Lattice QCD study on $\Lambda_c - N$ central and tensor potentials with physical masses

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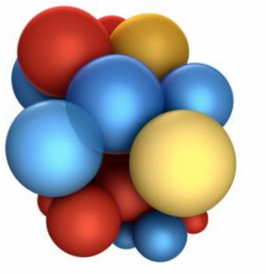
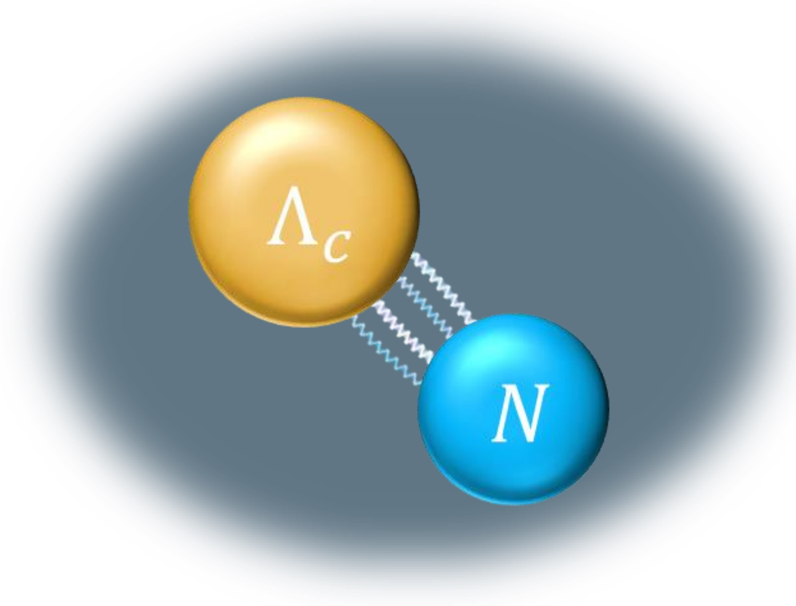
COLLABORATOR: TETSUO HATSUDA, TAKUMI DOI

FOR HAL QCD COLLABORATION

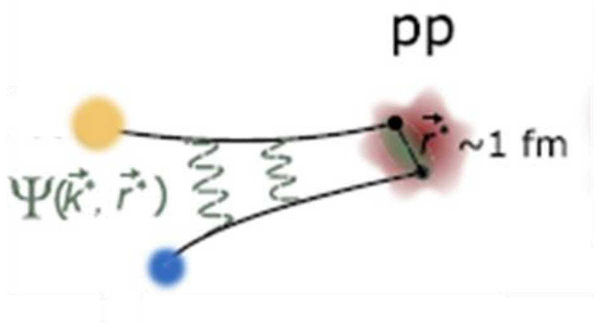
Introduction

Charm hypernucleus

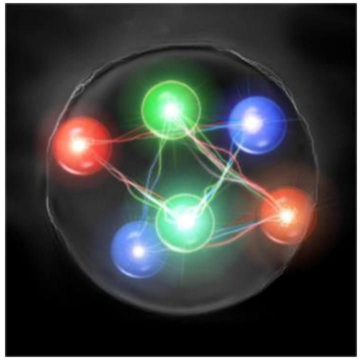


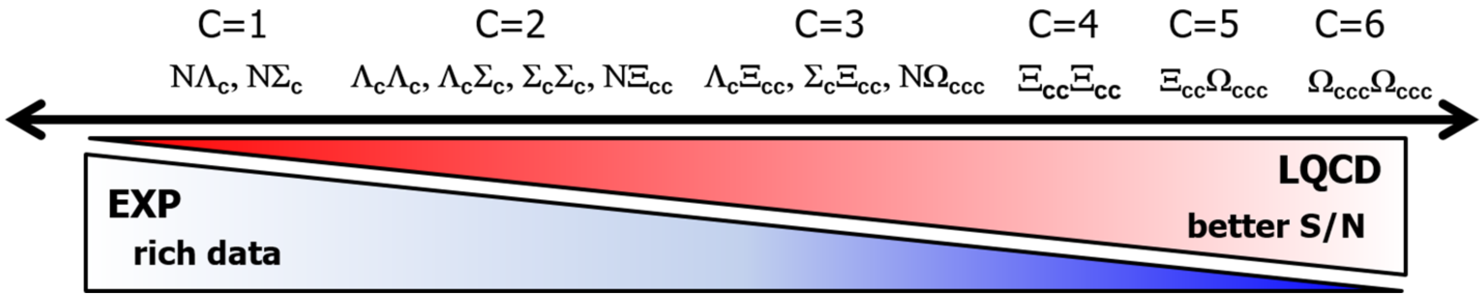


Femtoscopy



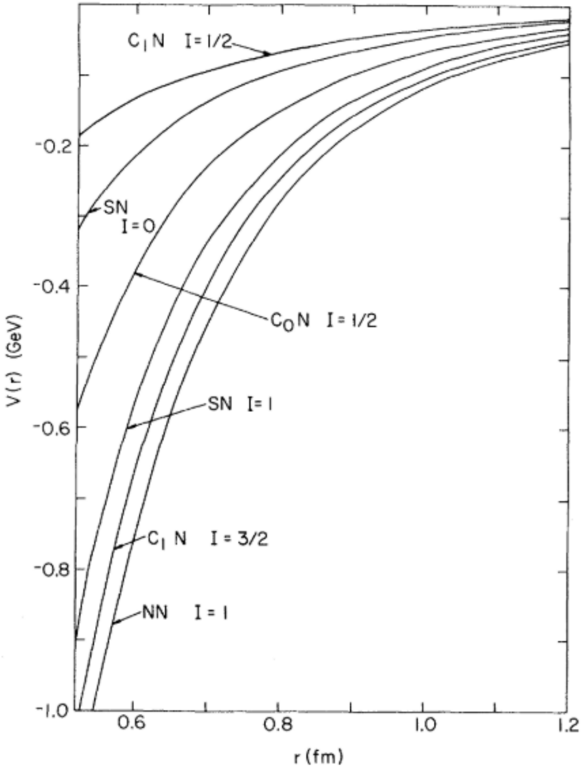
exotic hadron





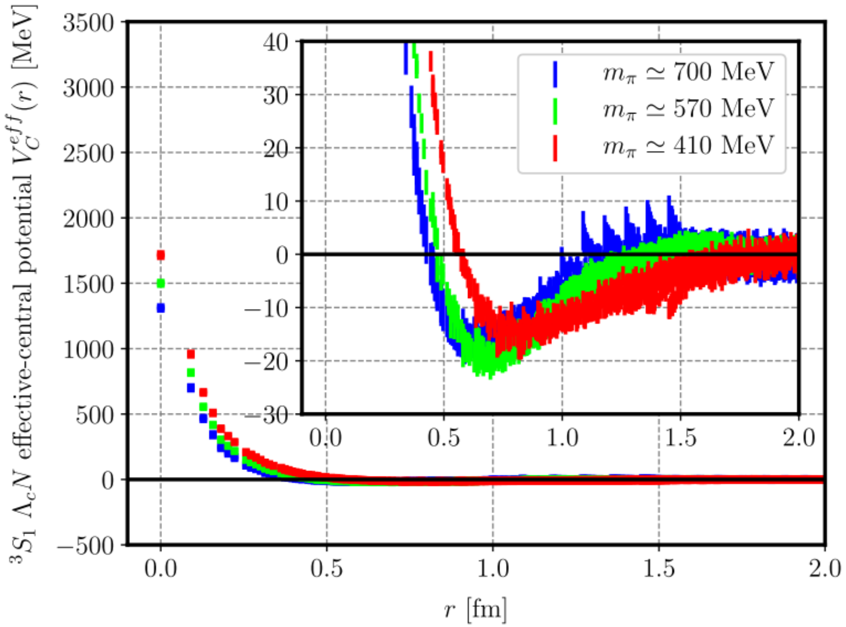
Research on $\Lambda_c - N$

Early investigation of $\Lambda_c - N$ mostly based on Boson Exchange Potential extended to SU(4) flavor symmetry



C. B. Dover and S. H. Kahana, Phys. Rev. Lett. **39**, 1506 (1977).

HAL QCD Collaboration also calculated $\Lambda_c - N$ interaction



T. Miyamoto *et al.*, Nucl. Phys. A **971**, 113 (2018).

Other theories:

- chEFT
- One-σ/ω-Exchange
- quark delocalization color screening model(QDCSM)

……

But above theories predict different:

- Old HAL result → $A \geq 12$
- QDCSM → $^3_{\Lambda_c}H$
- Two chEFT results diverge at tensor potential

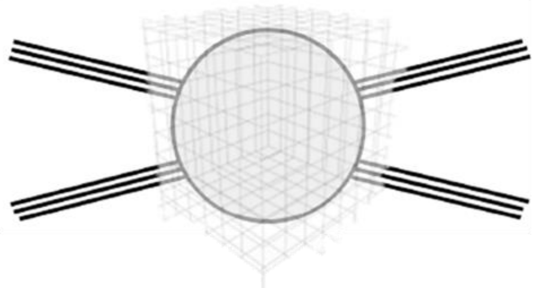
Physical Point Calculation is Needed!

HAL QCD method

HAL QCD method provide a First-Principles calculation method on hadron interaction.

$$\langle 0|B_1(\boldsymbol{x}+\boldsymbol{r})B_2(\boldsymbol{x})|B_1B_2,W_{\boldsymbol{k}}\rangle$$

$$\boxed{\phi_{\mathbf{k}}(\mathbf{r})}e^{-W_{\mathbf{k}}t} \equiv$$

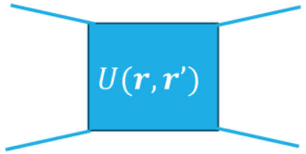


Nambu-Bethe-Salpeter (NBS) wave function

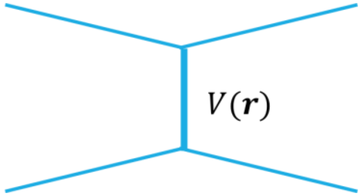
Lattice calculation

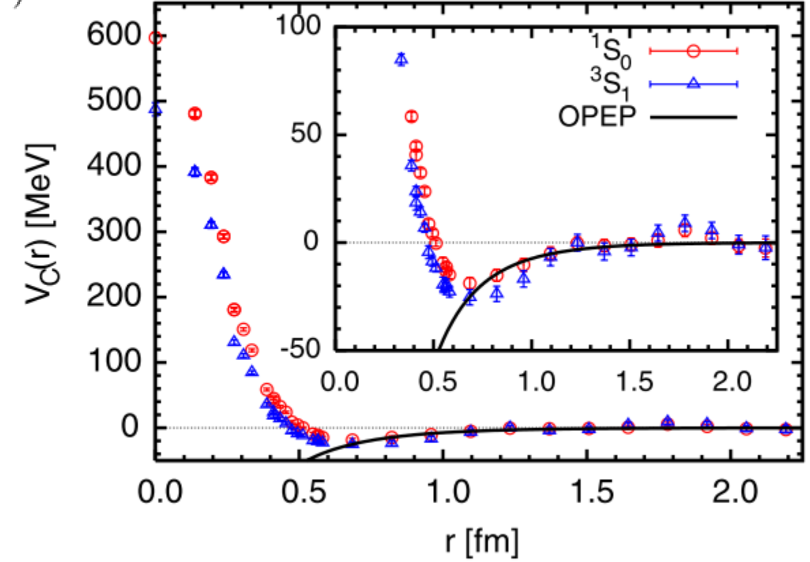
$$V^{\mathrm{LO}}(\mathbf{r},\nabla) = \frac{(E_k - H_0)\phi_k(r)}{\phi_k(r)}$$

$$(E_k - H_0)\phi_{\mathbf{k}}(\mathbf{r}) = \int d^3 r' U(\mathbf{r},\mathbf{r}')\phi_{\mathbf{k}}(\mathbf{r}')$$



$$U(\mathbf{r},\mathbf{r}') = V(\mathbf{r},\nabla)\delta^{(3)}(\mathbf{r}-\mathbf{r}')$$





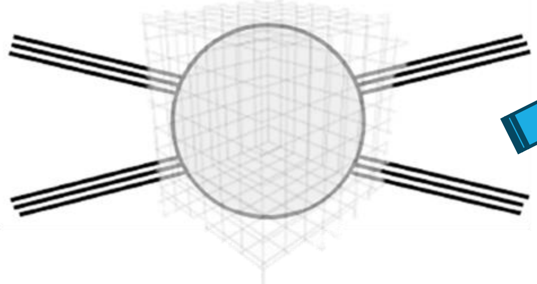
N. Ishii, S. Aoki, and T. Hatsuda, Phys. Rev. Lett. **99**, 5 (2007).
S. Aoki and T. Doi, Front. Phys. **8**, 1 (2020).

HAL QCD method

HAL QCD method provide a First-Principles calculation method on hadron interaction.

$$\boxed{\phi_{\mathbf{k}}(\mathbf{r})} e^{-W_{\mathbf{k}} t} \equiv \langle 0 | B_1(\boldsymbol{x}+\boldsymbol{r}) B_2(\boldsymbol{x}) | B_1 B_2, W_{\boldsymbol{k}} \rangle$$

Nambu-Bethe-Salpeter (NBS) wave function

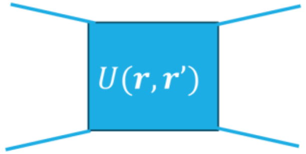


Lattice calculation

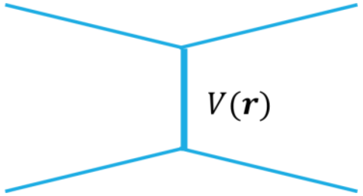
$$\xleftarrow{(W_{k1}-W_{k0})t \gg 1} F^J(\boldsymbol{r}, t-t_0) = \langle 0 | B_1(\boldsymbol{x}+\boldsymbol{r}, t) B_2(\boldsymbol{x}, t) \bar{J}_{B_1 B_2}(t_0) | 0 \rangle = \sum_n A_n^J \phi_{\boldsymbol{k}n}(\boldsymbol{r}) e^{-W_{\boldsymbol{k}n}(t-t_0)}$$

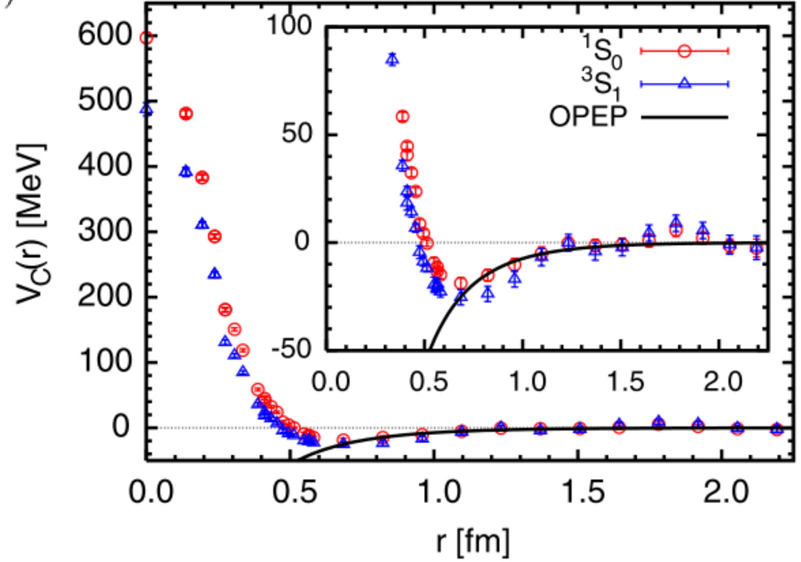
$$V^{\mathrm{LO}}(\mathbf{r}, \nabla) = \frac{(E_k - H_0)\phi_k(r)}{\phi_k(r)}$$

$$(E_k - H_0)\phi_{\mathbf{k}}(\mathbf{r}) = \int d^3 r' U(\mathbf{r}, \mathbf{r}')\phi_{\mathbf{k}}(\mathbf{r}')$$



$$U(\mathbf{r}, \mathbf{r}') = V(\mathbf{r}, \nabla)\delta^{(3)}(\mathbf{r}-\mathbf{r}')$$





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S. Aoki and T. Doi, Front. Phys. **8**, 1 (2020).

HAL QCD method

$$G_B(t-t_0) = \sum_{\boldsymbol{x}} \langle 0|N(\boldsymbol{x}, t-t_0)N(\boldsymbol{0}, t_0)|0\rangle \simeq Z_B e^{-m_B(t-t_0)} + \cdots$$

$$B_n^J = \frac{A_n^J}{Z_{B_1B_2}} \qquad \Delta W_{kn} = W_{kn} - m_{B_1} - m_{B_2}$$

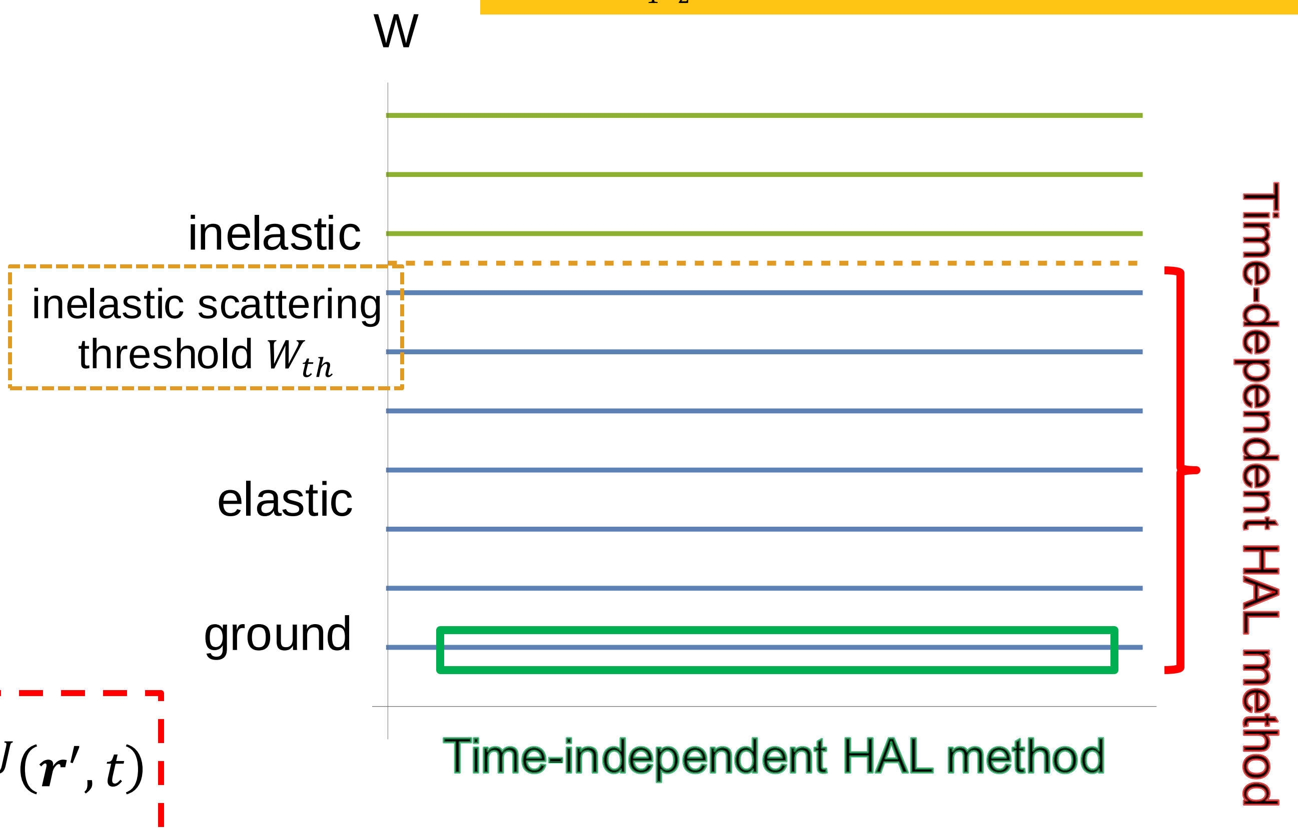
To loosen the restrictions, Time-dependent HAL method was introduced

$$R^J(\boldsymbol{r}, t) = \frac{F^J(\boldsymbol{r}, t)}{G_{B_1}(t)G_{B_2}(t)} = \sum_n B_n^J \phi_{\boldsymbol{kn}}(\boldsymbol{r}) e^{-\Delta W_{kn} t}$$

$$E_n \equiv \frac{k_n^2}{2\mu} = \Delta W_n + \frac{1+3\delta^2}{8\mu}(\Delta W_n)^2 + \mathcal{O}\left((\Delta W_n)^3\right)$$

$$\left(\frac{1+3\delta^2}{8\mu}\partial_t^2 - \partial_t - H_0\right) R^J(\boldsymbol{r}, t) = \int d^3\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}') R^J(\boldsymbol{r}', t)$$

As long as t satisfies the condition that $W_{th} t \gg 1$
We can approximately extract $V(r, \nabla)$



N. Ishii *et al.*, Phys. Lett. Sect. B Nucl. Elem. Part. High-Energy Phys. **712**, 437 (2012).
S. Aoki and T. Doi, Front. Phys. **8**, 1 (2020).

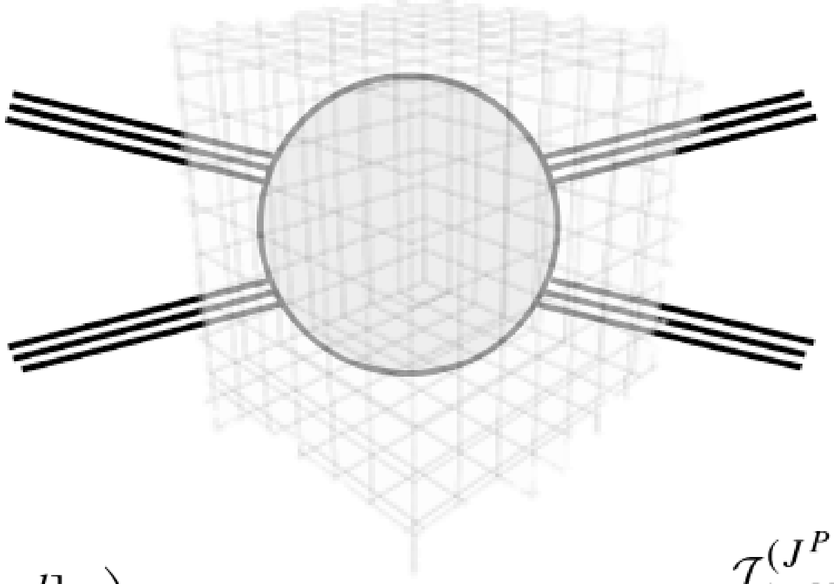
$\Lambda_c - N$ system

$$\langle 0 | N(\boldsymbol{r}+\boldsymbol{x}, t)\Lambda_c(\boldsymbol{x}, t)\mathcal{J}^{1^+}(t_0) | 0 \rangle$$

t $\longleftarrow$ t_0

$N(\vec{x})$ N^{wall}

$$F^{1^+}(\boldsymbol{r}, t-t_0) = \sum_{\vec{x}}$$



$\Lambda_c(\vec{r}+\vec{x})$ $\Lambda_c^{\mathrm{wall}}$

$$\mathcal{J}_{\Lambda_c N}^{(J^P)\mathrm{wall}}(t_0)$$

$$N \equiv \begin{pmatrix} p \\ n \end{pmatrix} = \begin{pmatrix} [ud]u \\ [ud]d \end{pmatrix},$$

$$\Lambda_c = \frac{1}{\sqrt{6}}([cd]u + [uc]d - 2[du]c)$$

Wall Source

$$q^{\mathrm{wall}}(t_0) \equiv \sum_{\vec{x}} q(\vec{x}, t_0)$$

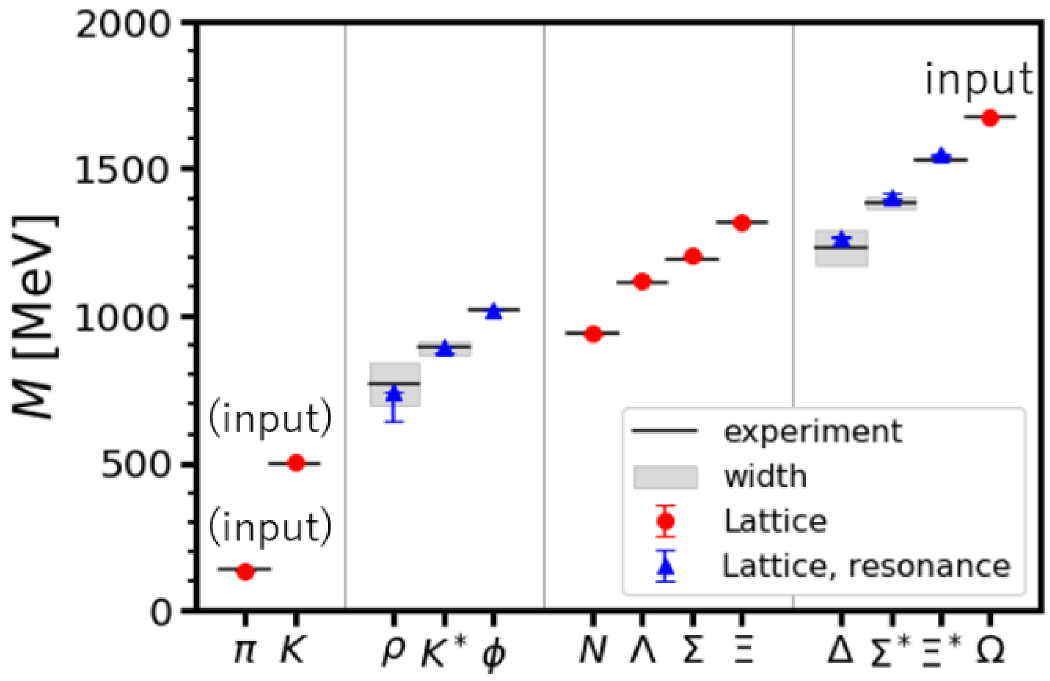
1. Using wall source
2. Calculating 4-point correlation $F^{J^P}(\boldsymbol{r}, t-t_0)$ with source projected to J^P state. Here $J^P = 1^+$
3. Calculate $R^{J^P}(\boldsymbol{r}, t-t_0)$ as time-dependent HAL method
4. Solve effective potential

T. Miyamoto *et al.*, Phys. Rev. D **101**, 74514 (2020).

Configurations

Here we using HAL-conf-2023 to do the calculation
which enable lattice simulations at the physical point, on a large lattice volume and with a large number of ensembles.

- ✓ $2+1$ flavor QCD configurations
- ✓ employing the Iwasaki gauge and $\mathrm{O}(a)$-improved Wilson-clover quark actions.
- ✓ $m_\pi \simeq 137\ MeV,\ m_K \simeq\ 502\ MeV$
- ✓ Size of the lattice is 96^4, corresponding to $(8.1\mathrm{fm})^4$ in physical units
- ✓ 8,000 trajectories
- ✓ $a^{-1} = 2338.8(1.5)(^{+0.2}_{-3.0})$ MeV
- ✓ $\Lambda_c - N$ is the first physical point simulation



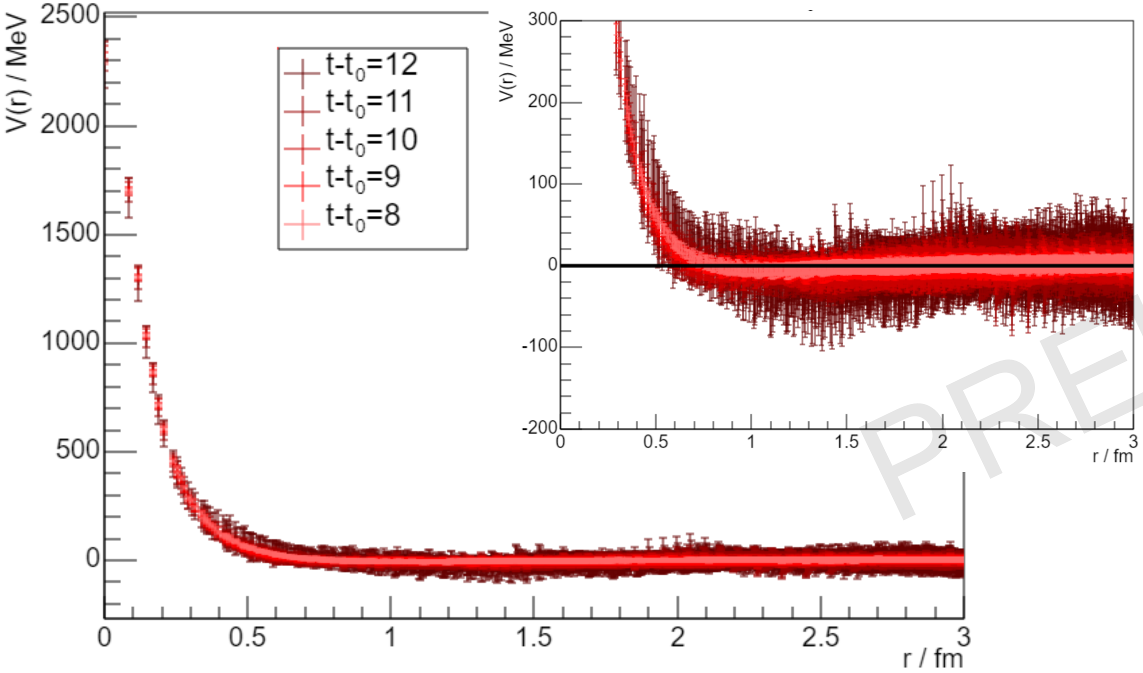
T. Aoyama *et al.*, arXiv:2406.16665.
E. Itou, *PoS(LATTICE2023)* p. 140.

$\Lambda_c - N$ potential

$$P^{(L=0)}R(\vec{r}, t-t_0) \equiv \frac{1}{24}\sum_{g\in SO(3,\mathbb{Z})} R(g^{-1}\vec{r}, t-t_0)$$

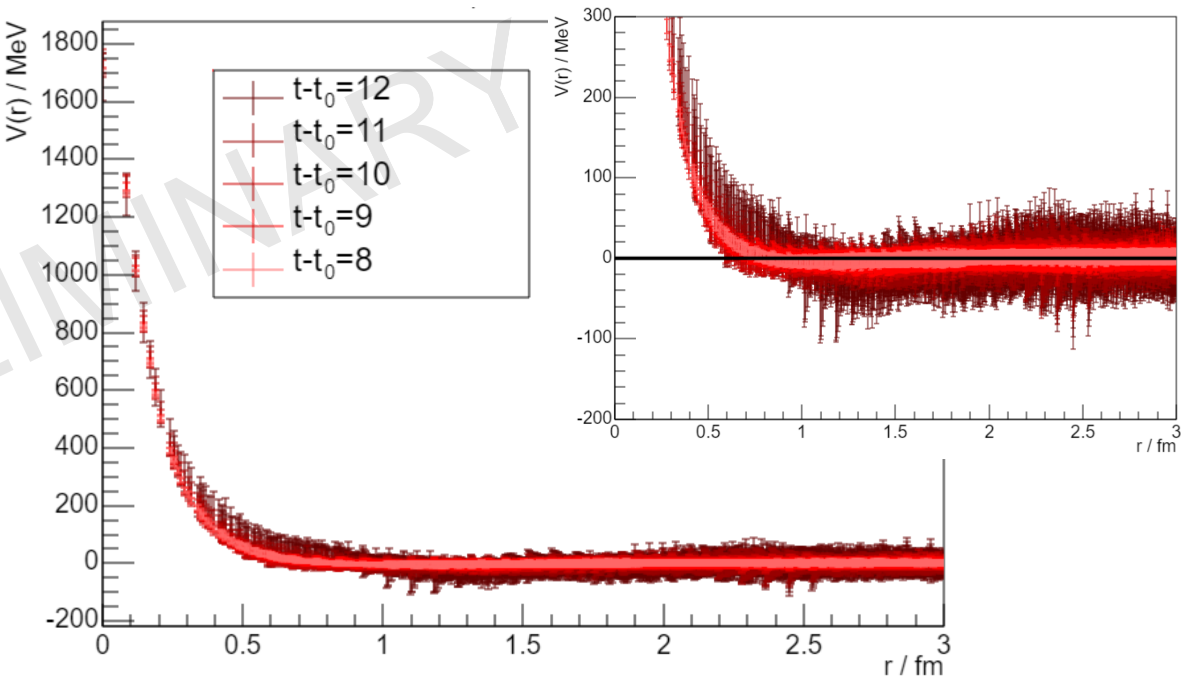
${}^1S_0$ central potential

$$R_{{}^1S_0}(\vec{r}, t-t_0) \equiv P^{(L=0)}P^{(S=0)}_{\beta\alpha}R_{\alpha\beta}(\vec{r}, t-t_0; J^P = 0^+)$$



${}^3S_1$ effective central potential

$$R_{{}^3S_1}(\vec{r}, t-t_0) \equiv P^{(L=0)}P^{(S=1)}_{\beta\alpha}R_{\alpha\beta}(\vec{r}, t-t_0; J^P = 1^+)$$



- Potential getting less attractive at physical point
- Consistent over time within the error bars

- Light Λ_c hypernucleus may not exist

$^3S_1 - {}^3D_1$ coupled channel calculation

To extract tensor force, $^3S_1 - {}^3D_1$ coupled channel effect should be considered.

Firstly, we need to separate the S-wave and D-wave.

$$P_S R_{\alpha\beta}(\vec{r}, t-t_0) \equiv P^{(L=0)} R_{\alpha\beta}(\vec{r}, t-t_0)$$
$$P_D R_{\alpha\beta}(\vec{r}, t-t_0) \equiv \left(1 - P^{(L=0)}\right) R_{\alpha\beta}(\vec{r}, t-t_0)$$

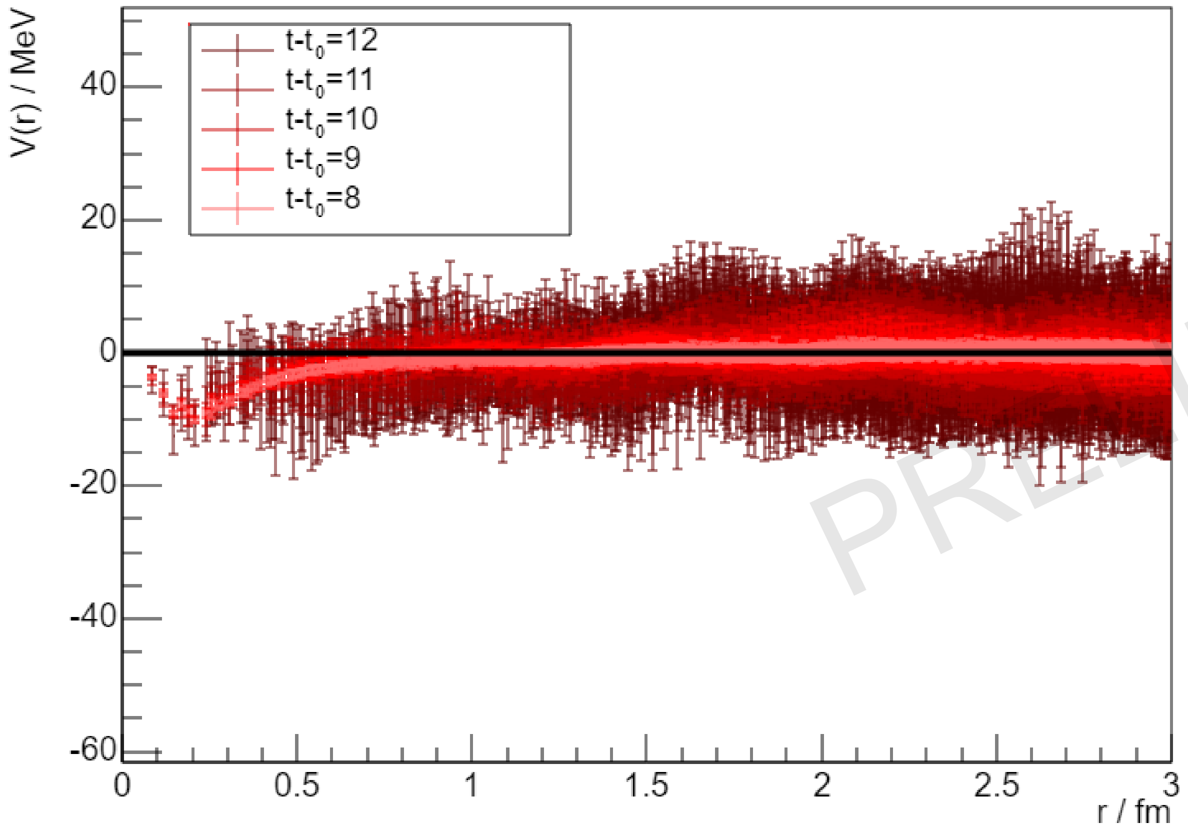
Then these two correlations should follow

$$\mathcal{K}\left[P_S R_{\alpha\beta}(\vec{r}, t-t_0)\right] = V_C^{(1^+)}(\vec{r})\left[P_S R_{\alpha\beta}(\vec{r}, t-t_0)\right] + V_T(\vec{r})\left[P_S\,(S_{12}R)_{\alpha\beta}\,(\vec{r}, t-t_0)\right],$$
$$\mathcal{K}\left[P_D R_{\alpha\beta}(\vec{r}, t-t_0)\right] = V_C^{(1^+)}(\vec{r})\left[P_D R_{\alpha\beta}(\vec{r}, t-t_0)\right] + V_T(\vec{r})\left[P_D\,(S_{12}R)_{\alpha\beta}\,(\vec{r}, t-t_0)\right]$$
$$\mathcal{K} \equiv \left(\frac{1+3\delta^2}{8\mu}\right)\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0$$

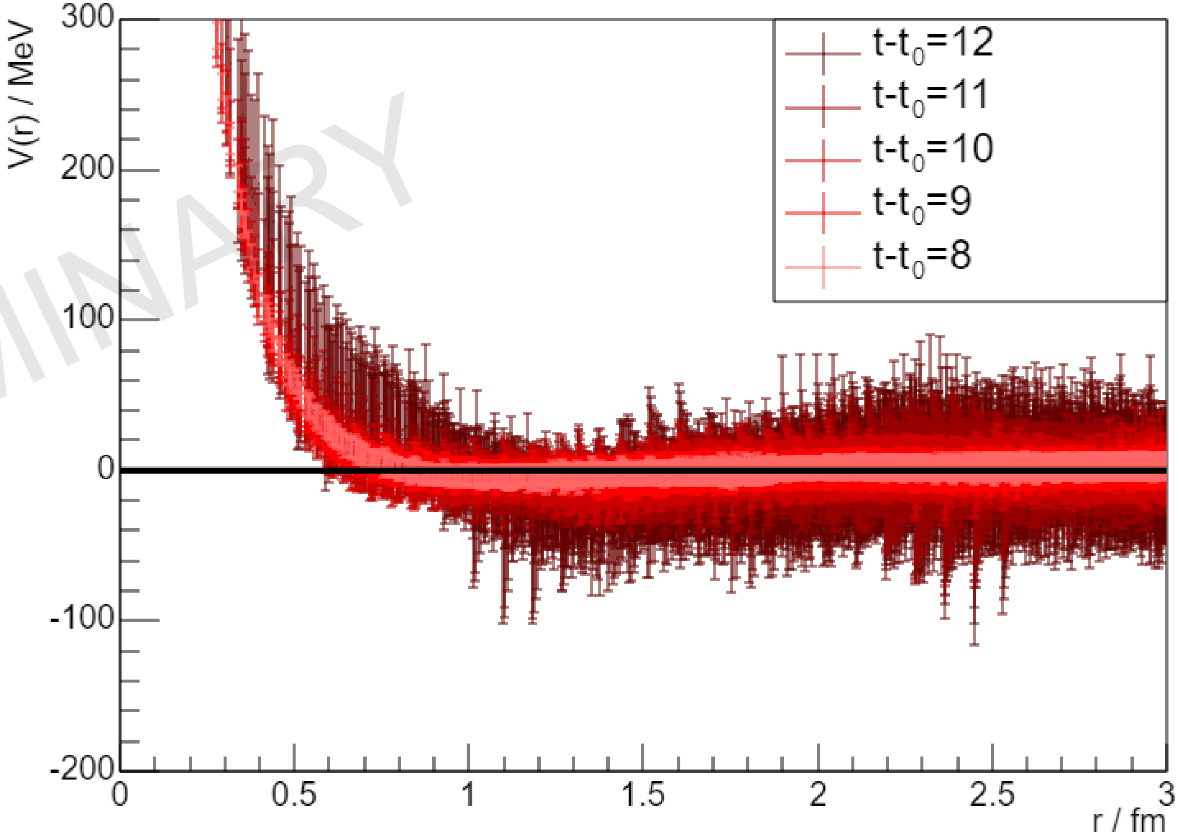
By solving above equation, we can calculate tensor force and central force of $J^P = 1^+$ state

Tensor and $J^P = 1^+$ central potential

Tensor potential



Central potential of $J^P = 1^+$



- Weak tensor potential
- central potential is similar to the $^3S_1$ central effective force

Tensor force has limited effect on $\Lambda_c - N$ $^3S_1$ state calculation

Summary & outlook

Different theories give different prediction on $\Lambda_c - N$ interaction

We perform the first physical point calculation on $\Lambda_c - N$ system by employing "HAL-conf-2023" generated by the HAL Collaboration

Using HAL QCD method to analysis $\Lambda_c - N$ interaction

Reported $^1S_0$ central potential, $^3S_1$ effective potential, tensor potential and $J^P = 1^+$central potential

To do in future:

- Fit the potential and calculate phase shift
- Try to calculate some experiment observations and find possible Λ_c hypernucleus

Thank you for your attention!

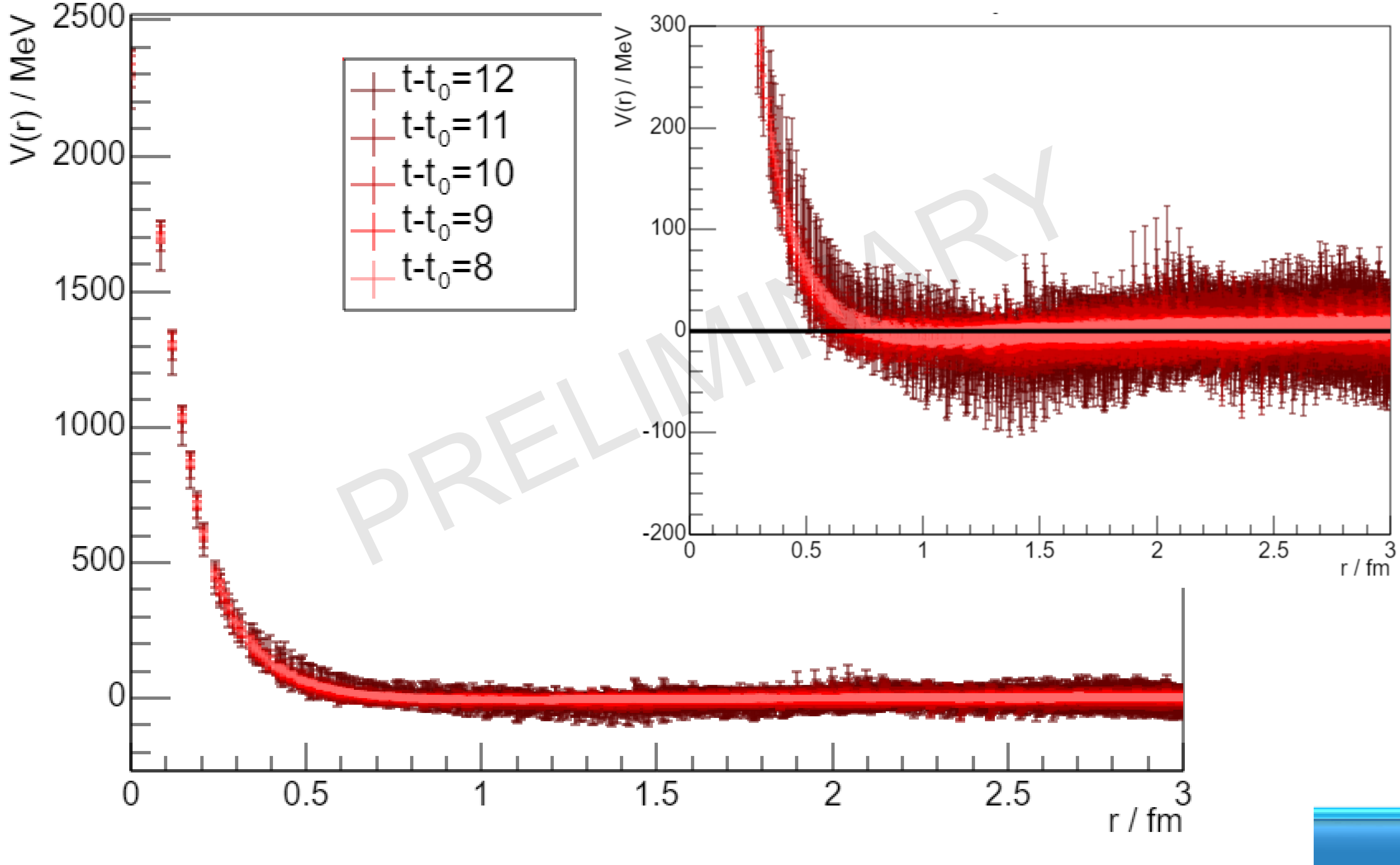
Any question is welcome!

Backup

$\Lambda_c - N\ {}^1S_0$ central potential

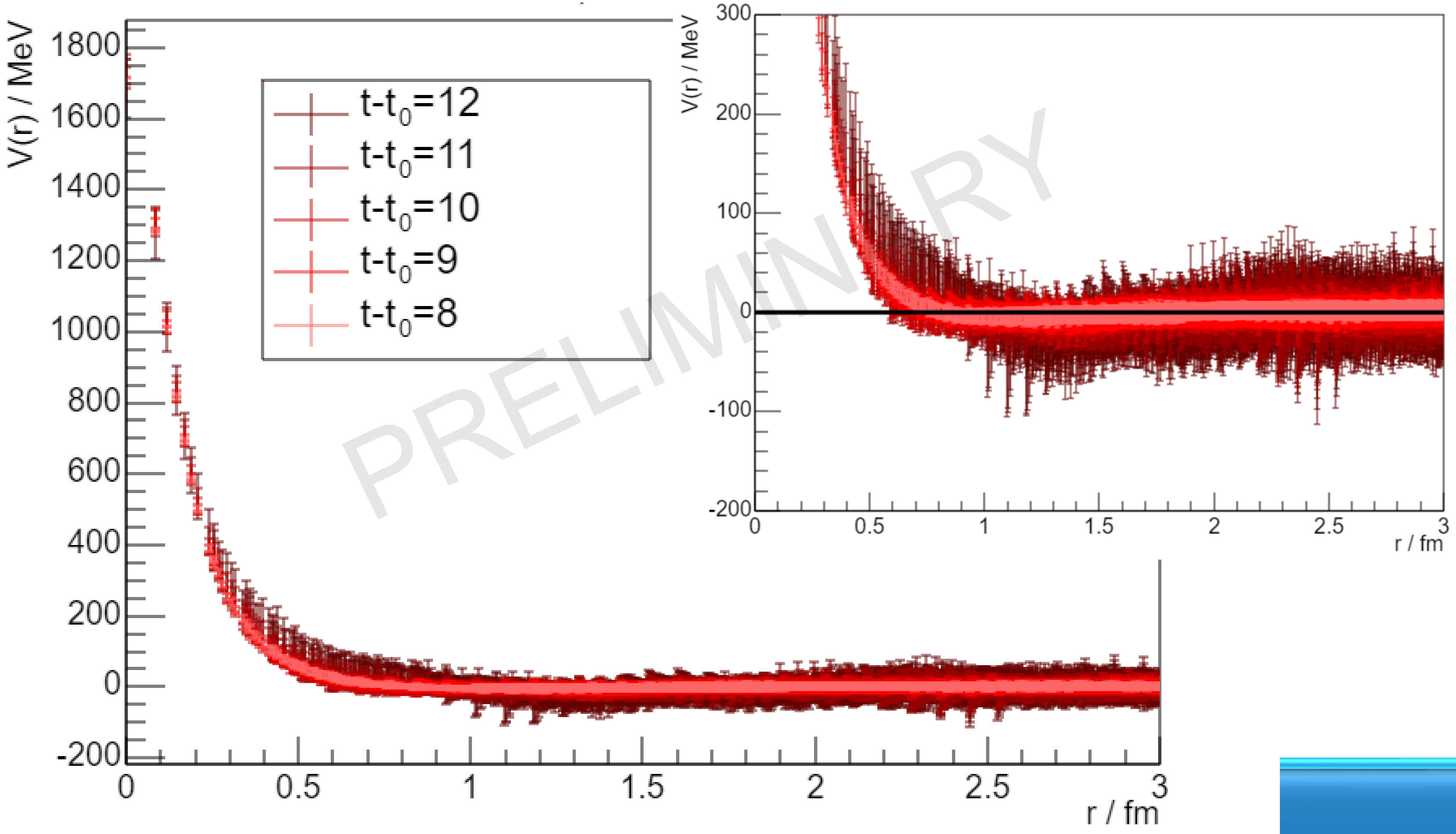
$$R_{^1S_0}(\vec{r}, t - t_0) \equiv P^{(L=0)} P^{(S=0)}_{\beta\alpha} R_{\alpha\beta}(\vec{r}, t - t_0; J^P = 0^+)$$

$$P^{(L=0)} R(\vec{r}, t - t_0) \equiv \frac{1}{24} \sum_{g \in SO(3,\mathbb{Z})} R(g^{-1}\vec{r}, t - t_0)$$



$\Lambda_c - N\ ^3S_1$ effective central potential

$$R_{^3S_1}(\vec{r}, t-t_0) \equiv P^{(L=0)} P^{(S=1)}_{\beta\alpha} R_{\alpha\beta}(\vec{r}, t-t_0; J^P = 1^+)$$



Potential

$$V(\mathbf{r}, \nabla) = \underbrace{V_0(r) + V_\sigma(r)(\sigma_1 \cdot \sigma_2) + V_T(r)S_{12}}_{\mathrm{LO}} + \underbrace{V_{\mathrm{LS}}(r)\mathbf{L} \cdot \mathbf{S}}_{\mathrm{NLO}} + O(\nabla^2),$$

$$V_{LO}(\vec{r}) = V_0(\vec{r}) + V_\sigma(\vec{r})(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T(\vec{r})S_{12};$$

$$S_{12} = 3\frac{(\vec{r} \cdot \vec{\sigma}_1)(\vec{r} \cdot \vec{\sigma}_2)}{|\vec{r}|^2} - (\vec{\sigma}_1 \cdot \vec{\sigma}_2),$$

Reference

C. B. Dover and S. H. Kahana, Phys. Rev. Lett. **39**, 1506 (1977).

T. Miyamoto *et al.*, Nucl. Phys. A **971**, 113 (2018).

J. Haidenbauer and G. Krein, Eur. Phys. J. A 2018 5411 **54**, 1 (2018).

J. Song *et al.*, Phys. Rev. C **102**, 1 (2020).

S. Wu *et al.*, Phys. Rev. C **109**, 014001 (2024).

R. Chen, A. Hosaka, and X. Liu, Phys. Rev. D **96**, 1 (2017).

J. Haidenbauer and G. Krein, arXiv:2101.07160 (2021).

N. Ishii, S. Aoki, and T. Hatsuda, Phys. Rev. Lett. **99**, 5 (2007).
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chEFT

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