeV) is

$$L_{1,A} = 3.5(1.3)\ \mathrm{fm}^3 \text{ (Ignoring both } \delta_{1B} \text{ and } \delta_{2B})$$

$$L_{1,A} = 3.4(1.5)\ \mathrm{fm}^3 \text{ (} \delta_{2B} \text{ included)}$$

$$L_{1,A} = 21(84)\ \mathrm{fm}^3 \text{ (Both } \delta_{1B} \text{ and } \delta_{2B} \text{ included)}$$

- NPLQCD's result at $m_\pi \sim 806$ MeV : $L_{1,A} = 3.9(0.2)(1.4)\mathrm{fm}^3$ (Ignoring both δ_{1B} and δ_{2B})

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The **major FV** correction comes from **one-body** contribution. Because the **two-body** contribution accounts for only about **2%** of the matrix elements.

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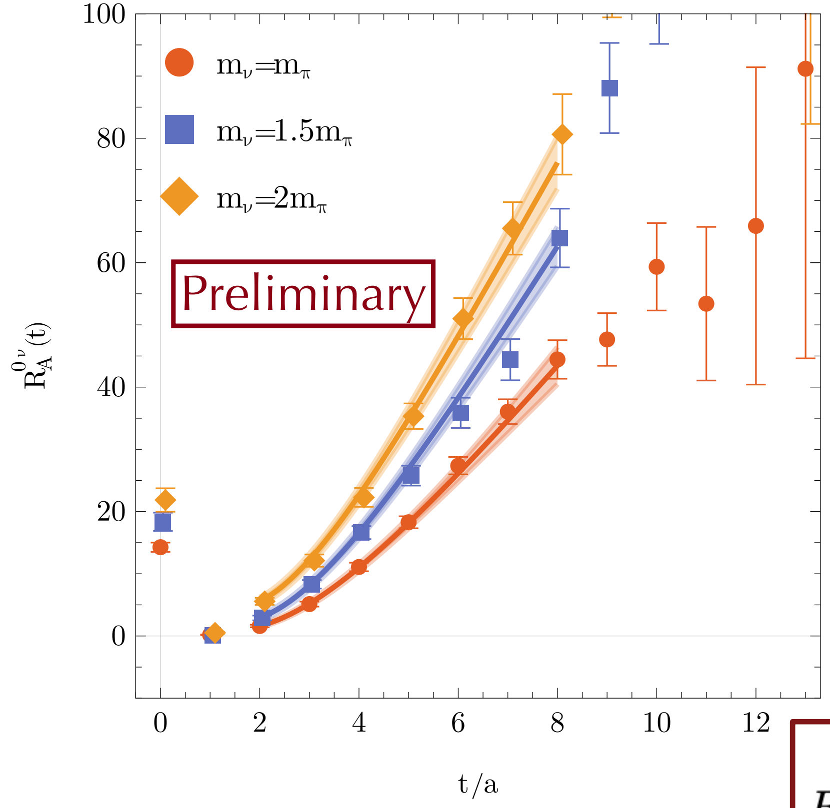
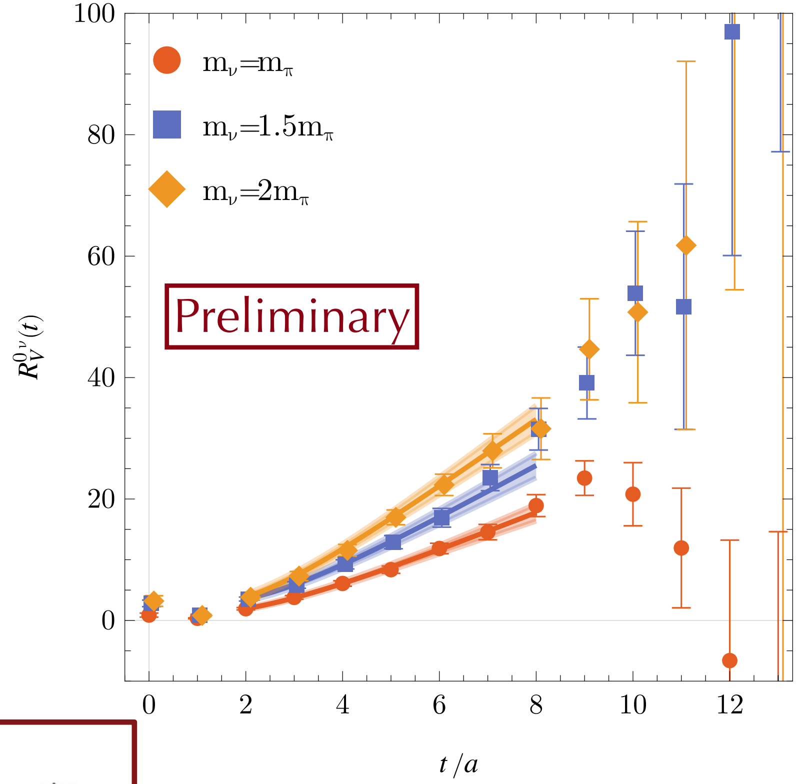
Neutrinoless Double Beta Decay (Preliminary)

- **Massive neutrino** scheme is used to remove the divergence of neutrino propagator.
- This scheme can also simplify finite volume analysis. Xu Feng et al., PRD 103 (2021) 3, 034508
- We noticed that the loop momentum can easily exceed the pion production threshold, so the match requires **chiral EFT**, which is currently not available.





$$R(t) \equiv \frac{C_{4pt}(t)}{C_{2pt}(t)} = M_{0\nu}Vt + be^{-\Delta t},$$

Conclusion

➢ Lattice Calculation of Proton-Proton Fusion Matrix Element

- **Bi-local interpolators** can effectively reduce the excited-state contamination.
- **Deuteron** is likely to be **a shallow bound state** at m_π $\sim$ **432 MeV**.
- **Diproton/dineutron** is likely to be **a shallow bound state** or **scattering state**.
- We obtained the **p-p fusion matrix element** and constrained corresponding **LEC** $\boldsymbol{L_{1,A}}$.
- The **FV correction of one-body contribution** has significantly influence on constraining $L_{1,A}$.

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➢ Outlooks

- We will use ensembles with larger spatial volume to control FV effects and other systematics.
- Improving the accuracy of two-nucleon FV spectrum is essential to the FV corrections.
- With these techniques, we are able to move on to more physical pion mass.

Thanks.