Lattice QCD calculation of the semileptonic decay $J/\psi \to D/D_s l\nu_l$

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J/ψ decay channels

- Decays involving hadronic resonances
- Decays into stable hadrons
- Radiative decays
- Dalitz decays
- Weak decays

Γ_{364}	$D^- e^+ \nu_e$ + c.c.	$< 7.1 \times 10^{-8}$	CL=90%	984
Γ_{365}	$\overline{D}^0 e^+ e^-$ + c.c.	$< 8.5 \times 10^{-8}$	CL=90%	987
Γ_{366}	$D_s^- e^+ \nu_e$ + c.c.	$< 1.3 \times 10^{-6}$	CL=90%	923
Γ_{367}	$D_s^{*-} e^+ \nu_e$ + c.c.	$< 1.8 \times 10^{-6}$	CL=90%	828
Γ_{368}	$D^- \pi^+$ + c.c.	$< 7.5 \times 10^{-5}$	CL=90%	977
Γ_{369}	$\overline{D}^0 \overline{K}^0$ + c.c.	$< 1.7 \times 10^{-4}$	CL=90%	898
Γ_{370}	$\overline{D}^0 \overline{K}^{*0}$ + c.c.	$< 2.5 \times 10^{-6}$	CL=90%	670
Γ_{371}	$D_s^- \pi^+$ + c.c.	$< 1.3 \times 10^{-4}$	CL=90%	915
Γ_{372}	$D_s^- \rho^+$ + c.c.	$< 1.3 \times 10^{-5}$	CL=90%	663

- Semileptoinc decay: $J/\psi \to D/D_s l \nu_l$ this work
- Phenomenological aspect: plenties of studies on hadronic and radiative decay, less on semileptonic decay(Br $< 10^{-8}$) $\longleftarrow$ limited by the experimental detection

J/ψ number by BESIII

Item	2017-2019	2012	2009
$N_{\rm sel}(\times 10^6)$	6912.03 ± 0.08	860.59 ± 0.03	180.84 ± 0.01
$N_{\rm bg}(\times 10^6)$	118.66 ± 0.05	15.32 ± 0.02	6.89 ± 0.04
$\epsilon_{\rm trig}$	1.00	1.00	1.00
$\epsilon_{\rm data}^{\psi(3686)}$	0.7680 ± 0.0005	0.7699 ± 0.0005	0.7707 ± 0.0001
$\epsilon_{\rm MC}^{\psi(3686)}$	0.7693 ± 0.0002	0.7709 ± 0.0002	0.7723 ± 0.0002
$\epsilon_{\rm MC}^{J/\psi}$	0.7756 ± 0.0001	0.7776 ± 0.0001	0.7780 ± 0.0001
$f_{\rm cor}$	1.0082 ± 0.0007	1.0086 ± 0.0008	1.0074 ± 0.0003
$N_{J/\psi}(\times 10^6)$	8774.0 ± 0.2	1088.5 ± 0.1	224.0 ± 0.1

BESIII,CPC46,074001(2022)

- Total J/ψ samples: $1.0087(44) \times 10^{10}$
- It is time to study the J/ψ rare decay.

Experimental searches

- Completed measurements

channels	Upper limit	J/ψ number	Refs
$J/\psi \to D_s e \nu_e$	4.9×10^{-5}	5.8×10^{7}	PLB639,418(2006)
$J/\psi \to D_s e \nu_e$	1.3×10^{-6}	2.3×10^{8}	PRD90,112014(2014)
$J/\psi \to D e \nu_e$	7.1×10^{-8}	1.01×10^{10}	JHEP06,157(2021)
$J/\psi \to D \mu \nu_\mu$	5.6×10^{-7}	1.01×10^{10}	JHEP01,126(2024)

BES & BESIII collaboration

- Future measurements ?

channels	Upper limit	J/ψ number	Refs
$J/\psi \to D_s e \nu_e$	—	1.01×10^{10}	BESIII
$J/\psi \to D_s \mu \nu_\mu$	—	1.01×10^{10}	BESIII
$J/\psi \to D_s e \nu_e$	—	$\sim 10^{12}$	STCF
$J/\psi \to D_s \mu \nu_\mu$	—	$\sim 10^{12}$	STCF

Phenomenological studies

$J/\psi \to$	$D_s e \nu_e \cdot 10^{-10}$	$D_s \mu \nu_\mu \cdot 10^{-10}$	$D e \nu_e \cdot 10^{-11}$	$D \mu \nu_\mu \cdot 10^{-11}$	Ref
QCDSR	$1.8^{+0.7}_{-1.5}$	$1.7^{+0.7}_{-0.5}$	$0.73^{+0.43}_{-0.22}$	$0.71^{+0.42}_{-0.22}$	EPJC54,107(2008).
BS	$3.67^{+0.52}_{-0.44}$	$3.54^{+0.50}_{-0.43}$	$2.03^{+0.29}_{-0.25}$	$1.98^{+0.28}_{-0.24}$	JPG44,045004(2017)
CCQM	3.3	3.2	1.71	1.66	PRD92,074030(2015)
BSW	$10.4^{+0.90}_{0.75}$	$9.93^{+0.95}_{-0.65}$	$6.0^{+0.8}_{-0.7}$	$5.8^{+0.8}_{-0.6}$	AHEP2013,706543 (2013)
CLFQ	$10.21^{+0.89}_{-1.55}$	$9.59^{+0.90}_{-1.47}$	$6.10^{+0.20}_{-0.25}$	$5.78^{+0.22}_{-0.20}$	EPJC84,65(2024)

a. QCD sum rules (QCDSR)
b. Bethe-Salpeter (BS)
c. Confined covariant quark model (CCQM)
d. Covariant light-front quark model(CLFQM)
e. Bauer-Stech-Wirbel(BSW)

- Significant discrepancy between different models, and they are almost unable to be used to extract CKM matrix element $V_{cs(d)}$ by combining with the future experiment.
- A genuine nonperturbative lattice calculation is essential

$J/\psi \to D/D_s l\nu_l$

- The amplitude

$$i\mathcal{M} = -i\frac{G_F}{\sqrt{2}}V_{cs(d)}\epsilon_\alpha(p')H_{\mu\alpha}(p,p')g_{\mu\nu}\bar{u}_l\gamma_\nu(1-\gamma_5)u_{\nu_l}$$

with the nonperturbative hadronic interaction ZPC46,93(1990)

$$\begin{aligned} H_{\mu\alpha}(p,p') &\equiv \langle D/D_s(p)|J_\mu^W|J/\psi_\alpha(\epsilon,p')\rangle \\ &= F_1(q^2)g_{\mu\alpha} + \frac{F_2(q^2)}{Mm}p'_\mu p_\alpha + \frac{F_3(q^2)}{m^2}p_\mu p_\alpha - \frac{iF_0}{Mm}\epsilon_{\mu\alpha\rho\sigma}p'_\rho p_\sigma \end{aligned}$$

- The decay width

$$\begin{aligned} \Gamma &= \frac{G_F^2 V_{cs(d)}^2}{12M^2}\frac{1}{32\pi^3}\int_{m_l^2}^{(M-m)^2} dq^2 \times \Big[c_0(E_l^+ - E_l^-) \\ &+ \frac{c_1}{2}((E_l^+)^2 - (E_l^-)^2) + \frac{c_2}{3}((E_l^+)^3 - (E_l^-)^3)\Big] \end{aligned}$$

with $E_l^\pm = \frac{1}{2M}\left[q^2 + m_l^2 - \frac{1}{2q^2}\left((q^2 - M^2 + m^2)(q^2 + m_l^2) \mp 2M|\vec{p}|(q^2 - m_l^2)\right)\right]$

Extracting F_0 on the lattice

- Euclidean hadronic function in the infinite volume

$$
\begin{aligned}
H_{\mu\nu}(\vec{x},t) &= \langle 0|\phi_h(\vec{x},t)J_\mu^W(0)|J/\psi_\nu(\epsilon,p')\rangle, t>0 \\
&\doteq \int \frac{d^3\vec{p}}{(2\pi)^3}\frac{1}{2E_h}e^{-E_h t+i\vec{p}\cdot\vec{x}}\langle 0|\phi_h(0)|\phi_h(\vec{p})\rangle\langle\phi_h(\vec{p})|J_\mu^W(0)|J/\psi_\nu(p')\rangle
\end{aligned}
$$

- Considering the parameterizations

$$
\begin{aligned}
\langle 0|\phi_h(0)|\phi_h(\vec{p})\rangle &= Z_h \\
\langle\phi_h(\vec{p})|J_\mu^V(0)|J/\psi_\nu(\epsilon,p')\rangle &= \frac{F_0}{Mm}\epsilon_{\mu\alpha\rho\sigma}p'_\rho p_\sigma
\end{aligned}
$$

- The spatial Fourier transform of $V_{\mu\nu}$, defined by $H_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$

$$
\tilde{V}_{\mu\nu}(\vec{p},t) \doteq \frac{F_0(q^2)}{Mm}\frac{Z_h}{2E_h}e^{-E_h t}\epsilon_{\mu\nu\rho\sigma}p'_\rho p_\sigma
$$

- Constructing a scalar function by multiplying $\epsilon_{\mu\nu\rho\sigma}p'_\rho p_\sigma$ on two sides

Extracting F_0 on the lattice

- Scalar function method

$$F_0(q^2) = \frac{mE_h}{Z_h} e^{E_h t} \int d^3\vec{x} \frac{j_1(|\vec{p}||\vec{x}|)}{|\vec{p}||\vec{x}|} \epsilon_{\mu\nu\alpha 0} x_\alpha V_{\mu\nu}(\vec{x}, t)$$

with $V_{\mu\nu}(\vec{x}, t) \equiv \langle 0|D/D_s(\vec{x}, t) J^V_\mu(0)|J/\psi_\nu(p')\rangle$ calculated on lattice

- A similar scheme has been used for high-precision calculation
  - $\Gamma(\eta_c \to 2\gamma) = 6.67(16)(6)\mathrm{keV}$, Y.M et al,Science Bulletin68,1880(2023)
    2.9 σ tension with the PDG value, verified by HPQCD PRD108,014513(2023)
  - $\Gamma(D_s^* \to D_s\gamma) = 0.0549(54)\mathrm{keV}$ Y.M et al,PRD109,074511(2024)
    A more improved precision compared to previous result by HPQCD, PRL112,212002(2014)
  - π^0-pole's contribution to HLBL, Tian Lin's talk 14:55 Mon.

Extracting $F_i (i = 1, 2, 3)$ on the lattice

- Similarly, we have

$$F_1(q^2) = \frac{2E_h e^{E_h t}}{3m^2 Z_h} \left[E_h^2 I_2 - E_h |\vec{p}|^2 (I_3 + I_4) - m^2 I_1 - |\vec{p}|^2 I_5)\right]$$
$$F_2(q^2) = \frac{2E_h e^{E_h t}}{m Z_h} \left[E_h I_2 - E_h^2 I_4 - E_h I_5 - |\vec{p}|^2 I_3\right]$$
$$F_3(q^2) = \frac{2E_h e^{E_h t}}{3m^2 Z_h} \left[E_h^2 I_2 + 3m_h^2 (E_h I_4 + I_5) - m^2 I_1 - |\vec{p}|^2 (E_h I_3 + E_h I_4 + I_5)\right]$$

$$I_1 = \int d^3\vec{x} j_0(|\vec{p}||\vec{x}|) \delta_{\mu\nu} A_{\mu\nu}(\vec{x}, t)$$
$$I_2 = \int d^3\vec{x} j_0(|\vec{p}||\vec{x}|) A_{00}(\vec{x}, t)$$
$$I_3 = \int d^3\vec{x} \frac{j_1(|\vec{p}||\vec{x}|)}{|\vec{p}||\vec{x}|} x_i A_{0i}(\vec{x}, t)$$
$$I_4 = \int d^3\vec{x} \frac{j_1(|\vec{p}||\vec{x}|)}{|\vec{p}||\vec{x}|} x_i A_{i0}(\vec{x}, t)$$
$$I_5 = \int d^3\vec{x} \left\{ \frac{j_1(|\vec{p}||\vec{x}|)}{|\vec{p}||\vec{x}|} \delta_{ij} - |\vec{p}|^2 \frac{j_2(|\vec{p}||\vec{x}|)}{(|\vec{p}||\vec{x}|)^2} x_i x_j \right\} A_{ij}(\vec{x}, t)$$

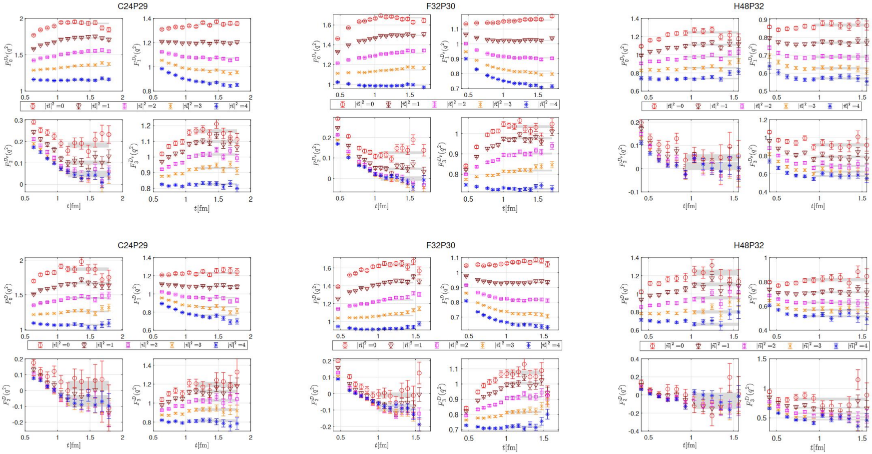
- Hadroinc function $A_{\mu\nu}(\vec{x}, t) \equiv \langle 0 | D/D_s(\vec{x}, t) J_\mu^A(0) | J/\psi_\nu(p') \rangle$ is calculatable directly.
- The E_h, Z_h are extracted from two-point function $C_2(\vec{p}, t) = \frac{Z_h^2}{2E_h} \left(e^{-E_h t} + e^{-E_h (T-t)}\right)$ using a single-state fitting.

Lattice setup

Ensemble	C24P29	F32P30	H48P32
a(fm)	0.10530(18)	0.07746(18)	0.05187(26)
$a\mu_s$	-0.2400	-0.2050	-0.1700
$a\mu_c$	0.4479	0.2079	0.0581
$L^3 \times T$	$24^3 \times 72$	$32^3 \times 96$	$48^3 \times 144$
$N_{\text{cfg}} \times N_{\text{src}}$	450×72	719×96	100×72
m_π(MeV)	292.7(1.2)	303.2(1.3)	317.2(0.9)
t	6-17	6-20	8-30
Z_V	0.79814(23)	0.83548(12)	0.86855(04)
Z_A	0.85442(85)	0.88161(64)	0.90113(36)

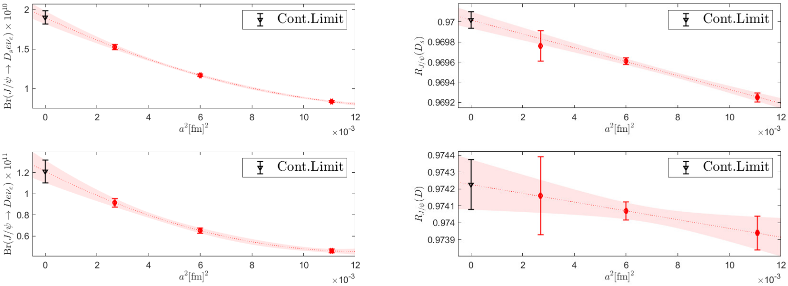
- (2+1)-flavor Wilson-clover gauge ensembles by CLQCD collaboration CLQCD,PRD109,054507(2024)
- Similar pion mass ~ 300 MeV, volume ~ 2.5 fm, more fine lattice spacing $\Rightarrow$ continuum limit
- Charm quark mass $a\mu_c$ is tuned by physical J/ψ mass

Form factors



- Correlated fit to a constant at suitable time region $\sim [0.8, 1.7]$ fm for all ensembles
- A polynomial form $F_i(q^2) = d_i^{(0)} + d_i^{(1)} \cdot q^2 + d_i^{(2)} \cdot q^4$ describes lattice data well

Decay width



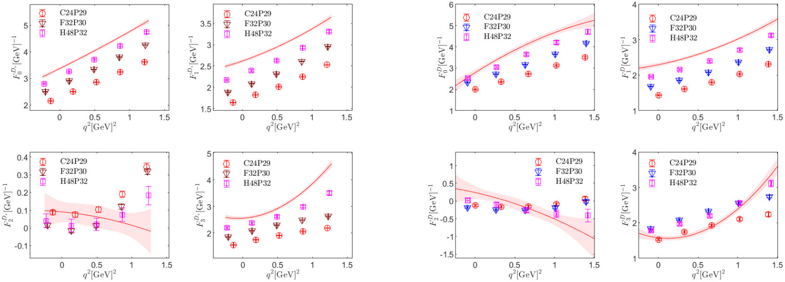
- The branching fraction with $V_{cs} = 0.975(6), V_{cd} = 0.221(4)$ Y.M et al, 2407.13568

$$\begin{aligned} \mathrm{Br}(J/\psi \to D_s e \nu_e) &= 1.90(6)_{\mathrm{stat}}(5)_{V_{cs}} \times 10^{-10} \\ \mathrm{Br}(J/\psi \to D e \nu_e) &= 1.21(6)_{\mathrm{stat}}(9)_{V_{cd}} \times 10^{-11} \end{aligned}$$

- The ratio between μ and e

$$\begin{aligned} R_{J/\psi}(D_s) &= 0.97002(8)_{\mathrm{stat}} \\ R_{J/\psi}(D) &= 0.97423(15)_{\mathrm{stat}} \end{aligned}$$

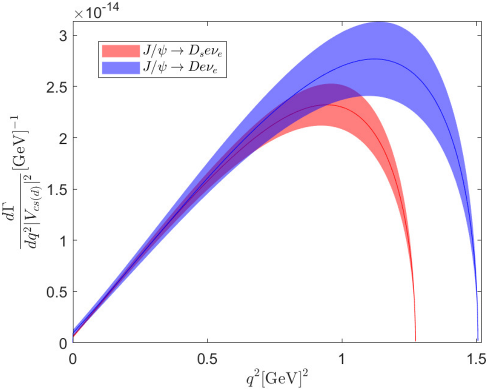
Differential decay width



- The polynomial q^2-expansion

$$F^{D/D_s}(a^2, q^2) = \sum_{n=0}^{n_{\max}} (c_n + d_n a^2 + f_n a^4) q^{2n}$$

Differential decay width



- The experimental inputs $m_{J/\psi} = 3.09690(1)$ GeV, $m_{D_s} = 1.96834(7)$ GeV, and $m_D = 1.86966(5)$ GeV
- A potential test by future Super Tau Charm Facility with expected 10^{12} J/ψ samples Front. Phys. (Beijing) 19, 14701(2024)

Relationship with traditional approach

- The traditional parameterization for $P \to V$ semileptonic decay
  Rev.Mod.Phys 67,893(1995)

$$\langle V(\epsilon,p)|J^W_\mu|P(p')\rangle = \epsilon_{\mu\alpha\beta\delta}\epsilon_\alpha(p',\lambda)p'^\beta p^\delta \frac{2V(q^2)}{M+m} + 2MA_0(q^2)\frac{\epsilon(p',\lambda)\cdot q}{q^2}q_\mu$$
$$+ \quad (M+m)A_1(q^2)\left[\epsilon_\mu(p',\lambda) - \frac{\epsilon(p',\lambda)\cdot q}{q^2}q_\mu\right] - A_2(q^2)\frac{\epsilon(p',\lambda)\cdot q}{M+m}\left[p'_\mu + p_\mu - \frac{M^2-m^2}{q^2}q^\mu\right]$$

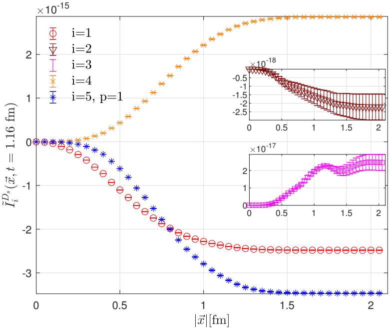
  - with a kinematic constraint $A_0(0) = \frac{m+M}{2M}A_1(0) - \frac{M-m}{2M}A_2(0)$

- Relationship with the form factor $F_i(i=0,1,2,3)$

$$A_0 = \frac{1}{2M}\left(F_1 - \frac{M^2-m^2+q^2}{2mM}F_2 - \frac{M^2-m^2-q^2}{2m^2}F_3\right)$$
$$A_1 = \frac{F_1}{m+M}, \quad A_2 = \frac{m+M}{2m^2M}(mF_2+MF_3), \quad V = \frac{(m+M)}{2mM}F_0$$

  - Numerical simulations get $F_2(0) \simeq 0$, leading to the kinematic constraint on A_0, A_1, A_2 above

Finite-volume effects



$$\tilde{I}_0 = \epsilon_{\mu\nu\alpha 0} x_\alpha V_{\mu\nu}(\vec{x}, t)$$
$$\tilde{I}_1 = \delta_{ij} A_{ij}(\vec{x}, t)$$
$$\tilde{I}_2 = A_{00}(\vec{x}, t)$$
$$\tilde{I}_3 = x_i A_{0i}(\vec{x}, t)$$
$$\tilde{I}_4 = x_i A_{i0}(\vec{x}, t)$$
$$\tilde{I}_5 = \frac{j_2(|\vec{p}||\vec{x}|)}{(|\vec{p}||\vec{x}|)^2} x_i x_j A_{ij}(\vec{x}, t)$$

- The contribution of $|\vec{x}| \gtrsim 1.5$ fm is much small, finite-volume effects are under controll
- Combining with our checks on $\eta_c \to 2\gamma$ and $D_s^* \to D_s\gamma$ in the same way, the replacement $H^L_{\mu\nu}(\vec{x}, t) \to H_{\mu\nu}(\vec{x}, t)$ is straightforward when the intermediate state is charm or heavier.

Discussion

- The method can be applied to various $P \to V$ semileptonic decay, for example, $D_s \to \phi$, $D \to K^*$, $B \to K^*$, $B \to D^*$, $B \to J/\psi$, $\cdots$
- When the final state is a light hadron, e.g. ϕ, K^*, the exponentially suppressed finite-volume effects may not be ignored. In that case, one can use, for example, the infinite volume reconstruction method to deal with the problem.
  Xin-Yu Tuo et al,PRD105,054518(2022)
- With the potential input of future experiments, the lepton flavor universality can be checked and the CKM matrix element $V_{cs(d)}$ can also be extracted.

Conclusion and outlook

- Conclusion
  - We present the first lattice calculation of $J/\psi \to D/D_s$ semileptonic decay using Wilson-clover gauge ensembles by CLQCD.
  - Branching fraction of $J/\psi \to D/D_s$ are determined as

$$\begin{aligned}
\mathrm{Br}(J/\psi \to D_s e\nu_e) &= 1.90(6)_{\mathrm{stat}}(5)_{V_{cs}} \times 10^{-10} \\
\mathrm{Br}(J/\psi \to D e\nu_e) &= 1.21(6)_{\mathrm{stat}}(9)_{V_{cd}} \times 10^{-11} \\
R_{J/\psi}(D_s) &= 0.97002(8)_{\mathrm{stat}} \\
R_{J/\psi}(D) &= 0.97423(15)_{\mathrm{stat}}
\end{aligned}$$

  - The method can be generally applied to various $P \to V$ semileptonic decay.
- Outlook
  - The effects from the neglected disconnected diagrams, the quenching of the charm quark, and nonphysical light quark masses are considered in the future.

Thank you for attention!