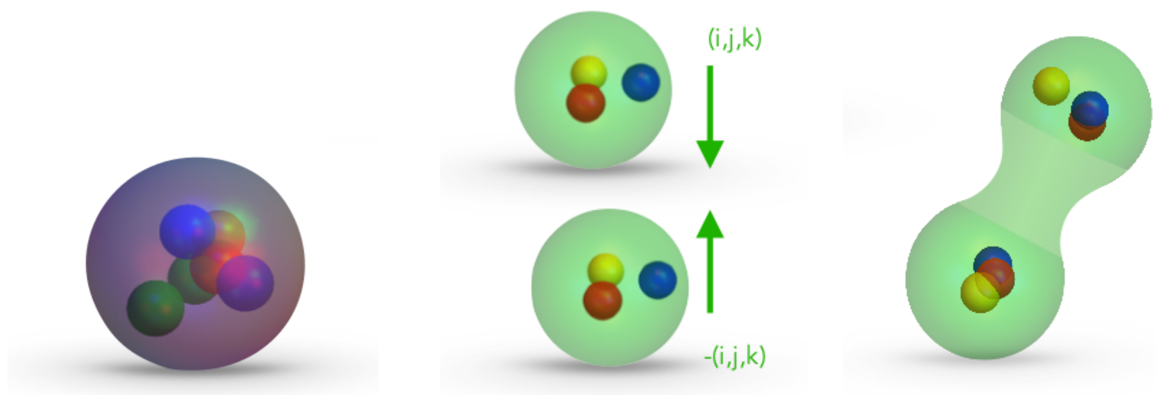
Universitat de Barcelona

NPLQCD

Studying lattice artifacts in multi-baryon systems



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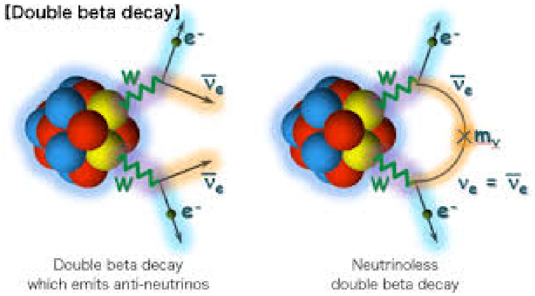
Why Nuclear Physics using LQCD?

- **Understand** emergence of nuclear complexity from the quarks and gluons.
- Particle content of SM has been observed, but SM cannot explain:
  - Dark matter,
  - Neutrino masses,
  - etc.
- Nuclear isotopes often employed as targets in BSM physics searches: often large source of systematic uncertainty.
  - LQCD has potential to be relevant to a broad set of experimental programs.

DEEP UNDERGROUND NEUTRINO EXPERIMENT



- Use LQCD as “virtual lab” to study more exotic systems: hypernuclear physics.
  - Information on YY and YN important for neutron star EoS.

Nuclear Systems are Challenging

- Contraction cost
- Exponentially bad signal-to-noise (StN),
- Small energy gaps

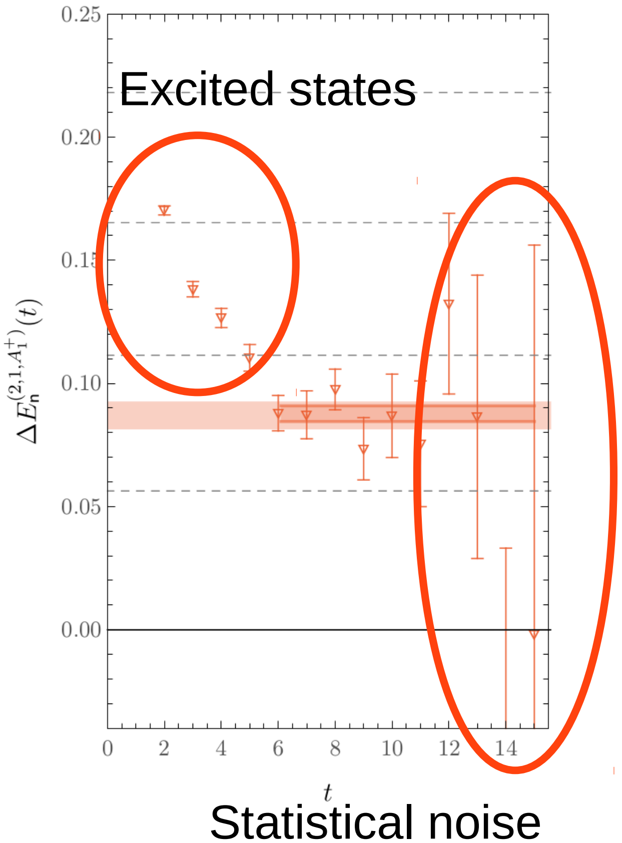
$$C_{\chi\chi}(t) = |Z_{0\chi}|^2 \, e^{-tE_0} + |Z_{1\chi}|^2 \, e^{-tE_1} + \ldots$$

$$\Delta E = E_1 - E_0 \sim \frac{1}{L^2}$$

We would like to understand systematics associated with:

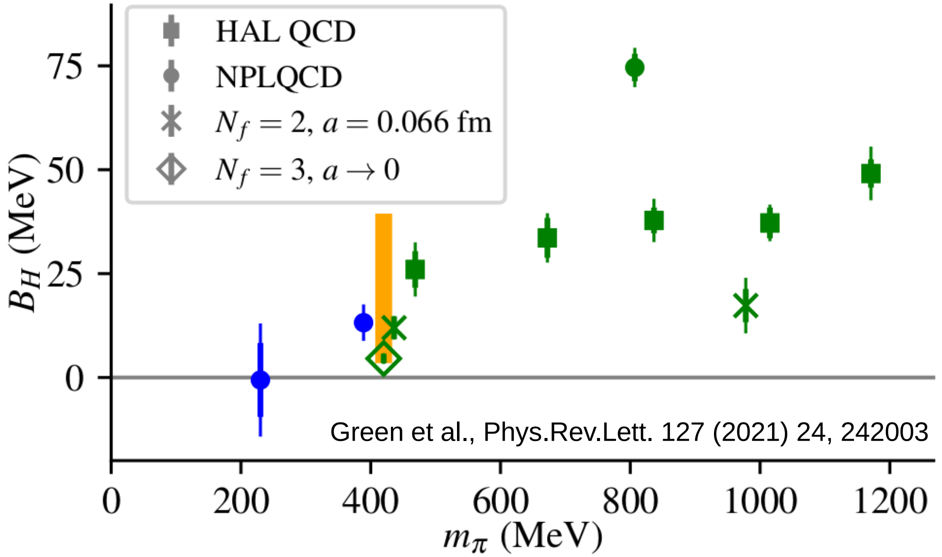
- Non-physical quark masses
- Finite volume
- Lattice discretization
- Excited state contamination
- Operator dependence

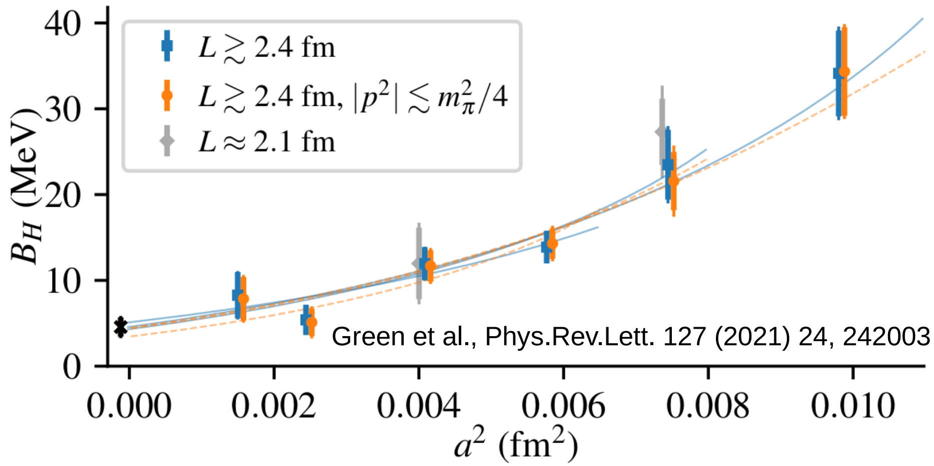
$$E_{\text{eff}} = \ln \frac{C_{\chi\chi'}(t)}{C_{\chi\chi'}(t+1)}$$



H-Dibaryon

- Are there other dibaryon bound states?
- Jaffe (1976): Yes – Lambda-Lambda
- S=-2 hexaquark
- Experimentally challenging.
- Lattice can contribute… provided we understand systematics





Challanging analysis!

Green et al strategy:

- Fit to logarithm of ratio (**non-convex**)

$$R_n(t) = \frac{\tilde{C}_{\Lambda\Lambda}(t)}{C_\Lambda^{\vec{p}_1}(t)C_\Lambda^{\vec{p}_2}(t)}$$

- Exponentially small StN at large times

Different approaches to the same challenge

Asymmetric Correlator

- ✔ Relatively cheap to calculate
- ✗ Different operators at source and sink: not gauranteed to be positive-definite
- ✗ Effective mass can approach from below

Lanczos*

- ✔ Potential for rigorous two-sided error on energy
- ✗ Novel method – less known about systematics etc.

*M. Wagman, "Lanczos, the transfer matrix, and the signal-to-noise problem" 2406.20009 [hep-lat]

Variational method

- ✔ Rigorous variational upper bounds
- ✗ Expensive to compute matrix of correlators

$$\mathrm{H}_{\mathrm{QCD}} \left|n\right\rangle = E_n \left|n\right\rangle$$

HALQCD

- ✔ Physically intuitive: "Nuclear potentials"
- ✗ Requires only elastic scattering states contribute to correlator.

Direct Fits to Correlation Matrices

- ✔ Get the "most" out of correlation matrix
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"Lanczos for matrix elements",
Daniel Hackett
Friday, 12:35

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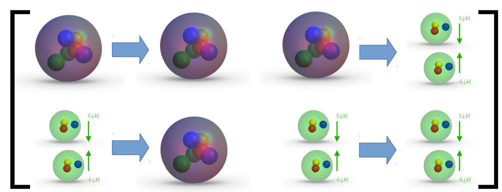
The Variational Method

- (Hermitian) matrix of correlators:

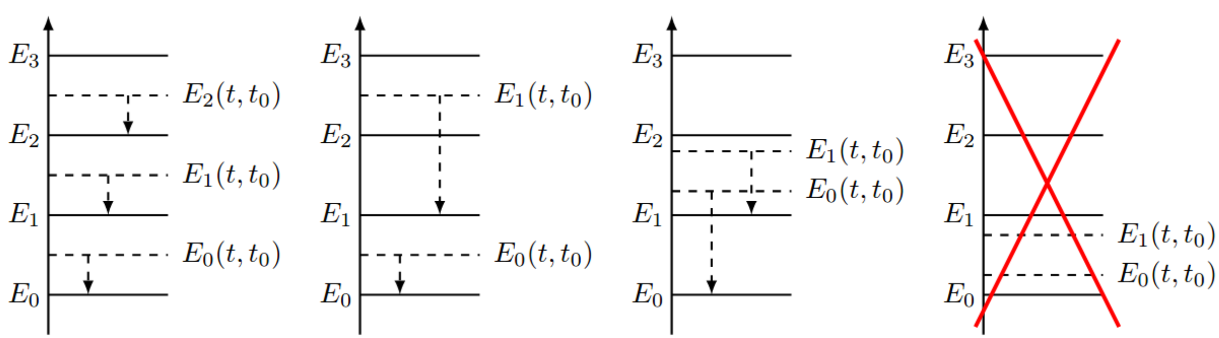
$$\mathbf{C}(t) =$$ 

Most recent work: up to 46x46 correlation matrix!

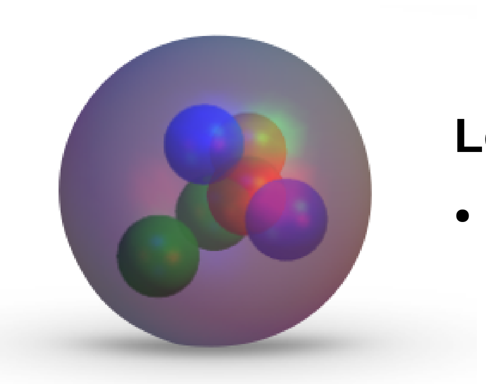
- Solve Generalized Eigenvalue Problem (GEVP):

$$\mathbf{C}(t)\vec{v}_{\mathsf{n}}(t,t_0) = \lambda_{\mathsf{n}}(t,t_0)\mathbf{C}(t_0)\vec{v}_n(t,t_0)$$

- Cauchy Interlacing theorem: GEVP Eigenvalues provide **rigorous (stochastic) variational upper bounds on energy levels.** Provides **lower bound on number of states**

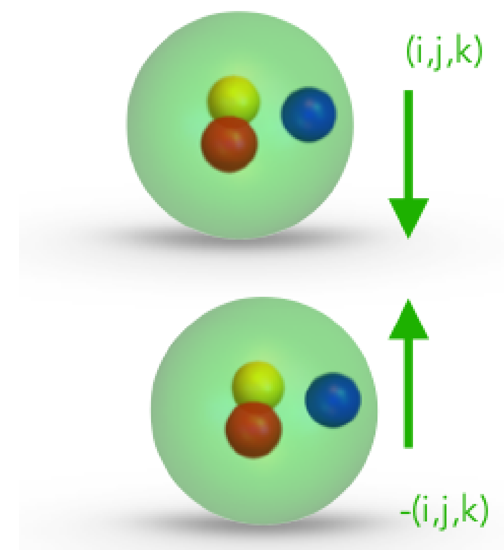


Current Operator Set



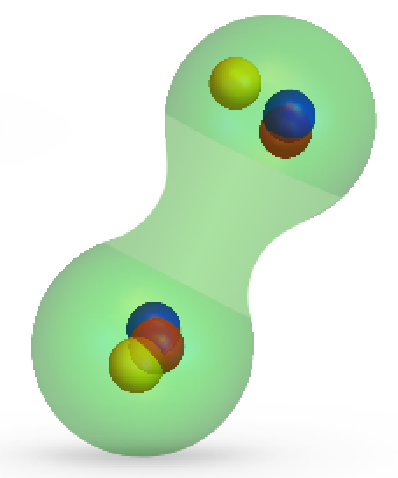
Local hexaquark operators

- Six Gaussian smeared quarks at a point



Dibaryon Operators

- Two spatially-separated plane-wave baryons with relative momenta
- Relative momentum: up to four units → 5 operators



Quasi-local Operators

- Two exponentially localized baryons
- NN -EFT motivated deuteron-like structure

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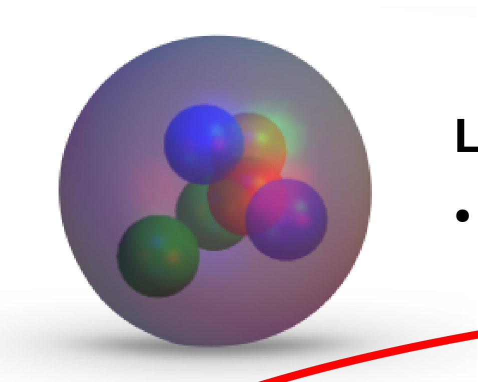

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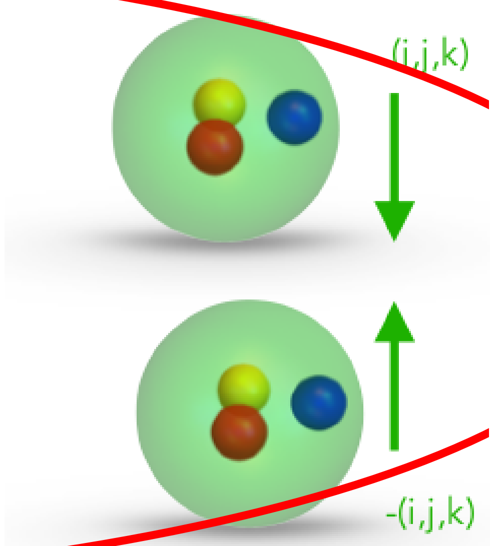

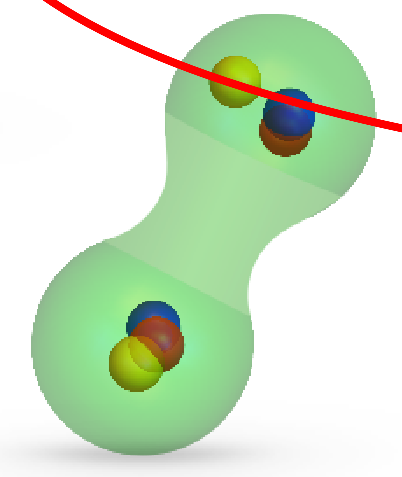

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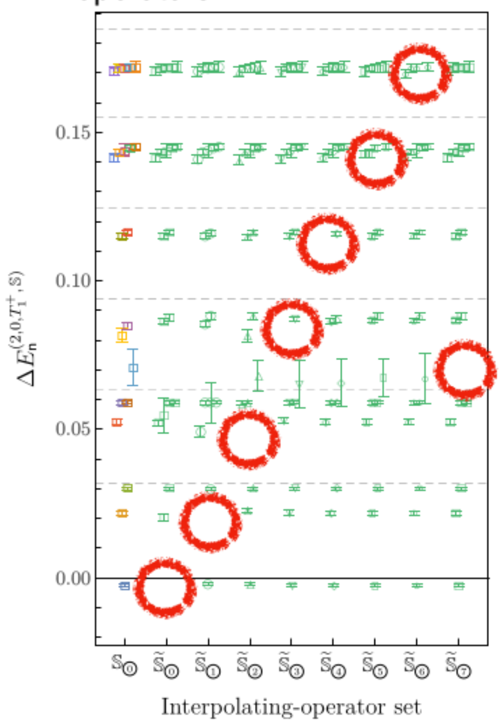
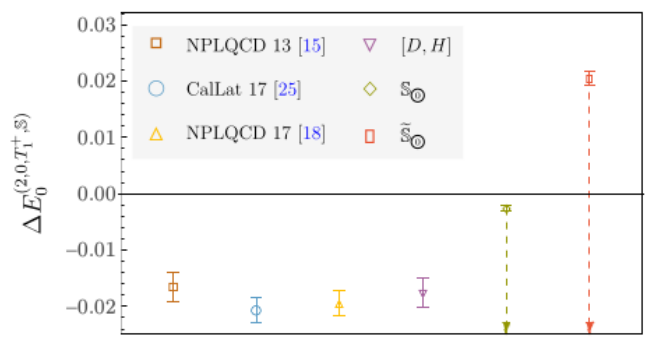
Interpolating operator dependence

Removing interpolating operators leads to "missing energy levels" for states dominantly overlapping with omitted operators



Variational upper bounds obtained using different interpolating operator sets are consistent



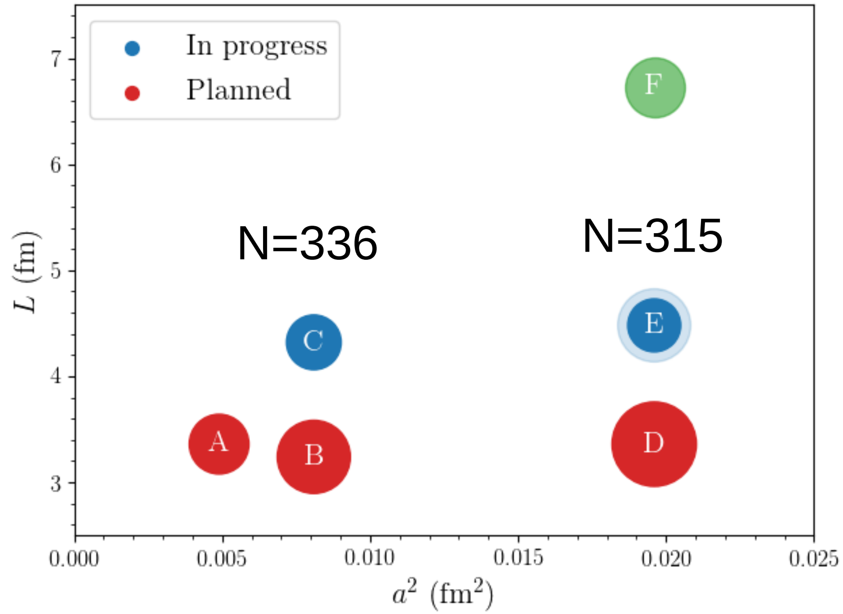
Ground-state energy **estimates** using different interpolating-operator sets show large discrepancies

M. Wagman, Lattice 2022

Moral: Should be careful not to over-interpret results. Results consistent if viewed as variational bounds.

Details of Ensembles Used

Ensemble	β	L/a	T/a	am_q	c_{sw}	a (fm)	N_{sparse}	L (fm)	T (fm)
A	6.5	48	96	-0.1788	1.1701	0.072	8	3.46	6.91
B	6.3	36	64	-0.2050	1.2054	0.086	6	3.10	5.50
C	6.3	48	64	-0.205	1.2054	0.086	6	4.13	5.50
D	6.1	24	48	-0.245	1.2493	0.145	4	3.48	6.96
E	6.1	32	48	-0.245	1.2493	0.145	4	4.64	6.96
F	6.1	48	64	-0.245	1.2493	0.145	4	6.96	9.28



In this talk: Data at two lattice spacings

In the coming months: Add data at smaller physical volume

Goal: Study finite volume and lattice artifacts for S=-2 systems at m_π = 806 MeV.

Use knowledge to **inform near-physical point calculations**.

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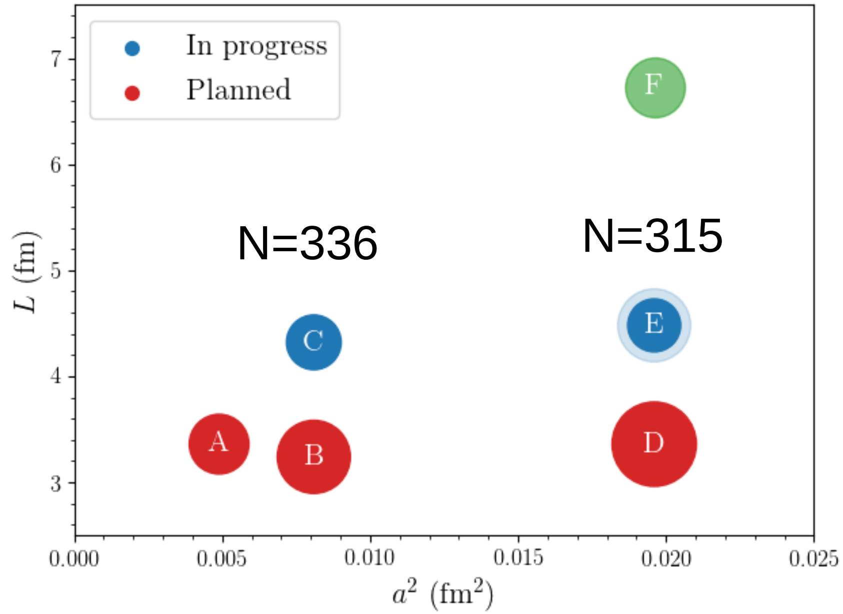

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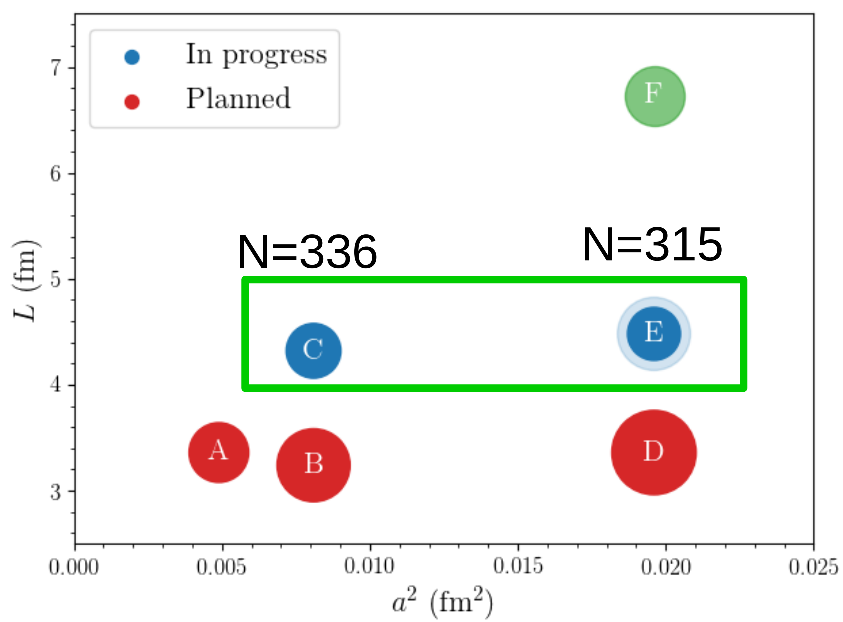

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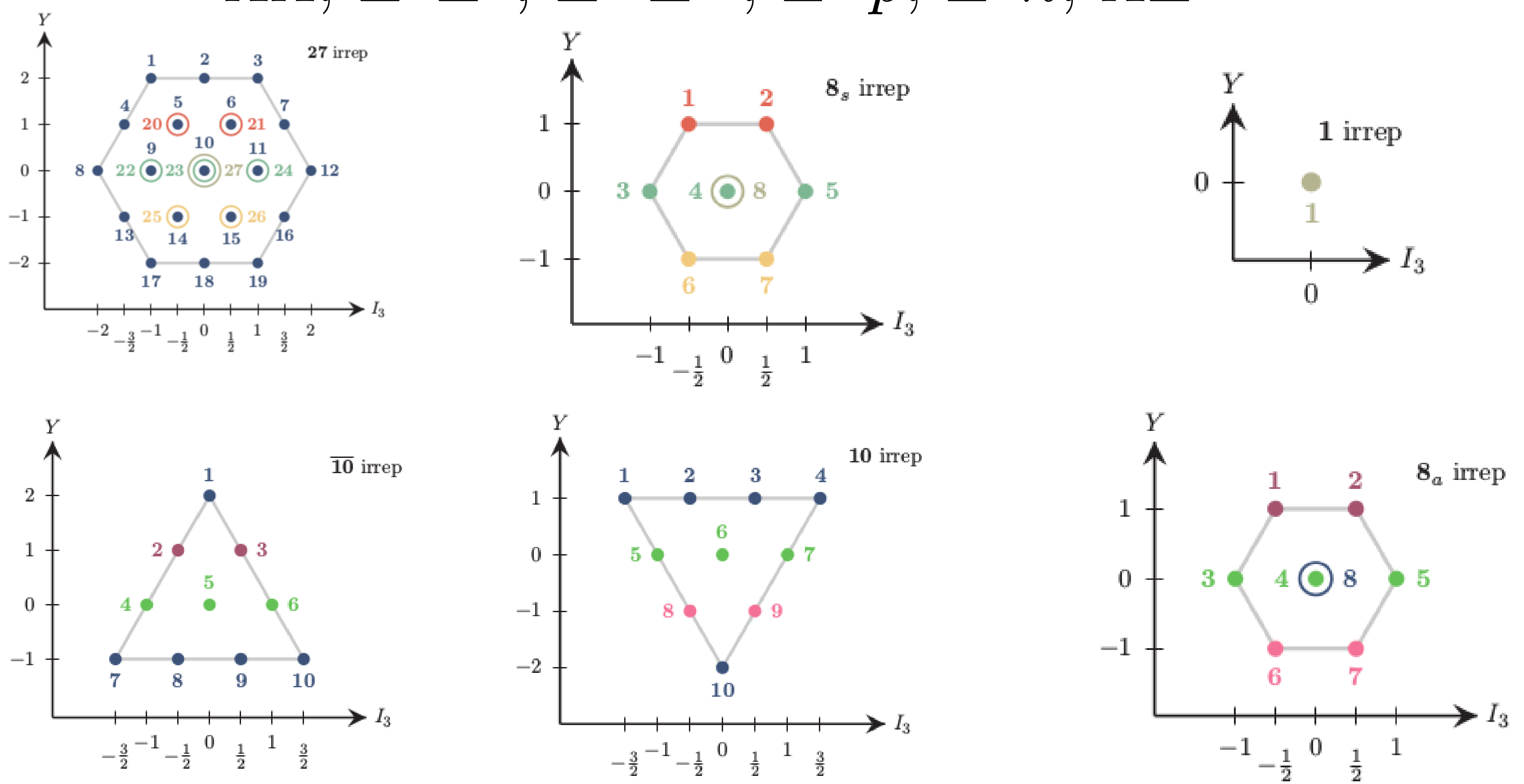
SU(3) Symmetry

- SU(3) flavor symmetric point: octet baryons are mass-degenerate

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{27} \oplus \mathbf{8}_s \oplus \mathbf{1} \oplus \overline{\mathbf{10}} \oplus \mathbf{10} \oplus \mathbf{8}_a$$

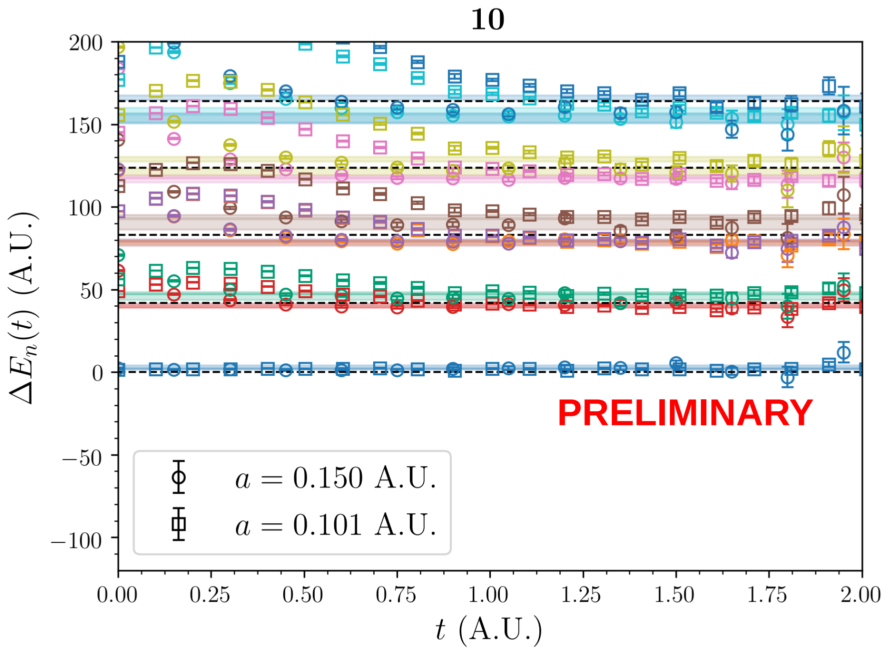
- Study 6 two-baryon flavor wave functions:

$$\Lambda\Lambda,\ \Sigma^0\Sigma^0,\ \Sigma^+\Sigma^-,\ \Xi^-p,\ \Xi^0n,\ \Lambda\Sigma^0$$



M. Illa, 2021 “Approaching nuclear interactions with lattice QCD”

Effective Mass Plots (J=1)



* Fit to 1B and 2B correlation functions. Then compute difference

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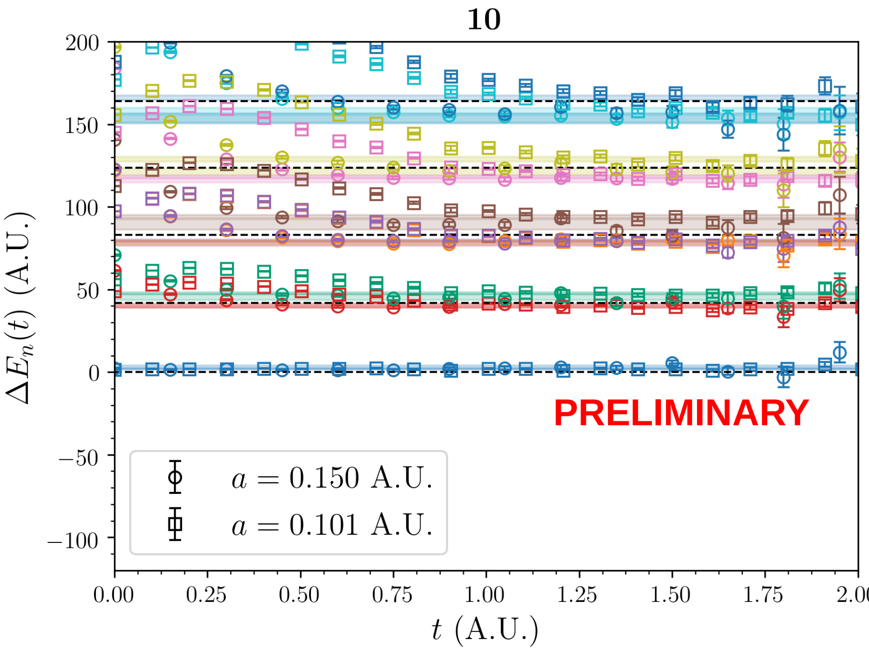

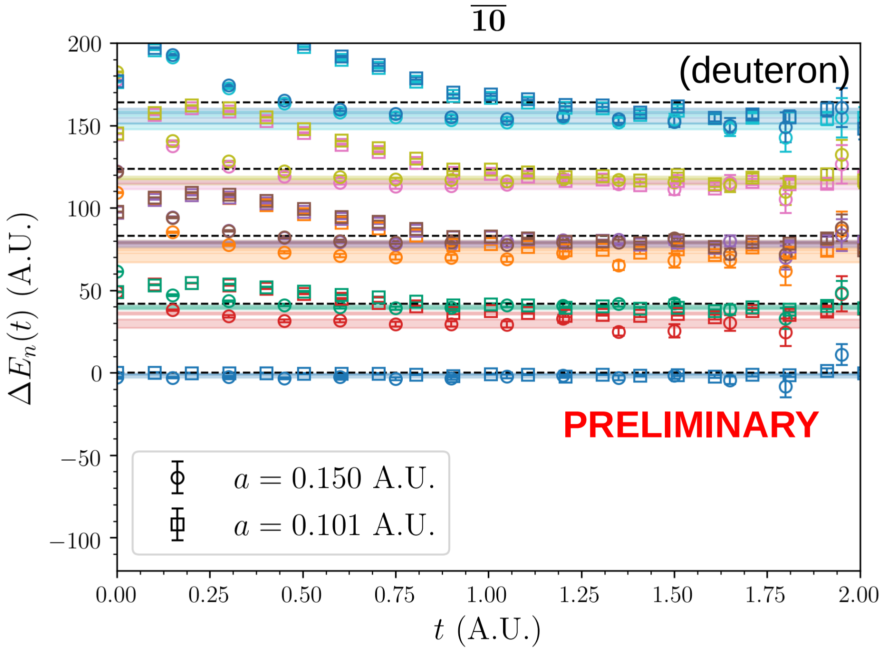

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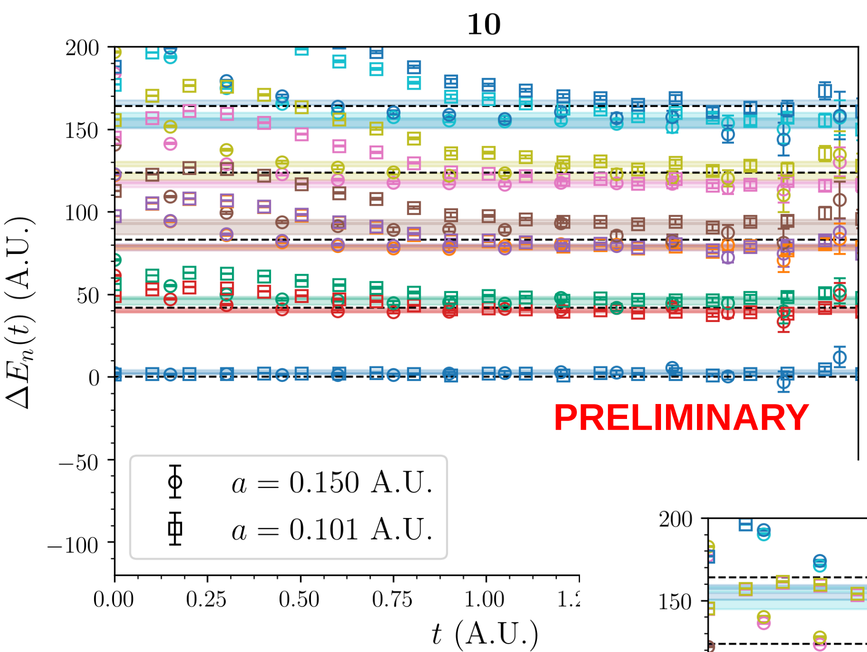

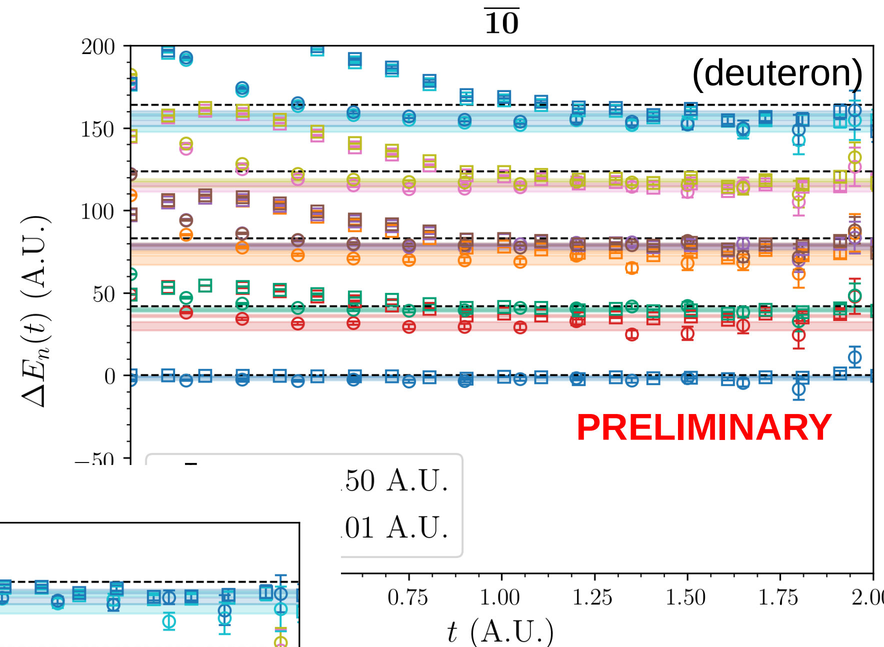

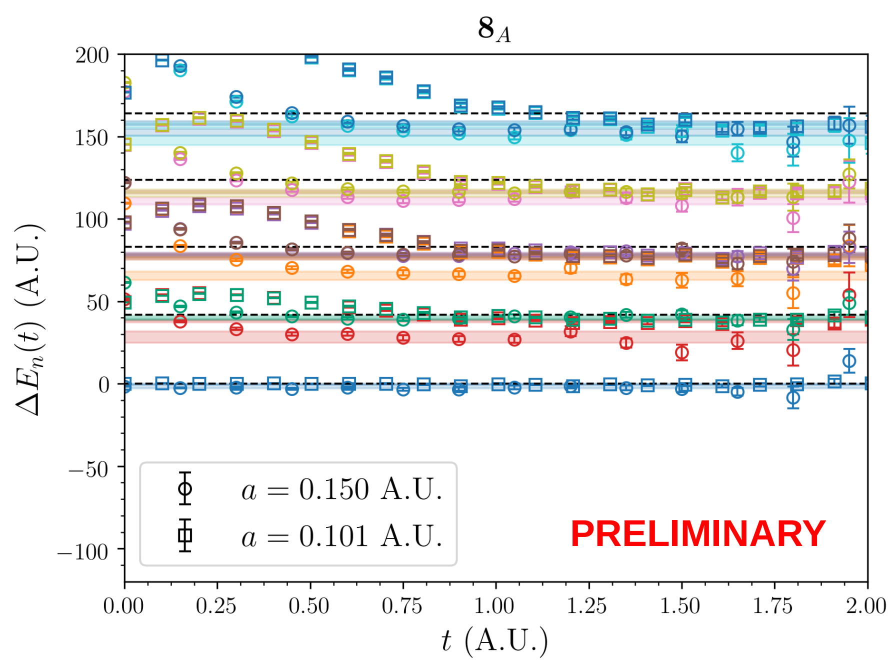

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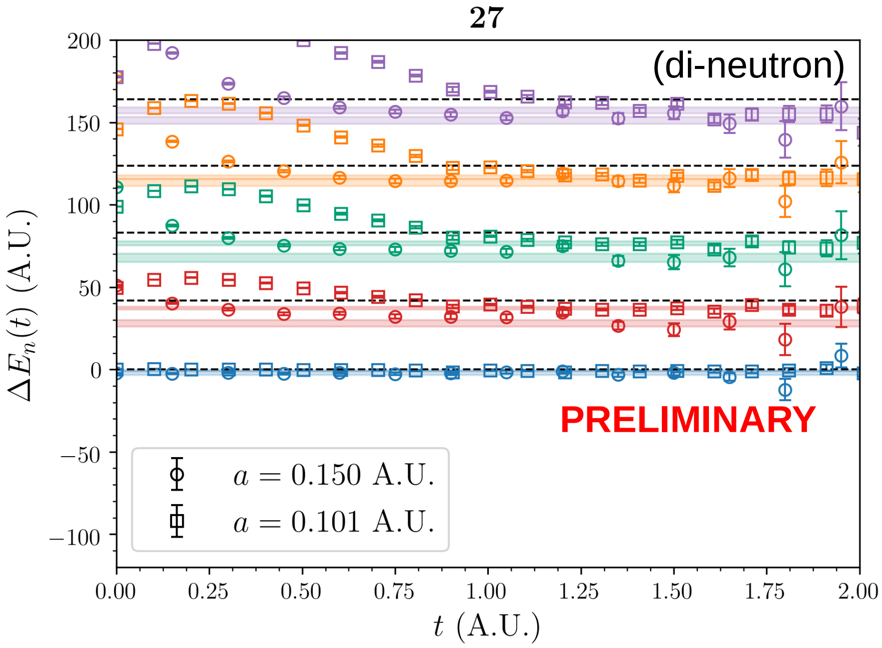

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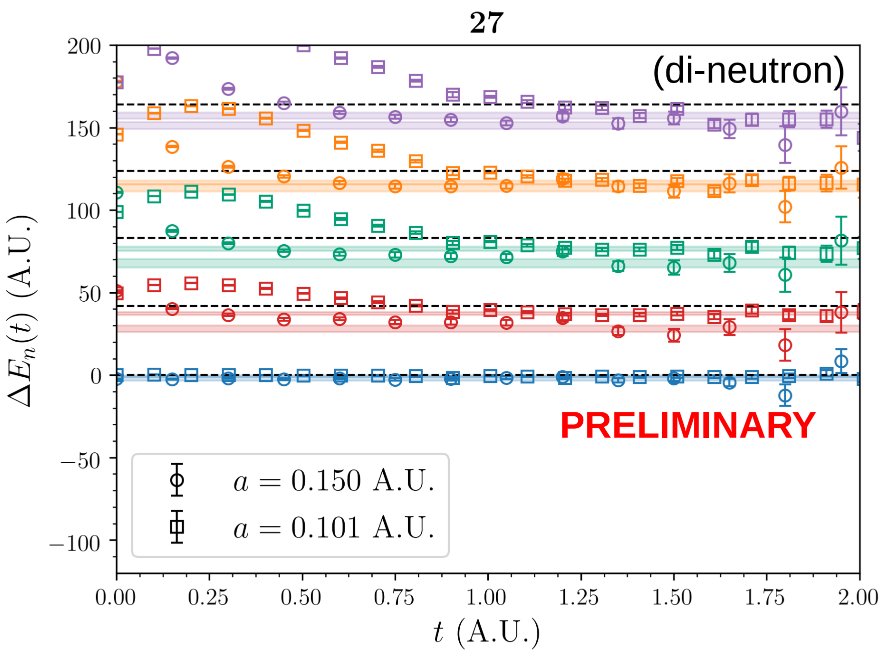

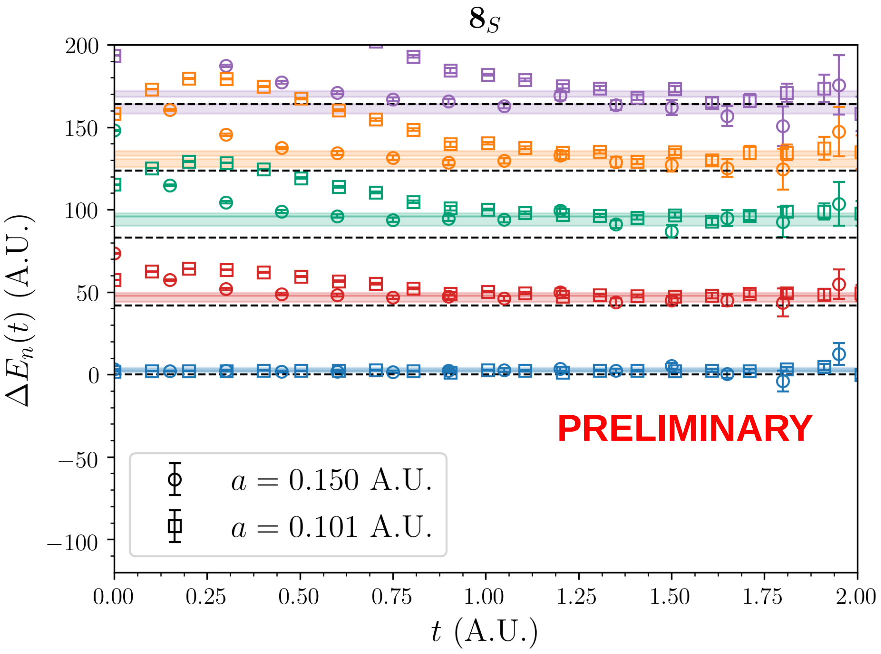

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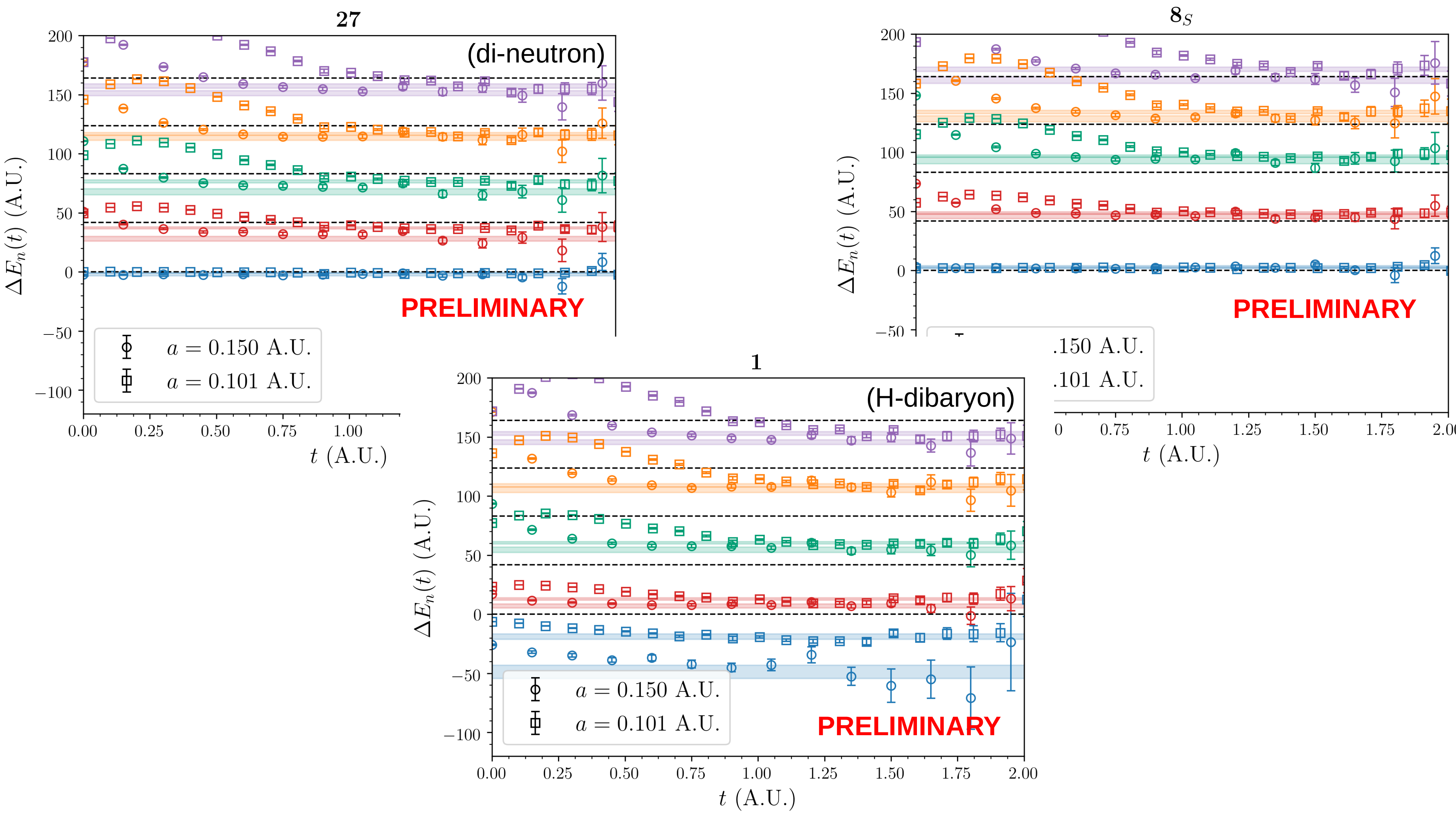
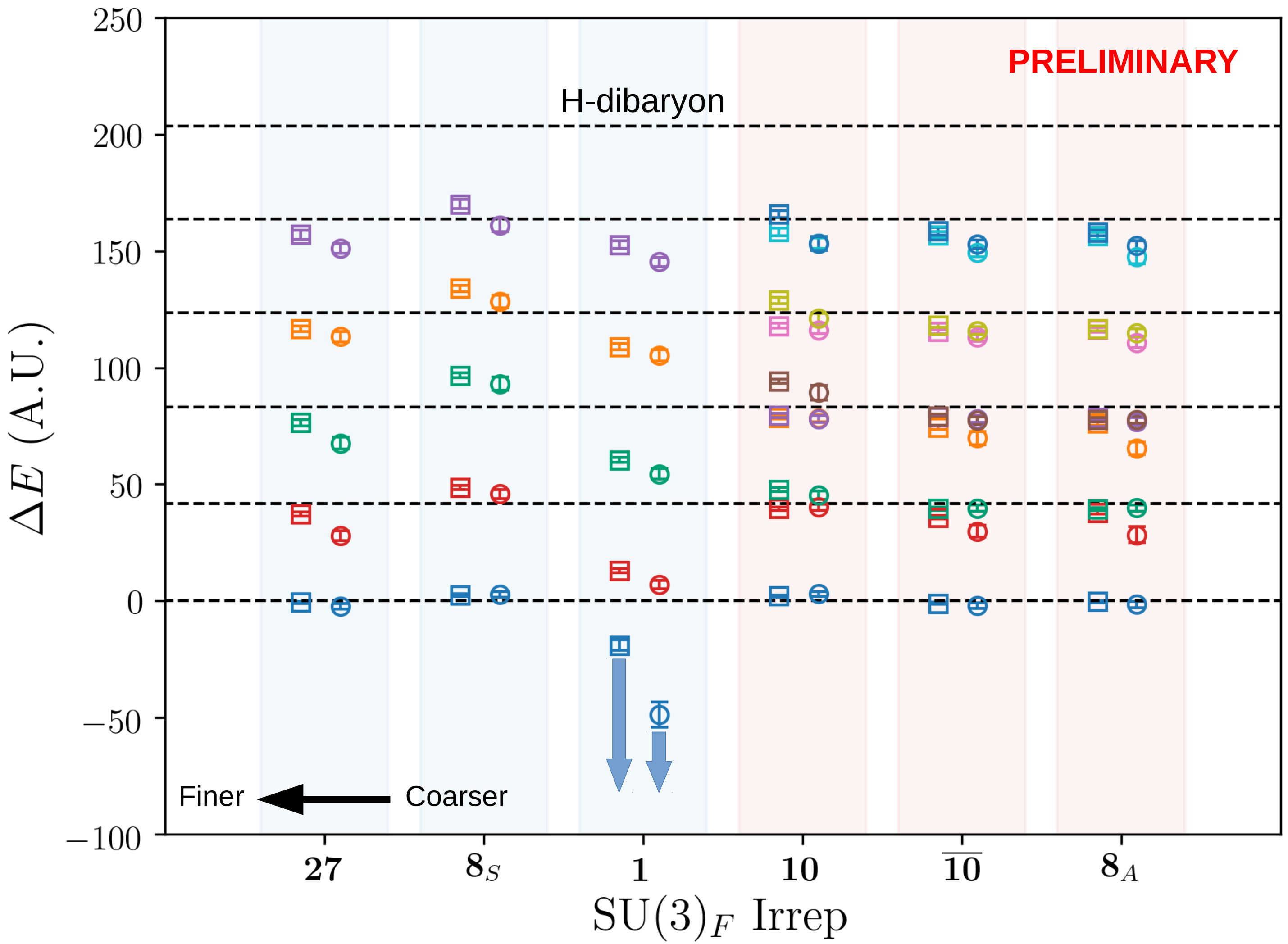


Summary Plot



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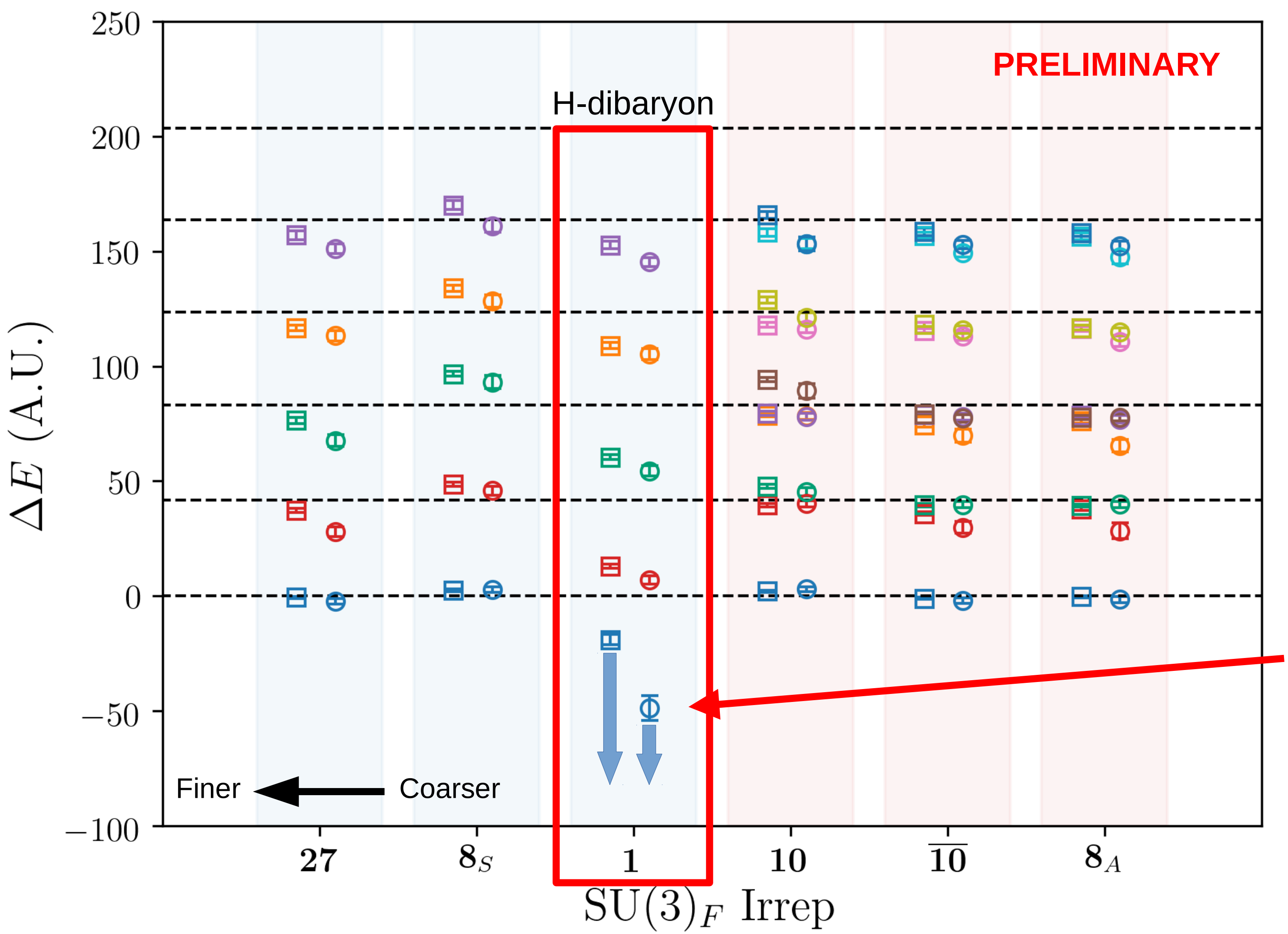

Viewed as bounds, these are **consistent.**
If saturated, then large lattice artifacts

Conclusions

- LQCD has potential to be relevant to a broad set of experimental programs, **provided** we understand systematic errors.
- The variational method is **one** approach to study the (finite volume) energy eigenvalues of the QCD Hamiltonian.
- Computed variational bounds for all SU(3) irreps.
- **If viewed as bounds**, results from two lattice spacings **are consistent**.
- **If bounds have saturated**, evidence for lattice artifacts in flavor singlet (H-dibaryon).
- To make a determination, we think more evidence is required:
  - Explore the impact of other operators on variational bounds
  - Compute (Lanczos, brute force, something else) correlation function to larger Euclidean times.
- With data at two physical volumes, can study interplay of lattice artifacts and finite volume effects.

However, Challenging Data Analysis...

Analysis strategy (Green et al):

1) Fits to logarithm of ratio (**non-convex**)

$$R_n(t) = \frac{\tilde{C}_{\Lambda\Lambda}(t)}{C_{\Lambda}^{\vec{p}_1}(t)\,C_{\Lambda}^{\vec{p}_2}(t)}$$

2) Large statistical uncertainties in region where single-state dominates nucleon correlator.