

# Dark Matter Scattering

arXiv:2405.06506

With F. Zierler & A. Maas

Yannick Dengler



NAWI Graz  
Natural Sciences

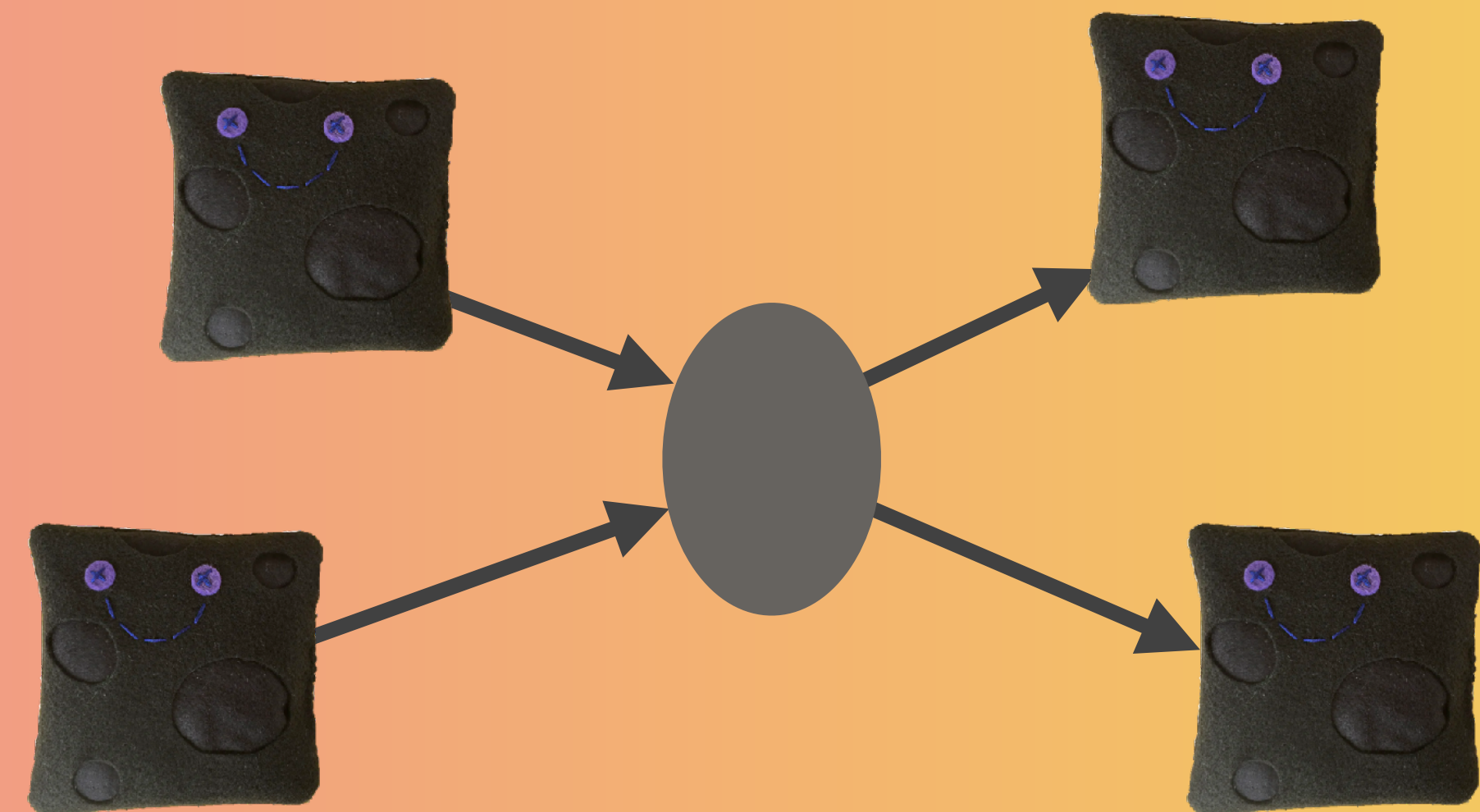


FWF

# LATTICE 2024



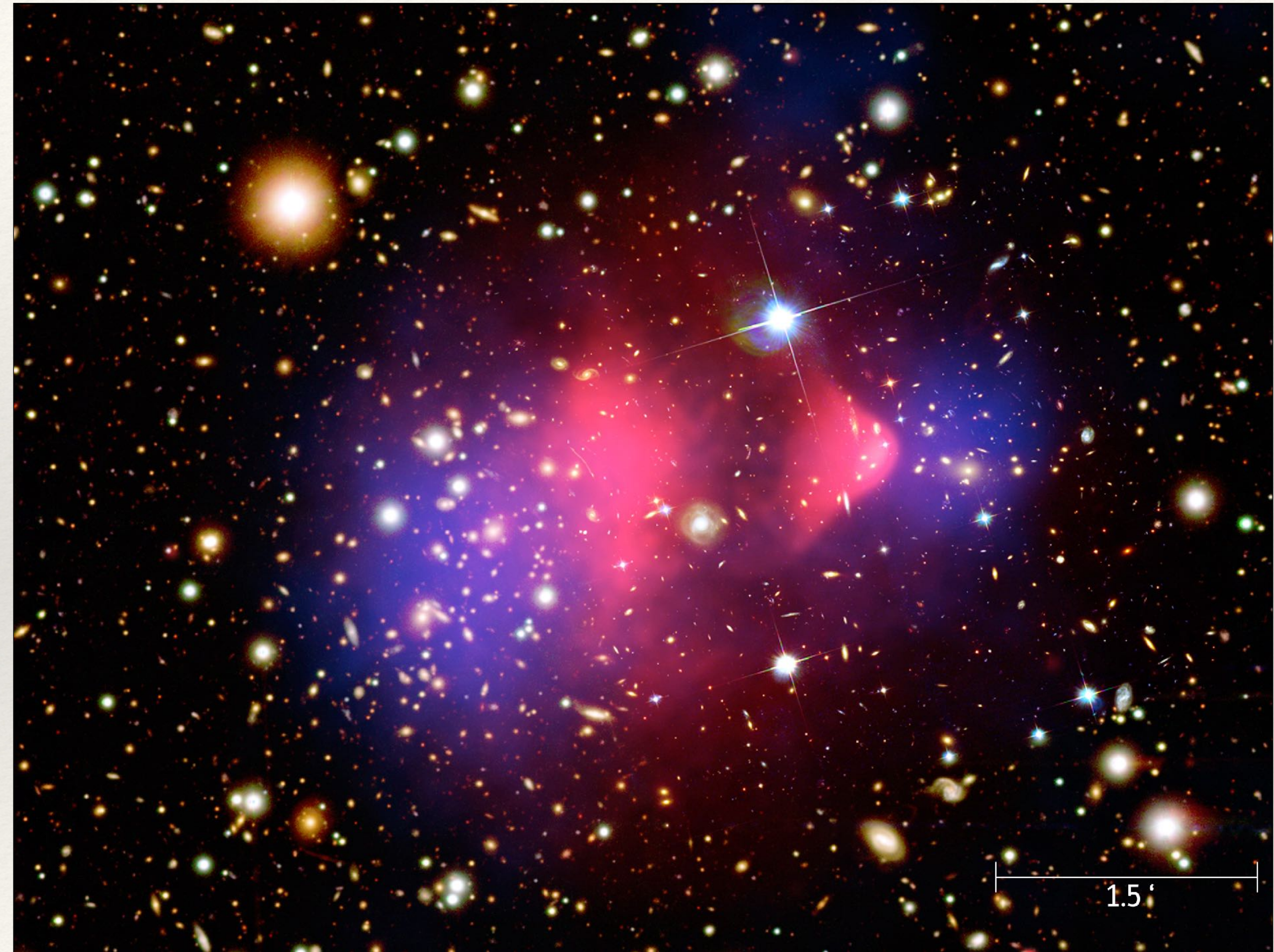
# LIVERPOOL





# Dark Matter

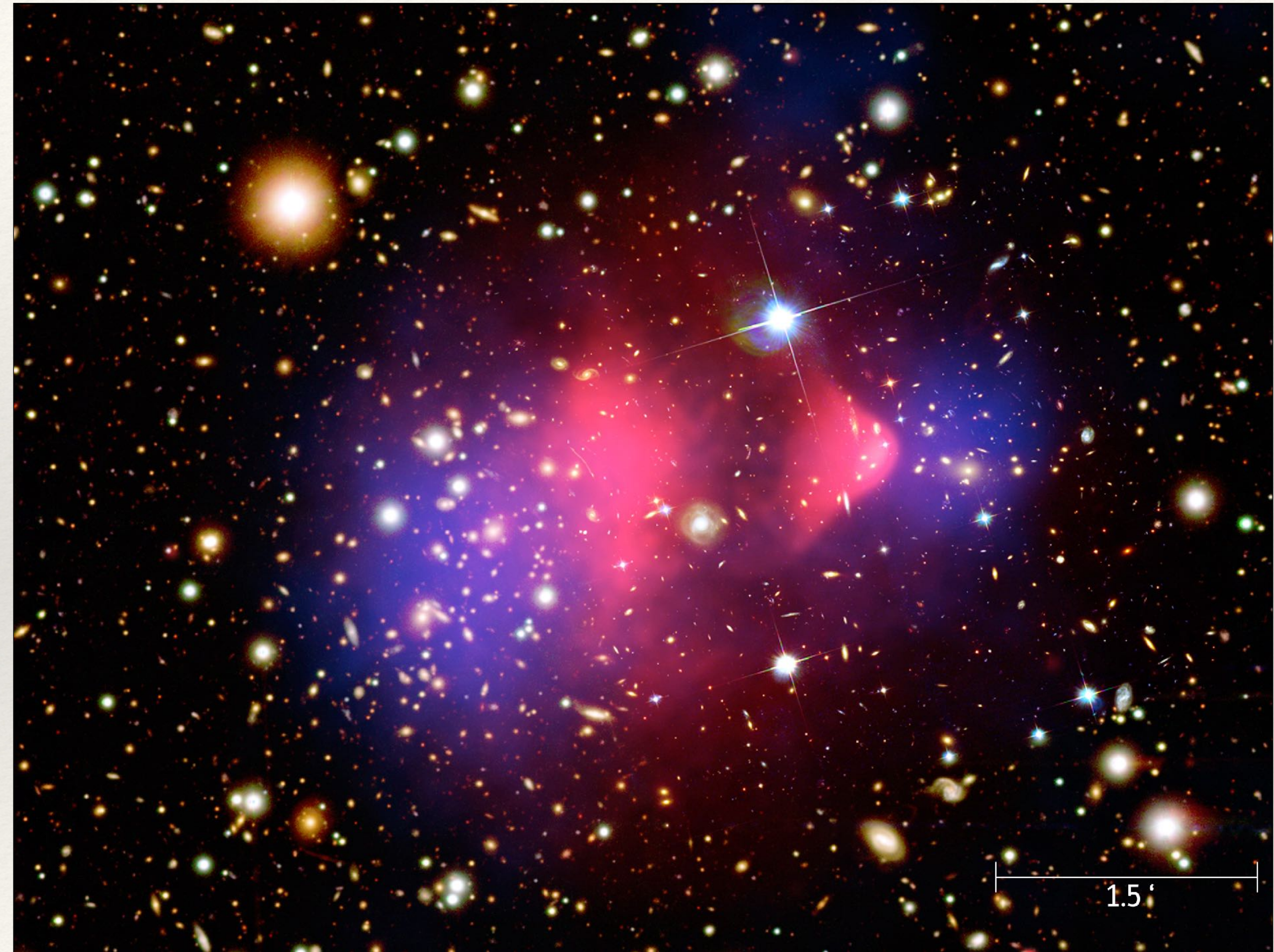
- ❖ One of the biggest mysteries in physics





# Dark Matter

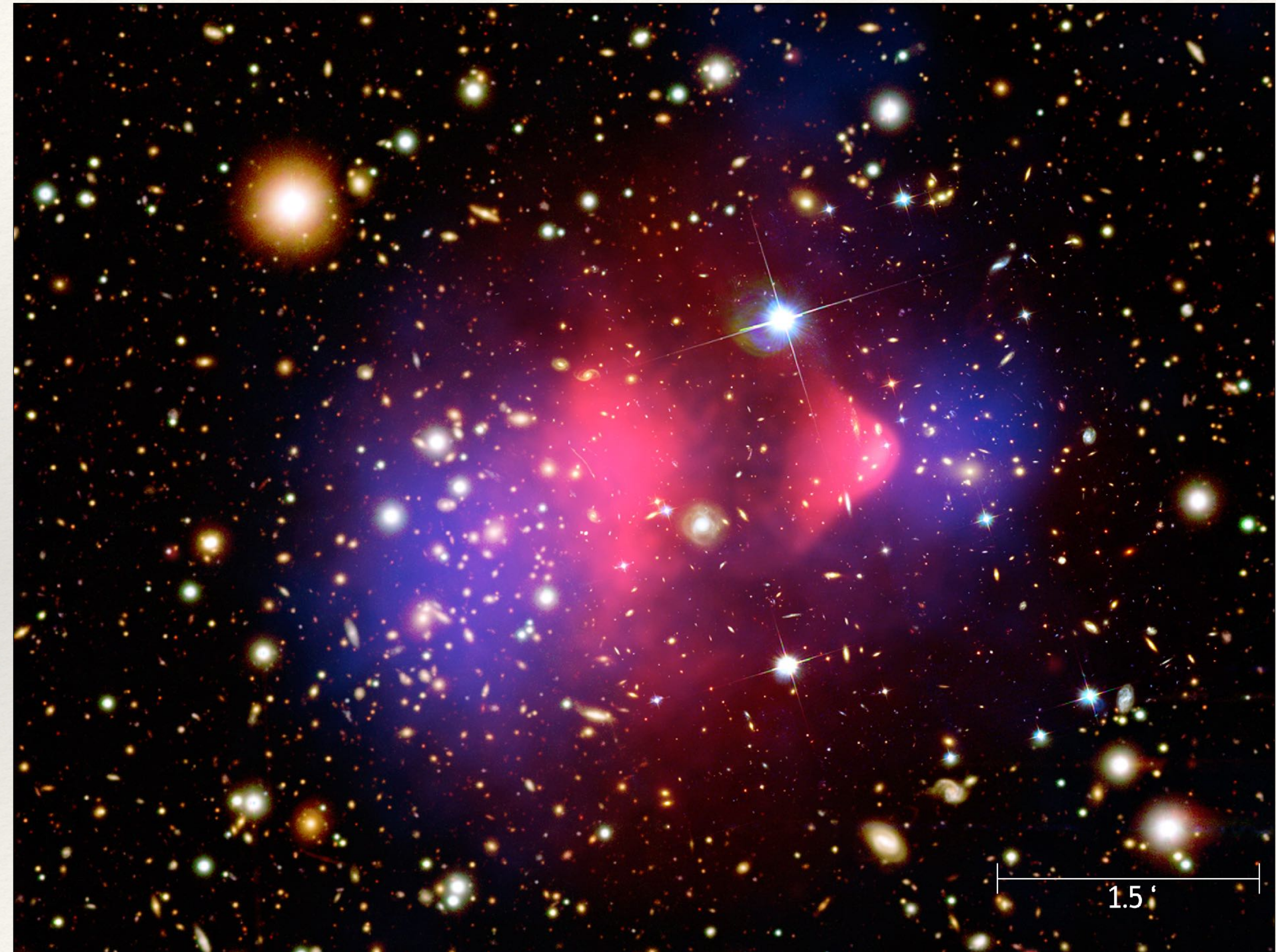
- ❖ One of the biggest mysteries in physics
- ❖ Evidence for particle DM:
  - ❖ i.e. "Bullet cluster"





# Dark Matter

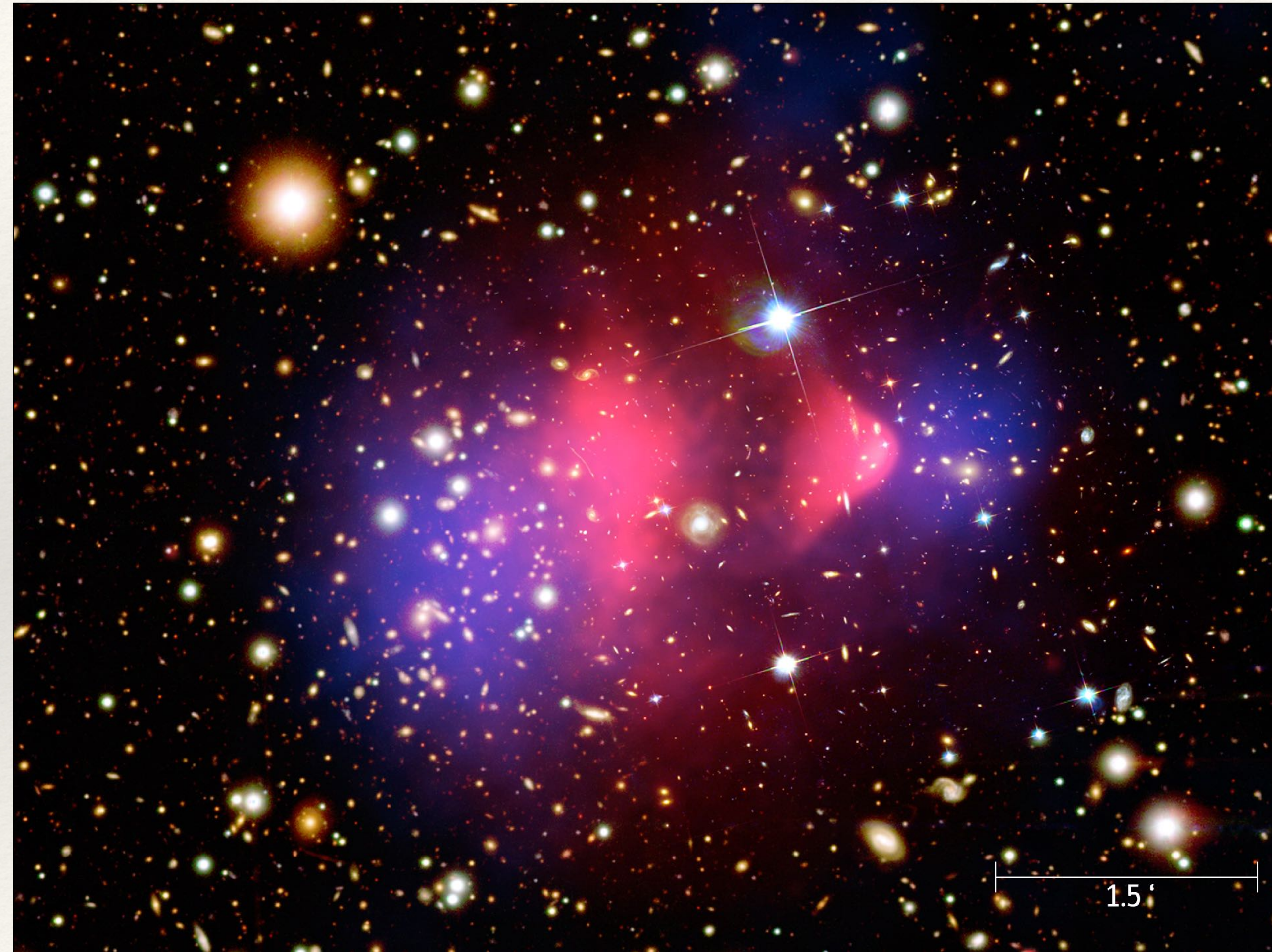
- ❖ One of the biggest mysteries in physics
- ❖ Evidence for particle DM:
  - ❖ i.e. "Bullet cluster"
- ❖ Properties:
  - ❖ Massive, stable, "invisible"





# Dark Matter

- ❖ One of the biggest mysteries in physics
- ❖ Evidence for particle DM:
  - ❖ i.e. "Bullet cluster"
- ❖ Properties:
  - ❖ Massive, stable, "invisible"
- ❖ Interaction?
  - ❖ With SM: no (low)
  - ❖ Self: perchance?

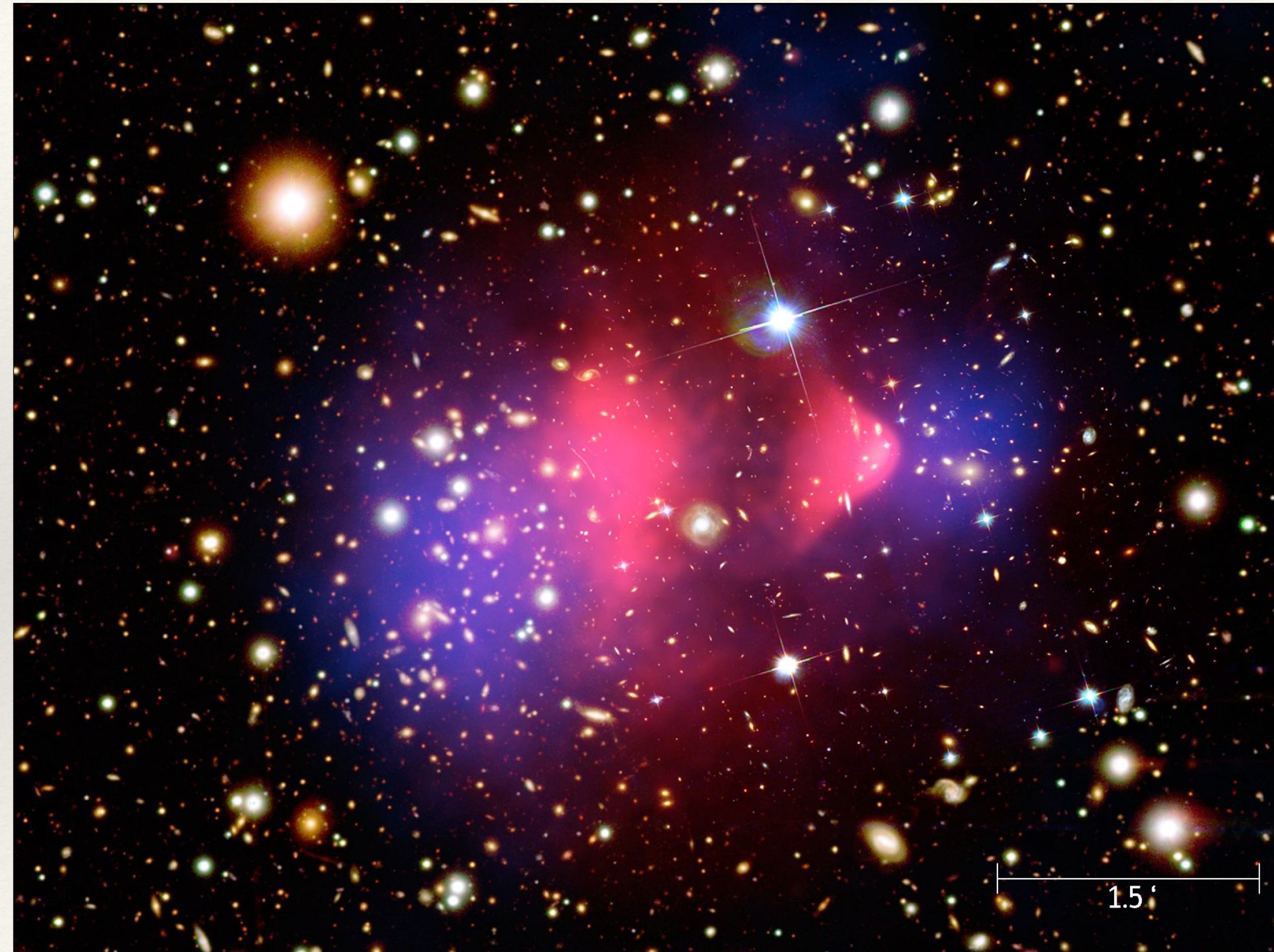




# Dark Matter

## What can lattice do?

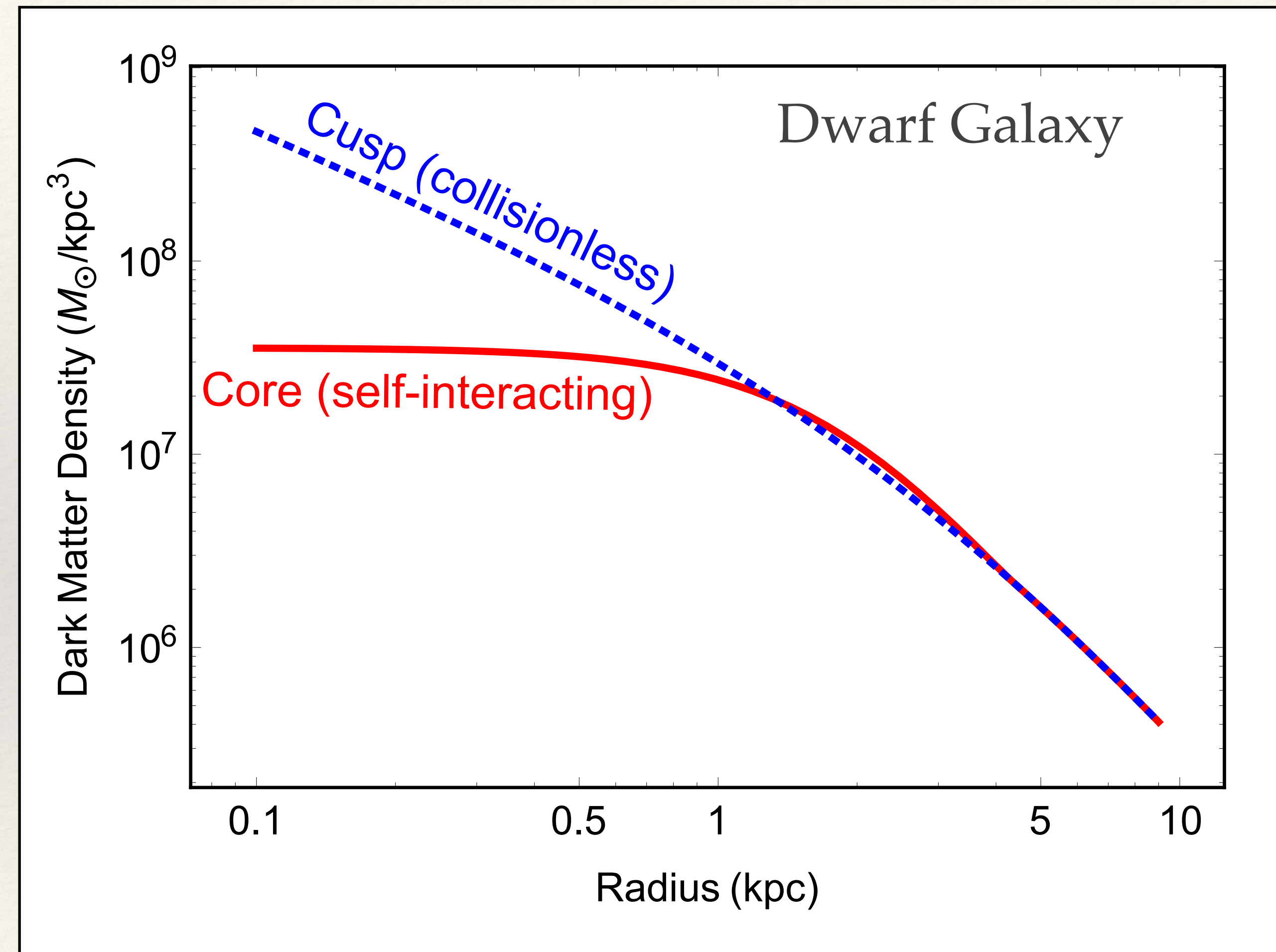
- ❖ Test/limit effective theories
- ❖ Provide first-principles verification of dark matter models
- ❖ Compare directly to i.e. astro data





# Self-interaction

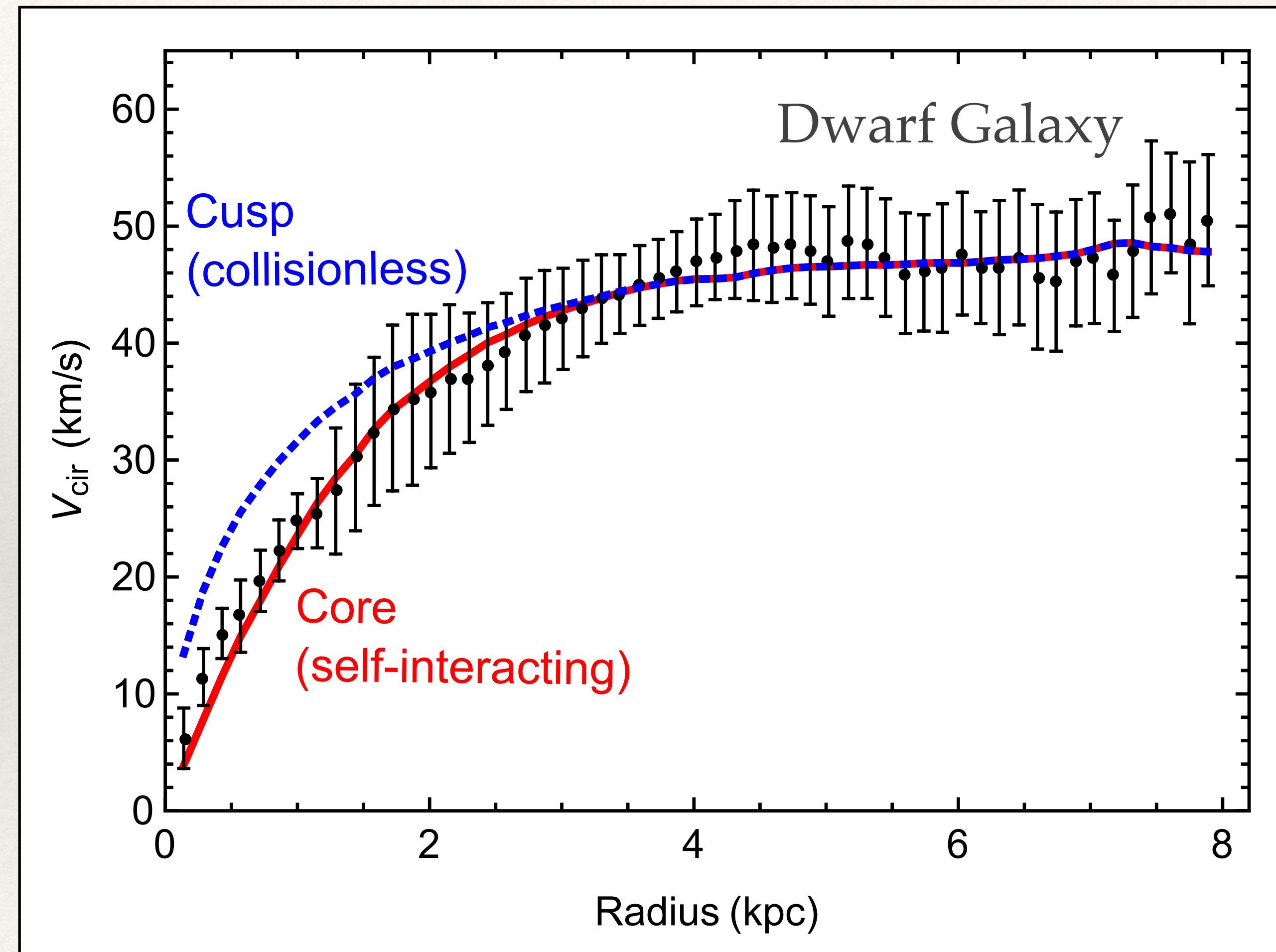
- ❖ "Small structure problems"





# Self-interaction

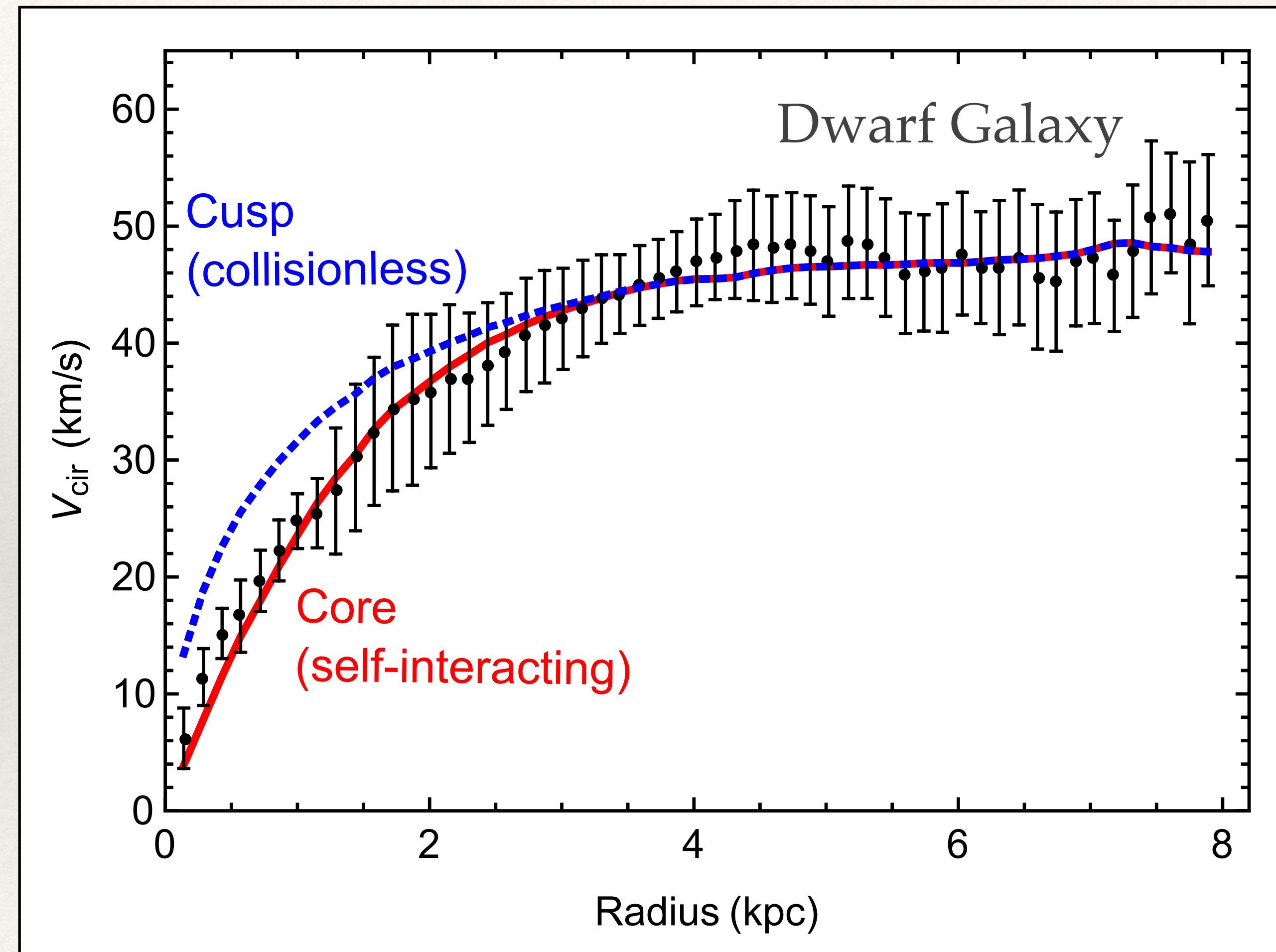
- ❖ "Small structure problems"
- ❖ Core-like shape preferred
- ❖ Hints towards self-interaction





# Self-interaction

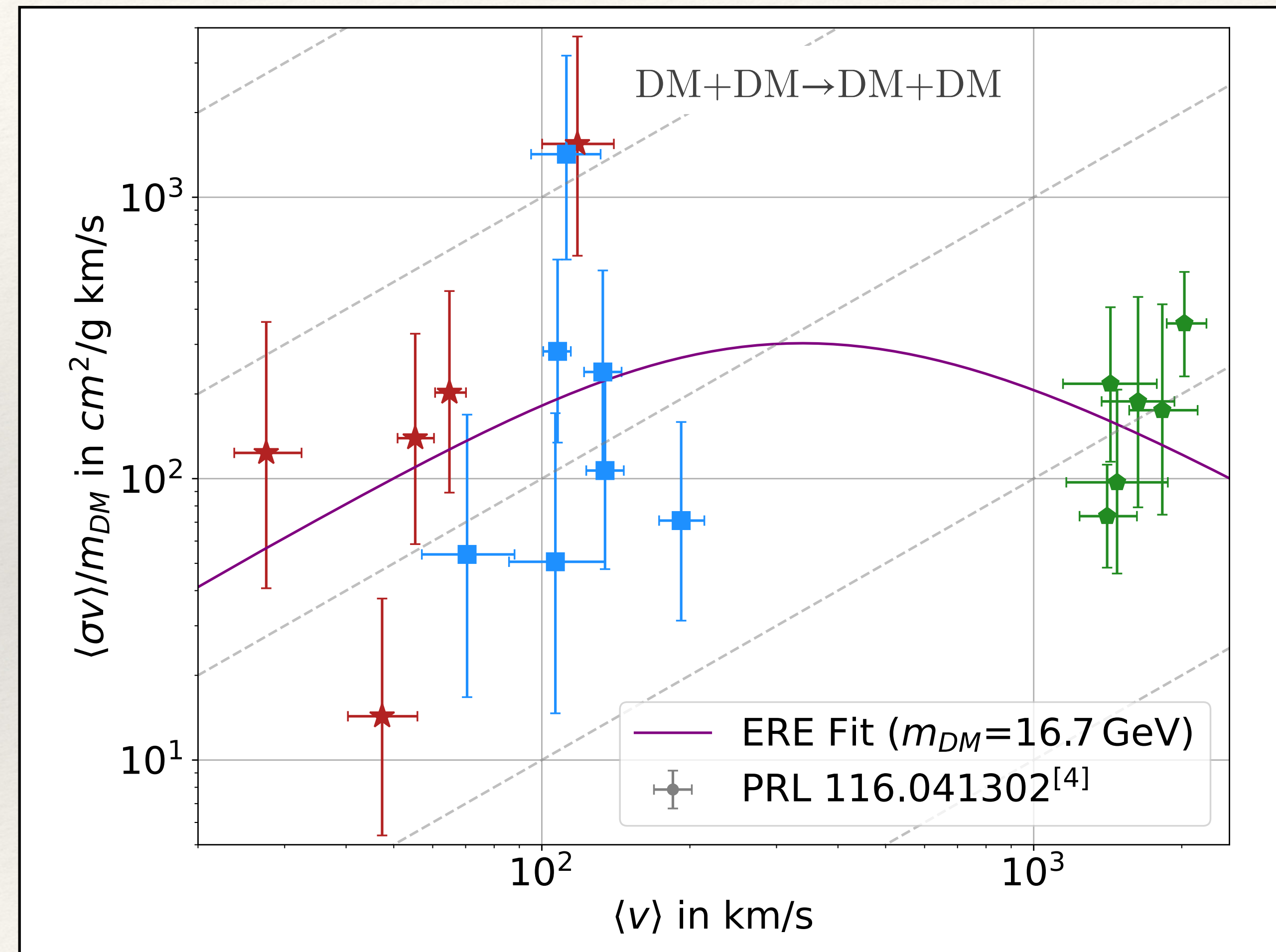
- ❖ "Small structure problems"
- ❖ Core-like shape preferred
  - ❖ Hints towards self-interaction
- ❖ Upper bounds on cross-section





# Velocity-dependent cross-section

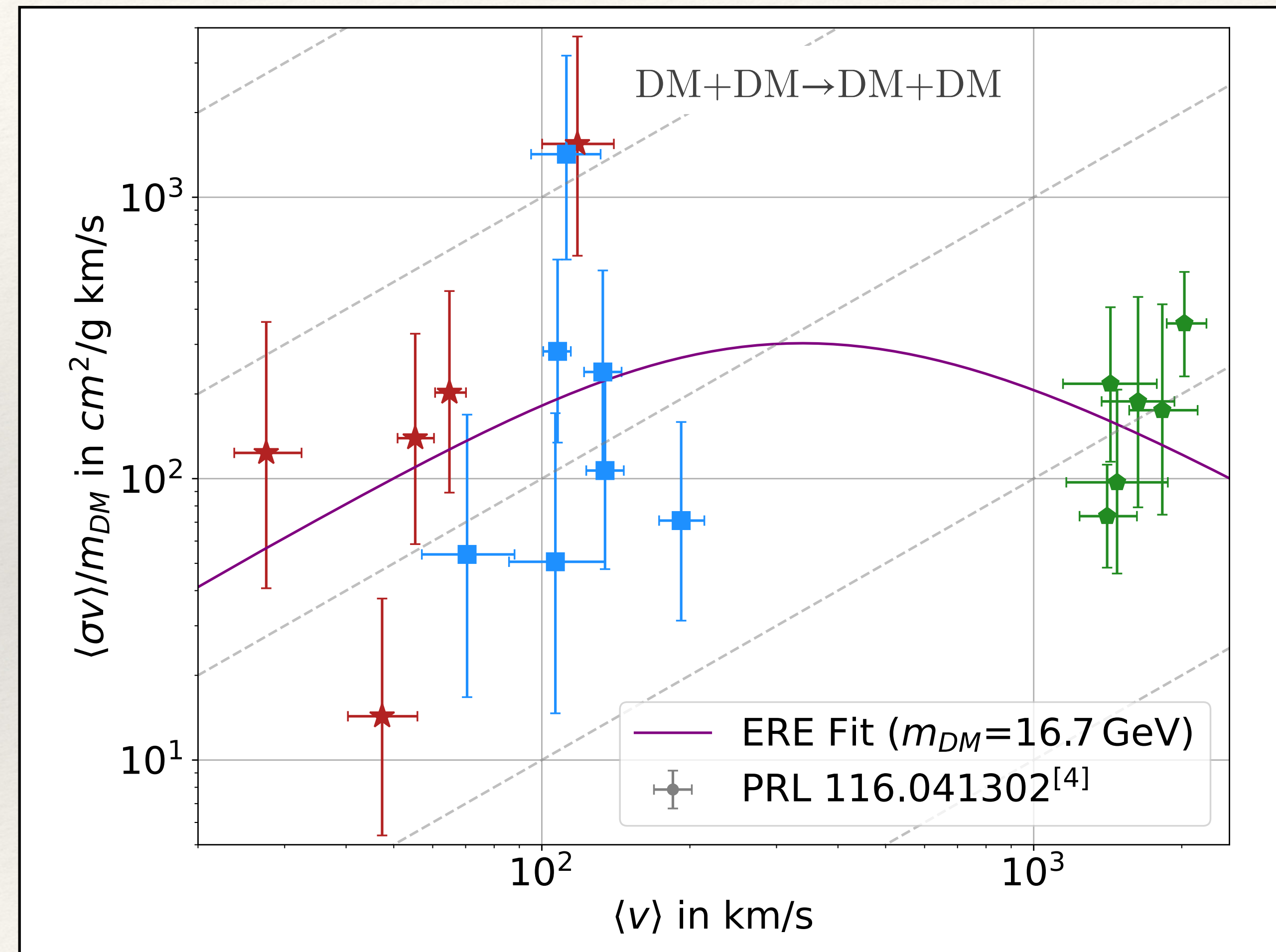
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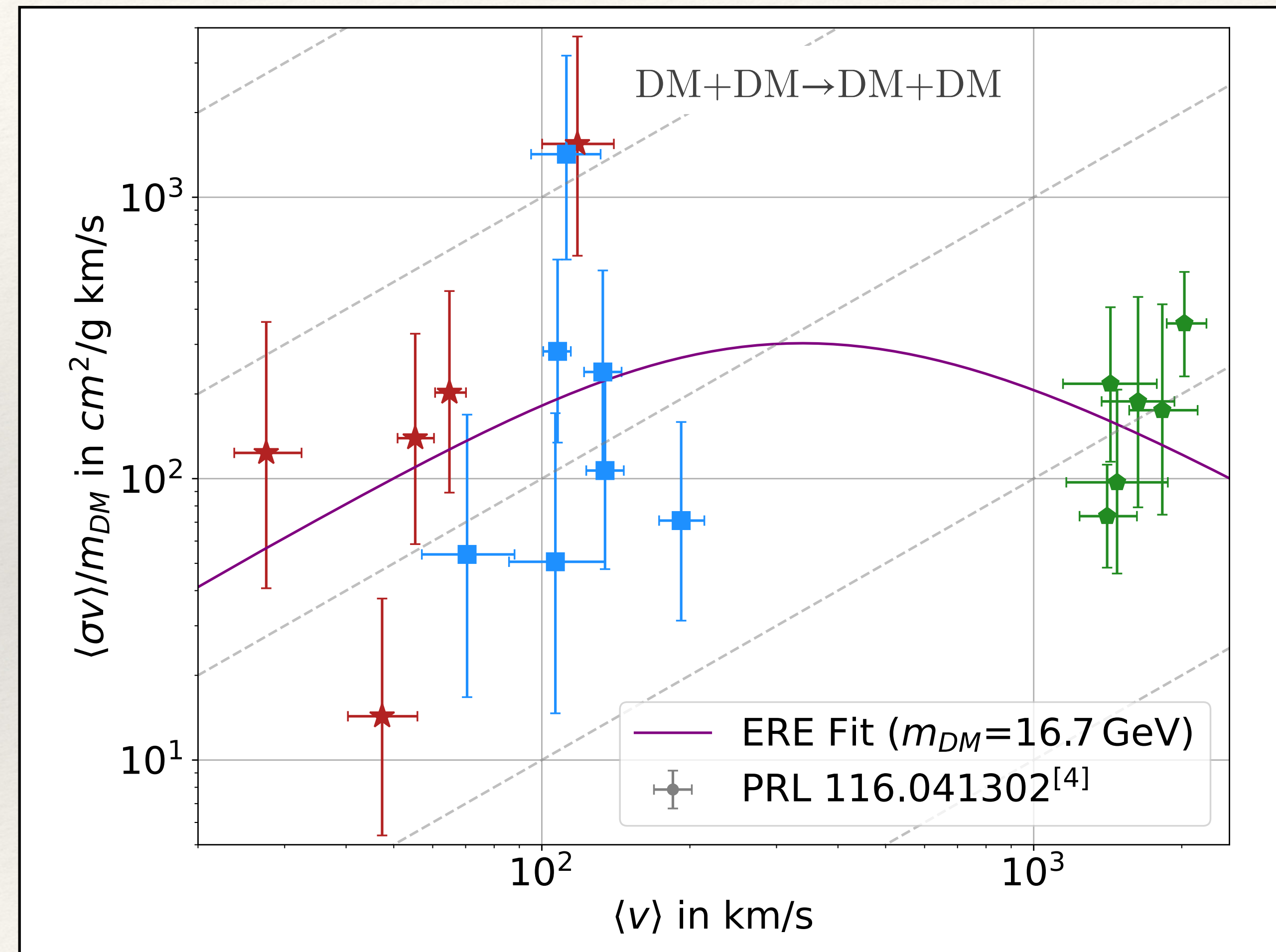
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- ❖ Mild velocity dependence @ non-relativistic velocities





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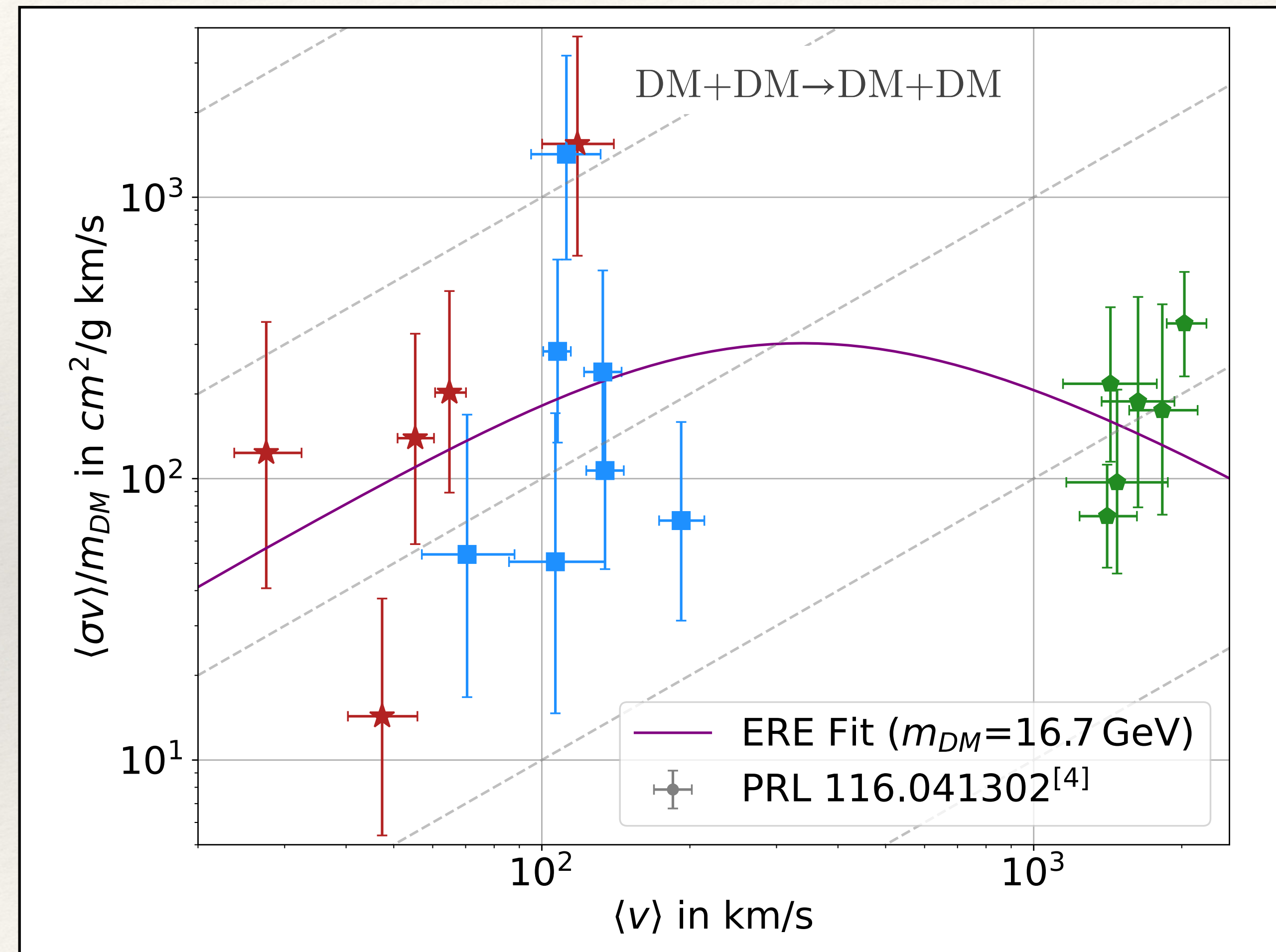
- ❖ "Dark matter halos as particle colliders"
- ❖ Mild velocity dependence @ non-relativistic velocities
- ❖ Relies on simulations of dark halos
  - ❖ model-dependent





# Velocity-dependent cross-section

- ❖ DM in halo thermalized



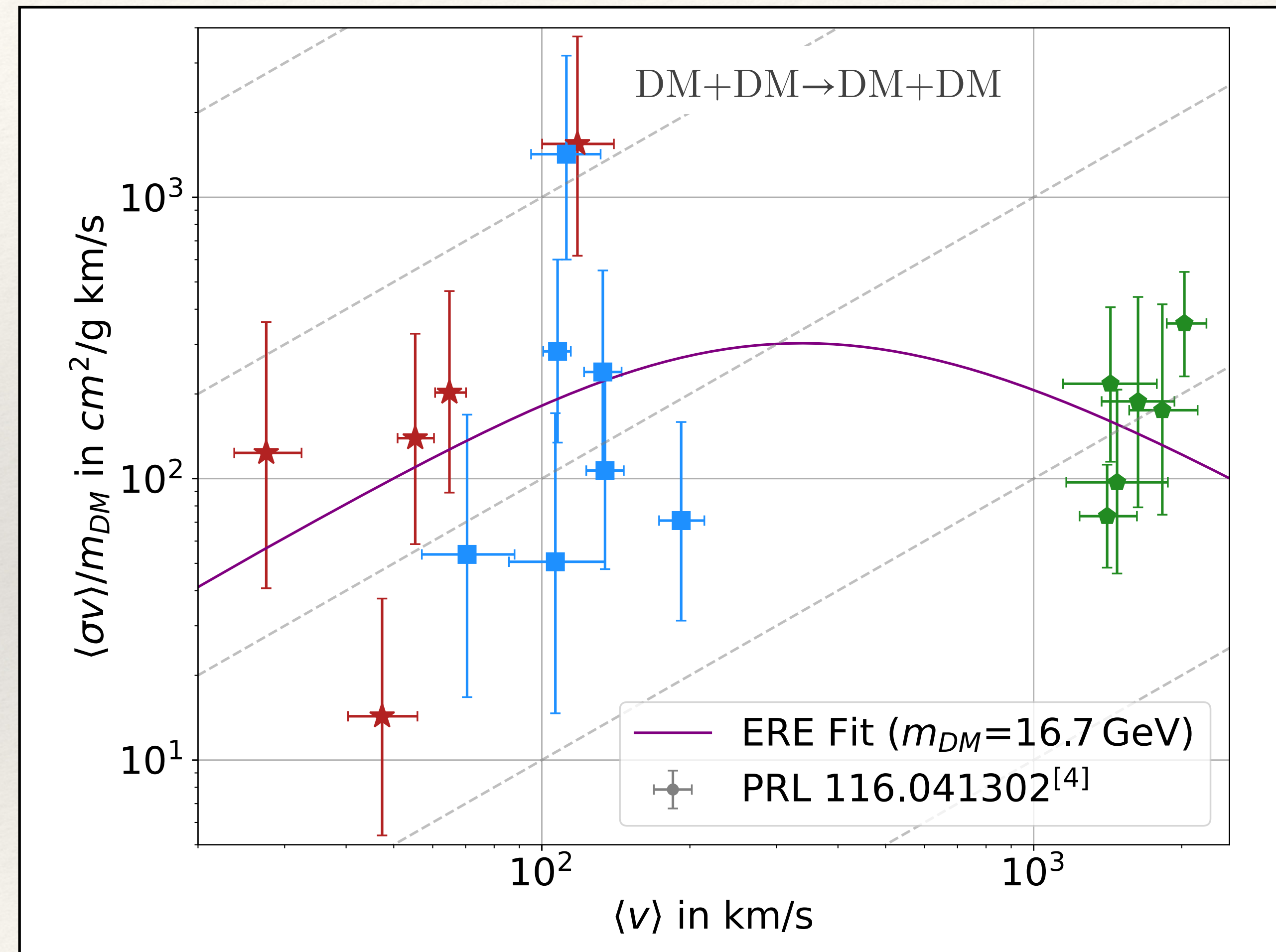


# Velocity-dependent cross-section

❖ DM in halo thermalized

$$\langle \sigma v \rangle = \int_0^{v_{esc}} dv \sigma(v) v f(v)$$

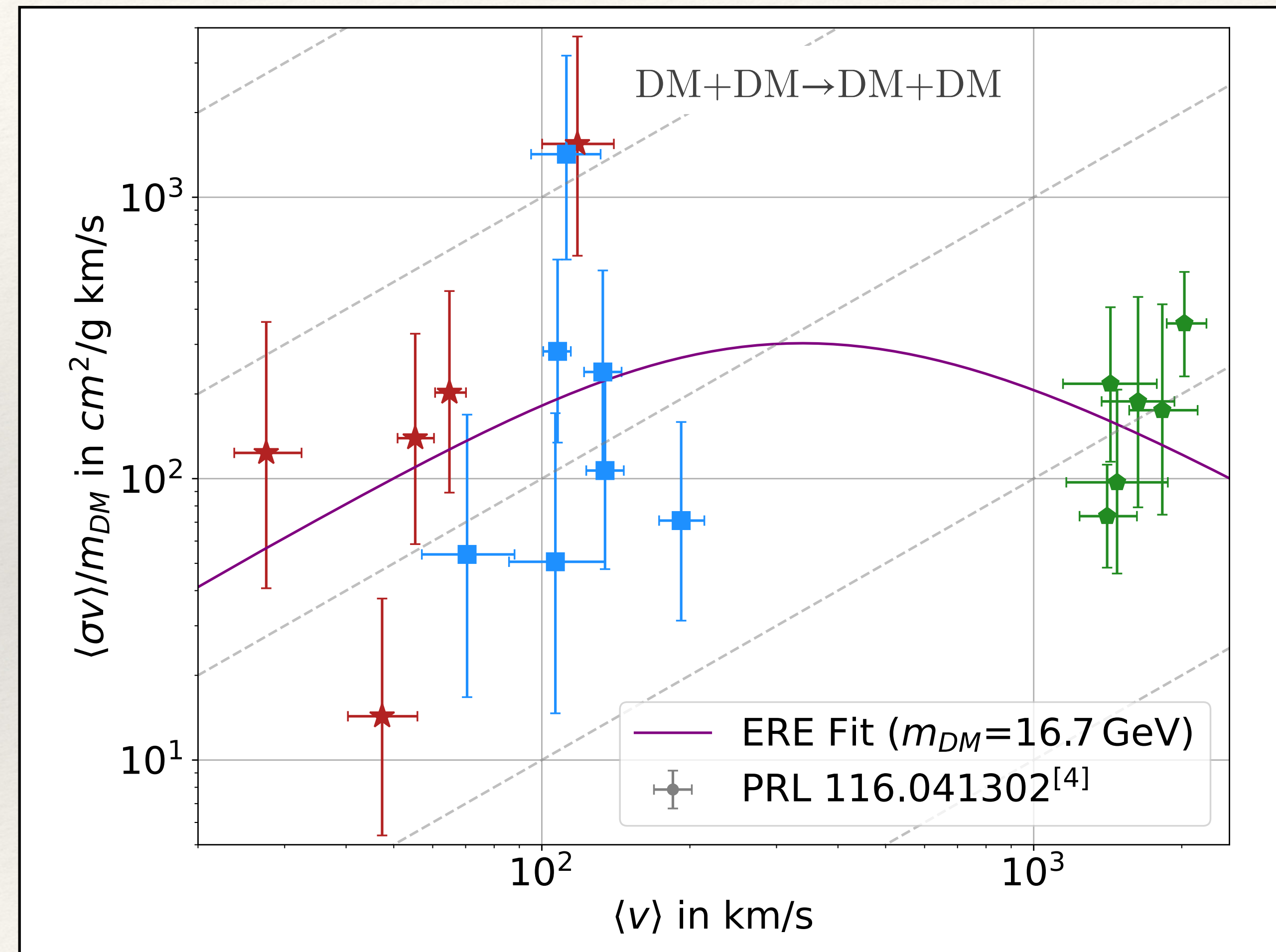
❖  $v$  - rel. velocity,  $f(v)$  - Maxwellian





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- ❖  $v$  - rel. velocity,  $f(v)$  - Maxwellian
- ❖ Can be done on the lattice
- ❖  $\sigma(v)$  needed





# Velocity-dependent cross-section

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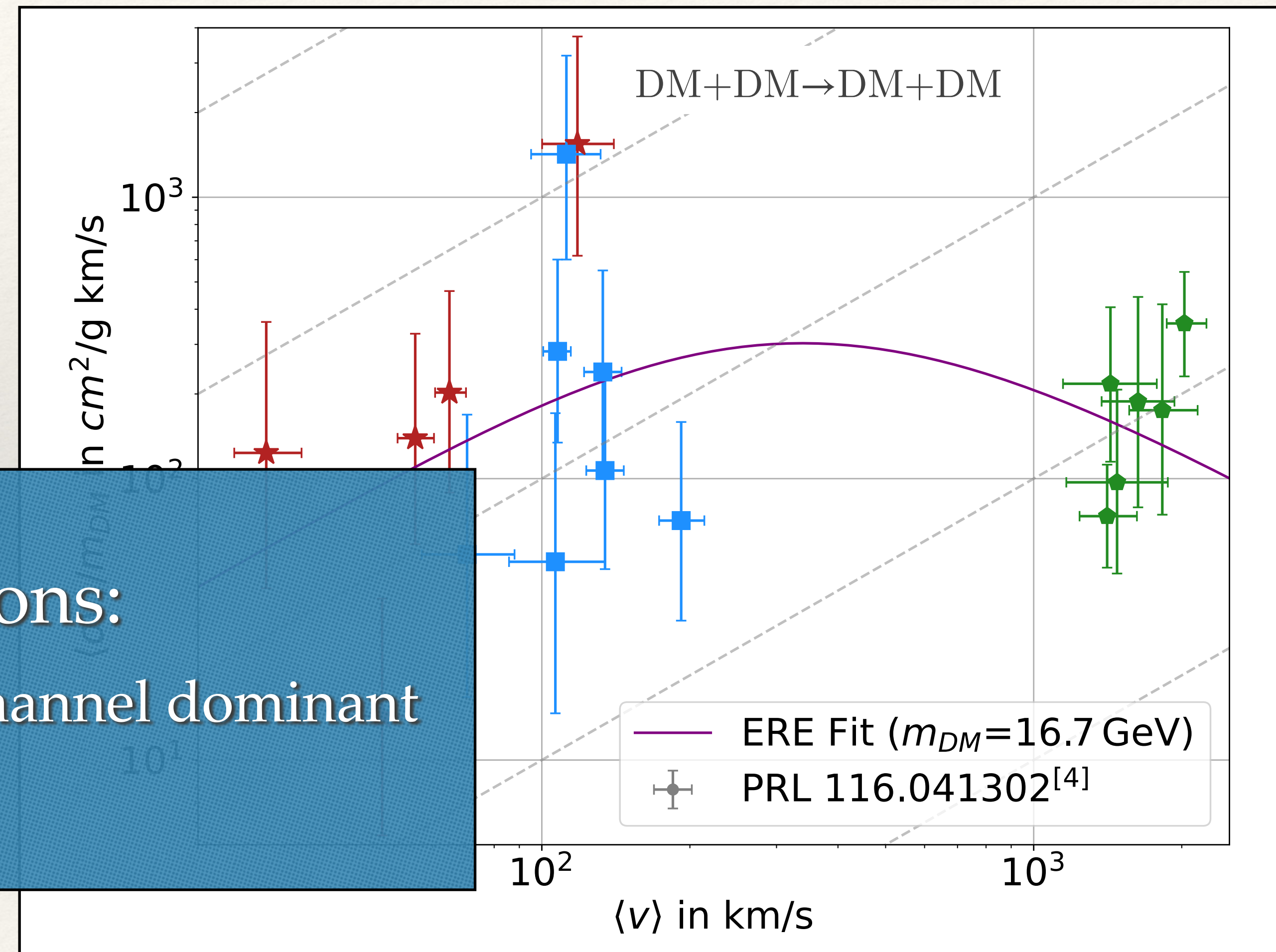
❖  $v$  - rel. velocity

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## Assumptions:

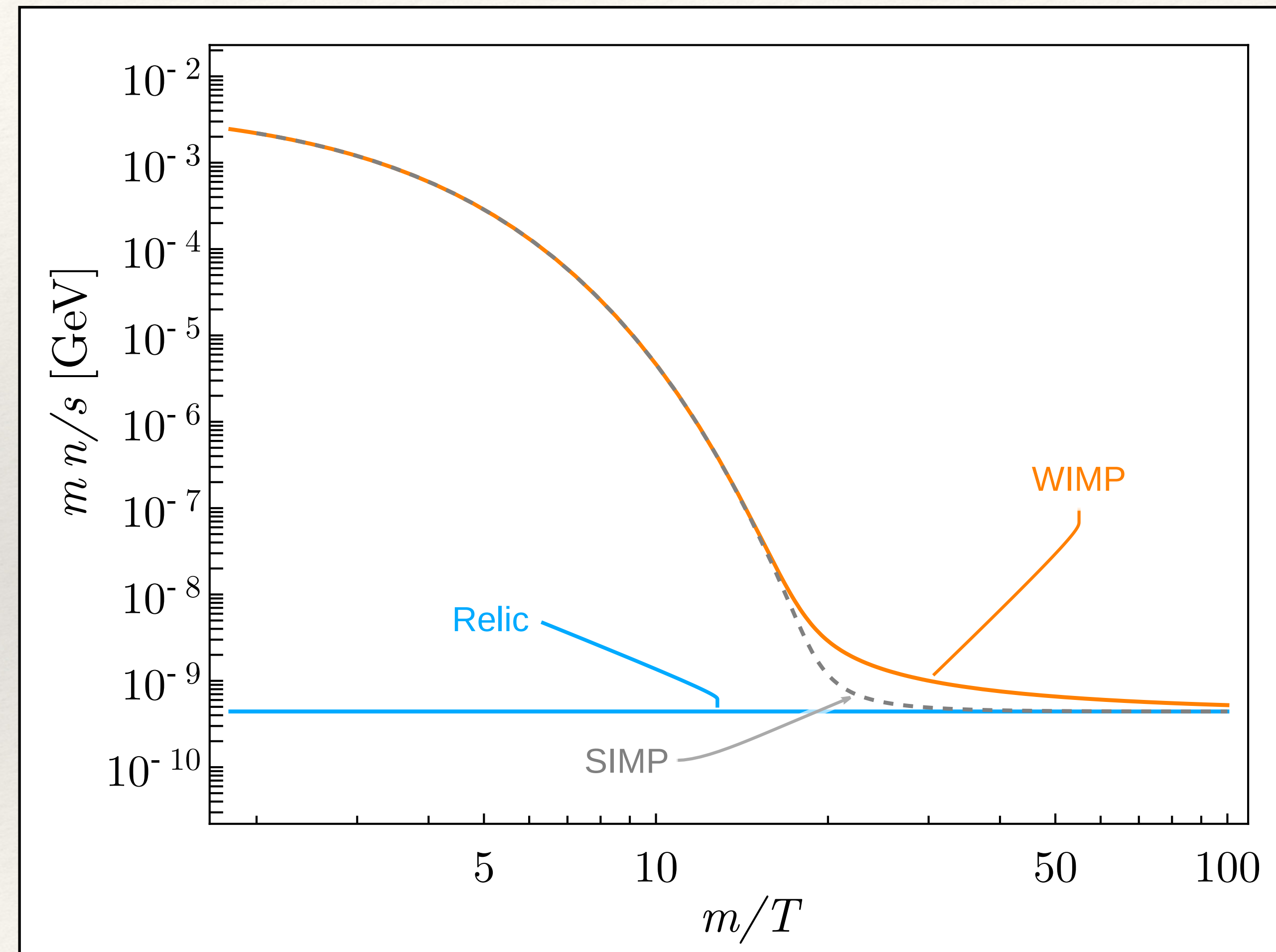
1. Maximal scattering channel dominant
2. s-wave scattering





# Relic density

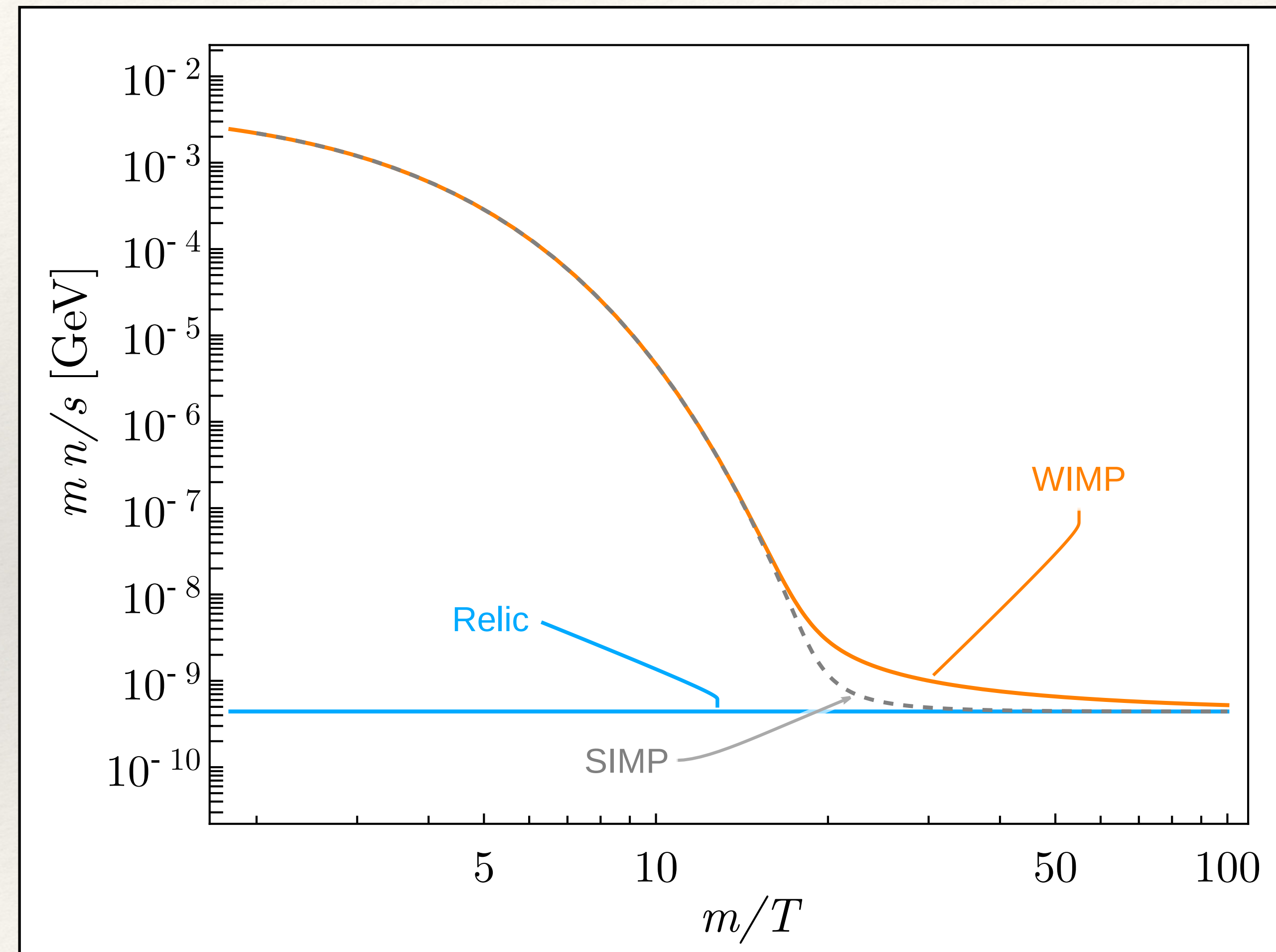
- ❖ Possibility: Dark matter as a thermal relic from the early universe





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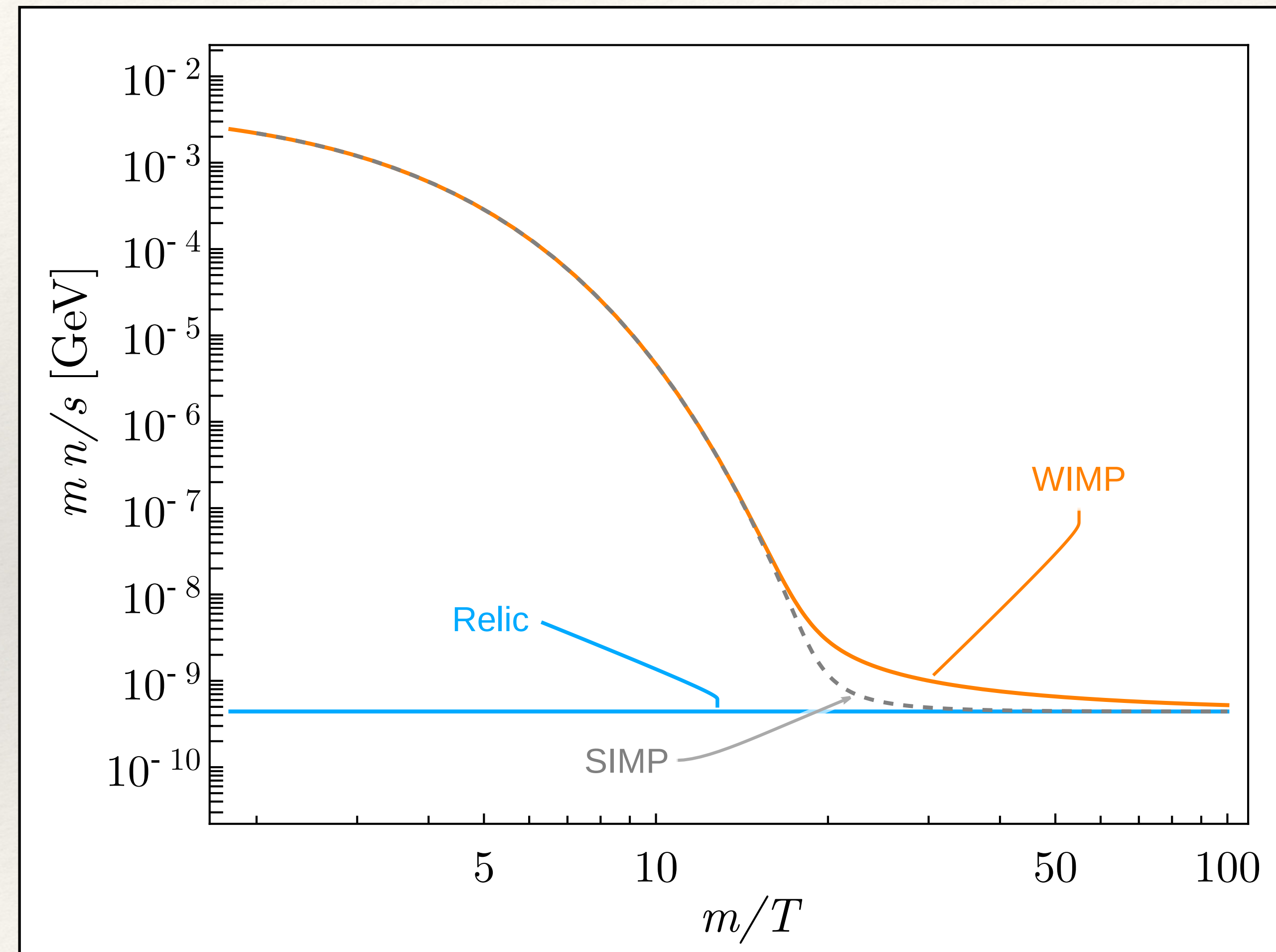
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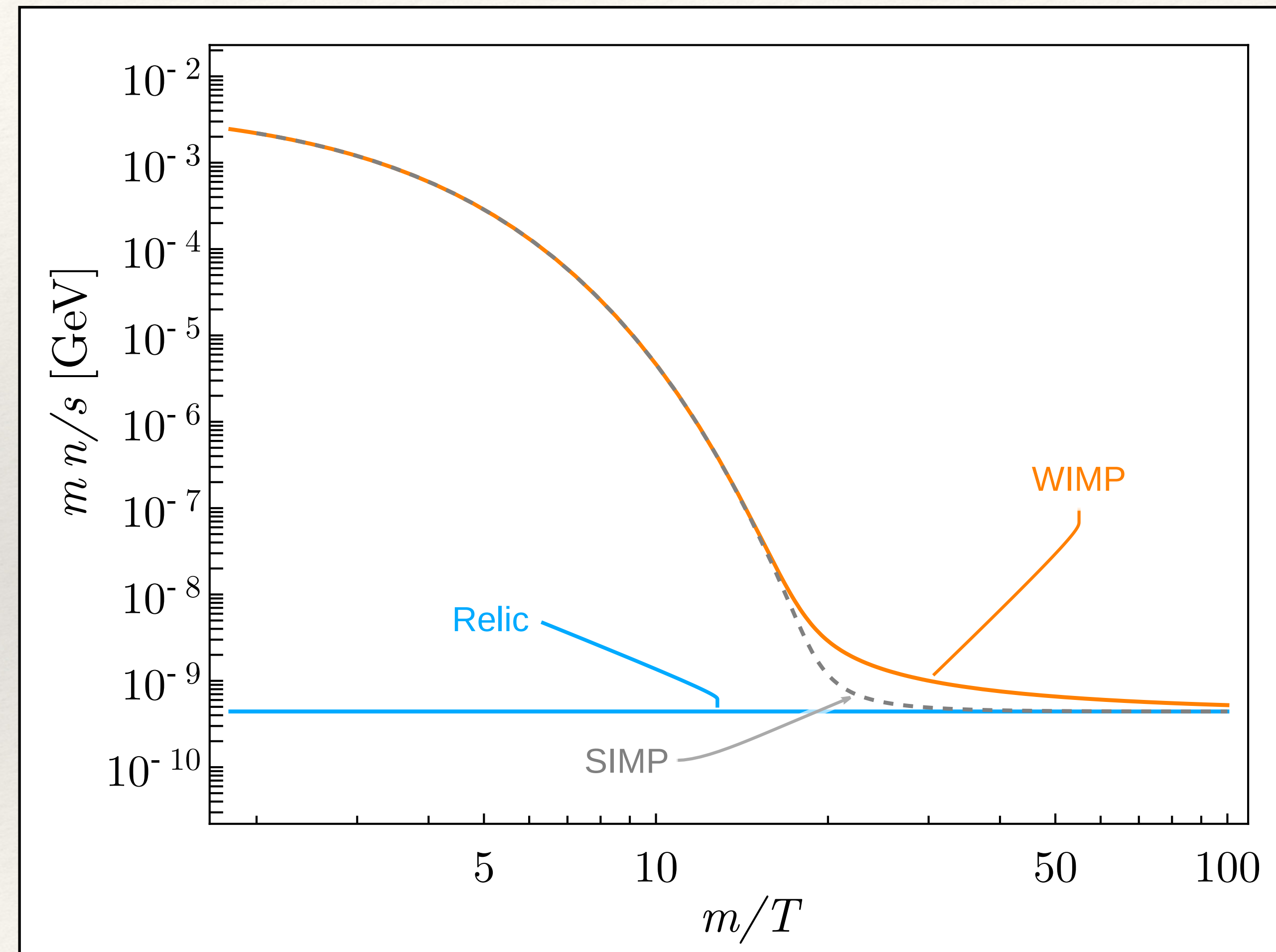
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- ❖ Temperature decreases  $\rightarrow$  interaction "freezes out"





# Relic density

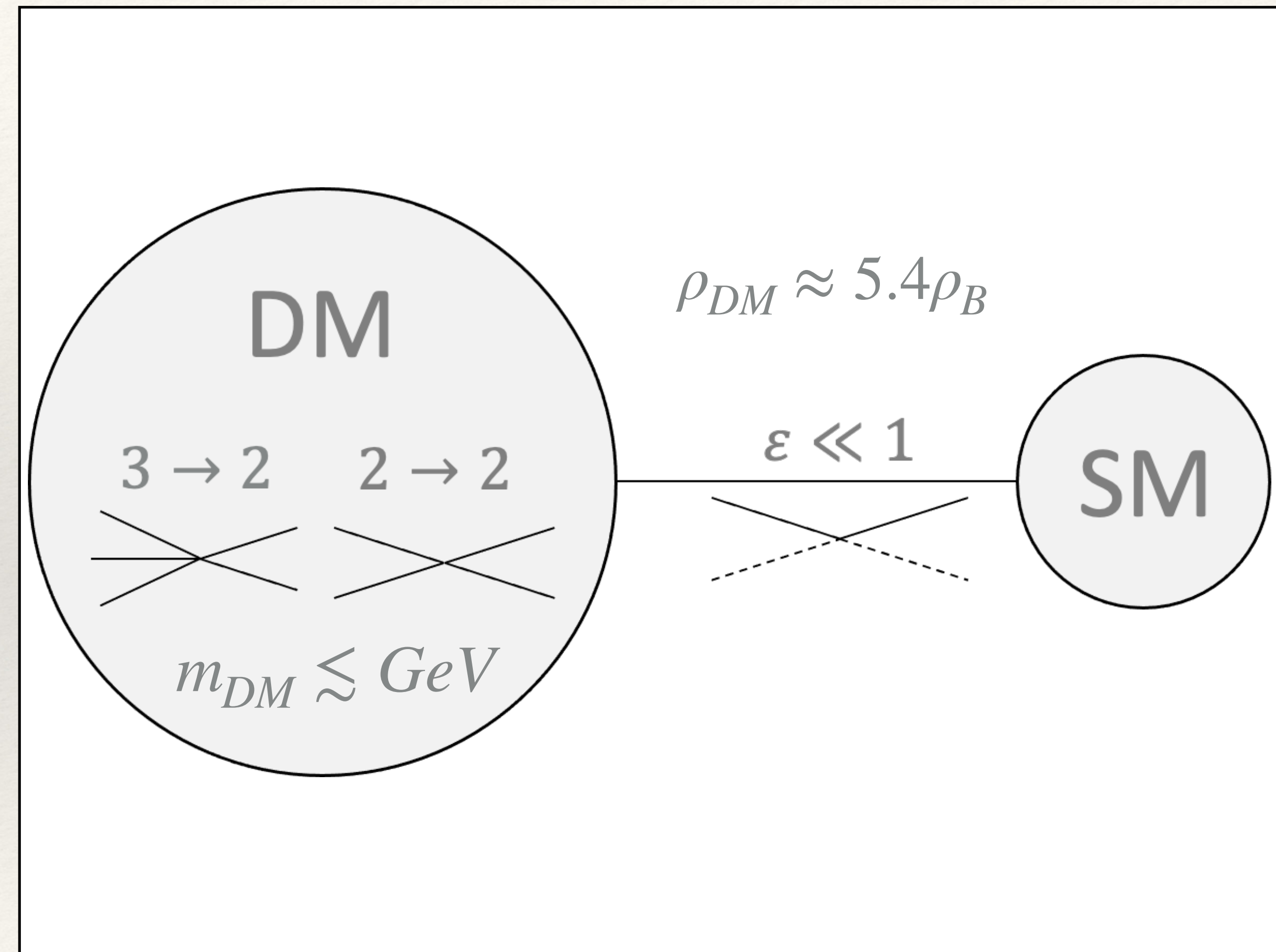
- ❖ Possibility: Dark matter as a thermal relic from the early universe
- ❖ Handle on the dark matter abundance
- ❖ Temperature decreases  $\rightarrow$  interaction "freezes out"
- ❖ Example:
  - ❖ WIMP:  $DM + DM \rightarrow SM + SM$





# Strongly Interacting Massive Particles

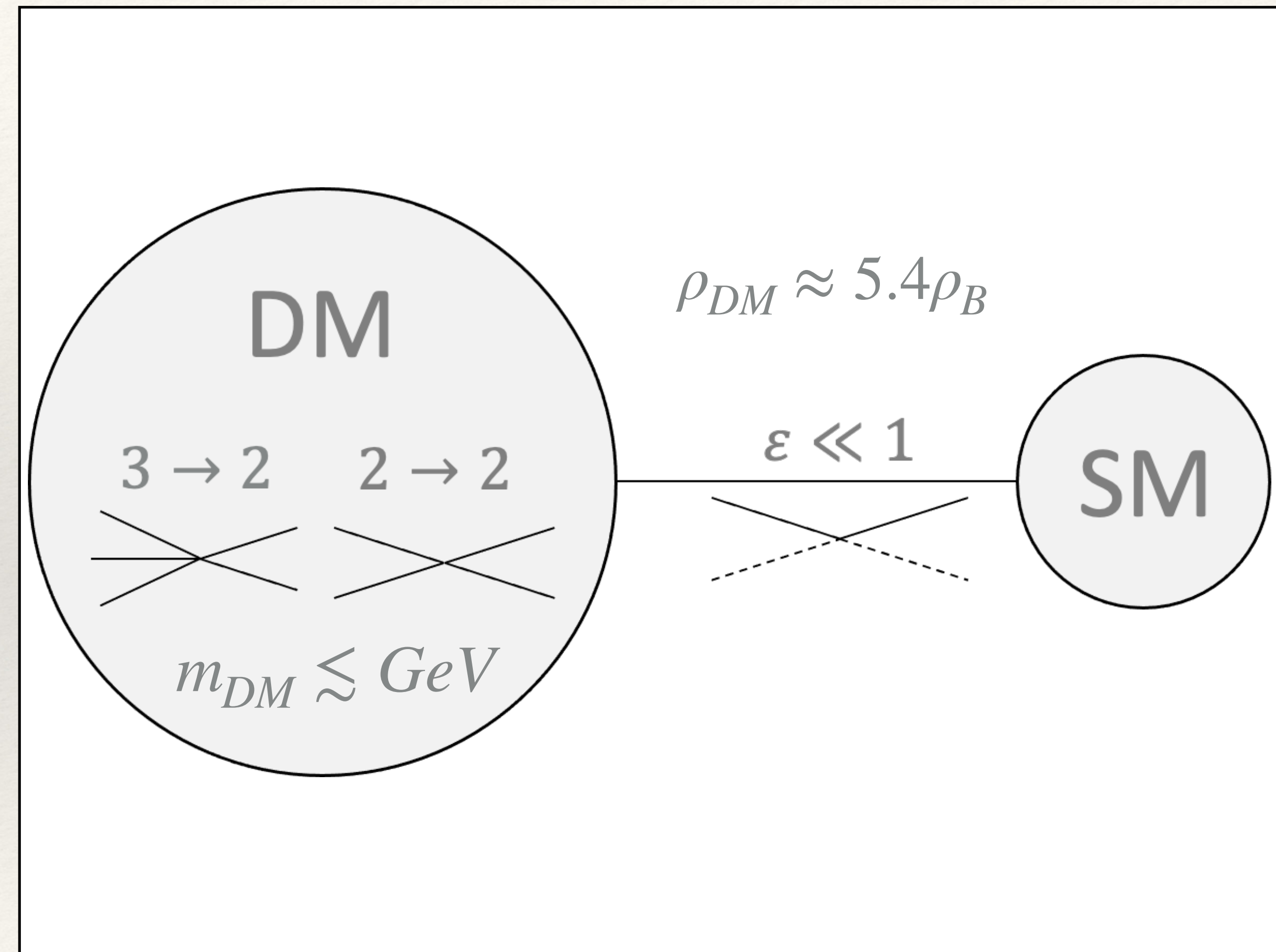
- ❖ Alternative freeze-out paradigm





# Str**o**ngly Interacting Massive Particles

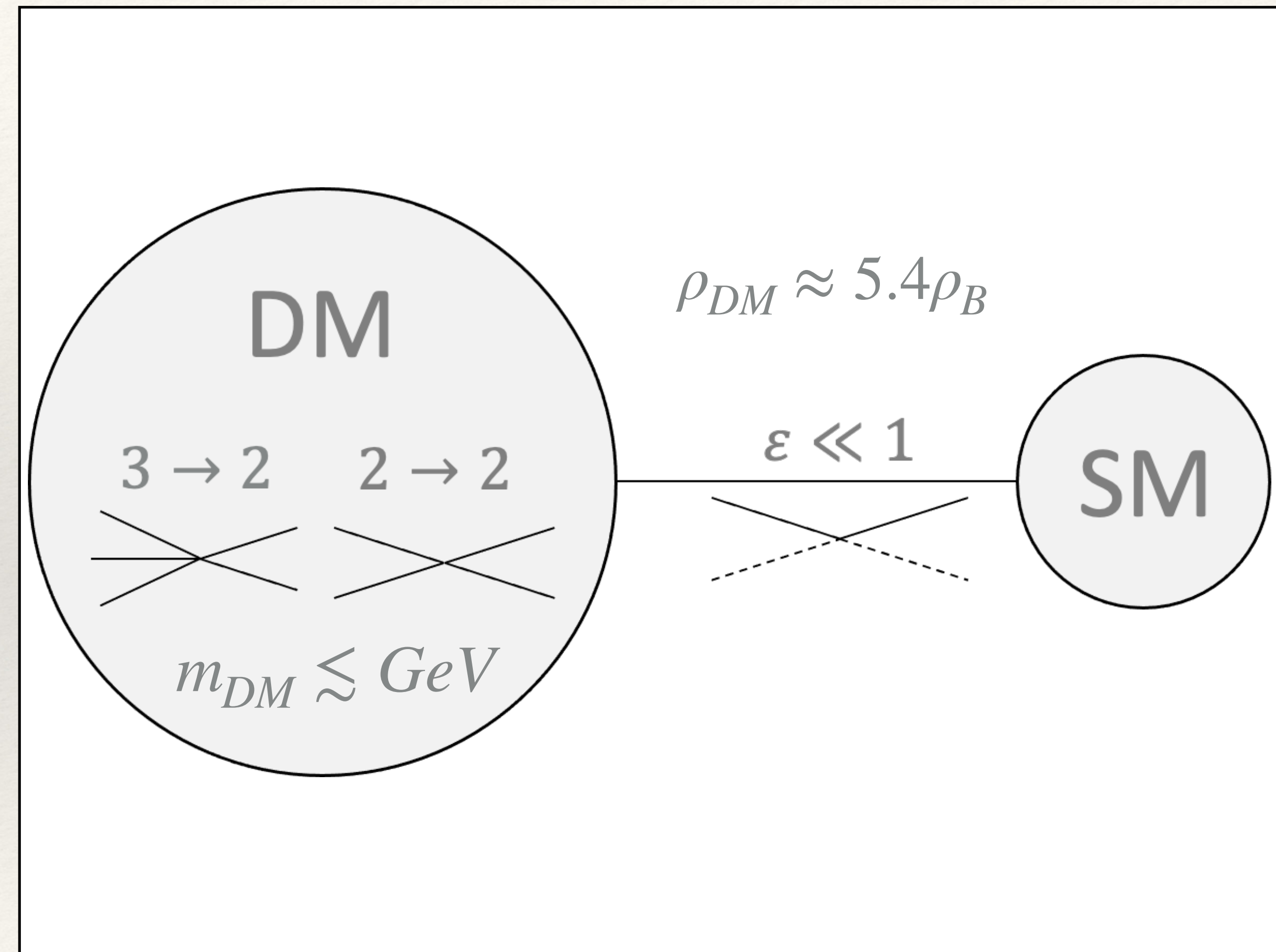
- ❖ Alternative freeze-out paradigm
- ❖ Number lowering process in the dark sector
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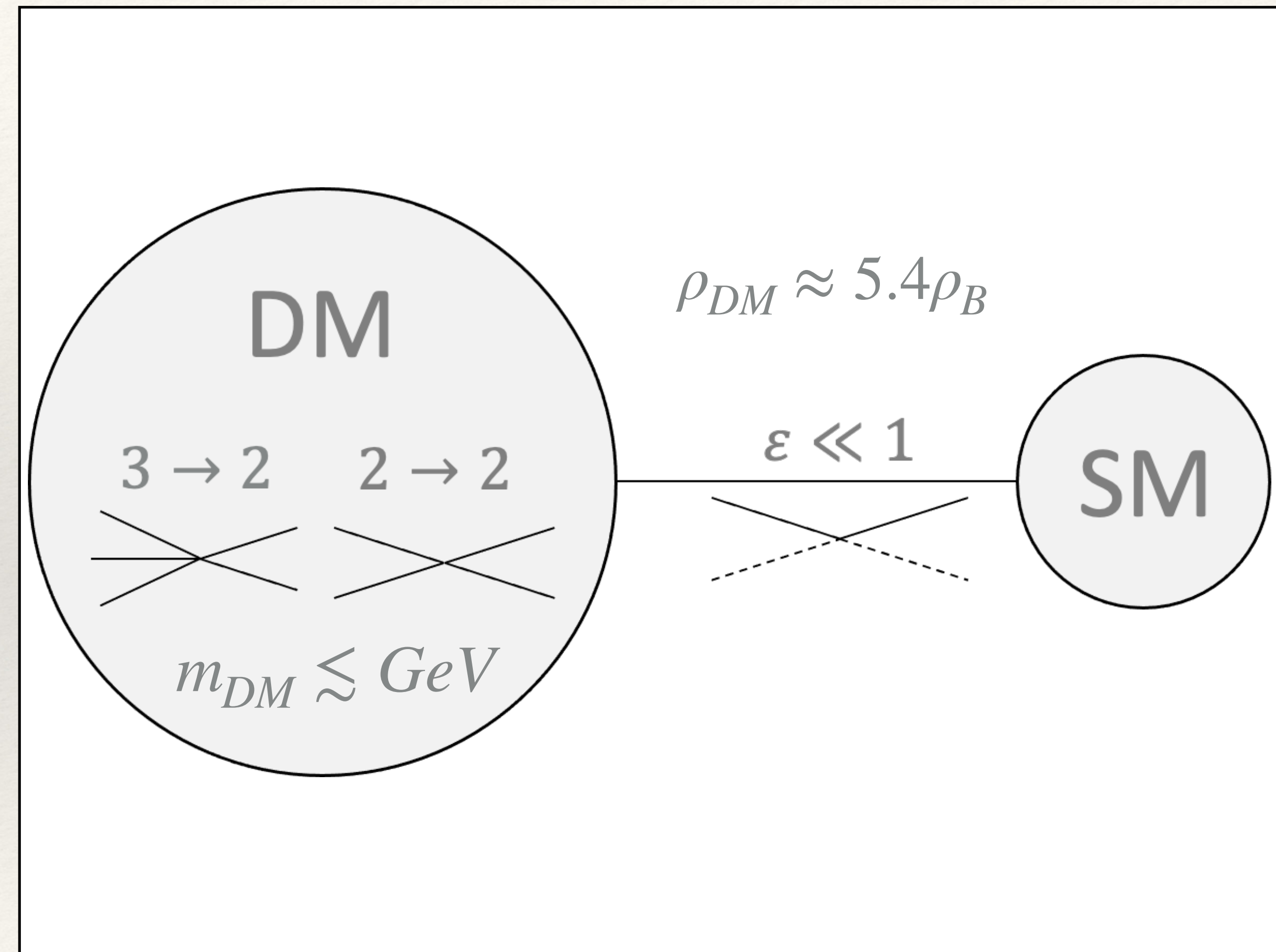
- ❖ Alternative freeze-out paradigm
- ❖ Number lowering process in the dark sector
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- ❖ Coupling to the SM sector needed to prevent heat-up
  - ❖ Mediator enables direct detection





# Strongly Interacting Massive Particles

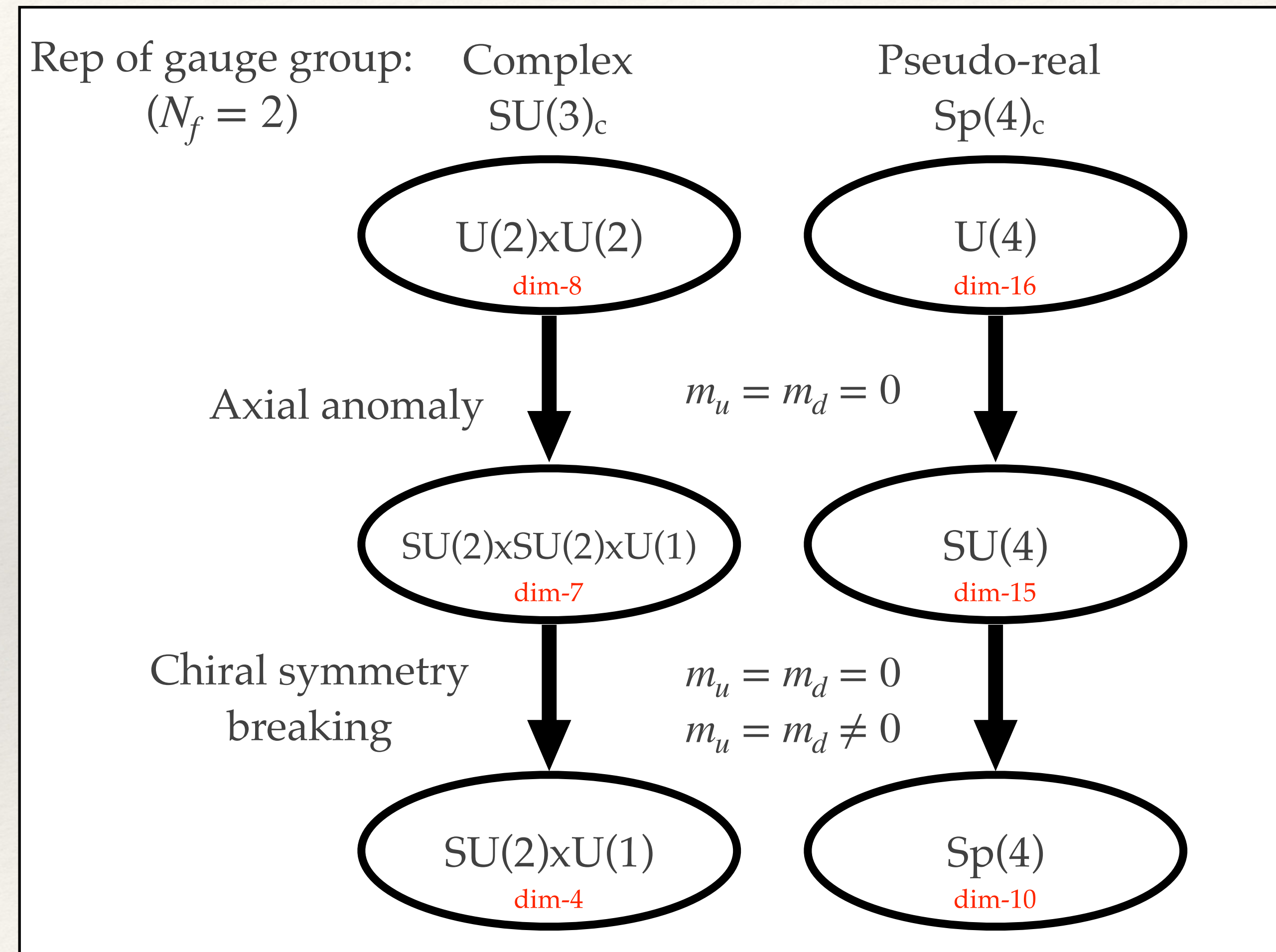
- ❖ Alternative freeze-out paradigm
- ❖ Number lowering process in the dark sector
  - ❖ Addresses self-interaction
- ❖ Coupling to the SM sector needed to prevent heat-up
  - ❖ Mediator enables direct detection
- ❖ Underlying UV theory?





# Minimal realisation

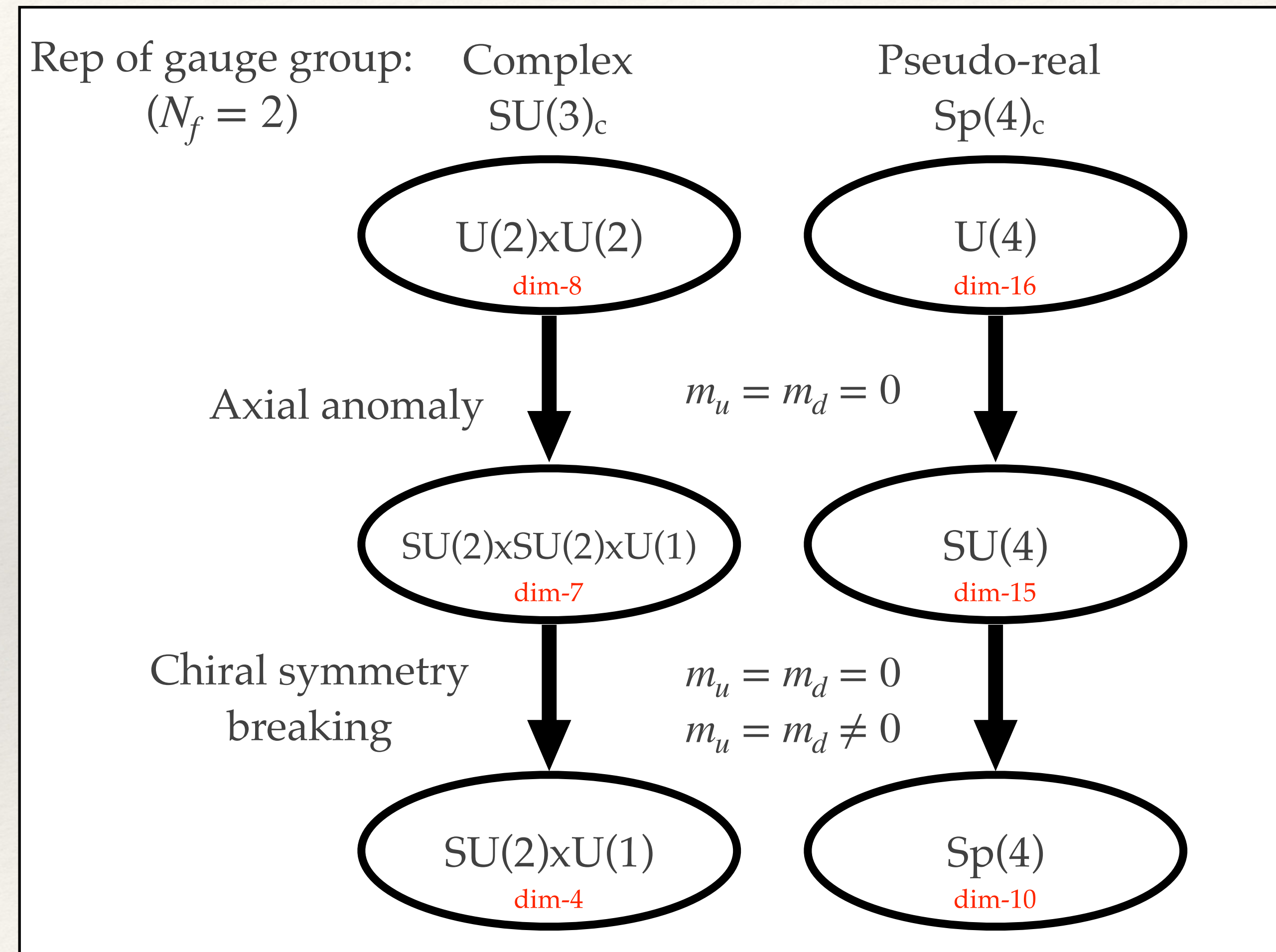
- ❖  $Sp(4)$  flavour symmetry





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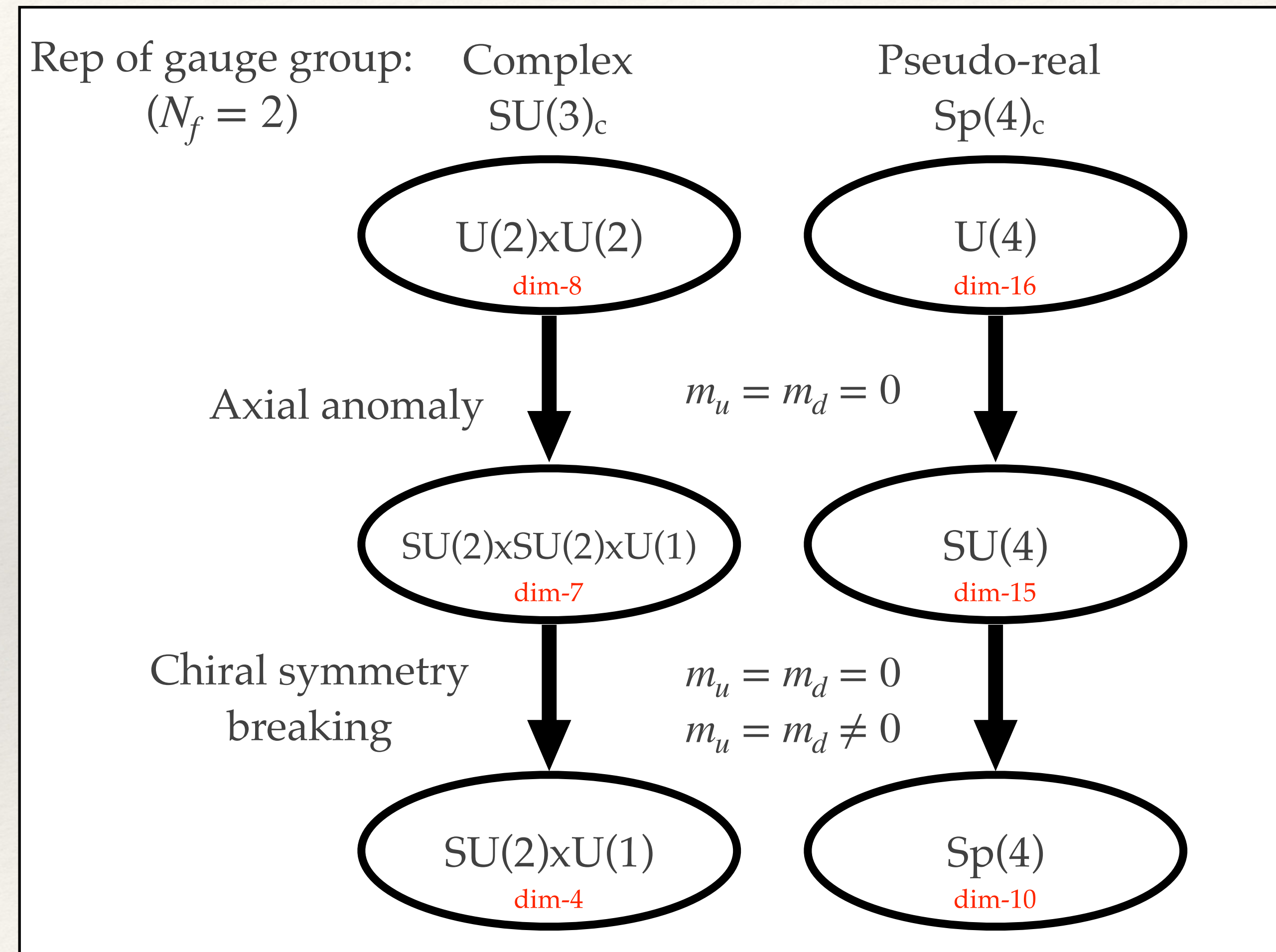
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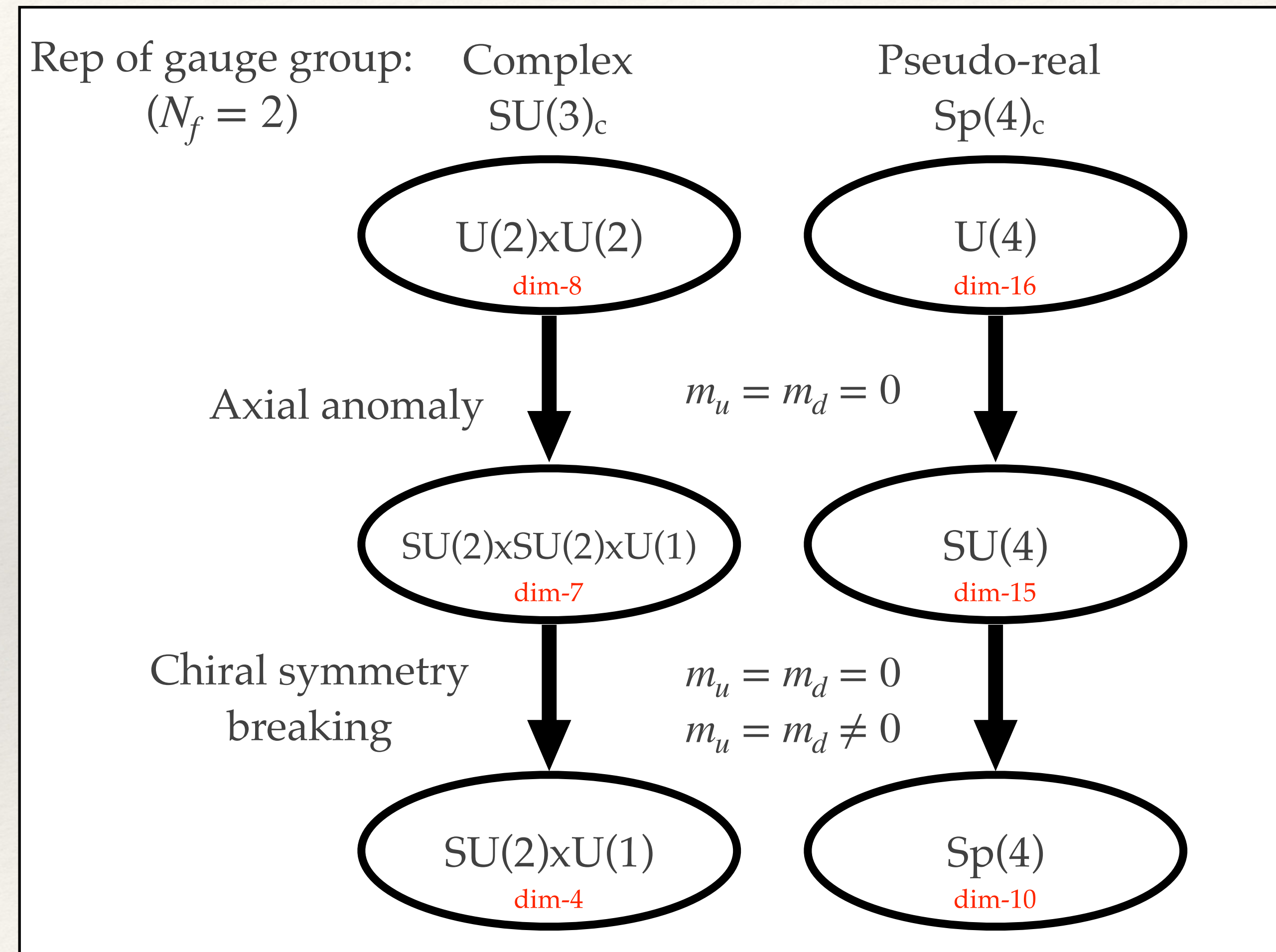
- ❖  $Sp(4)$  flavour symmetry
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- ❖ Mixing of left- and right handed components (Weyl-fermions)
  - ❖ Symmetry is enlarged





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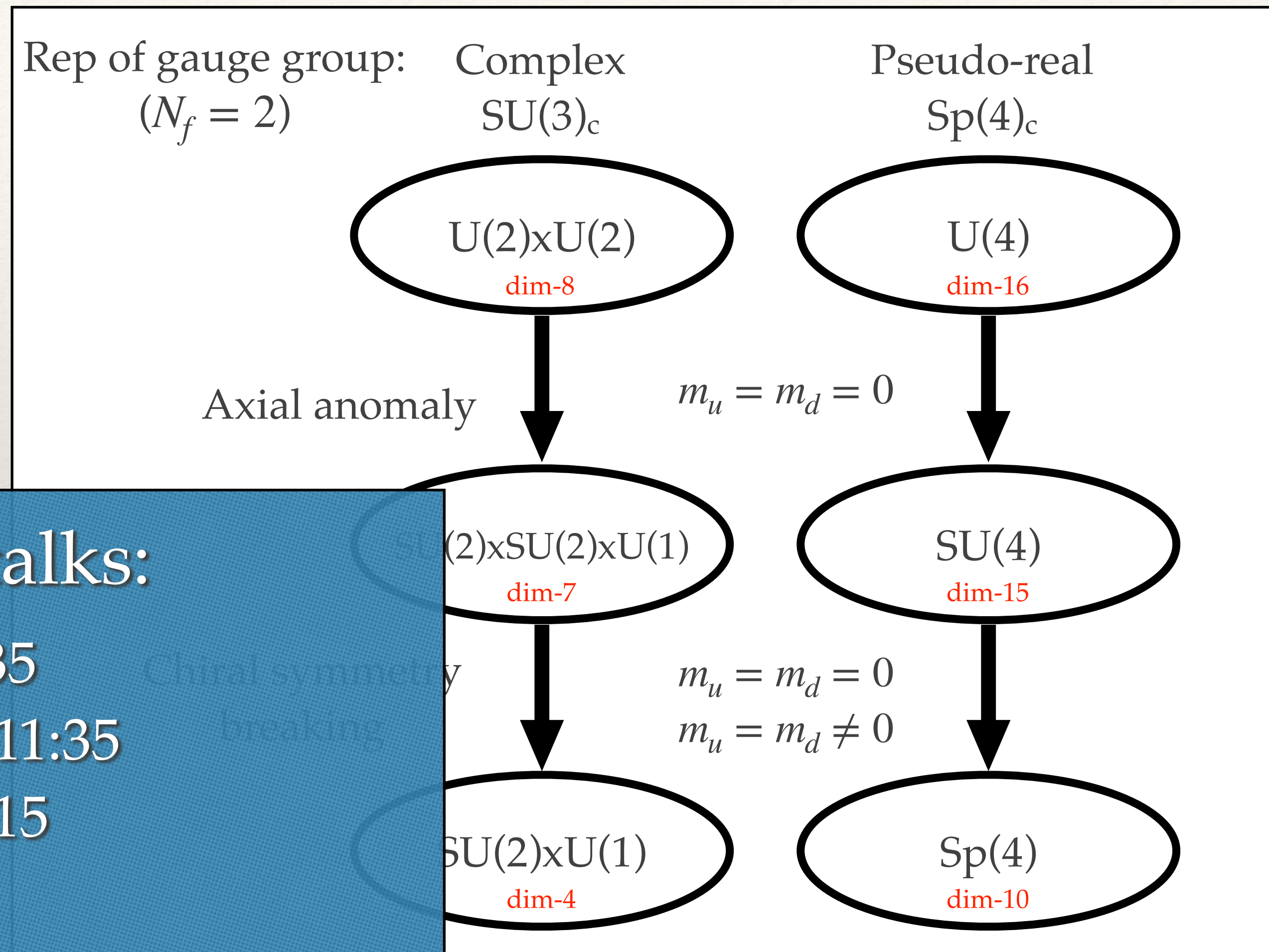
- ❖  $Sp(4)$  flavour symmetry
- ❖ Pseudo-real rep of gauge group with  $N_f = 2$
- ❖ Mixing of left- and right handed components (Weyl-fermions)
  - ❖ Symmetry is enlarged
- ❖ Result: 5 pNGBs
  - ❖  $3 \rightarrow 2$  process possible
  - ❖ WZW description in ChPT





# Minimal realisation

- ❖  $Sp(4)$  flavour symmetry
- ❖ Pseudo-real rep of gauge group with  $N_f = 2$
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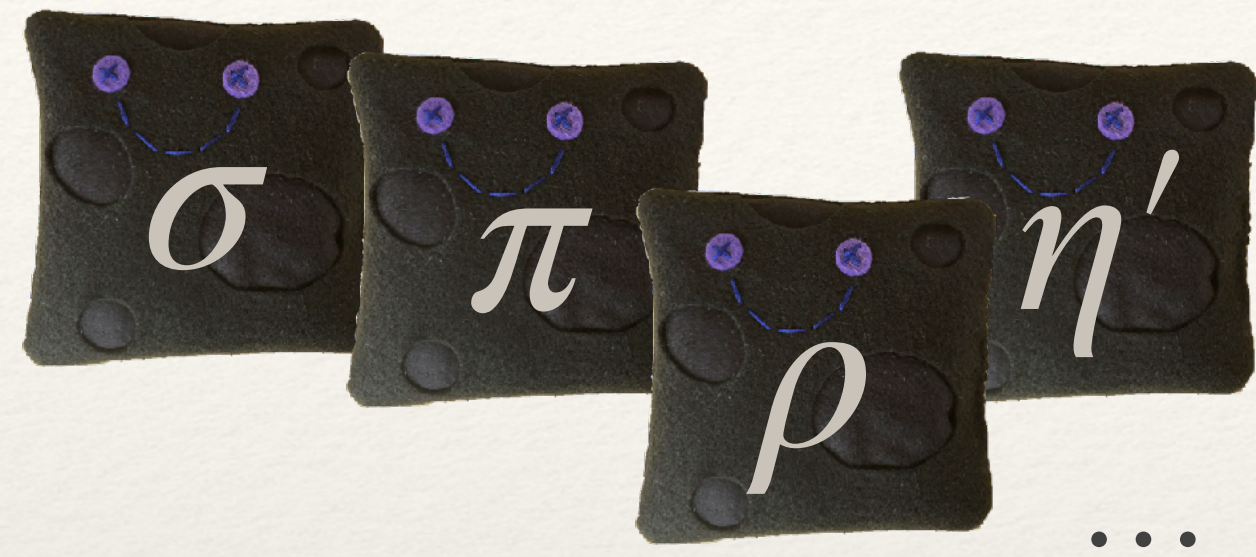


## Other $Sp(4)$ talks:

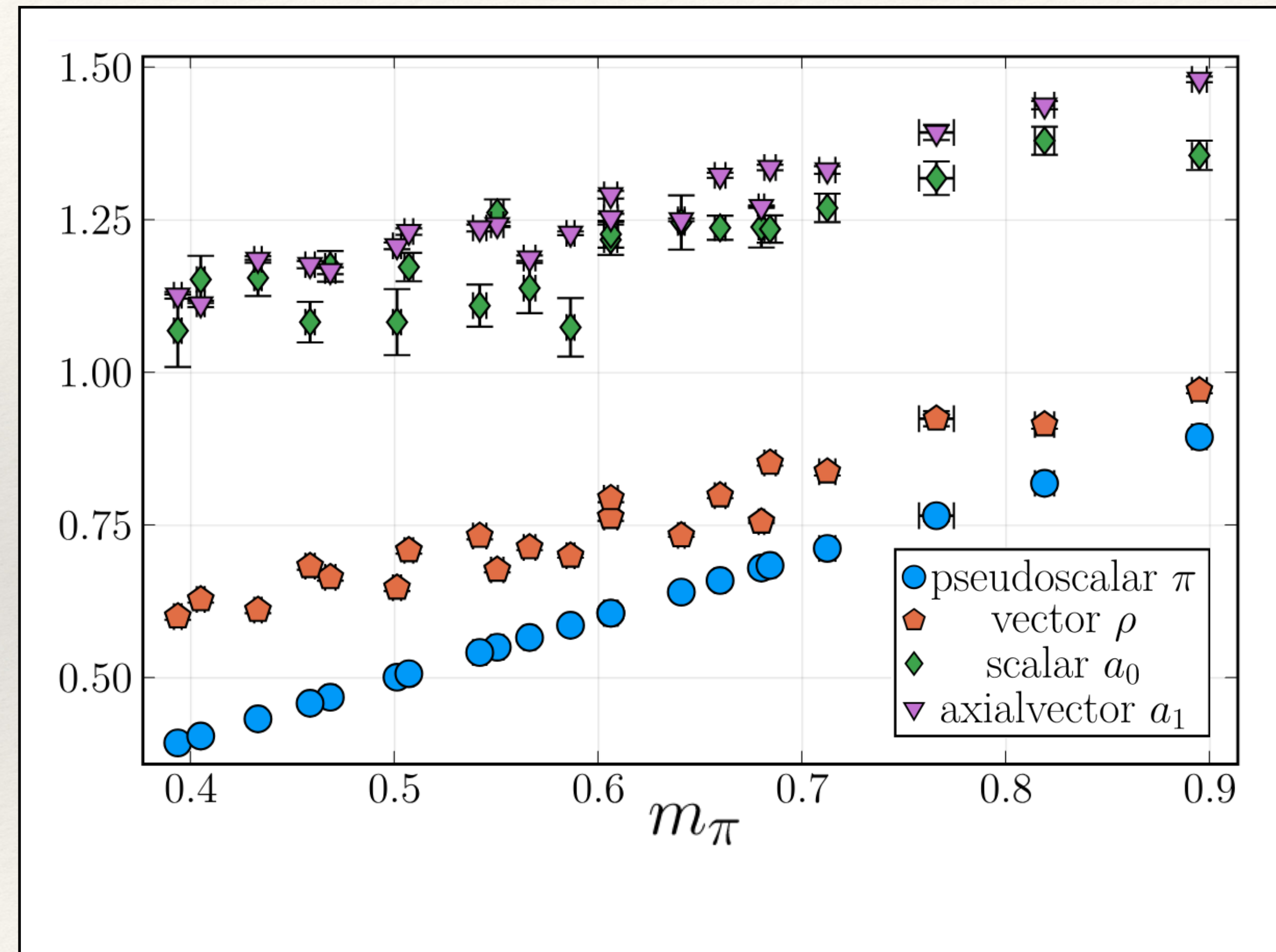
1. Fabian Zierler: Tue 11:35
2. Niccolo Forzano: Wed 11:35
3. David Mason: Wed 12:15
4. Ho Hsiao: Fri 11:15



# Particle phenomenology



❖ Zoo of dark hadrons

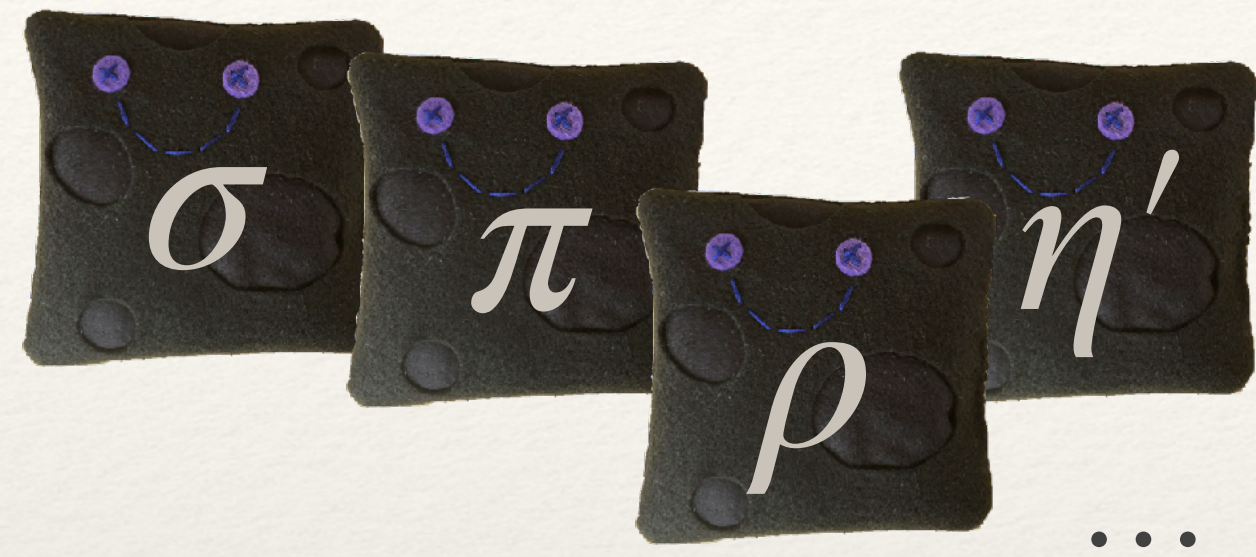


Bennett et al.: arXiv:1909.12662 (2019)

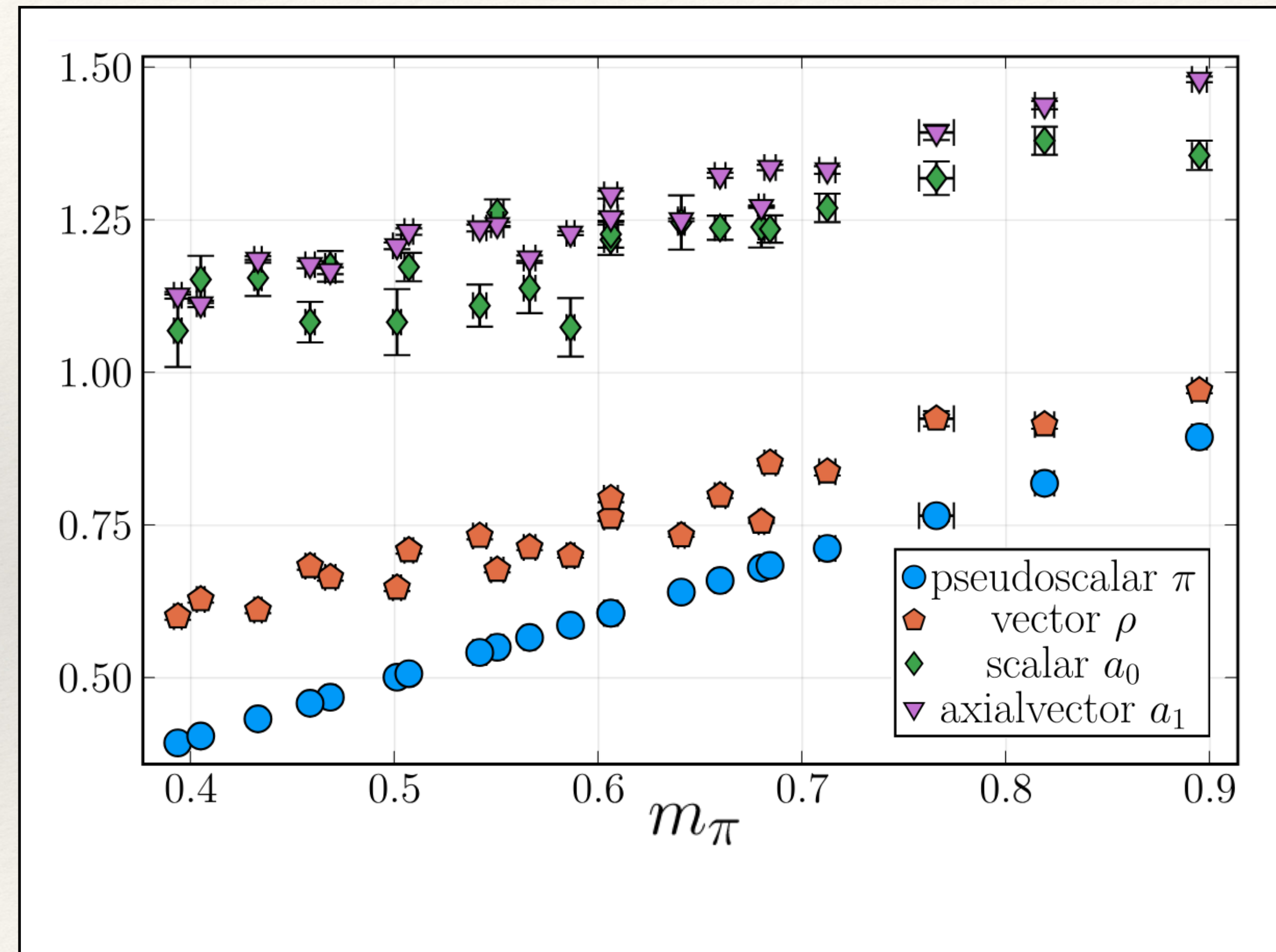
F. Zierler: PHD thesis



# Particle phenomenology



- ❖ Zoo of dark hadrons
- ❖ 5 Pions & 10 Rhos lightest non-singlets

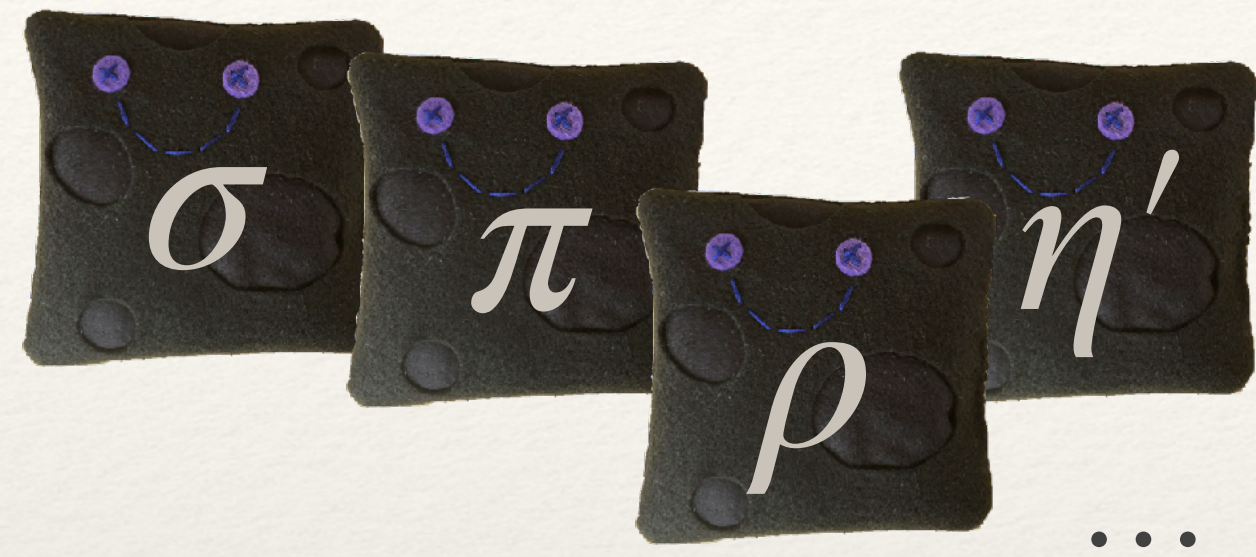


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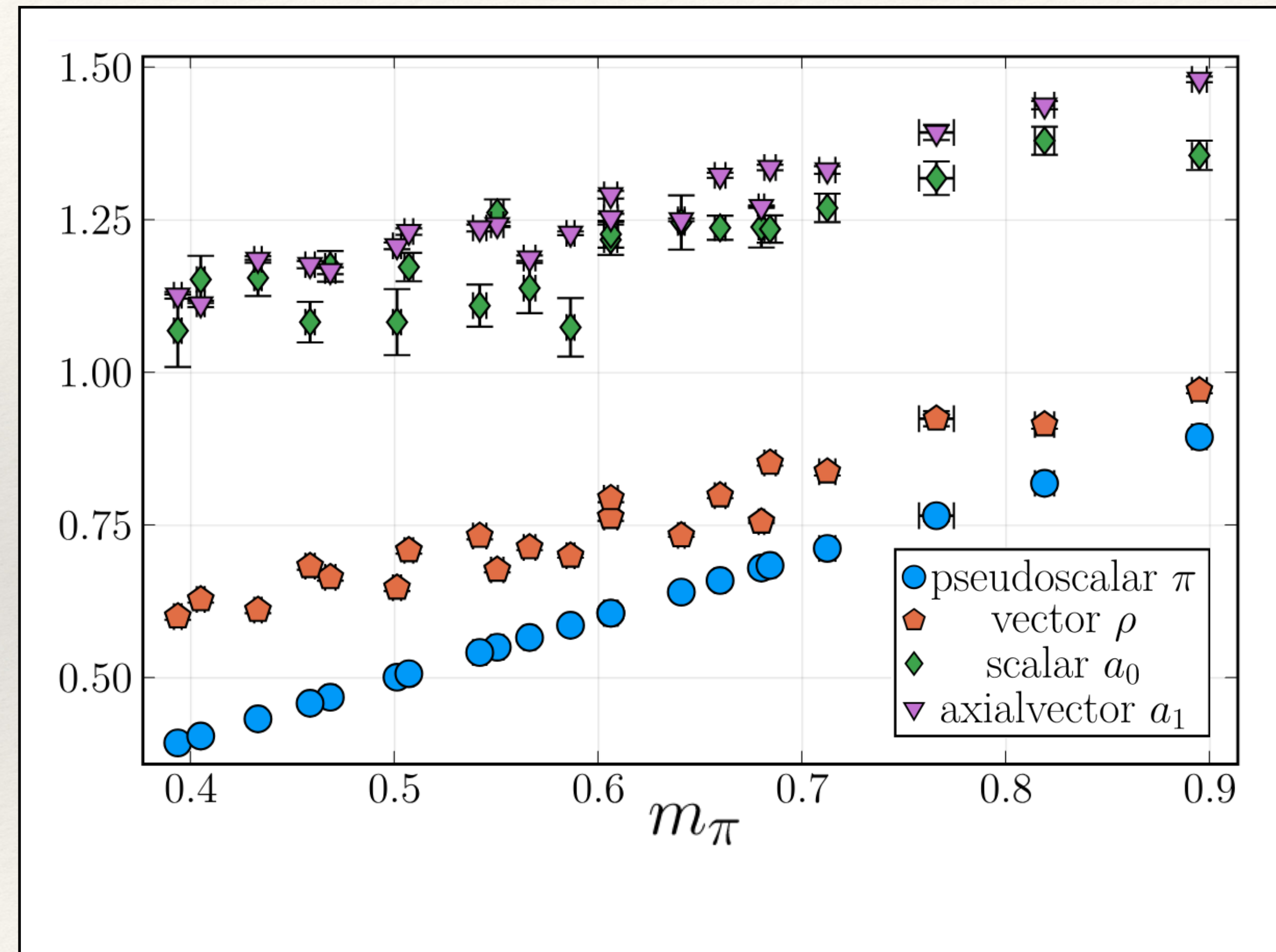
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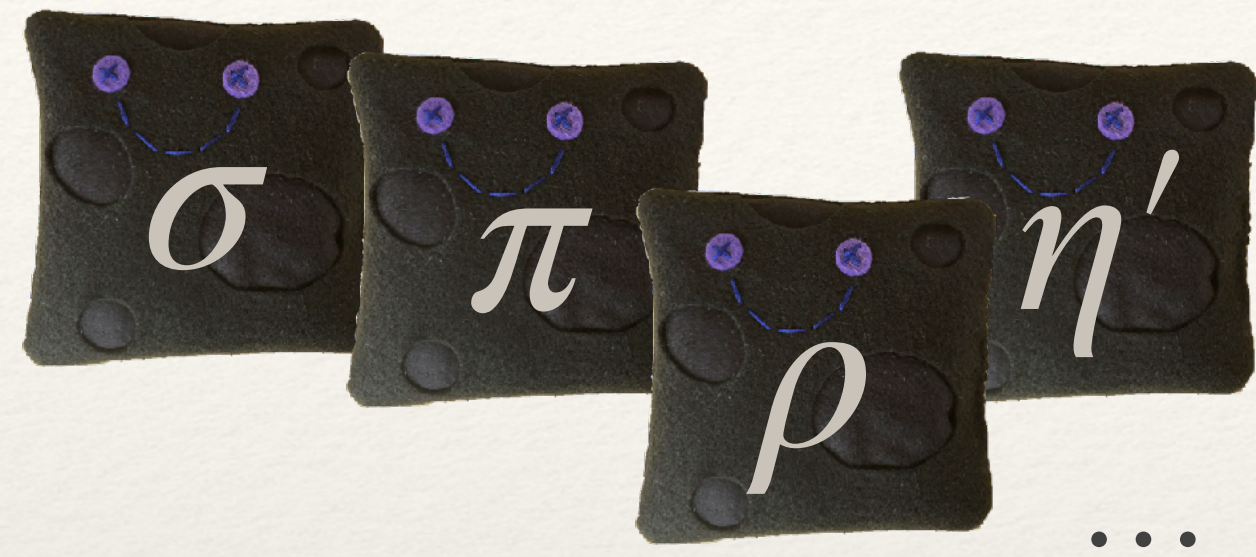


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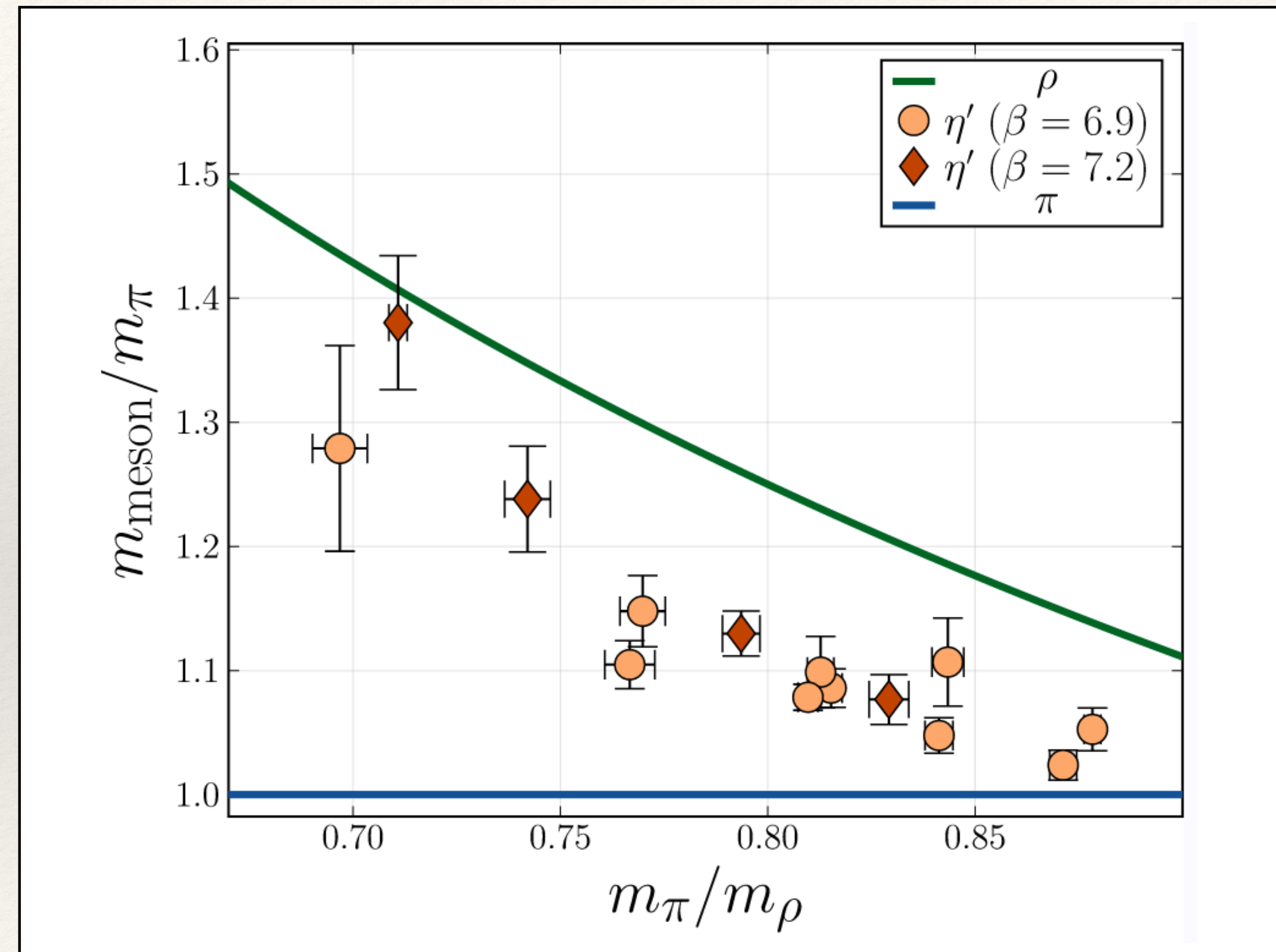
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# Particle phenomenology



- ❖ Zoo of dark hadrons
- ❖ 5 Pions & 10 Rhos lightest non-singlets
- ❖ No fermionic bound states
- ❖ Light  $\eta'$  relevant for  $\pi\pi$  scattering
  - ❖ Limits ChPT validity



Bennett et al.: arXiv:1909.12662 (2019)

F. Zierler: PHD thesis



# Scattering phenomenology

❖ 3  $\pi\pi$  scattering channels:

$Sp(4)_f$
$5 \otimes 5 = 1 \oplus 10 \oplus 14$ $\pi\pi \rightarrow \pi\pi \text{ (dim = 1, 10, 14)}$



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$5 \otimes 5 = 1 \oplus 10 \oplus 14$ $\pi\pi \rightarrow \pi\pi \text{ (dim = 1, 10, 14)}$ $\pi\pi \rightarrow \rho \text{ (dim = 10)}$ $5 \otimes 5 \otimes 5 = 3(5) \oplus 10 \oplus 30 \oplus 35$ $\pi\pi\pi \rightarrow \pi\pi \text{ (dim = 10)}$ <p>etc.</p>



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  - ❖ 14-dim: "Maximal" -  $\pi^+\pi^+ \rightarrow \pi^+\pi^+$

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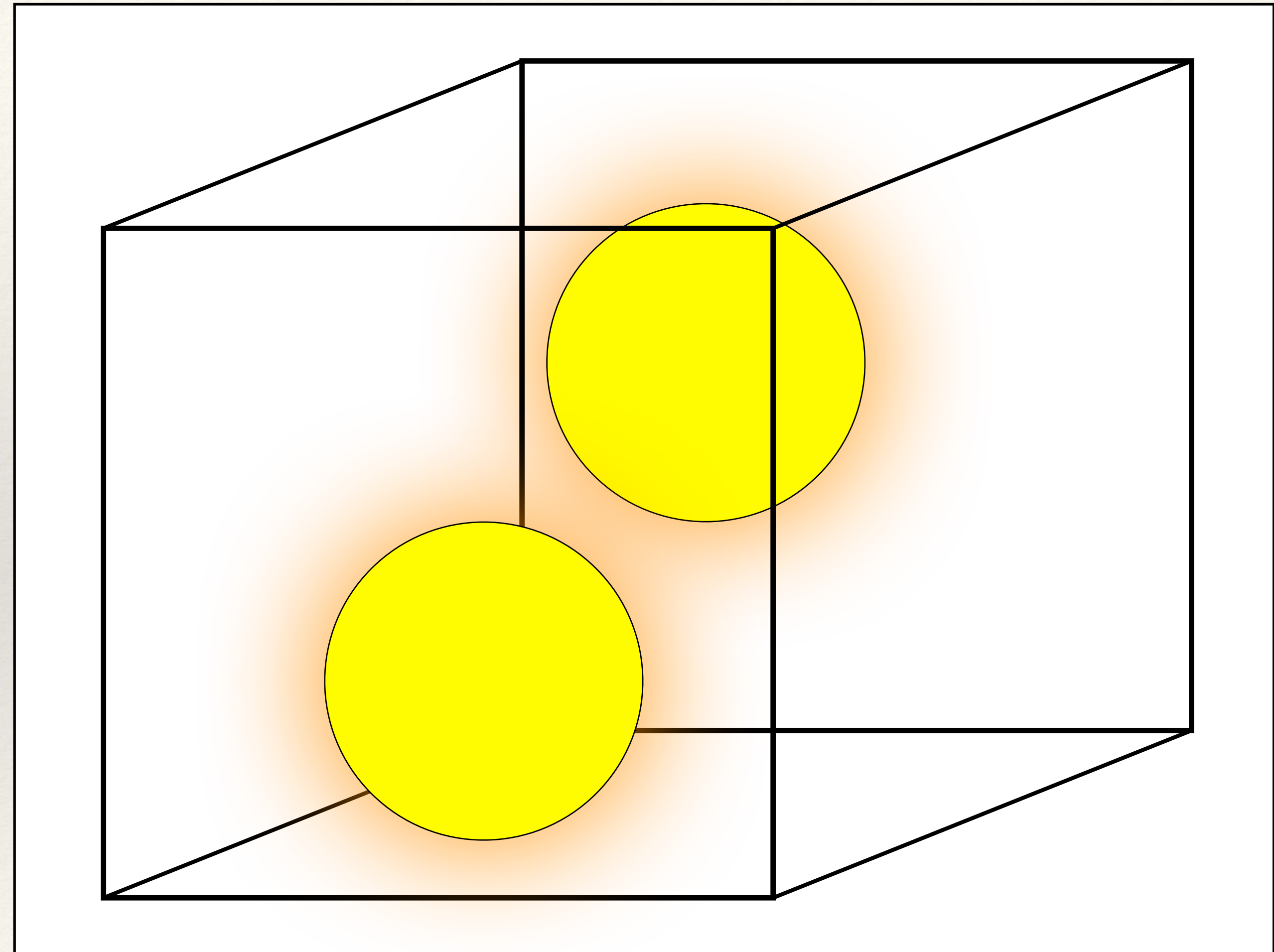
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  - ❖ 14-dim: "Maximal" -  $\pi^+\pi^+ \rightarrow \pi^+\pi^+$
- ❖ Contribution to  $\pi\pi \rightarrow \pi\pi$  scattering naively scales with dimension

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# Scattering on the lattice

- ❖ Standard Lüscher
  - ❖ Relate finite volume energy levels with infinite volume scattering properties



Lüscher et al.: Commun. Math. Phys. 104/105 (1986)

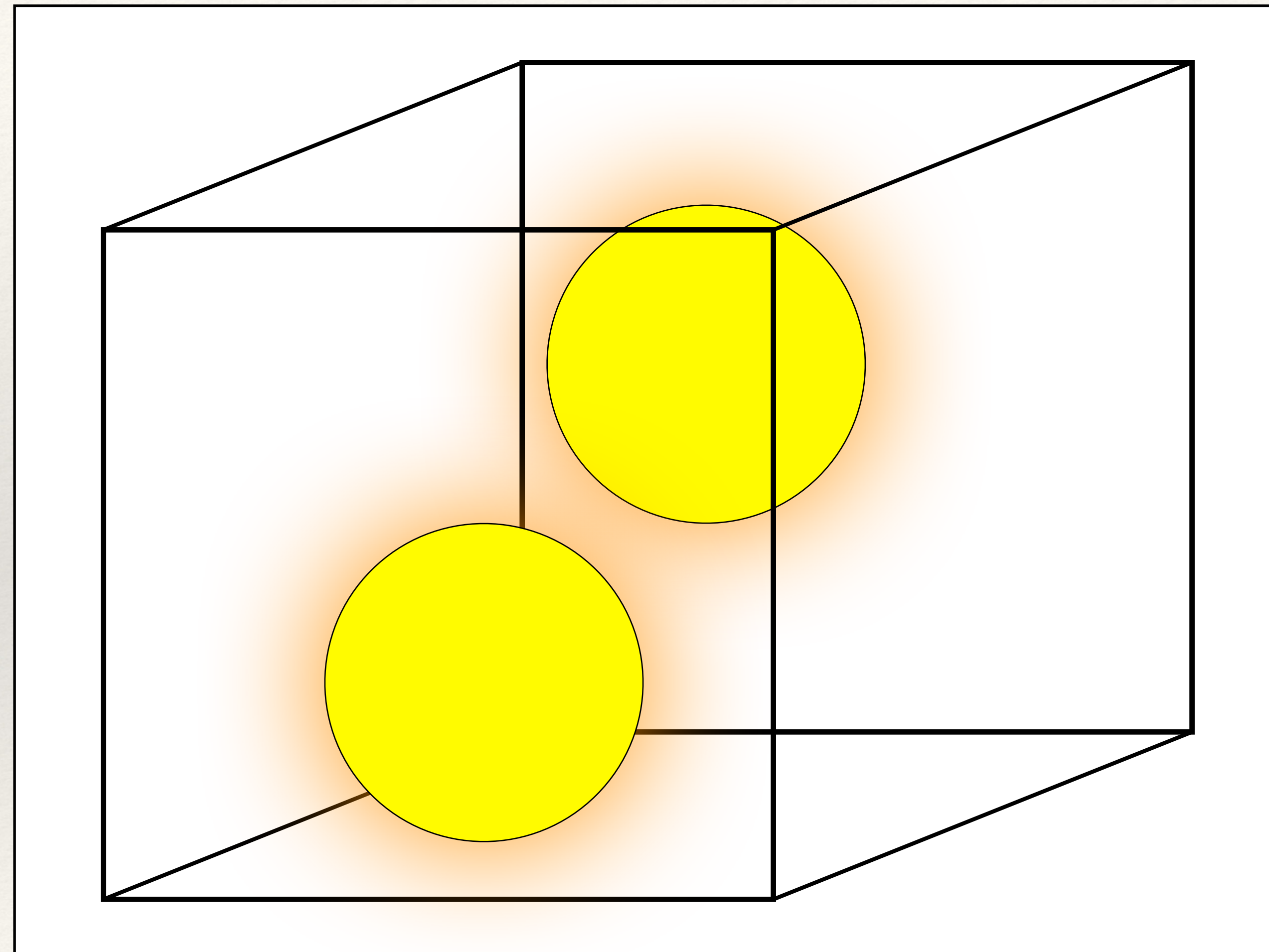
Briceño et al.: Rev.Mod.Phys. 90 (2018) 2, 025001



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- ❖ Standard Lüscher
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- ❖ Zero total momentum:

$$\text{❖ } \tan(\delta) = \frac{\pi^{\frac{3}{2}} q}{\mathcal{Z}_{00}^{\vec{0}}(1, q^2)}, \quad q = \frac{L}{2\pi} p$$



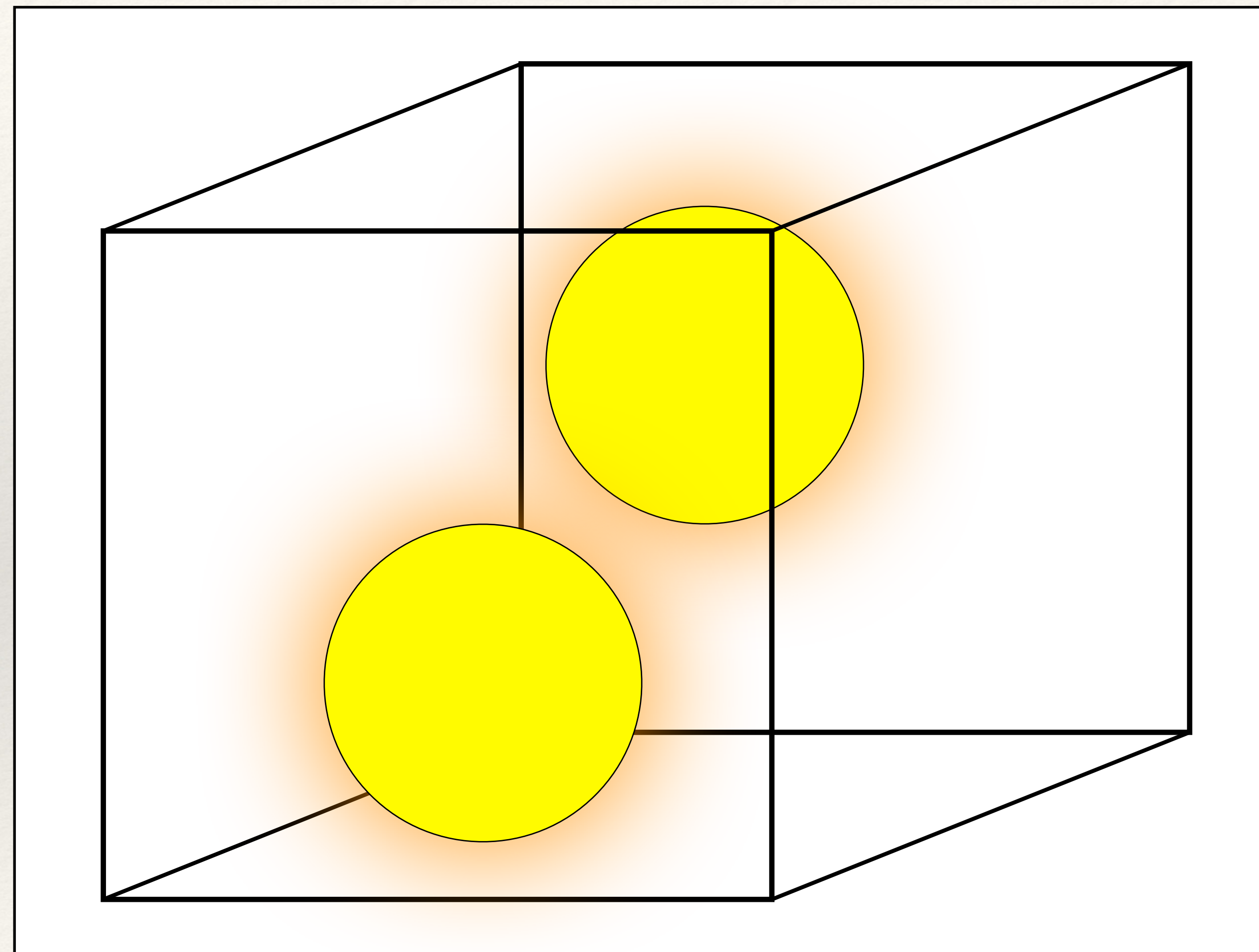
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- ❖ Result: Energy-dependent phase-shift





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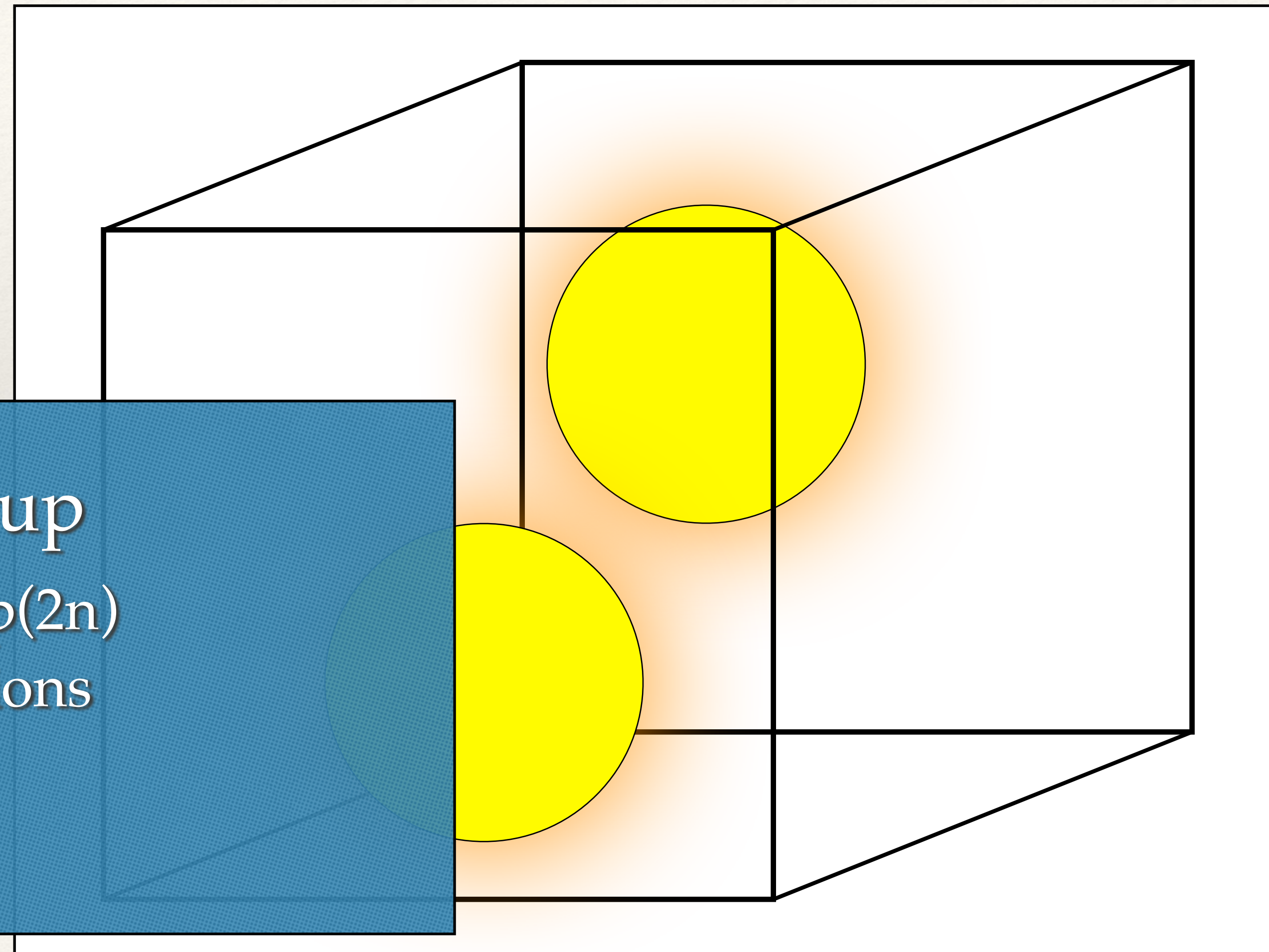
- ❖ Zero total momentum

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- ❖ Result: Energy-

## Lattice setup

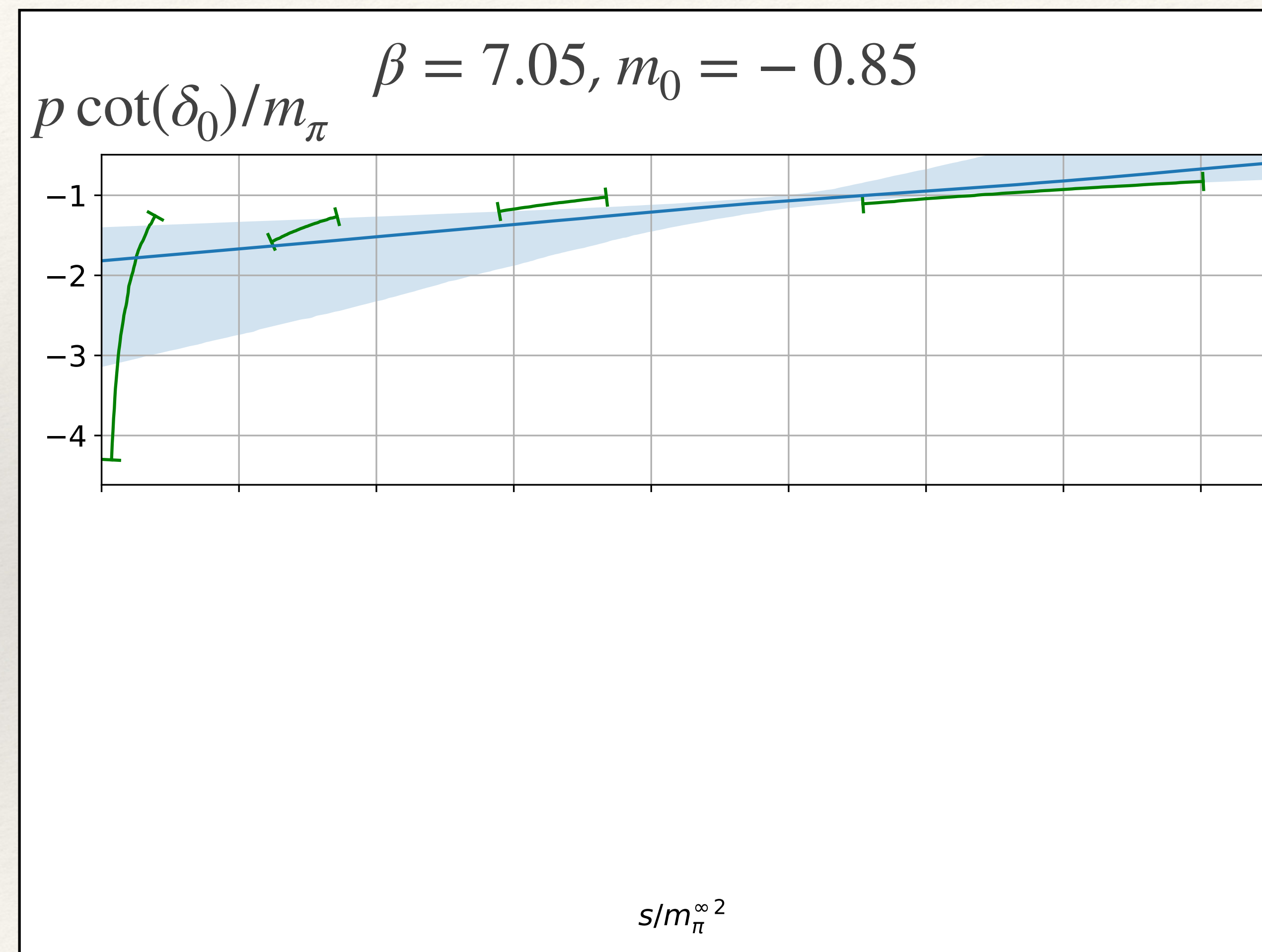
1. HiRep extension for  $Sp(2n)$
2. Standard Wilson fermions
3.  $\frac{m_\pi}{m_\rho} = 0.70 - 0.87$





# Phase shift

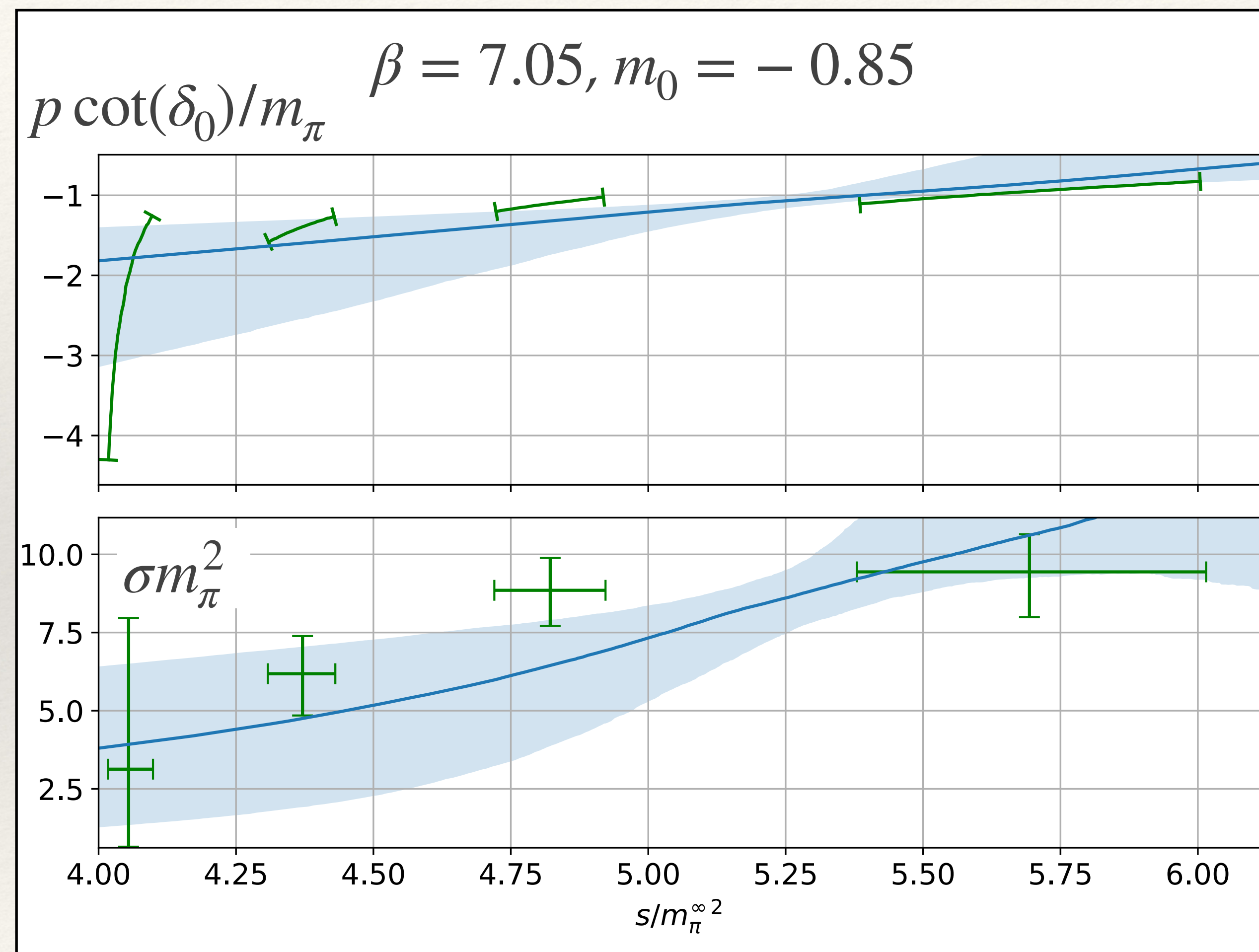
- ❖ Effective range expansion:
  - ❖ Expand phase shift in  $\mathcal{O}(p^2)$





# Phase shift

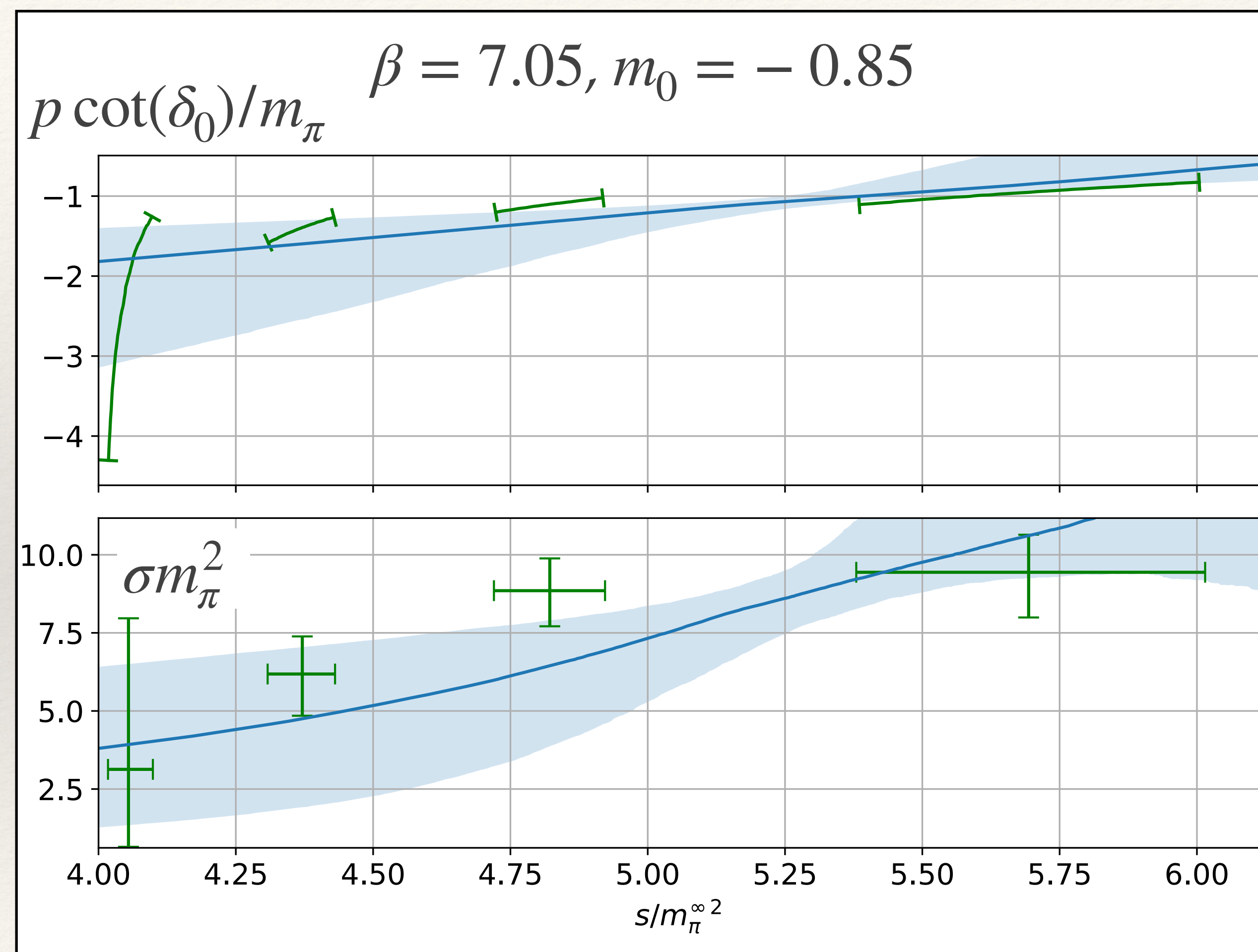
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- ❖ Access to  $\sigma(s)$





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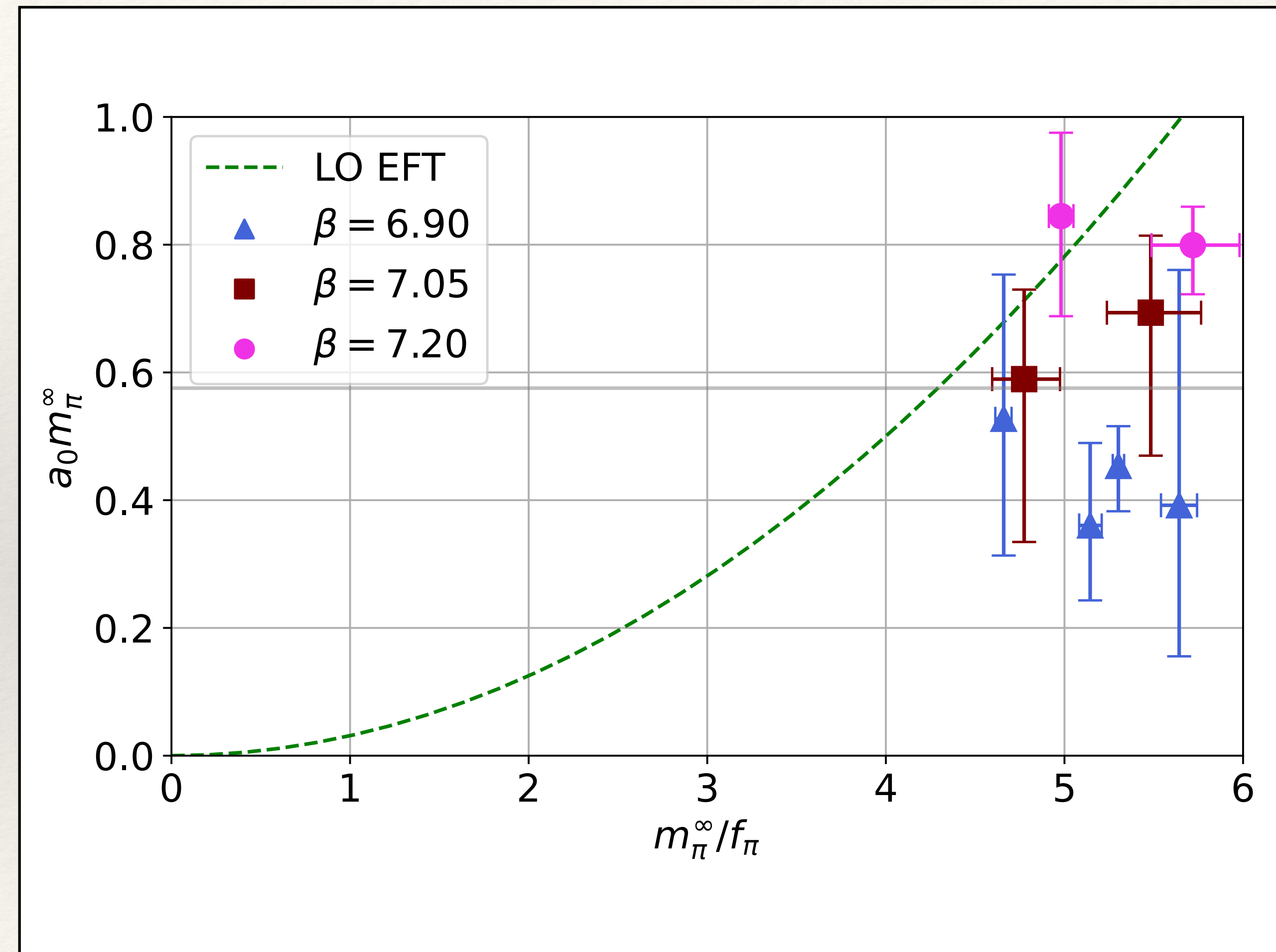
- ❖ Effective range expansion:
  - ❖ Expand phase shift in  $\mathcal{O}(p^2)$
- ❖ Access to  $\sigma(s)$
- ❖ No quantitative effect from using different models for the fit





# $\chi$ -pT comparison

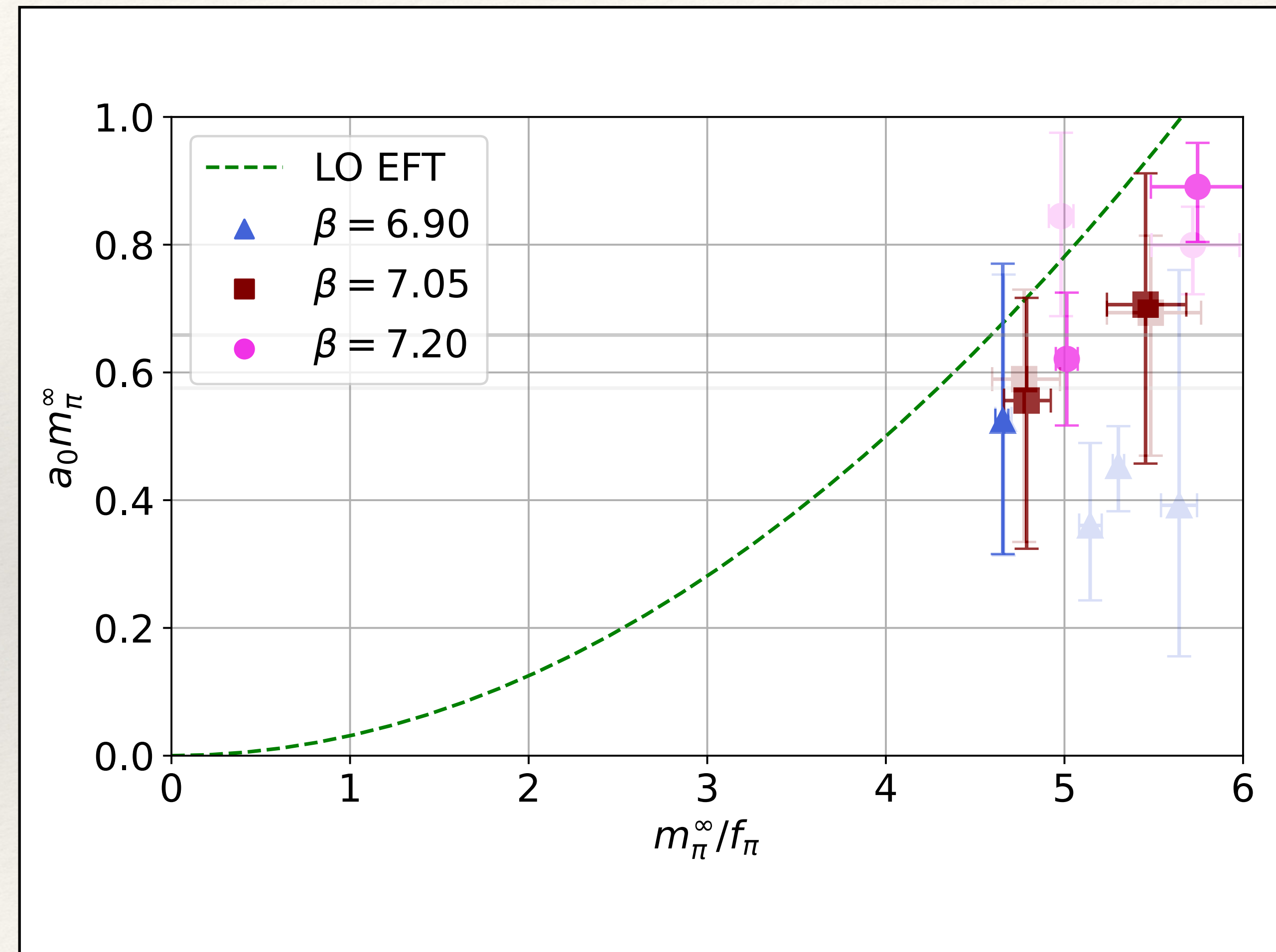
❖ Prediction:  $a_0 m_\pi = \frac{1}{32} \left( \frac{m_\pi}{f_\pi} \right)^2$





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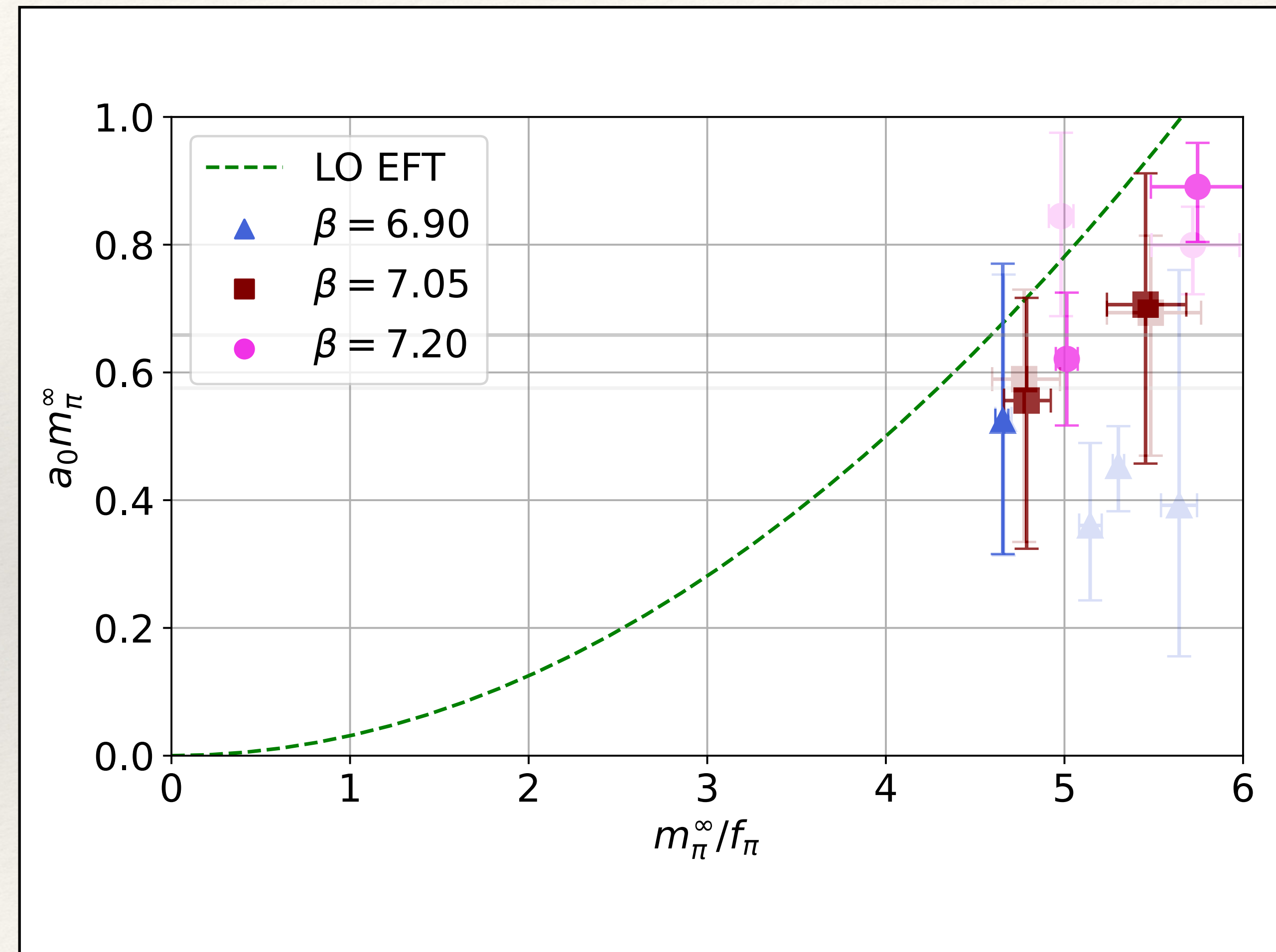
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- ❖ Potential systematics





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- ❖ Prediction:  $a_0 m_\pi = \frac{1}{32} \left( \frac{m_\pi}{f_\pi} \right)^2$
- ❖ Potential systematics
- ❖ Promising for ChPT
- ❖ NLO?

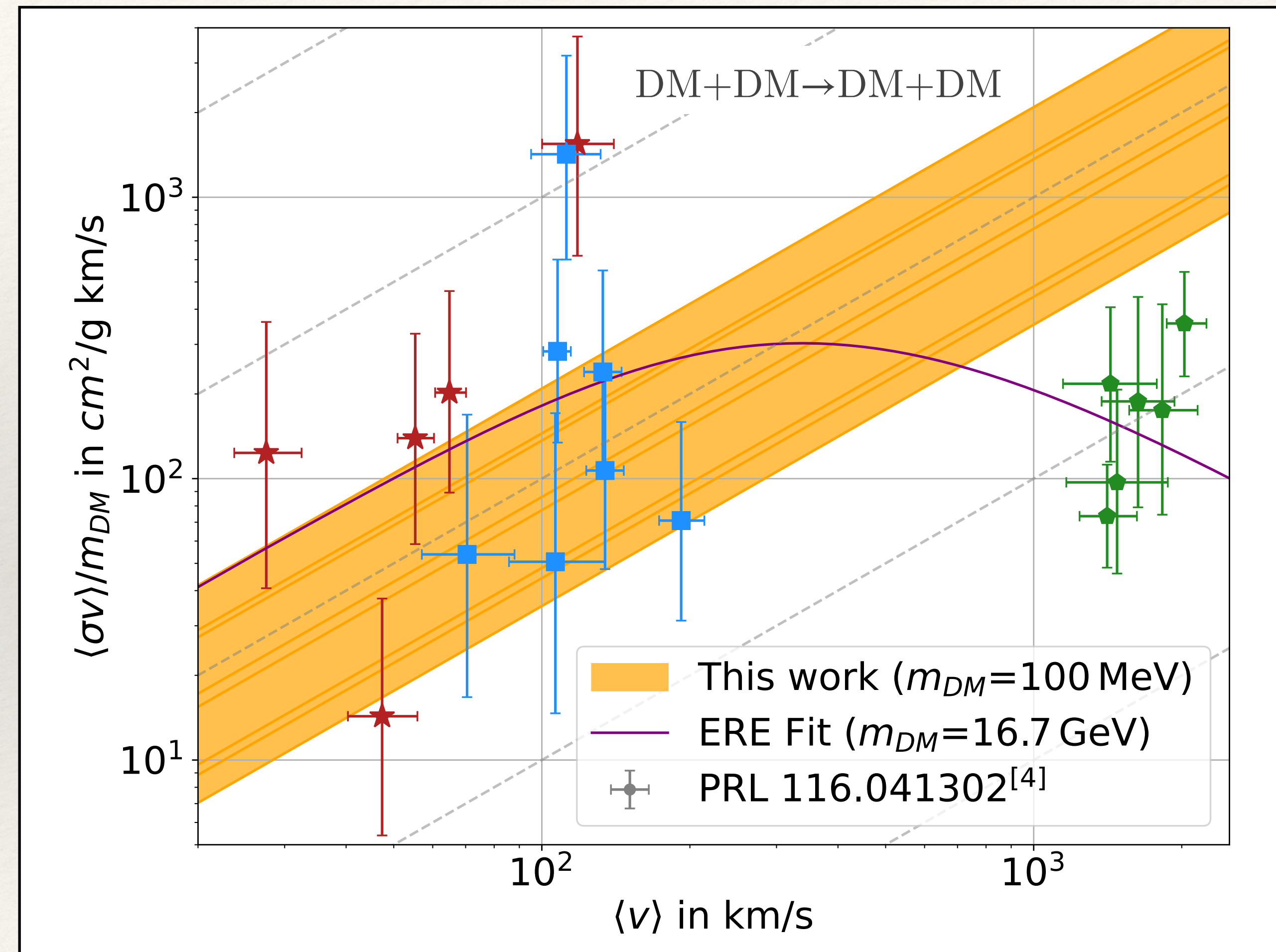




# Velocity-weighted cross-section

$$\langle \sigma v \rangle = \int_0^{v_{esc}} dv \sigma(v) v f(v)$$

- ❖ No sign for a velocity dependence
- ❖ Discrepancy in  $a_0 m_{DM}$

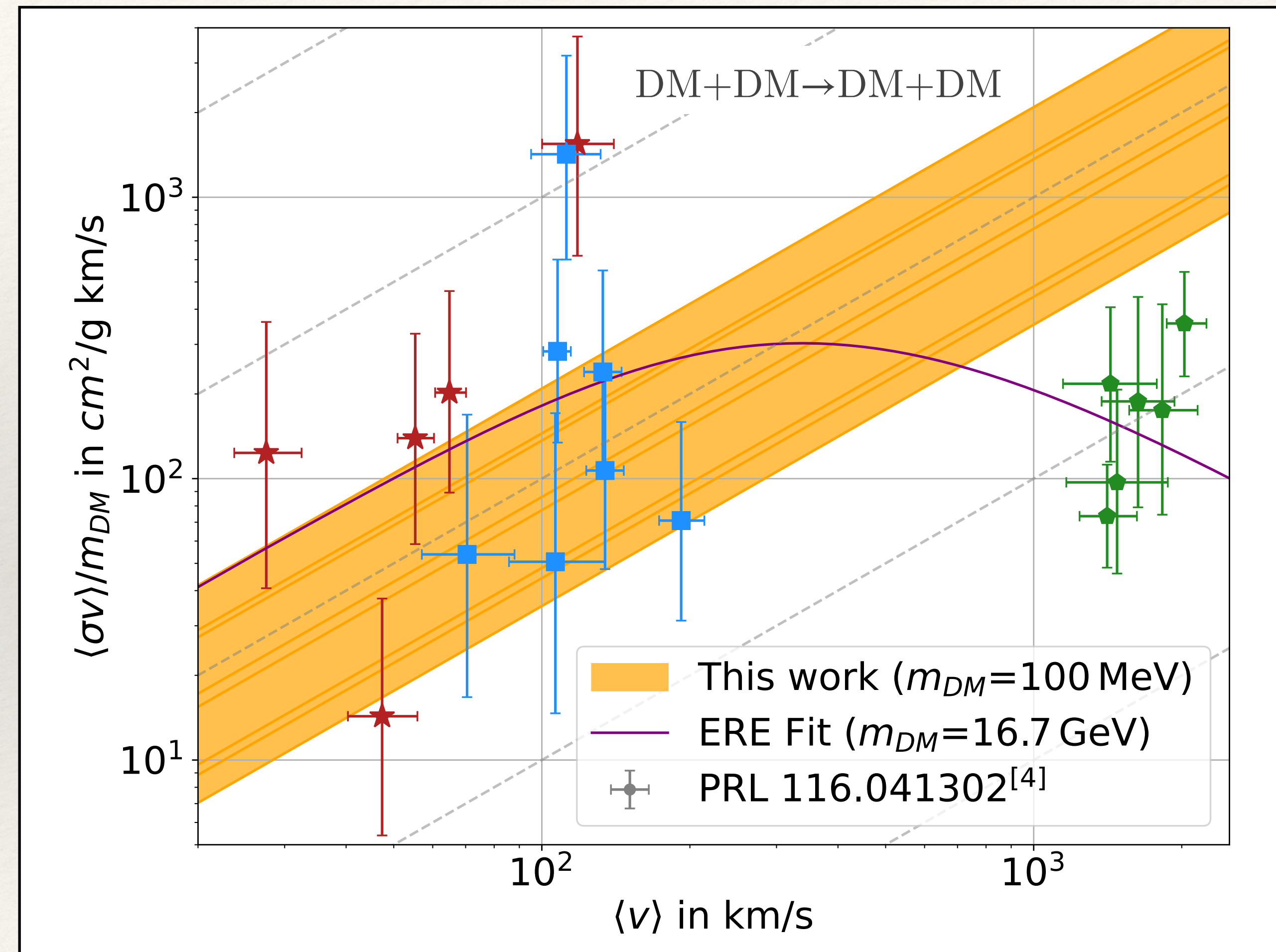




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- ❖  $m_{DM} \sim 100$  MeV predicted by SIMP

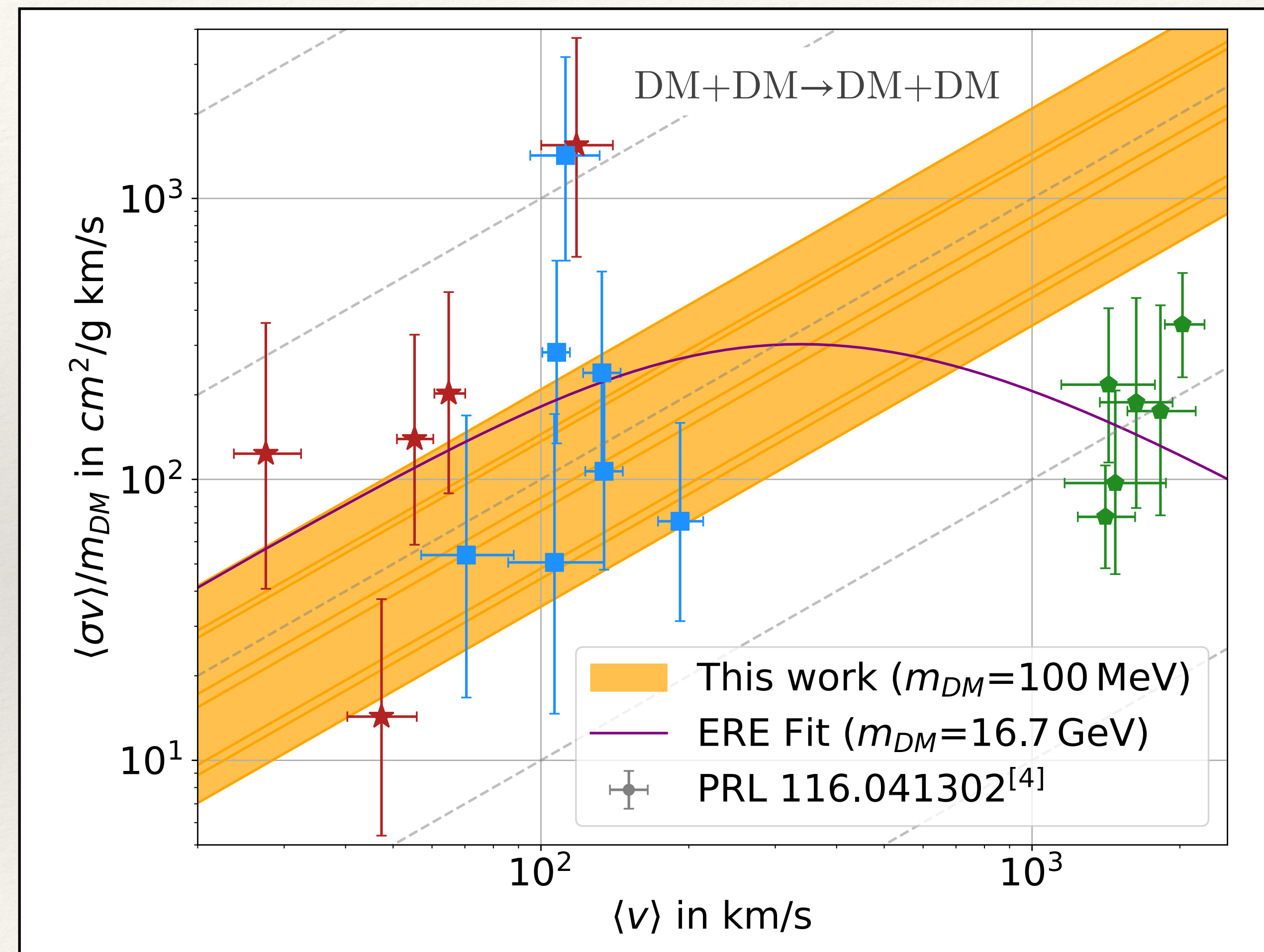




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$$\langle \sigma v \rangle = \int_0^{v_{esc}} dv \sigma(v) v f(v)$$

- ❖ No sign for a velocity dependence
- ❖ Discrepancy in  $a_0 m_{DM}$
- ❖  $m_{DM} \sim 100$  MeV predicted by SIMP
- ❖ Sp(4) not ruled out





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# Summary & Outlook

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- ❖  $\pi\pi$ -scattering in the most common channel
- ❖ Decent agreement with astro-data with  $m_{DM} = 100$  MeV



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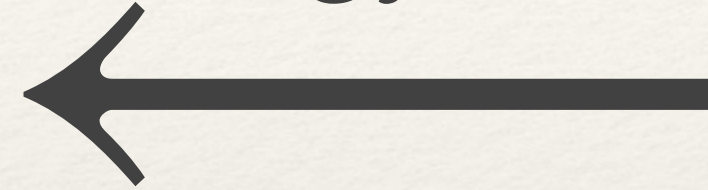
Thank you!



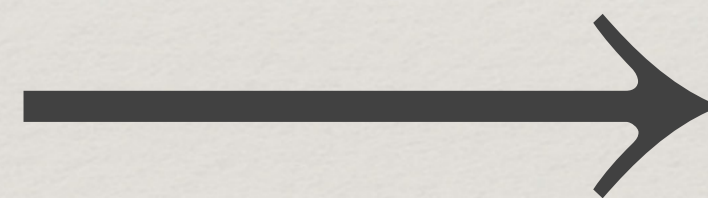
# Result tables

$\beta$	$am_0$	$N_L$	$N_T$	$n_{\text{config}}$	$m_\pi/m_\rho$	$am_\pi$	$aE_{\pi\pi}$	$af_\pi$	$\langle P \rangle$
6.9	-0.87	10	20	976	0.8744(43)	0.7425(12)	1.4961(22)	0.1313(25)	0.550680(46)
6.9	-0.87	12	24	400	0.8754(41)	0.7414(15)	1.4891(27)	0.13195(91)	0.550441(54)
6.9	-0.87	16	32	100	0.8762(28)	0.74060(96)	1.4820(23)	0.1324(35)	0.550525(61)
6.9	-0.9	8	16	651	0.795(11)	0.6241(25)	1.2799(48)	0.1014(44)	0.557959(99)
6.9	-0.9	8	24	402	0.811(12)	0.6267(26)	1.2806(53)	0.0969(20)	0.55796(10)
6.9	-0.9	10	20	1273	0.7998(38)	0.5738(12)	1.1602(26)	0.1044(21)	0.557172(40)
6.9	-0.9	12	24	2904	0.8110(22)	0.56409(54)	1.1339(14)	0.10484(87)	0.557009(18)
6.9	-0.9	14	24	942	0.8115(27)	0.56222(63)	1.1280(16)	0.10599(58)	0.556981(26)
6.9	-0.9	16	32	546	0.8156(28)	0.56275(57)	1.1283(12)	0.1064(13)	0.556921(25)
6.9	-0.9	18	36	356	0.8135(24)	0.56121(58)	1.1245(12)	0.10576(91)	0.556987(24)
6.9	-0.91	12	24	1268	0.7698(77)	0.4920(10)	0.9950(27)	0.0949(13)	0.559351(28)
6.9	-0.91	14	24	513	0.7756(81)	0.4857(12)	0.9781(29)	0.0945(23)	0.559409(34)
6.9	-0.91	16	32	435	0.7658(65)	0.48610(86)	0.9765(19)	0.0948(23)	0.559353(27)
6.9	-0.92	12	24	63	0.738(61)	0.416(19)	0.885(14)	0.0853(68)	0.56145(14)
6.9	-0.92	14	24	550	0.699(10)	0.3926(14)	0.7914(40)	0.0724(19)	0.562096(34)
6.9	-0.92	16	32	176	0.670(11)	0.3894(14)	0.7848(35)	0.0821(15)	0.562116(42)
6.9	-0.92	24	32	467	0.7035(31)	0.38649(51)	0.7734(12)	0.08260(35)	0.562077(14)
7.05	-0.835	8	24	402	0.777(13)	0.6585(60)	1.345(15)	0.0481(47)	0.577085(74)
7.05	-0.835	12	24	313	0.790(11)	0.4616(15)	0.9424(32)	0.0792(14)	0.575237(39)
7.05	-0.835	14	24	619	0.7877(91)	0.4417(17)	0.9085(30)	0.0793(20)	0.575368(25)
7.05	-0.835	20	36	100	0.7945(61)	0.4380(10)	0.8792(27)	0.0796(31)	0.575269(29)
7.05	-0.85	12	24	84	0.611(33)	0.3778(57)	0.786(22)	0.0582(39)	0.577835(69)
7.05	-0.85	14	24	167	0.716(26)	0.3496(25)	0.7236(67)	0.0675(17)	0.577429(44)
7.05	-0.85	16	32	101	0.660(17)	0.3375(17)	0.6892(41)	0.0669(11)	0.577413(41)
7.05	-0.85	24	36	100	0.7118(64)	0.33076(97)	0.6638(23)	0.0684(19)	0.577371(24)
7.2	-0.78	8	24	401	0.8770(81)	0.8089(42)	1.617(11)	0.0402(50)	0.590527(59)
7.2	-0.78	10	20	195	0.648(16)	0.5508(48)	1.1345(88)	0.0497(20)	0.589788(65)
7.2	-0.78	12	24	150	0.835(19)	0.4382(34)	0.9024(84)	0.0569(14)	0.589547(56)
7.2	-0.78	14	24	425	0.7762(83)	0.3857(14)	0.7951(35)	0.06569(73)	0.589362(26)
7.2	-0.78	16	32	265	0.7930(90)	0.3809(11)	0.7703(31)	0.0645(11)	0.589253(22)
7.2	-0.78	24	36	508	0.7852(30)	0.36963(39)	0.74360(79)	0.0646(26)	0.5892779(85)
7.2	-0.794	12	24	101	0.732(26)	0.3932(63)	0.823(13)	0.0389(24)	0.590837(54)
7.2	-0.794	14	24	234	0.691(31)	0.3234(26)	0.6888(66)	0.0533(14)	0.590422(39)
7.2	-0.794	16	32	101	0.796(27)	0.3097(17)	0.6463(50)	0.0570(13)	0.590330(40)
7.2	-0.794	28	36	504	0.7163(57)	0.28524(35)	0.57582(97)	0.05689(71)	0.5904516(67)

Energy levels



Effective range expansion

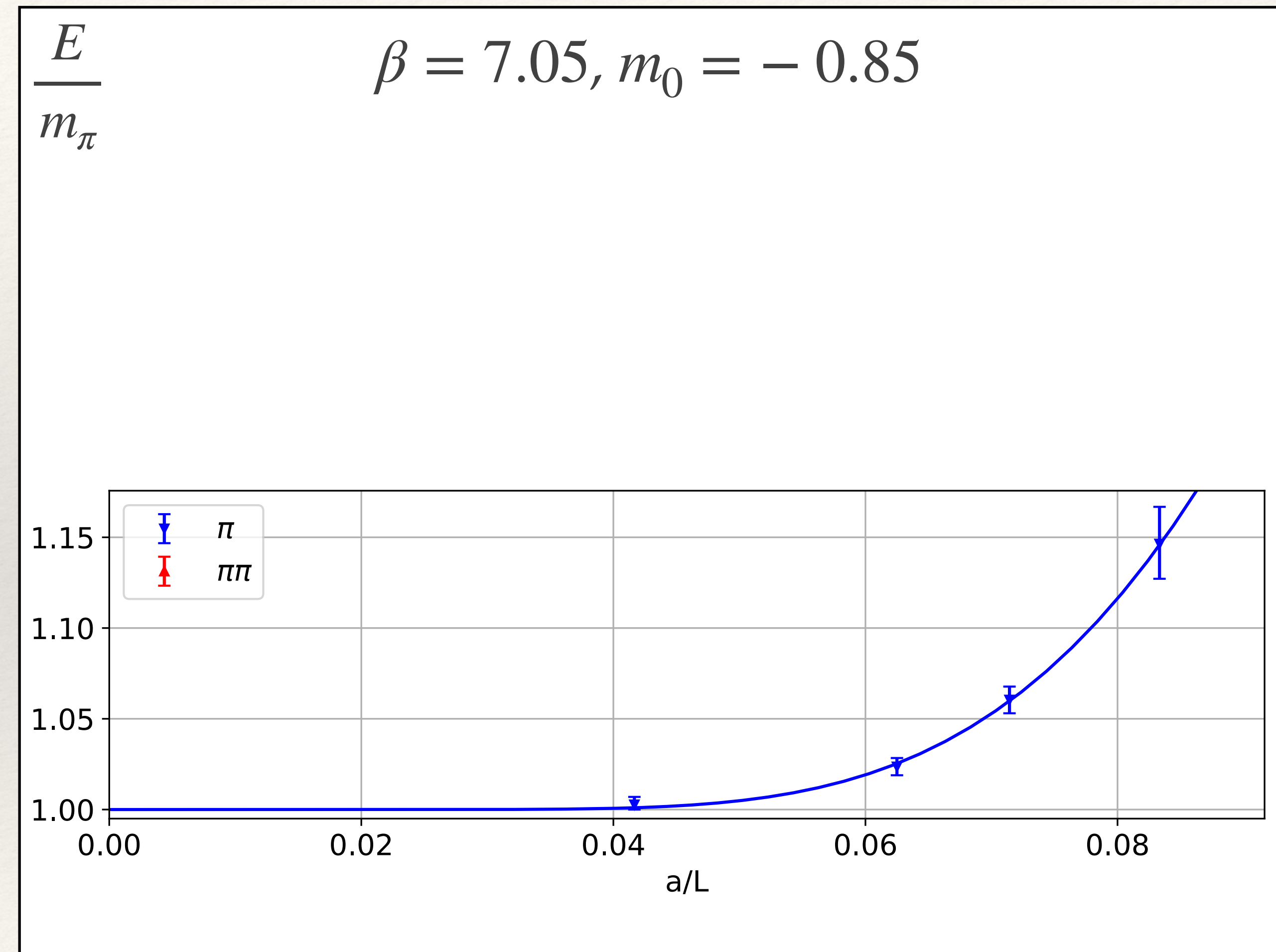


$\beta$	$am_0$	$m_\pi^\infty \times 10^4$	$a_0 m_\pi$	$r_0 m_\pi$
6.9	-0.87	$7401_{-9}^{+9}$	$0.39_{-0.25}^{+0.35}$	$59_{-110}^{+329}$
6.9	-0.9	$5608_{-4}^{+4}$	$0.45_{-0.07}^{+0.06}$	$9.3_{-2.2}^{+3.5}$
6.9	-0.91	$4845_{-9}^{+9}$	$0.36_{-0.12}^{+0.12}$	$42_{-27}^{+57}$
6.9	-0.92	$3845_{-31}^{+18}$	$0.52_{-0.21}^{+0.23}$	$6.7_{-3.8}^{+9.5}$
7.05	-0.835	$4373_{-9}^{+9}$	$0.70_{-0.21}^{+0.12}$	$1.9_{-0.4}^{+1.1}$
7.05	-0.85	$3297_{-13}^{+11}$	$0.60_{-0.25}^{+0.14}$	$3.7_{-1.7}^{+7.4}$
7.2	-0.78	$3696_{-4}^{+4}$	$0.80_{-0.08}^{+0.06}$	$2.0_{-0.2}^{+0.3}$
7.2	-0.794	$2837_{-14}^{+12}$	$0.84_{-0.16}^{+0.13}$	$1.3_{-0.4}^{+0.7}$



# Energy levels

- ❖ Infinite volume pion mass





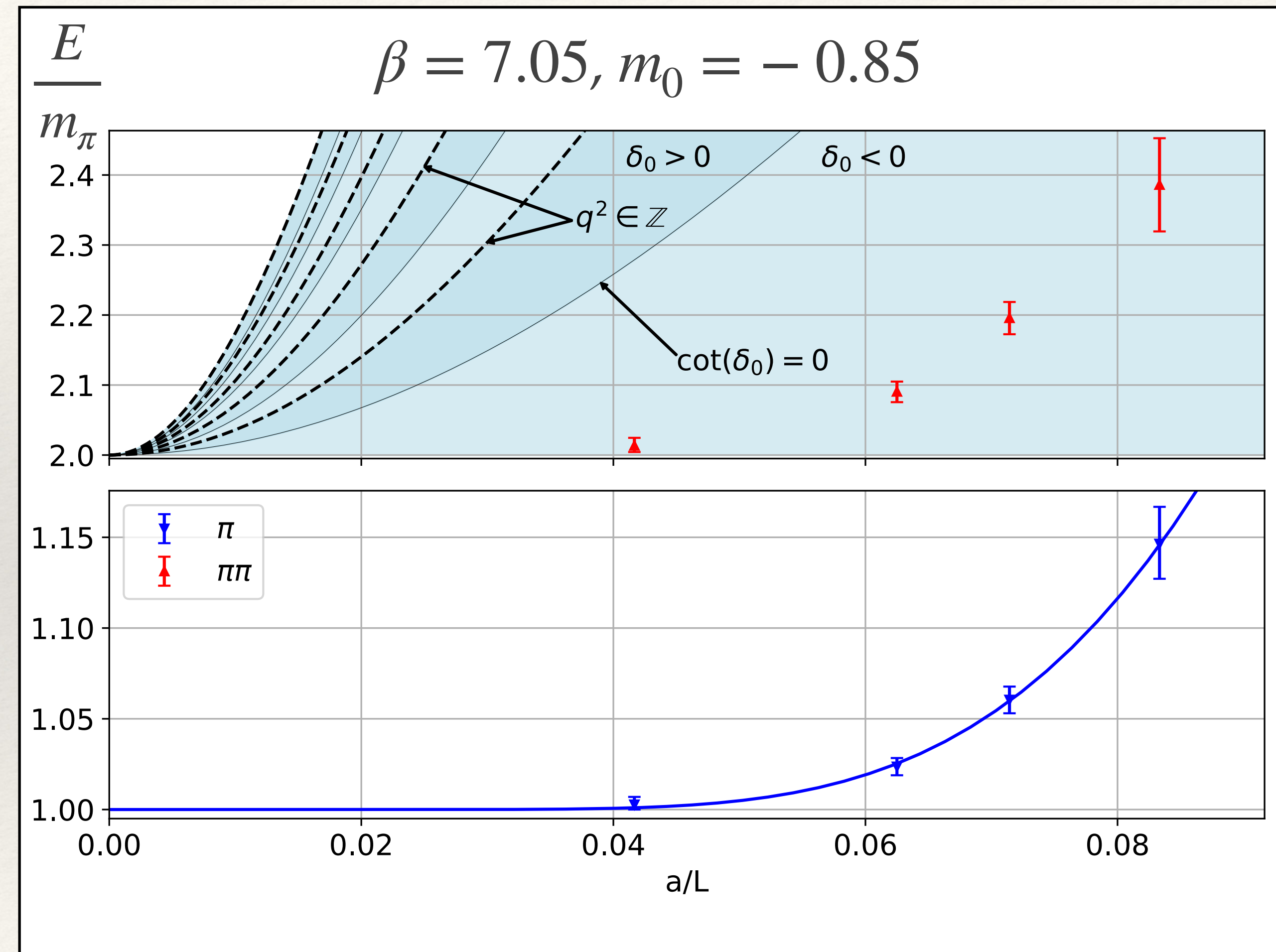
# Energy levels

- ❖ Infinite volume pion mass

$$q = \frac{L}{2\pi}P, \tan(\delta) = \frac{\pi^{\frac{3}{2}}q}{\mathcal{L}_{00}^{\vec{0}}(1,q^2)}$$

- ❖ Non-interacting levels:  $q^2 \in \{1,2,\dots\}$

- ❖ Resonances:  $\mathcal{L}(1,q^2) = 0$



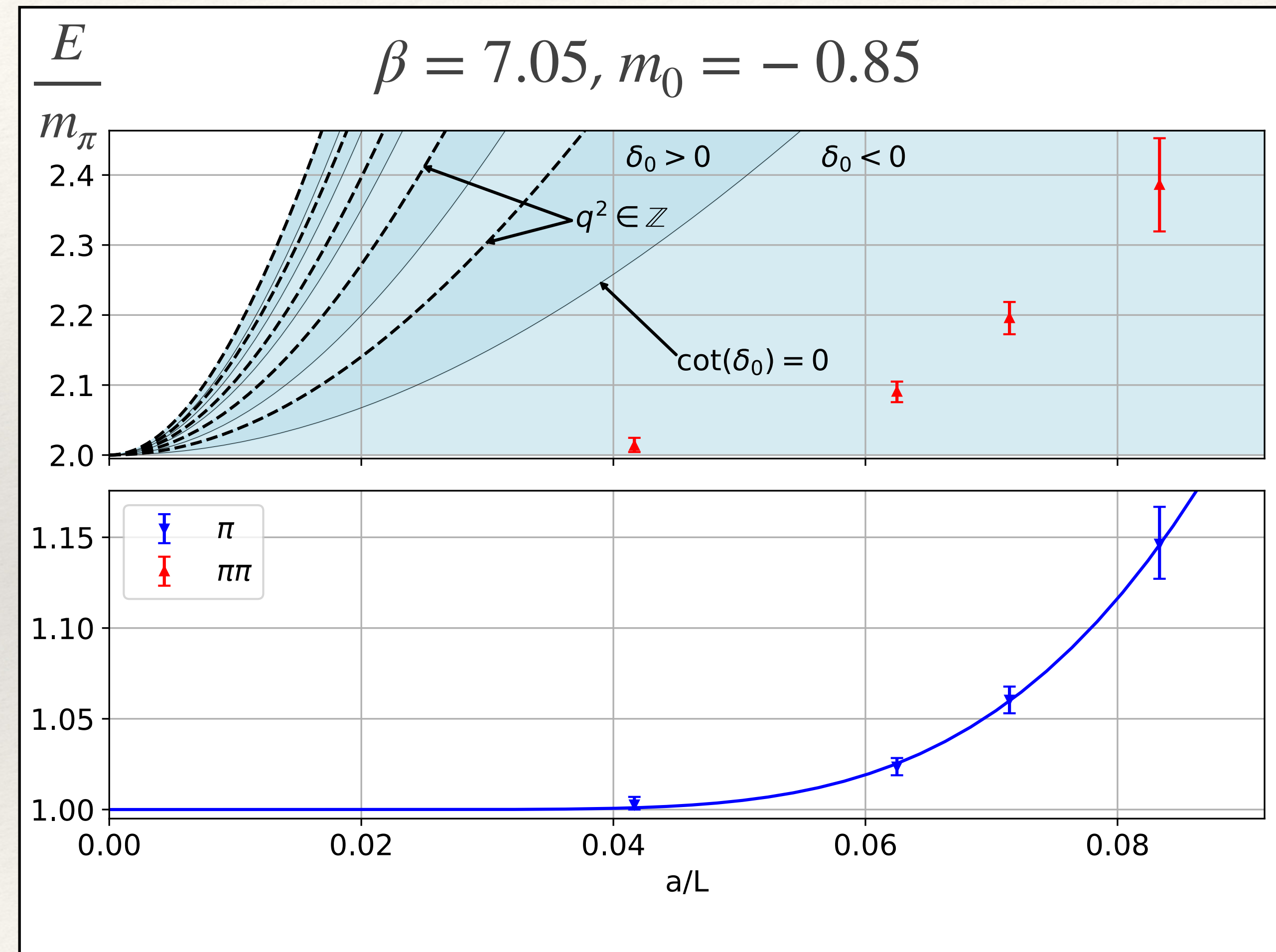


# Energy levels

- ❖ Infinite volume pion mass

$$q = \frac{L}{2\pi}P, \tan(\delta) = \frac{\pi^{\frac{3}{2}}q}{\mathcal{L}_{00}^{\vec{0}}(1, q^2)}$$

- ❖ Non-interacting levels:  $q^2 \in \{1, 2, \dots\}$
- ❖ Resonances:  $\mathcal{L}(1, q^2) = 0$
- ❖ One to one mapping of  $E_{\pi\pi}(L)$  to sign of phase shift

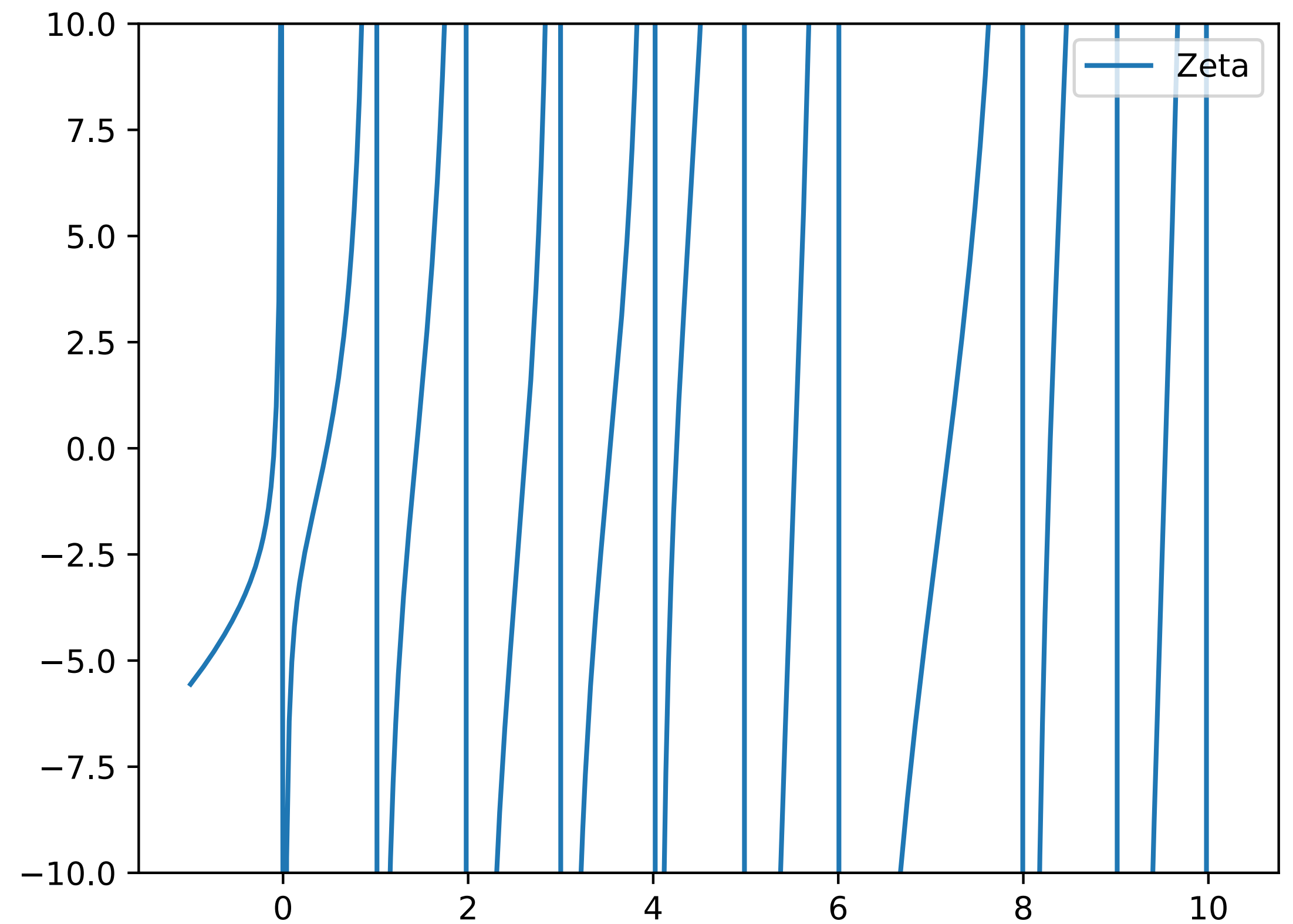




# The Zeta function

$$\mathcal{Z}_{Jm}^{\vec{d}}(r, q^2) = \sum_{\vec{x} \in P_{\vec{d}}} \frac{|\vec{x}|^J Y_{Jm}(\vec{x})}{(\vec{x}^2 - q^2)^r}$$

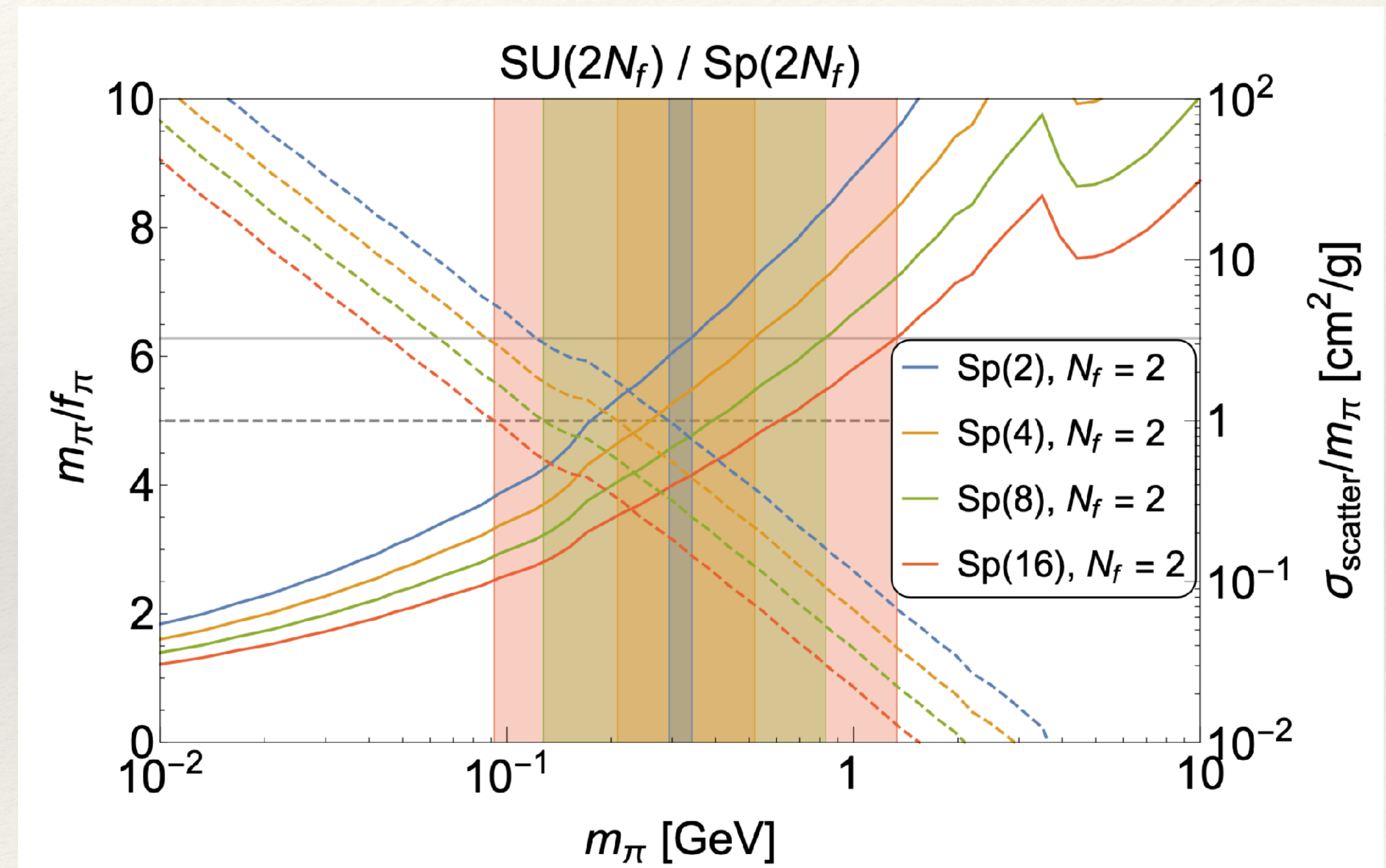
$$P_{\vec{d}} = \left\{ \vec{x} \in \mathbb{R}^3 \mid \vec{x} = \vec{y} + \frac{\vec{d}}{2}, \vec{y} \in \mathbb{Z}^3 \right\}$$





# Why not SU(2)

- ❖  $\text{Sp}(2) = \text{SU}(2)$
- ❖ "Large  $N_c$ " further away from the conformal window for fixed  $N_f$
- ❖ Studies of  $\text{SU}(2)$  exist but are less compatible




" $m_\pi$  less constraint for larger  $N_c$ "



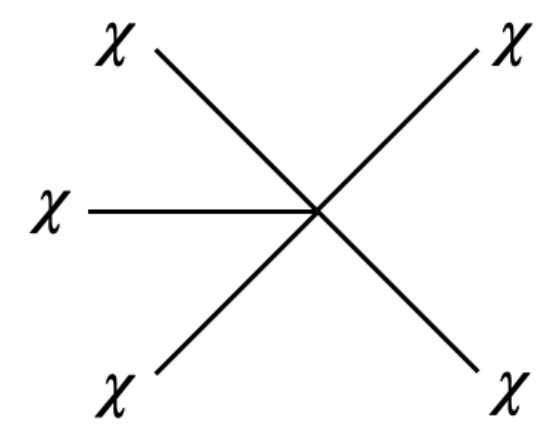
# Why confining gauge theory

- ❖ Large coupling needed
- ❖ Arises naturally in confining theories
- ❖ Hard to make it work with elementary particles



## Dark matter: connecting to particle physics

Hochberg et al. arXiv:1402.5143

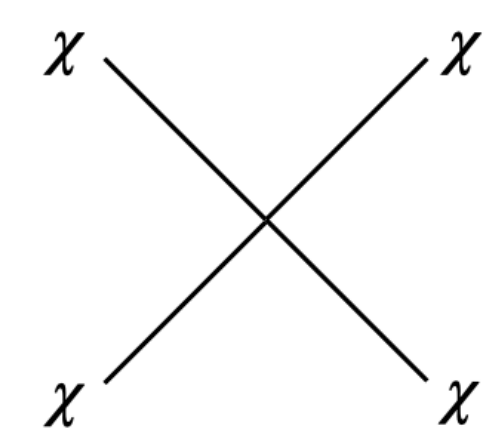


3 → 2 annihilations

$$\Gamma_{3 \rightarrow 2} \sim H$$

$$n_\chi^2 \langle \sigma v^2 \rangle_{3 \rightarrow 2} \sim \frac{T_{eq}^2 m_\chi^4}{x_F^6} \times \frac{\alpha_{eff}^3}{m_\chi^5} \sim H_F \sim \frac{T_F^2}{M_{Pl}}$$

$$m_\chi \sim \alpha_{eff} (T_{eq}^2 M_{Pl})^{1/3} < \alpha_{eff} \times \mathcal{O}(100) \text{ MeV}$$



2 → 2 self-interactions

$$\frac{\sigma_{\chi\chi}}{m_\chi} \sim a_{int} \frac{\text{barn}}{\text{GeV}} \sim \frac{\alpha_{eff}}{m_\chi^3}$$

$$m_\chi \geq 10 \left( \frac{a_{int}}{\alpha_{eff}} \right)^{1/3} \text{ MeV}$$

- Relic density and self-interactions require non-perturbative couplings and sub-GeV DM mass
- Very small region to reconcile both

S. Kulkarni
4
18 July 2024



# Small-scale structure problems

*Core-cusp problem:* High-resolution simulations show that the mass density profile for CDM halos increases toward the center, scaling approximately as  $\rho_{\text{dm}} \propto r^{-1}$  in the central region [47, 48, 49]. However, many observed rotation curves of disk galaxies prefer a constant “cored” density profile  $\rho_{\text{dm}} \propto r^0$  [50, 51, 52], indicated by linearly rising circular velocity in the inner regions. The issue is most prevalent for dwarf and low surface brightness (LSB) galaxies [53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65], which, being highly DM-dominated, are appealing environments to test CDM predictions.

*Diversity problem:* Cosmological structure formation is predicted to be a self-similar process with a remarkably little scatter in density profiles for halos of a given mass [49, 66]. However, disk galaxies with the same maximal circular velocity exhibit a much larger scatter in their interiors [67] and inferred core densities vary by a factor of  $\mathcal{O}(10)$  [68].

*Missing satellites problem:* CDM halos are rich with substructure, since they grow via hierarchical mergers of smaller halos that survive the merger process [69]. Observationally, however, the number of small galaxies in the Local Group are far fewer than the number of predicted subhalos. In the MW, simulations predict  $\mathcal{O}(100 - 1000)$  subhalos large enough to host galaxies, while only 10 dwarf spheroidal galaxies had been discovered when this issue was first raised [70, 71]. Nearby galaxies in the field exhibit a similar underabundance of small galaxies compared to the velocity function inferred through simulations [36, 72, 73].

*Too-big-to-fail problem (TBTf):* In recent years, much attention has been paid to the most luminous satellites in the MW, which are expected to inhabit the most massive subhalos in CDM simulations. However, it has been shown that these subhalos are too dense in the central regions to be consistent with stellar dynamics of the brightest dwarf spheroidals [74, 75]. The origin of the name stems from the expectation that such massive subhalos are too big to fail in forming stars and should host observable galaxies. Studies of dwarf galaxies in Andromeda [76] and the Local Group field [77] have found similar discrepancies.



# Sp(4) particle spectrum

Label ( $M$ )	Interpolating operator ( $\mathcal{O}_M$ )	Meson	$J^P$	$Sp(4)$
PS	$\overline{Q^i} \gamma_5 Q^j$	$\pi$	$0^-$	5
S	$\overline{Q^i} Q^j$	$a_0$	$0^+$	5
V	$\overline{Q^i} \gamma_\mu Q^j$	$\rho$	$1^-$	10
T	$\overline{Q^i} \gamma_0 \gamma_\mu Q^j$	$\rho$	$1^-$	10(+5)
AV	$\overline{Q^i} \gamma_5 \gamma_\mu Q^j$	$a_1$	$1^+$	5
AT	$\overline{Q^i} \gamma_5 \gamma_0 \gamma_\mu Q^j$	$b_1$	$1^+$	10(+5)



# Operators and correlators

❖ 14-dim:  $\pi$  &  $\pi\pi$ :

$$\langle \bar{\mathcal{O}}_{\pi}(n) \mathcal{O}_{\pi}(m) \rangle_F = \text{Diagram with nodes } n \text{ and } m \text{ and two curved arrows between them, one above and one below, both pointing from } m \text{ to } n.$$

$$\langle \bar{\mathcal{O}}_{\pi\pi}(n) \mathcal{O}_{\pi\pi}(m) \rangle_F = \text{Diagram with two nodes } n \text{ and } m \text{ and two curved arrows between them, one above and one below, both pointing from } m \text{ to } n, \text{ minus a diagram with two nodes } n \text{ and } m \text{ and two straight lines between them, one above and one below, crossing each other, both pointing from } m \text{ to } n.$$

❖ 10-dim:  $\rho$  &  $\pi\pi$ :

$$C_{11}(t) = \text{Diagram with two nodes } p \text{ and } 0 \text{ and two curved arrows between them, one above and one below, both pointing from } 0 \text{ to } p, \text{ minus a diagram with two nodes } p \text{ and } 0 \text{ and two curved arrows between them, one above and one below, both pointing from } p \text{ to } 0, \text{ plus a square diagram with nodes } p \text{ and } 0 \text{ at the corners and four straight lines forming the sides, with arrows pointing clockwise.}$$

$$+ \text{Diagram with two nodes } p \text{ and } 0 \text{ and two curved arrows between them, one above and one below, both pointing from } 0 \text{ to } p, \text{ plus a square diagram with nodes } p \text{ and } 0 \text{ at the corners and four straight lines forming the sides, with arrows pointing counter-clockwise, minus a square diagram with nodes } p \text{ and } 0 \text{ at the corners and four straight lines forming the sides, with arrows pointing clockwise.}$$

$$C_{12}(t) = -C_{21}^*(t) = \text{Diagram with three nodes } p, 0, p \text{ and three straight lines forming a triangle, with arrows pointing clockwise, minus a diagram with three nodes } p, 0, p \text{ and three straight lines forming a triangle, with arrows pointing counter-clockwise.}$$

$$C_{22}(t) = \text{Diagram with two nodes } p \text{ and } p \text{ and two curved arrows between them, one above and one below, both pointing from } p \text{ to } p.$$



# Energy levels on the lattice

- ❖ Each operator in a specified quantum number channel contains the full energy spectrum with some non-trivial (not possible to tell a priori) overlap
  - ❖ Solution: Try / use a lot of operators and perform variational analysis

$$C(t) = \langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle = \sum_k \langle 0 | \mathcal{O} | k \rangle \langle k | \mathcal{O}^\dagger | 0 \rangle \exp^{-tE_k}$$

$$\lim_{t \rightarrow \infty} C(t) = e^{-tm}$$

- ❖ Correlation functions can be expressed as diagrams



# Variational Analysis

$$C_{ij}(t) = \left\langle \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \right\rangle$$

- ❖ Build cross-correlation matrix
- ❖ The Eigenvalues of this matrix disentangle the energy levels
- ❖  $\lambda_k(t) \propto e^{tE_k}$
- ❖ Works best with large operator basis



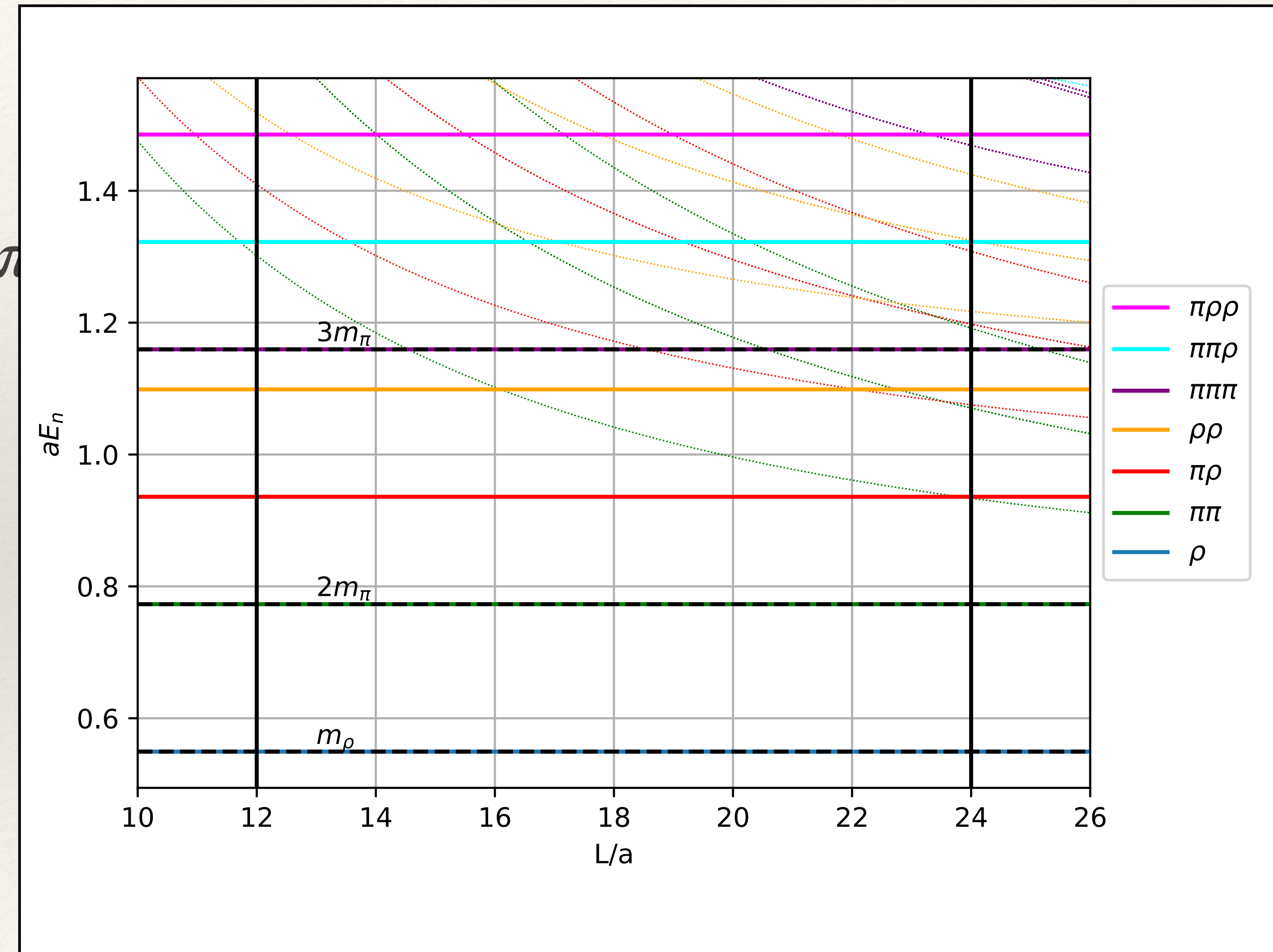
# Trivial energy levels

- ❖ Lot of scattering states possible
- ❖ Possible operators:  $\rho, \pi\pi, \pi\rho, \rho\rho, \pi\pi\pi, \pi\rho\rho$

- ❖ 
$$E = \sum_i \sqrt{m_i^2 + p_i^2}$$

- ❖ Trivial momenta in finite volume:

- ❖ 
$$p = \frac{2\pi |\vec{n}|}{L}, \vec{n} \in \mathbb{Z}^3$$





# Lüscher method

**Zero momentum  $\mathbf{P} = (\mathbf{0}, \mathbf{0}, \mathbf{0})$**   
 (for irrep  $T_1^-$  in  $O_h$ ) [3]:

$$\tan \delta(q) = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1; q^2)}$$

**Nonzero momentum  $\mathbf{P} = (\mathbf{0}, \mathbf{0}, \mathbf{1}) \frac{2\pi}{L}$**   
 (for irrep  $A_2^-$  in  $D_{4h}$ ) [7]:

$$\tan \delta(q) = \frac{\gamma \pi^{3/2} q^3}{q^2 \mathcal{Z}_{00}^{\mathbf{d}}(1; q^2) + \sqrt{\frac{4}{5}} \mathcal{Z}_{20}^{\mathbf{d}}(1; q^2)}$$

$\mathbf{P}=(1,1,0)$

(for irrep  $B_1^-$  in  $D_{2h}$ )

$$\tan \delta(q) = \frac{\gamma \pi^{3/2} q^3}{q^2 \mathcal{Z}_{00}^{\mathbf{d}}(1; q^2) - \sqrt{\frac{1}{5}} \mathcal{Z}_{20}^{\mathbf{d}}(1; q^2) + i \sqrt{\frac{3}{10}} (\mathcal{Z}_{22}^{\mathbf{d}}(1; q^2) - \mathcal{Z}_{\bar{2}\bar{2}}^{\mathbf{d}}(1; q^2))}$$



# Phenomenology of scattering channels

- $Sp(4)_f$
- ❖ 14-dim:
    - ❖ (Probably) contributes most to  $\pi\pi$ -scattering
    - ❖ 14 out of 25 possible combinations of Pions
- $$5 \otimes 5 = 1 \oplus 10 \oplus 14$$
- $$10 \otimes 5 = 5 \oplus 10 \oplus 35$$
- $$5 \otimes 5 \otimes 5 = 3(5) \oplus 10 \oplus 30 \oplus 35$$
- $\pi\pi \rightarrow \pi\pi$  (I=0,1,2)
- $\pi\pi \rightarrow \rho$  (I=1)
- $\pi\pi \rightarrow \pi\pi\pi$  (I=1)
- $\pi\pi \rightarrow \pi\pi\rho$  (I=0,1,2)
- etc.



# Phenomenology of scattering channels

- ❖ 1-dim:
  - ❖ (Probably) no large contribution to  $\pi\pi$ -scattering
  - ❖ Mixes in other scattering channel
  - ❖ Numerically challenging
- $Sp(4)_f$   
 $5 \otimes 5 = 1 \oplus 10 \oplus 14$   
 $10 \otimes 5 = 5 \oplus 10 \oplus 35$   
 $5 \otimes 5 \otimes 5 = 3(5) \oplus 10 \oplus 30 \oplus 35$   
 $\pi\pi \rightarrow \pi\pi (I=0,1,2)$   
 $\pi\pi \rightarrow \rho (I=1)$   
 $\pi\pi \rightarrow \pi\pi\pi (I=1)$   
 $\pi\pi \rightarrow \pi\pi\rho (I=0,1,2)$   
 etc.



# Phenomenology of scattering channels

- ❖ 10-dim:
  - ❖ Mixing with the Rho
  - ❖  $\pi\pi\pi \rightarrow \pi\pi$
- ❖ Work in progress

$$Sp(4)_f$$

$$5 \otimes 5 = 1 \oplus 10 \oplus 14$$

$$10 \otimes 5 = 5 \oplus 10 \oplus 35$$

$$5 \otimes 5 \otimes 5 = 3(5) \oplus 10 \oplus 30 \oplus 35$$

$$\pi\pi \rightarrow \pi\pi (I=0,1,2)$$

$$\pi\pi \rightarrow \rho (I=1)$$

$$\pi\pi \rightarrow \pi\pi\pi (I=1)$$

$$\pi\pi \rightarrow \pi\pi\rho (I=0,1,2)$$

etc.



# Phenomenology of scattering channels

- ❖ **14-dim:**

- ❖ Makes up most  $\pi\pi$  scattering (14/25)
- ❖ Easiest on the lattice

- ❖ **10-dim:**

- ❖ Mixing with dark  $\rho$
- ❖  $\pi\pi\pi \rightarrow \pi\pi$

- ❖ **1-dim:**

- ❖ Mixing with other states

$$Sp(4)_f$$

$$5 \otimes 5 = 1 \oplus 10 \oplus 14$$

$$10 \otimes 5 = 5 \oplus 10 \oplus 35$$

$$5 \otimes 5 \otimes 5 = 3(5) \oplus 10 \oplus 30 \oplus 35$$

$$\pi\pi \rightarrow \pi\pi \text{ (I=0,1,2)}$$

$$\pi\pi \rightarrow \rho \text{ (I=1)}$$

$$\pi\pi \rightarrow \pi\pi\pi \text{ (I=1)}$$

$$\pi\pi \rightarrow \pi\pi\rho \text{ (I=0,1,2)}$$

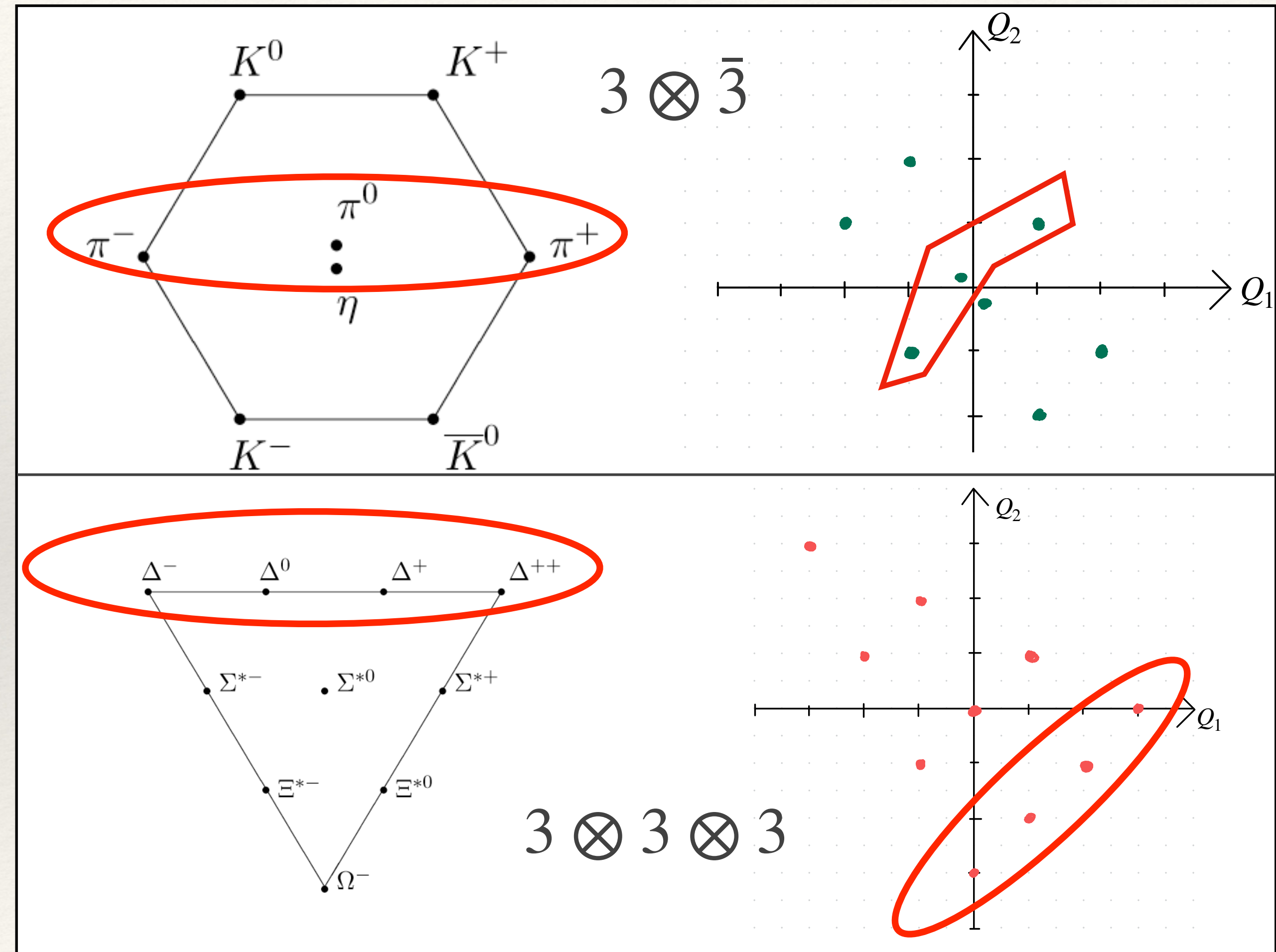
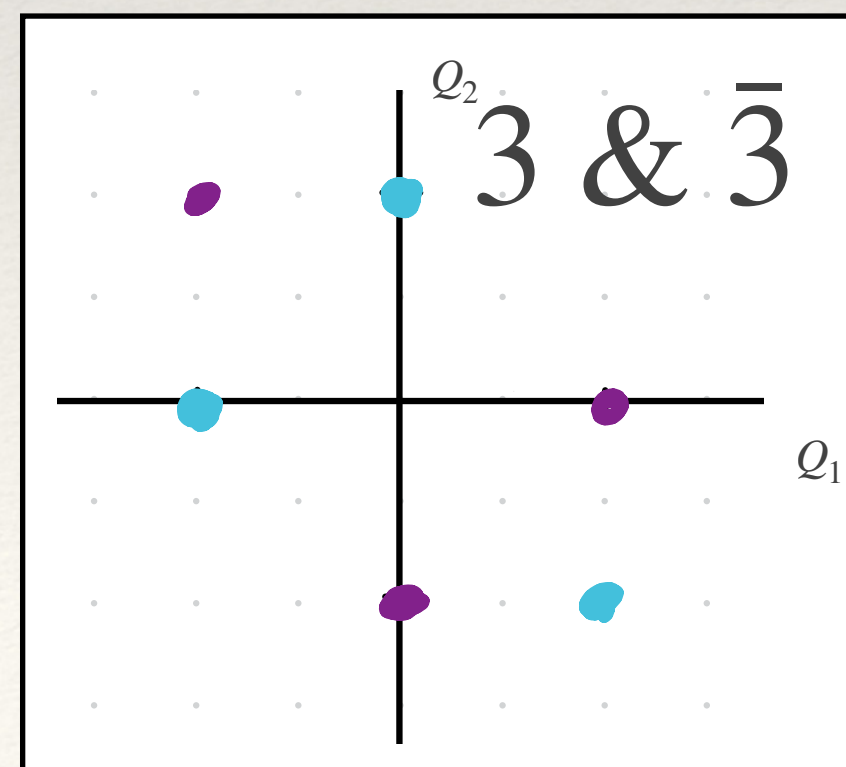
etc.



# Flavour quantum numbers

- ❖ Composite states live in irreps of the flavour symmetry
- ❖ Can be represented in diagrams given by the weight system
  - ❖ "Meson-octet" and "Baryon-Decuplet" in  $SU(3)_F$  (mass-degenerate)
  - ❖ Mass-degenerate  $\rightarrow$  perfect symmetry

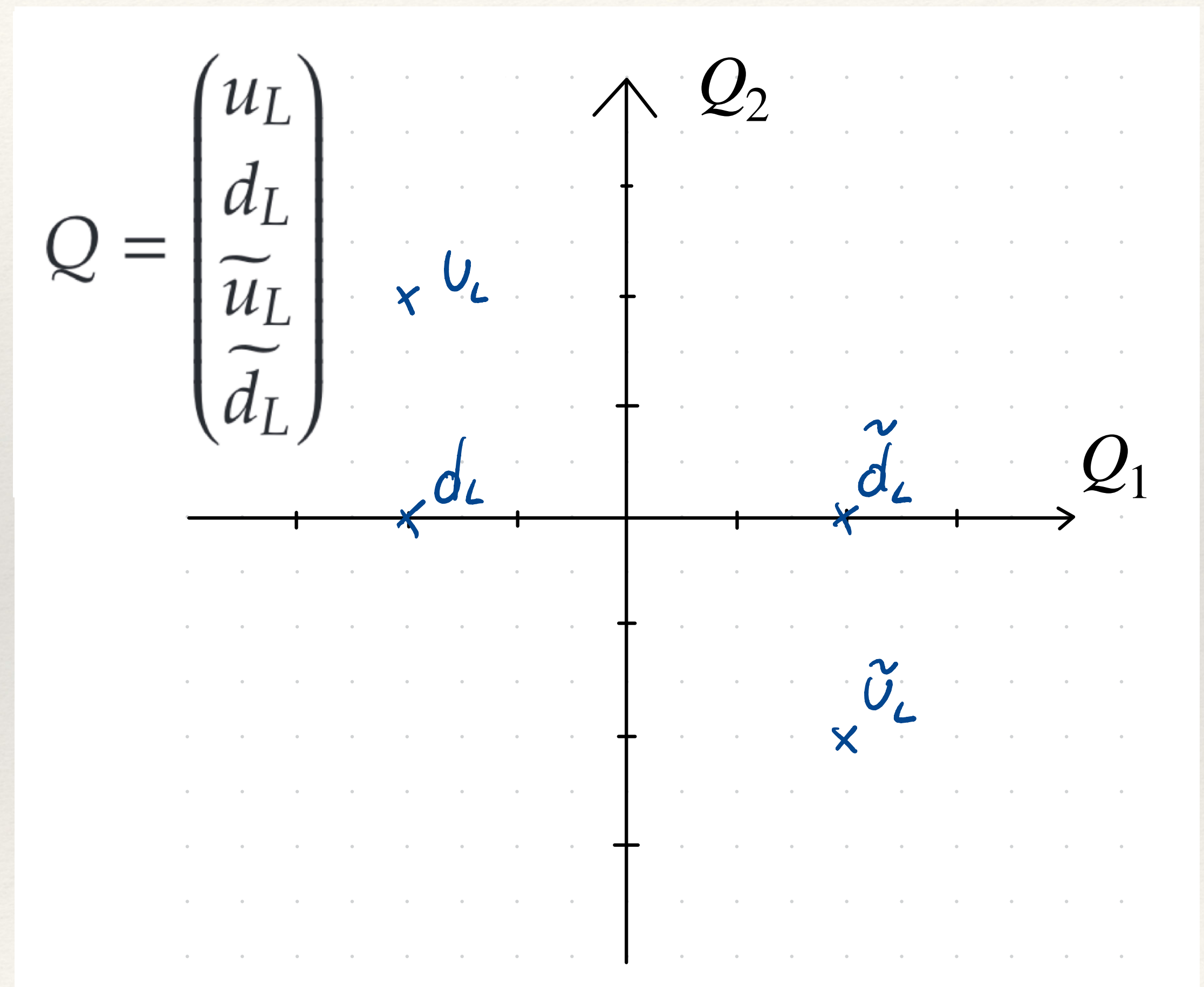
Weight system of the fundamental of  $SU(3)$ :





# Flavour quantum numbers in $Sp(4)$

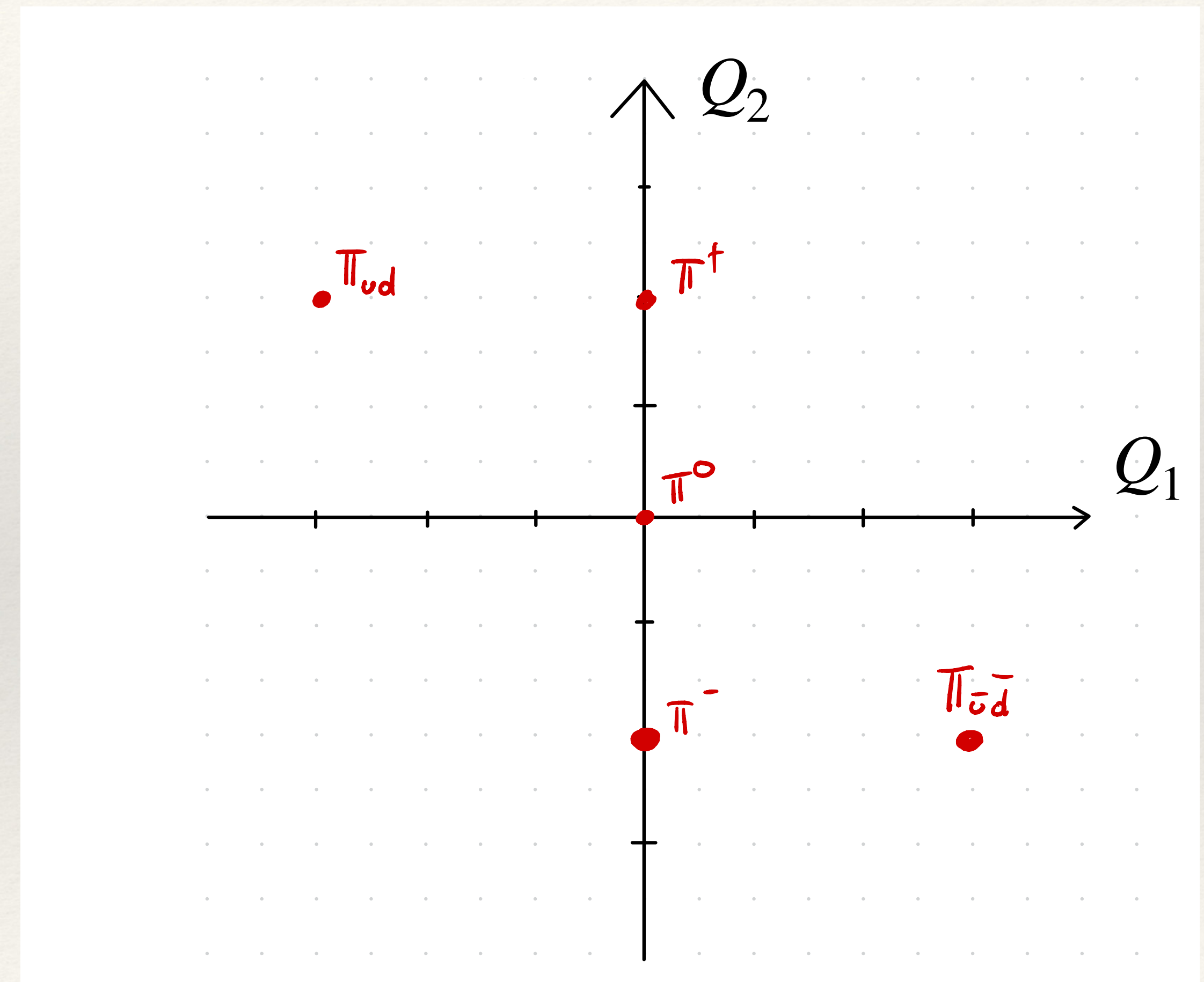
- ❖ Similar in  $Sp(4)$  for visualising scattering states
- ❖ Quarks in fundamental of  $Sp(4)$  (4-plet)
- ❖  $4 \otimes 4 = 1 \oplus 5 \oplus 10$ 
  - ❖ Pions in 5
  - ❖ Rhos in 10





# Pions form a 5-plet

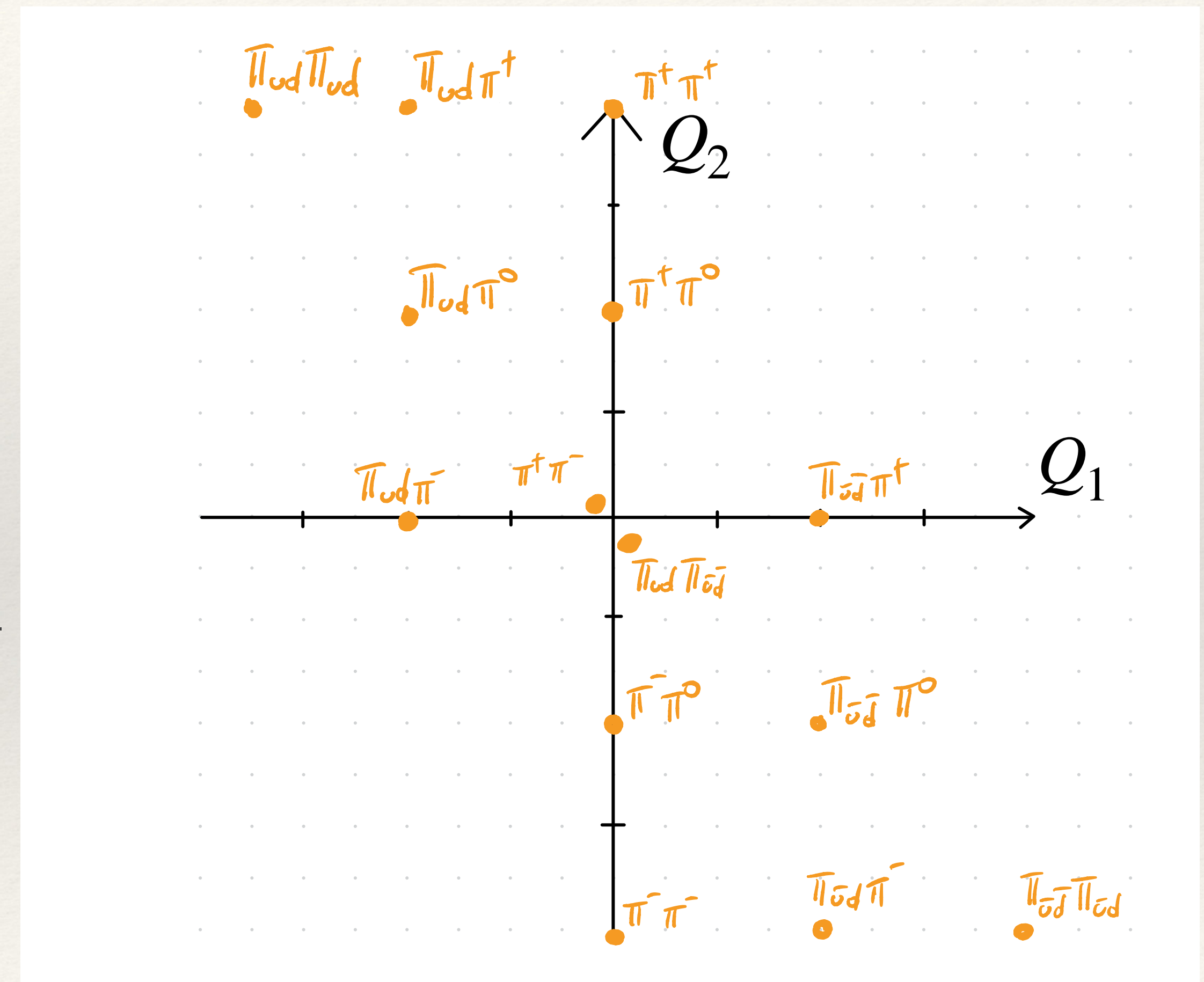
- ❖ Isomorphism:  $SO(5) = Sp(4)$
- ❖ Quark content can be read off
  - ❖  $\pi^+ = u\gamma_5\bar{d}$
- ❖ Scattering states:
  - ❖  $5 \otimes 5 = 1 \oplus 10 \oplus 14$





# The 14-plet

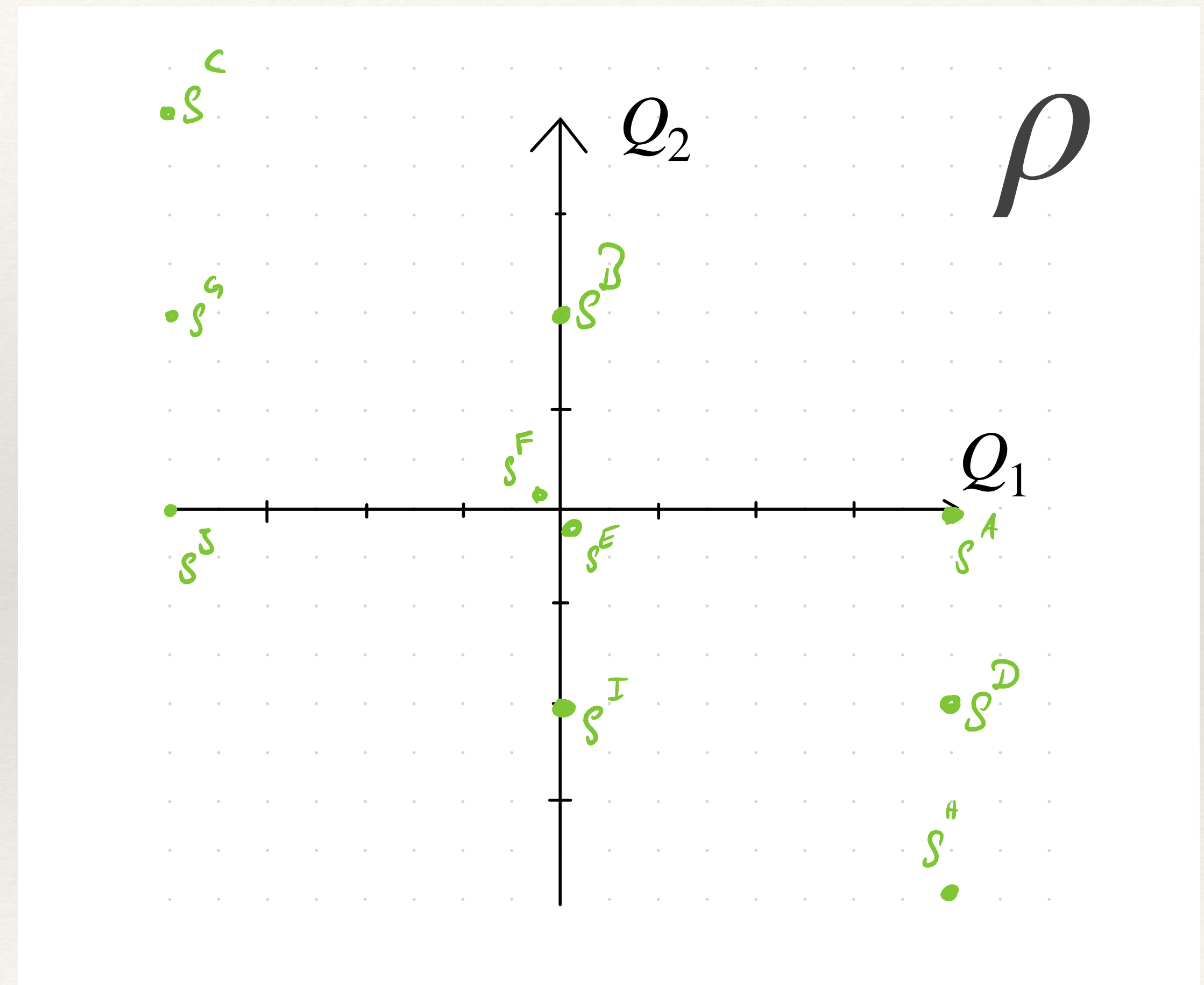
- ❖ Reminder:  $5 \otimes 5 = 1 \oplus 10 \oplus 14$
- ❖  $\pi^+ \pi^+$  is unique to the 14
- ❖  $\mathcal{O}_{\pi\pi}^{14} = \pi^+ \pi^+ = \pi^- \pi^- = \Pi_{ud} \Pi_{ud} = \Pi_{\bar{u}\bar{d}} \Pi_{\bar{u}\bar{d}}$





# The 10-plet

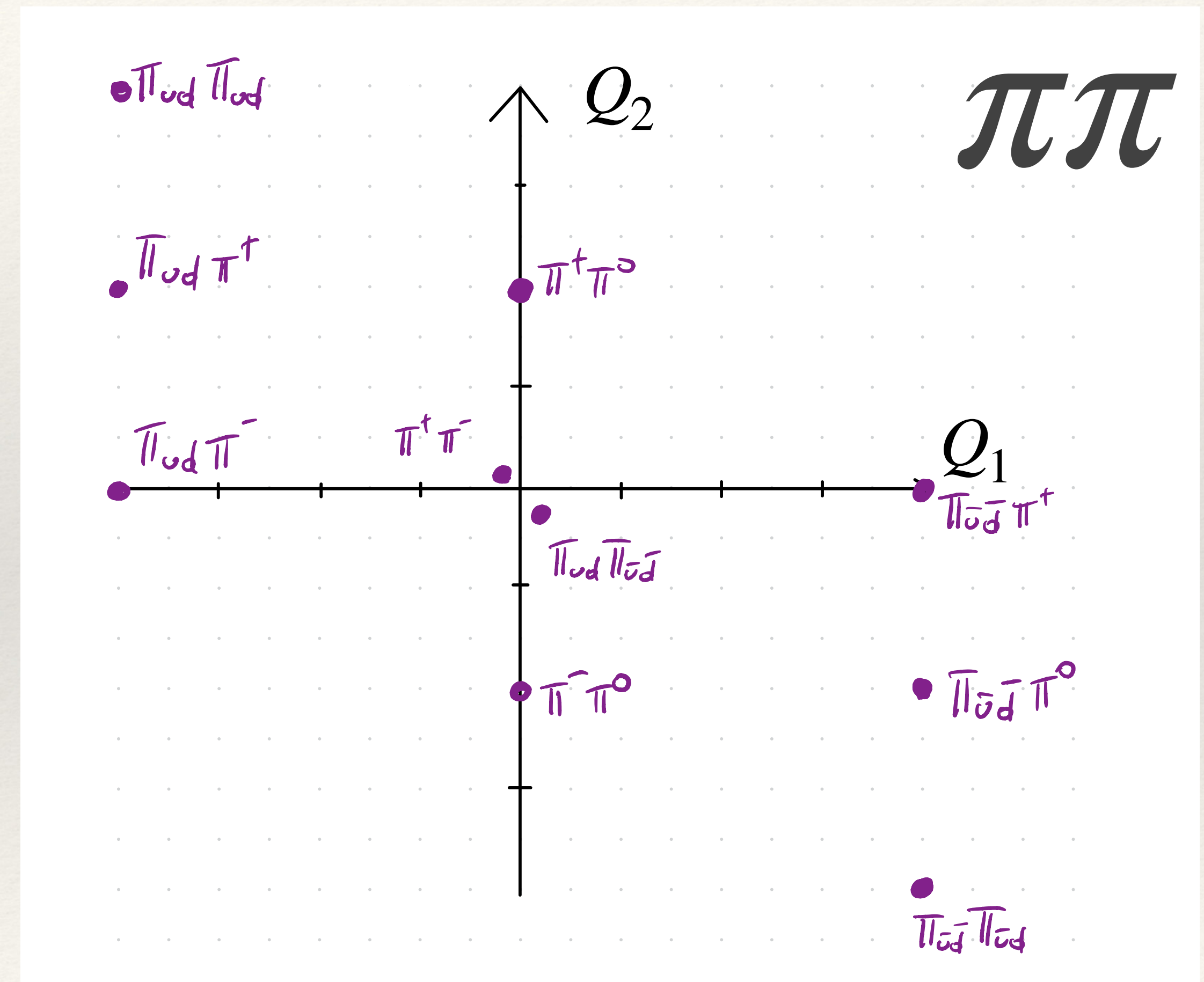
- ❖ Contains  $\rho, \pi\pi, \pi\pi\pi$
- ❖ Trick from before does not work
- ❖ Use young-diagrams





# The 10-plet

- ❖ Contains  $\rho, \pi\pi, \pi\pi\pi$
- ❖ Trick from before does not work
- ❖ Use young-diagrams





# Dark Matter

- ❖ Collection of phenomena with no explanation in the standard model (SM)
  - ❖ Rotation curves, structure formation, etc.
- ❖ Possible explanations:
  - ❖ Modified gravity
  - ❖ Non observable form of matter
  - ❖ Particle beyond the SM

