Light 1^{-+} Hybrid Decay in $N_f = 2$ Lattice QCD

Si-Yang Chen
In collaboration with : Juzheng Liang, Ying Chen, Chunjinag Shi, Wei Sun.

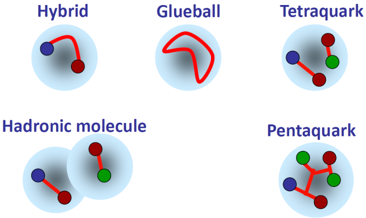
Institute of High Energy Physics

Aug. 1, 2024

中国科学院高能物理研究所
Institute of High Energy Physics
Chinese Academy of Sciences

1^{-+} mesons as candidate of hybrid

- Mesons classicificaion:
  - quark model mesons($\bar{q}q$)
  - glueball($gg...$)
  - hybrid($\bar{q}qg$)
  - ...
- 1^{-+} is exotic quantum number.
- Candidate of 1^{-+} hybrid
  - iso-scalar: $\eta_1(1855)$
  - iso-vector: $\pi_1(1600)$, $\pi_1(1400)$, $\pi_1(2015)$



Experimental status

- $\eta_1(1855)$ was observed by BESIII in $J/\Psi \to \gamma\eta\eta'$ channel(BESIII, PRL.129,192002(2022))
- Resonance parameters:
  - $m_{\eta_1} = 1855 \pm 9^{+6}_{-1}$ MeV
  - $\Gamma_{\eta_1} = 188 \pm 18^{+3}_{-8}$ MeV

- Various experimental report of 1^-1^{-+} states on $\eta\pi, \eta'\pi, b_1\pi, f_1\pi$ and $\rho\pi$ channel
- Experiment are not consist with each other for long time.
- In 2018, JPAC's analysis on COMPASS data(JPAC, PRL.122, 042002 (2019)) leads to a single pole around $(m, \Gamma) = (1564 \pm 89, 492 \pm 115)$ MeV

Previous Lattice Calculation about π_1

A.J.Woss (HSC Collaboration), PRD 103, 054502 (2021)

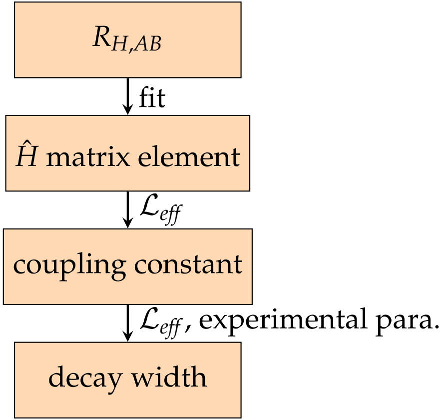
Lüscher's formalism :

- Finite volumn spectrum
- Lüscher's quantization condition : $det(\mathbf{1} + i\boldsymbol{\rho}\mathbf{t}(\mathbf{1} + i\mathcal{M}))$

	Thr./MeV	$\lvert c_i^{phys}\rvert$/MeV	Γ_i/MeV
$\eta\pi$	688	$0 \to 43$	$0 \to 1$
$\rho\pi$	910	$0 \to 203$	$0 \to 20$
$\eta'\pi$	1098	$0 \to 173$	$0 \to 12$
$b_1\pi$	1375	$799 \to 1559$	$139 \to 529$
$K^*\bar{K}$	1386	$0 \to 87$	$0 \to 2$
$f_1(1285)\pi$	1425	$0 \to 363$	$0 \to 24$
$\rho\omega\{^1P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$\rho\omega\{^3P_1\}$	1552	$\lesssim 32$	$\lesssim 0.09$
$\rho\omega\{^5P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$f_1(1420)\pi$	1560	$0 \to 245$	$0 \to 2$
			$\Gamma = \sum_i \Gamma_i = 139 \to 590$

Lattice Methodology(C. McNeile & C. Michael, PLB 556 (2003) 177)

$$
\begin{aligned}
C_{X,Y}(t) &= \langle \mathcal{O}_X(t)\mathcal{O}_Y(0)\rangle \\
&= \langle X|e^{-\hat{H}t}|Y\rangle \\
R_{H,AB}(t) &= \frac{C_{H,AB}(t)}{\sqrt{C_{HH}(t)C_{AB,AB}(t)}} \\
&= \frac{\langle H|AB\rangle - \langle H|\hat{H}|AB\rangle t}{\sqrt{\langle H|H\rangle\langle AB|AB\rangle}} + O(t^2)
\end{aligned}
$$



Lorentz structure and transition matrix element

$$
\begin{aligned}
\mathcal{L}_{HAP} &= g m_H H^\mu A_\mu P \\
\mathcal{L}_{HPP'} &= i g H^\mu (P \overleftrightarrow{\partial}_\mu P') \\
\mathcal{L}_{HVP} &= \frac{g}{m_H} \epsilon_{\mu\nu\rho\sigma} (\partial^\mu H^\nu)(\partial^\rho V^\sigma) P \\
\mathcal{L}_{HVV'} &= H^\nu (g V^\mu \partial_\mu V'_\nu \\
&+ g' V'^\mu \partial_\mu V_\nu + g_0 V_\mu \overleftrightarrow{\partial}_\nu V'^\mu)
\end{aligned}
$$

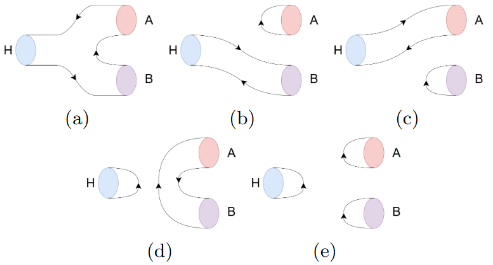
$$
\begin{aligned}
\overline{|\mathcal{M}(h \to AP)|^2} &= \frac{1}{3} g_{AP}^2 m_H^2 \left(3 + \frac{k_{\rm ex}^2}{m_A^2}\right), \\
\overline{|\mathcal{M}(h \to PP)|^2} &= \frac{4}{3} g_{PP}^2 k_{\rm ex}^2, \\
\overline{|\mathcal{M}(h \to VP)|^2} &= \frac{2}{3} g_{VP}^2 k_{\rm ex}^2, \\
\overline{|\mathcal{M}(h \to VV)|^2} &= \frac{4}{3} g_{VV}^2 k_{\rm ex}^2 \frac{m_H^2}{m_V^2}.
\end{aligned}
$$

Flavor structure and the quark contraction

$$X = \begin{pmatrix} \frac{\pi_X^0+\eta_X^{(l)}}{\sqrt{2}} & \pi_X^+ \\ \pi_X^- & \frac{-\pi_X^0+\eta_X^{(l)}}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{L}_{HAB}^{(-)} = \frac{g^{(-)}}{2} Tr(H[A,B])$$

$$\mathcal{L}_{HAB}^{(+)} = \frac{g}{2} Tr(H\{A,B\}) - g_h TrHTr(AB) - g_A TrA\, Tr(BH) - g_B TrB\, Tr(HA) + g_{full} TrHTrATrB$$



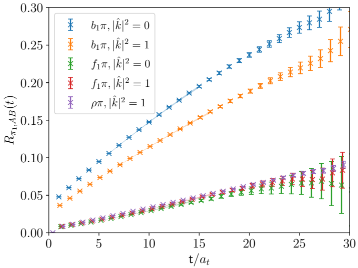
1. Motivation
2. Formalism
3. **Numerical Detail**
4. Summary

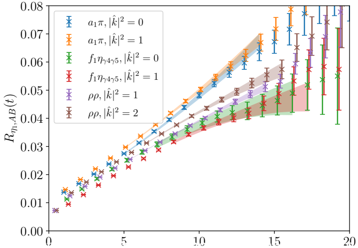
Lattice Setup

- Lattice action: Tadpole improved gauge action. Tadpole improved anisotropic Clover action for $N_f = 2$ degenerate sea quarks.
- Distillation(M. Peardon et al. PRD80(2009)054506) is used to calculate both the connected and disconnected diagrams.

IE	$N_x^3 \times N_t$	a_t^{-1}(GeV)	ξ	m_π(MeV)	N_V	$N_{\rm cfg}$
L16M415	$16^3 \times 128$	7.219	5.0	417	70	401

Coupling constant in $N_f = 2$ Lattice QCD





mode	g_{AB}	$\bar{g}_{AB}$
$\pi_1 \to b_1\pi(\hat{k}^2 = 0)$	4.41(8)	4.50(18)
$\pi_1 \to b_1\pi(\hat{k}^2 = 1)$	4.59(10)	
$\pi_1 \to f_1\pi(\hat{k}^2 = 0)$	0.80(5)	0.98(26)
$\pi_1 \to f_1\pi(\hat{k}^2 = 1)$	1.16(11)	
$\pi_1 \to \rho\pi(\hat{k}^2 = 1)$	4.37(6)	4.37(6)
$\eta_1 \to a_1\pi(\hat{k}^2 = 0)$	1.10(14)	1.43(50)
$\eta_1 \to a_1\pi(\hat{k}^2 = 1)$	1.76(19)	
$\eta_1 \to f_1\eta(\hat{k}^2 = 0)$	2.24(62)	2.41(87)
$\eta_1 \to f_1\eta(\hat{k}^2 = 1)$	2.58(77)	
$\eta_1 \to \rho\rho(\hat{k}^2 = 1)$	2.76(23)	2.93(50)
$\eta_1 \to \rho\rho(\hat{k}^2 = 2)$	3.10(42)	

Discussion of 3 flavor decay width

- $SU(3)$ flavor symmetric:
  quartet $\rightarrow$ nonet
- The mixing of singlet and octet, K_1 in $1^{++}(1^{+-})$ octet are set according to previous Lattice or Phenomenological study.

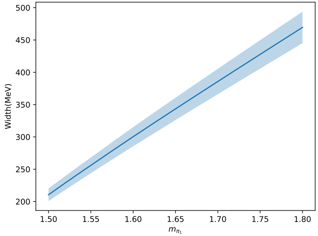
$$\begin{pmatrix} \eta_1^{(L)} \\ \eta_1^{(H)} \end{pmatrix} = \begin{pmatrix} cos\theta_h & -sin\theta_h \\ sin\theta_h & cos\theta_h \end{pmatrix} \begin{pmatrix} \eta_1^{(8)} \\ \eta_1^{(1)} \end{pmatrix}$$

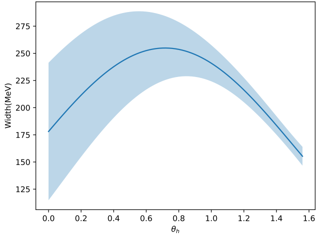
$$\begin{pmatrix} K_1(1270) \\ K_1(1400) \end{pmatrix} = \begin{pmatrix} cos\theta_K & sin\theta_K \\ -sin\theta_K & cos\theta_K \end{pmatrix} \begin{pmatrix} K_{1B} \\ K_{1A} \end{pmatrix}$$

- $\theta_P = -10^\circ, \theta_V = -54.7^\circ, \theta_A = -34^\circ, \theta_h = 32.1^\circ, \theta_K = 45^\circ$

Result(preliminary)

mode	g_{AB}	decay width
$\pi_1 \to b_1 + \pi$	4.46 ± 0.11	288.96 ± 14.77
$\pi_1 \to f_1(1285) + \pi$	0.72 ± 0.49	6.66 ± 9.07
$\pi_1 \to f_1(1420) + \pi$	0.34 ± 0.24	0.81 ± 1.17
$\pi_1 \to \rho + \pi$	4.37 ± 0.06	48.33 ± 1.30
$\pi_1 \to K^* + K$	3.09 ± 0.04	8.01 ± 0.22
$\pi_1 \to$All		352.77 ± 17.84
$\eta_1^{(L)} \to a_1 + \pi$	1.17 ± 0.86	28.99 ± 42.86
$\eta_1^{(L)} \to f_1(1285) + \eta$	0.38 ± 0.28	0.80 ± 1.20
$\eta_1^{(L)} \to K_1(1270) + K$	3.28 ± 0.09	122.02 ± 6.53
$\eta_1^{(L)} \to \rho + \rho$	2.55 ± 0.26	37.97 ± 7.76
$\eta_1^{(L)} \to \omega + \omega$	1.47 ± 0.15	11.59 ± 2.37
$\eta_1^{(L)} \to K^* + K^*$	1.03 ± 0.14	1.25 ± 0.34
$\eta_1^{(L)} \to K^* + K$	4.53 ± 0.06	34.46 ± 0.93
$\eta_1^{(L)} \to$All		237.09 ± 47.32
$\eta_1^{(H)} \to a_1 + \pi$	0.52 ± 0.54	8.92 ± 18.56
$\eta_1^{(H)} \to f_1(1285) + \eta$	0.85 ± 0.43	17.57 ± 17.72
$\eta_1^{(H)} \to f_1(1420) + \eta$	1.26 ± 0.52	29.23 ± 24.02
$\eta_1^{(H)} \to K_1(1270) + K$	2.16 ± 0.18	126.35 ± 21.18
$\eta_1^{(H)} \to K_1(1420) + K$	2.16 ± 0.18	98.20 ± 16.46
$\eta_1^{(H)} \to \rho + \rho$	1.27 ± 0.17	33.58 ± 9.03
$\eta_1^{(H)} \to \omega + \omega$	0.73 ± 0.10	10.66 ± 2.87
$\eta_1^{(H)} \to \phi + \phi$	2.10 ± 0.22	8.02 ± 1.65
$\eta_1^{(H)} \to K^* + K^*$	2.20 ± 0.24	87.93 ± 19.07
$\eta_1^{(H)} \to K^* + K$	2.85 ± 0.04	26.30 ± 0.71
$\eta_1^{(H)} \to$All		446.78 ± 95.31





1. Motivation
2. Formalism
3. Numerical Detail
4. **Summary**

Summary

- Light 1^{-+} meson decay property is calculated with MMP method. This is the first lattice calculation of η_1 decay.
- π_1 decay is calculated to be dominated by $b_1\pi$ channel, which is as the expectation of previous calculation with Lüscher's formalism.
- Not a small contribution of VV channel is shown in η_1 decay, which is expected to be suppressed in Flux tube model.

Thank You