$B \to D^{(*)}$ decays from HISQ and clover b-quark in the Fermilab interpretation

Pietro Butti[1] on behalf of Fermilab Lattice - MILC collaboration

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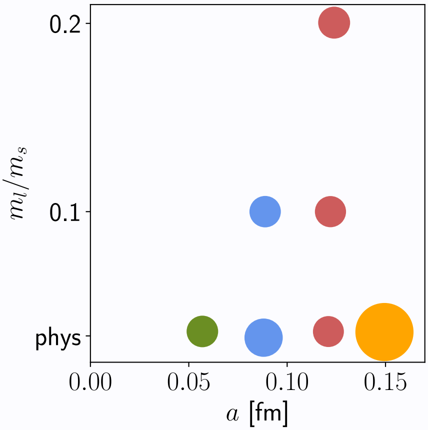
Semileptonic (heavy) B-decays

- $B \to D^{(*)} l\nu$: exclusive vs. inclusive [Eur.Phys.J.C 80 (2020) 10, 966]
- Differential decay rate from QFT ($w = v_{D^*} \cdot v_B$)

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}w} = |V_{cb}|^2 \times (\text{known factor}) \times \begin{cases} \{h_+(\mathrm{p}), h_-(\mathrm{p})\} \quad [B \to D] \\ \{h_{A_1}(\mathrm{p}), h_{A_2}(\mathrm{p}), h_{A_3}(\mathrm{p}), h_V(\mathrm{p})\} [B \to D^*] \end{cases}$$

- First lattice calculation at non-zero recoil [Eur.Phys.J.C 82 (2022)]
- Tension in successive lattice results (HPQCD, JLQCD) [plenary by T. Tsang [Mon., Jul 29th]]
- (other MILC contribution for $B \to \pi, K$ [poster by H. Jeong])

Fermilab-MILC campaign



- `gauge sector:` 1-loop improved Lüscher-Weisz action
- `sea:` HISQ
- `valence:` u,d,s $\rightarrow$ HISQ; c,b $\rightarrow$ clover Fermilab

[poster A. Lytle]
all-HISQ for $B_{(s)} \rightarrow D_{(s)}$, light

Data analysis overview

- 2-pts fits: Energies and overlap coefficients (✓)
- Build ratio of 3-pts functions and extract matrix elements (✓)
- Build bare lattice form factors (✓)
- Apply renormalization blinded factors
- Chiral+continuum limit
- z-expansion

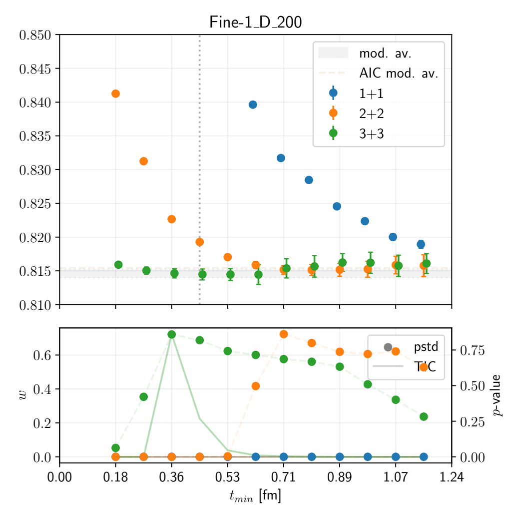
2-point correlation functions

- Staggered quarks induce opposite parity time-oscillating states

$$
\begin{aligned}
C_{Y_a \to Y_b}(\boldsymbol{p}, t) = & \sqrt{\frac{Z_a(\boldsymbol{p})}{2E_0(\boldsymbol{p})}} \left(e^{-E_0(\boldsymbol{p})t} + e^{-E_0(\boldsymbol{p})(N_t - t)} \right) \sqrt{\frac{Z_b(\boldsymbol{p})}{2E_0(\boldsymbol{p})}} \quad + \\
(-1)^{(t+1)} & \sqrt{\frac{Z'_a(\boldsymbol{p})}{2E_1(\boldsymbol{p})}} \left(e^{-E_1(\boldsymbol{p})t} + e^{-E_1(\boldsymbol{p})(N_t - t)} \right) \sqrt{\frac{Z'_b(\boldsymbol{p})}{2E_1(\boldsymbol{p})}} \quad + \\
& \ldots
\end{aligned}
$$

- Same analysis is performed in every jackknife bin
- Fully correlated fit w/ covariance rescaling+shrinking [J. Simone, Lattice17]
- Augmented-χ^2 minimization (Gaussian priors)
- Functional form with $N+N$ (phys. + oscill.) states
- Time range $\begin{cases} D & [t_{\min} \sim 0.4 \text{ fm}, t_{max} \sim 2.6 \text{ fm}] \\ D^* & [t_{\min} \sim 0.6 \text{ fm}, t_{max} \sim \text{ signal/noise} > 25\%] \end{cases}$

Stability test



Fix t_{min} and N_{states} such that

- FS corrected p-value test with a flat distribution across all fits
- Good values of $\chi^2/\chi^2_{\text{expected}}$ from [Comput.Phys.Commun. 285 (2023)]
- Final choice compatible with model average w/ Takeuchi IC [hep-lat:2302.06550]

Dispersion relation

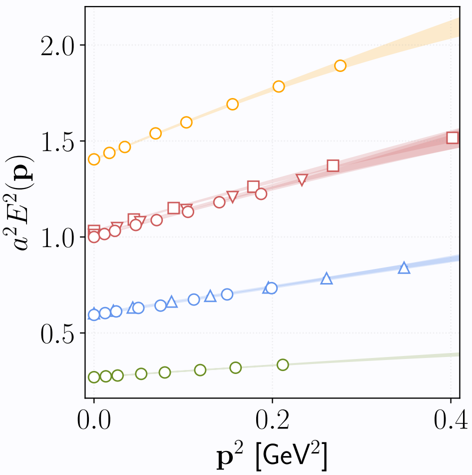
- Use fundamental state energies to fit the momentum dependence

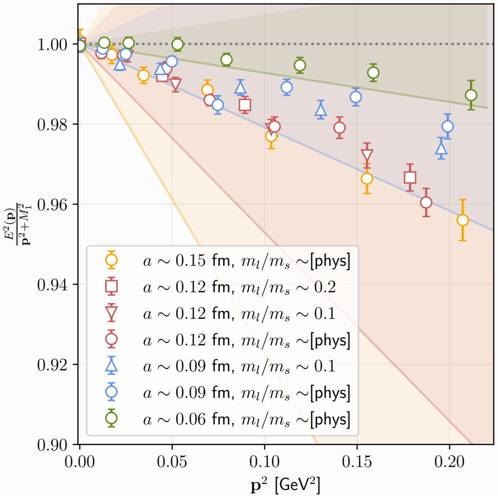
  [Phys. Rev. D 55, 3933 (1993)]

$$a^2 E_0(\boldsymbol{p})^2 = a^2 M_1^2 + \frac{M_1}{M_2}(a\boldsymbol{p})^2 + \frac{1}{4}\left[\frac{1}{(aM_2)^2} - \frac{aM_1}{(aM_4)^3}\right](a^2\boldsymbol{p}^2)^2 - \frac{aM_1 w_4}{3}\sum_{i=1}^{3}(ap_i)^4$$

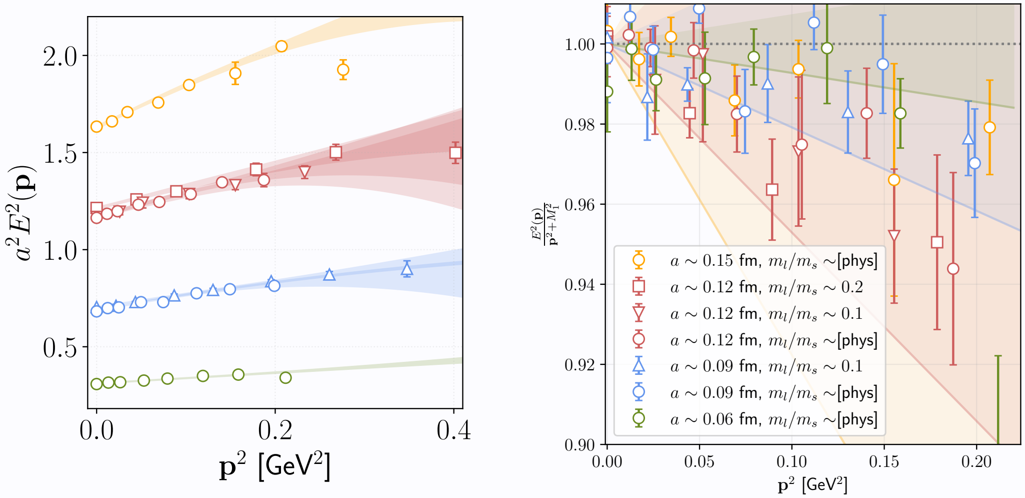
- Discretization errors expected to be $\mathcal{O}(\alpha_s \boldsymbol{p}^2)$

Dispersion relation (D)





Dispersion relation (D^*)



Ratio analysis

Form factors from 3pts func. ratios ($B \to D$)

Form factors defined as

$$h_+(w) = \sqrt{\bar{R}_+}\,\bar{Q}_+(\boldsymbol{p})\left[1 - \boldsymbol{R}_-(\boldsymbol{p}) \cdot \boldsymbol{x}_f(\boldsymbol{p})\right]$$

$$h_-(w) = \sqrt{\bar{R}_+}\,\bar{Q}_+(\boldsymbol{p})\left[1 - \frac{\bar{\boldsymbol{R}}_-(\boldsymbol{p}) \cdot \boldsymbol{x}_f(\boldsymbol{p})}{\boldsymbol{x}_f^2(\boldsymbol{p})}\right]$$

(barred quantities includes renormalization blinded factors)

$$R_+ = \frac{\left\langle D(0)\left|V^4_{cb}\right|B(0)\right\rangle\left\langle B(0)\left|V^4_{cb}\right|D(0)\right\rangle}{\left\langle D(0)\left|V^4_{cc}\right|D(0)\right\rangle\left\langle B(0)\left|V^4_{bb}\right|B(0)\right\rangle}$$

$$Q_+(\boldsymbol{p}) \equiv \frac{\left\langle D(\boldsymbol{p})\left|V^4\right|B(0)\right\rangle}{\left\langle D(0)\left|V^4\right|B(0)\right\rangle}$$

$$\boldsymbol{R}_-(\boldsymbol{p}) \equiv \frac{\langle D(\boldsymbol{p})|\boldsymbol{V}|B(0)\rangle}{\left\langle D(\boldsymbol{p})\left|V^4\right|B(0)\right\rangle}$$

$$\frac{w-1}{w+1} = \boldsymbol{x}_f(\boldsymbol{p}) \equiv \frac{\langle D(\boldsymbol{p})|\boldsymbol{V}|D(0)\rangle}{\left\langle D(\boldsymbol{p})\left|V^4\right|D(0)\right\rangle}$$

Form factors from 3pts func. ratios ($B \to D^*$)

$$h_{A_1}(w) = \frac{2\bar{R}_{A_1}}{w+1}$$

$$h_{A_2}(w) = \frac{2\bar{R}_{A_1}}{w^2-1}\left(wX_1 - \sqrt{w^2-1}\bar{X}_0 - 1\right)$$

$$h_{A_3}(w) = \frac{2\bar{R}_{A_1}}{w^2-1}\left(w - X_1\right)$$

$$h_V(w) = \frac{2\bar{R}_{A_1}}{\sqrt{w^2-1}}\bar{X}_V$$

$$R_{A_1}^2 = \frac{\langle D^*(\boldsymbol{p}_\perp)|A_j|B(0)\rangle\langle B(0)|A_j|D^*(\boldsymbol{p}_\perp)\rangle}{\langle D^*(0)|V^4|D^*(0)\rangle\langle B(0)|V^4|B(0)\rangle}$$

$$X_V = \frac{\langle D^*(\boldsymbol{p}_\perp)|V_j|B(0)\rangle}{\langle D^*(\boldsymbol{p}_\perp)|A_j|B(0)\rangle}$$

$$X_0 = \frac{\left\langle D^*\left(\boldsymbol{p}_\parallel\right)\left|A^4\right|B(0)\right\rangle}{\langle D^*(\boldsymbol{p}_\perp)|A_j|B(0)\rangle}$$

$$X_1 = \frac{\left\langle D^*\left(\boldsymbol{p}_\parallel\right)|A_j|B(0)\right\rangle}{\langle D^*(\boldsymbol{p}_\perp)|A_j|B(0)\rangle},$$

Methodology

$$C^{J^\mu}_{X_a\to Y_b}(\boldsymbol{p},t)=\sum_{n,m} s_n(t)s_m(T-t)\sqrt{Z_{Y_b,n}(\boldsymbol{p})}\frac{e^{-E_n(\boldsymbol{p})t}}{2E_n(\boldsymbol{p})}\left\langle Y_b,n,\boldsymbol{p}\left|J^\mu\right|X_a,m,0\right\rangle\sqrt{Z_{X_a,m}(0)}\frac{e^{-M_m(T-t)}}{2M_m},$$

- Use spectral decomposition to adjust for "leaking factors", e.g.

$$X_1(\boldsymbol{p},t,T)=\frac{C^{A_j}_{B_{1S}\to D^*_a}(\boldsymbol{p}_\parallel,t,T)}{C^{A_j}_{B_{1S}\to D^*_a}(\boldsymbol{p}_\perp,t,T)}\sqrt{\frac{Z_{D^*_a}(\boldsymbol{p}_\perp)}{Z_{D^*_a}(\boldsymbol{p}_\parallel)}}$$

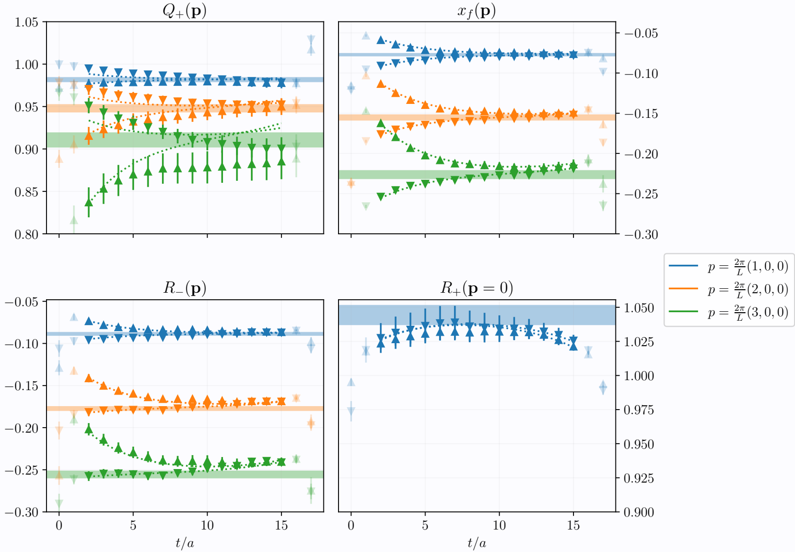
- Smooth out oscillating terms

$$R(t,T)=\frac{1}{2}R(t,T)+\frac{1}{4}R(t,T+1)+\frac{1}{2}R(t+1,T+1)$$

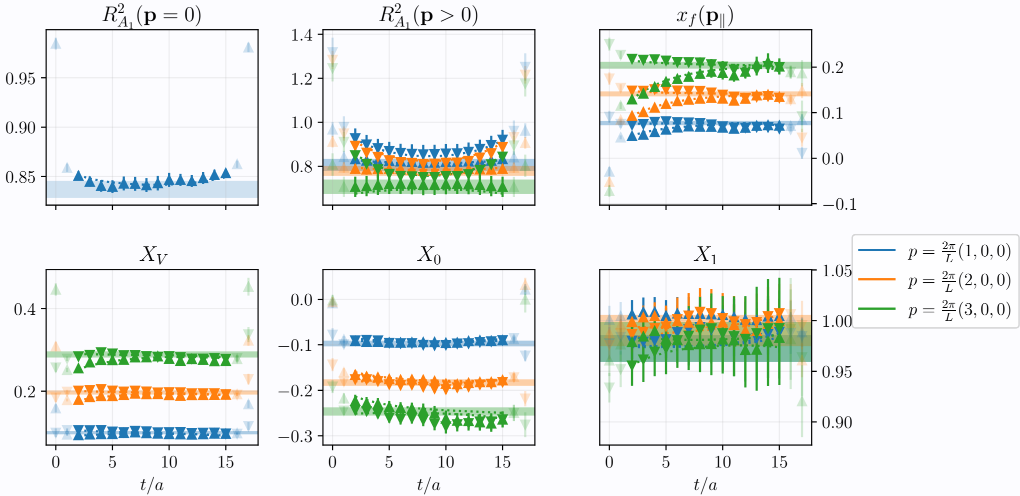
- Global fit w/ shared $\Delta E_{D^{(*)}}$, ΔM_B $\left(p^2 \ll M^2,\ \Delta E_{D^{(*)}} \approx \Delta E_B\right)$

$$R(\boldsymbol{p},t,T)=F_0+A(\boldsymbol{p})e^{-\Delta E_{\text{source}}t}+B(\boldsymbol{p})e^{-\Delta E_{\text{sink}}(T-t)}$$

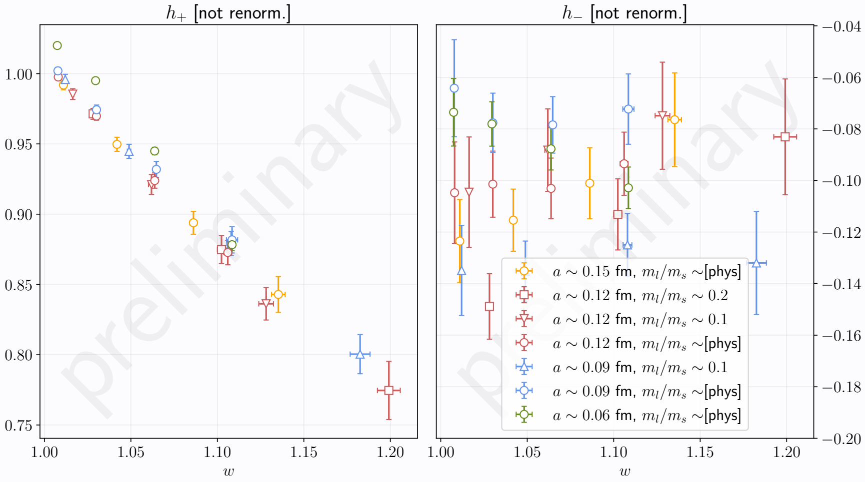
3pts functions ratios ($B \to D$)



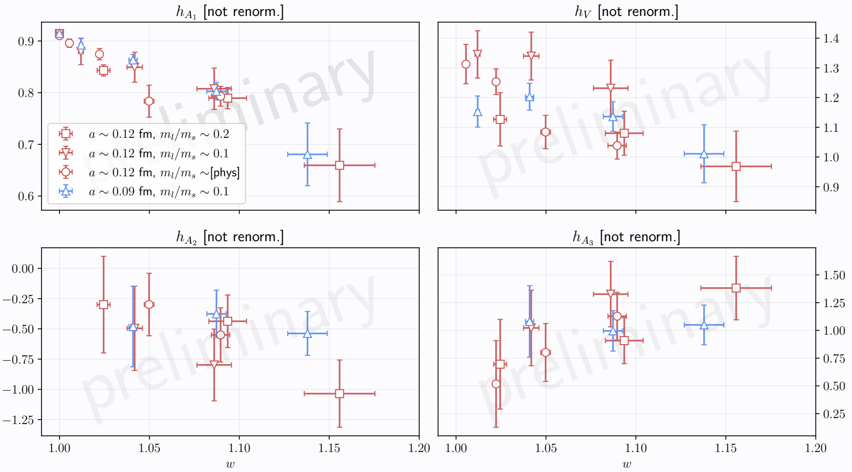
3pts functions ratios ($B \to D^*$)



Form factors ($B \to D$)



Form factors ($B \to D^*$)



Outlook

- We extracted fundamental state energies and matrix elements from staggered 2pts correlator stable fits.
- We obtained a consistent dispersion relation for lattice energies
- We were able to construct the relevant ratios and control leading dependence on excited states
- Renormalization (blinded) factors still to be added
- Missing: chiral+continuum extrapolation, z-expansion