Dark Matter Scattering

arXiv:2405.06506

With F. Zierler & A.Maas

Yannick Dengler

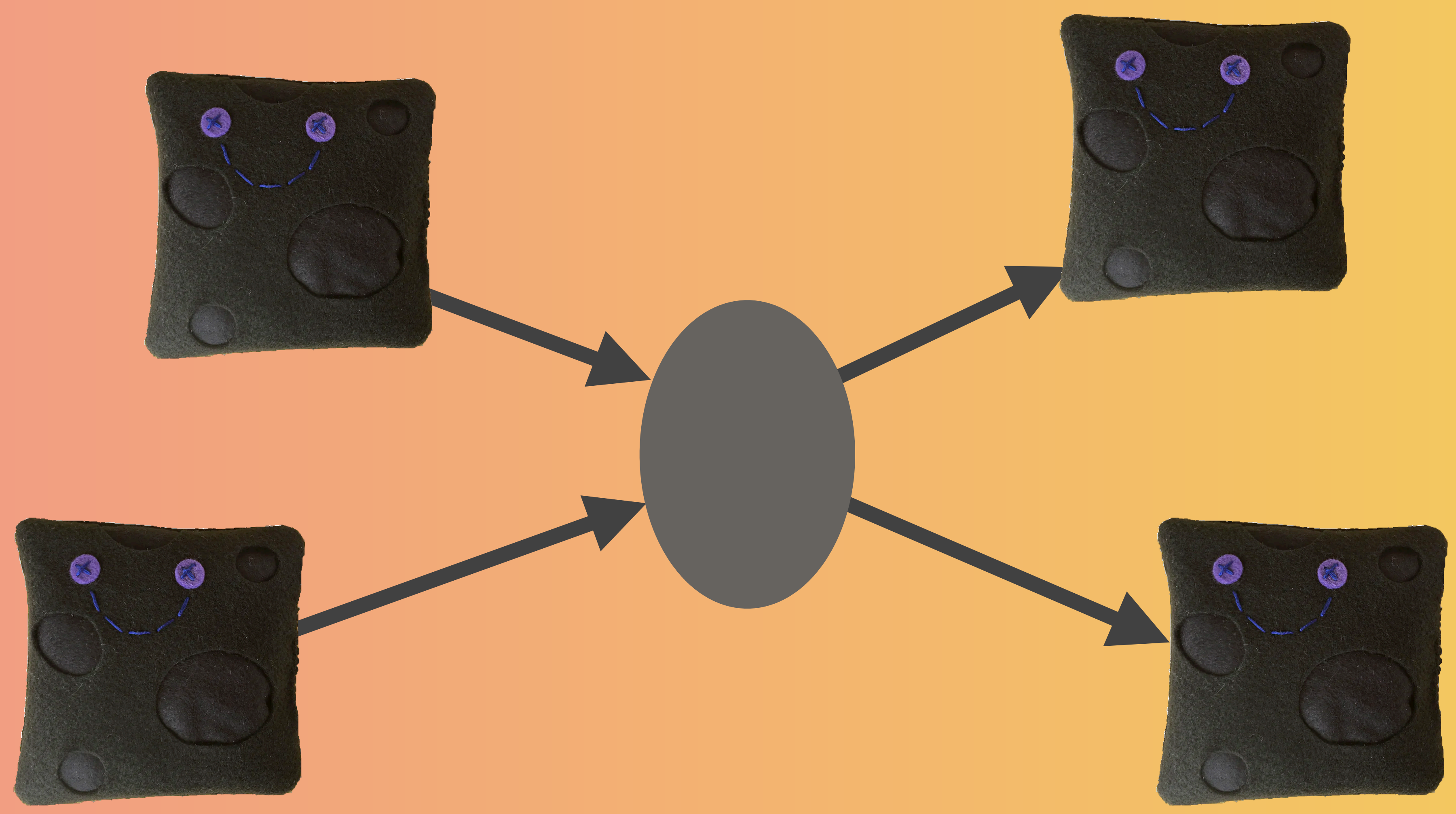
NAWI Graz Natural Sciences

UNI GRAZ

FWF

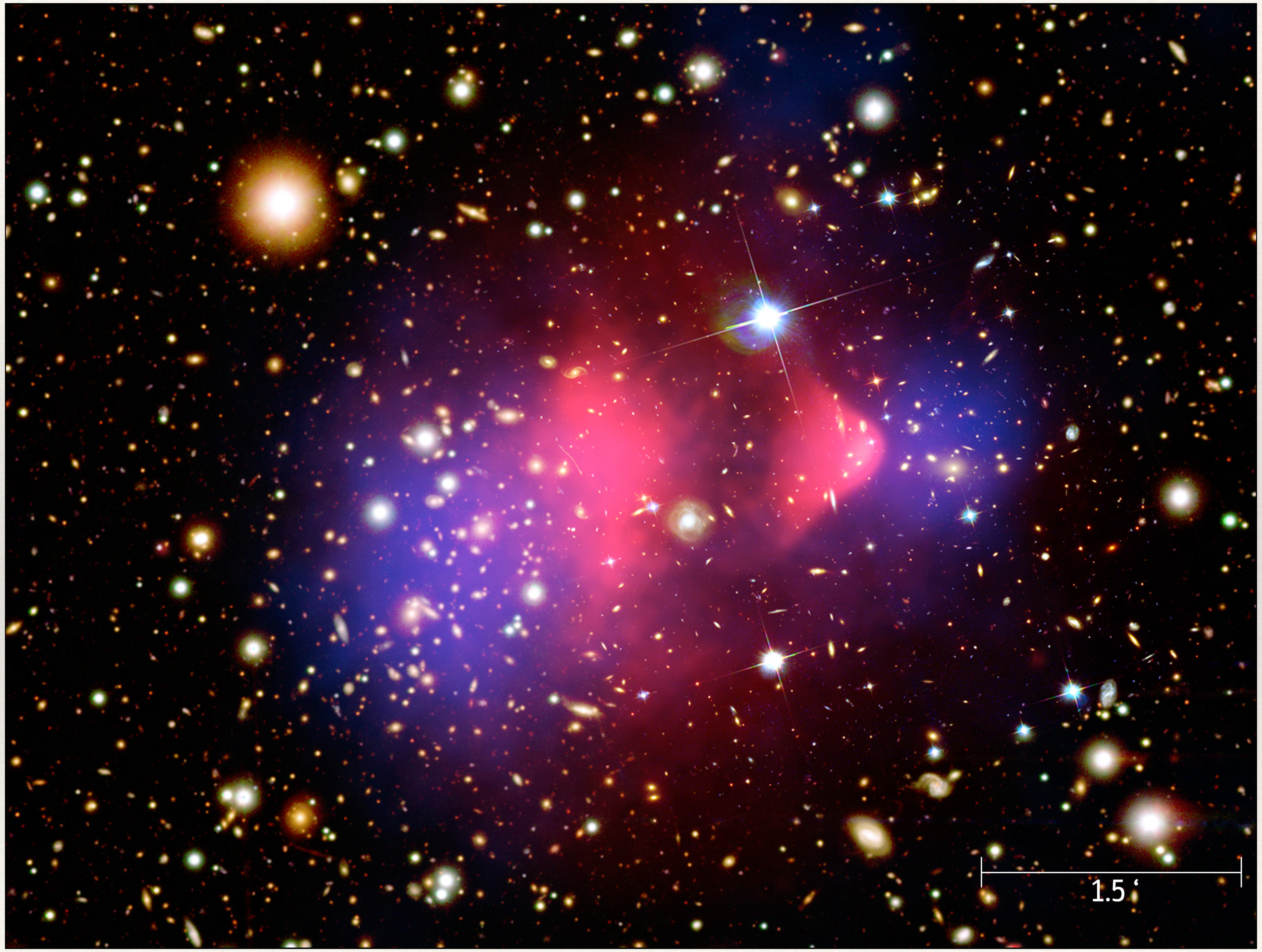
LATTICE 2024

LIVERPOOL



Dark Matter

- One of the biggest mysteries in physics
- Evidence for particle DM:
  - i.e. "Bullet cluster"
- Properties:
  - Massive, stable, "invisible"
- Interaction?
  - With SM: no (low)
  - Self: perchance?

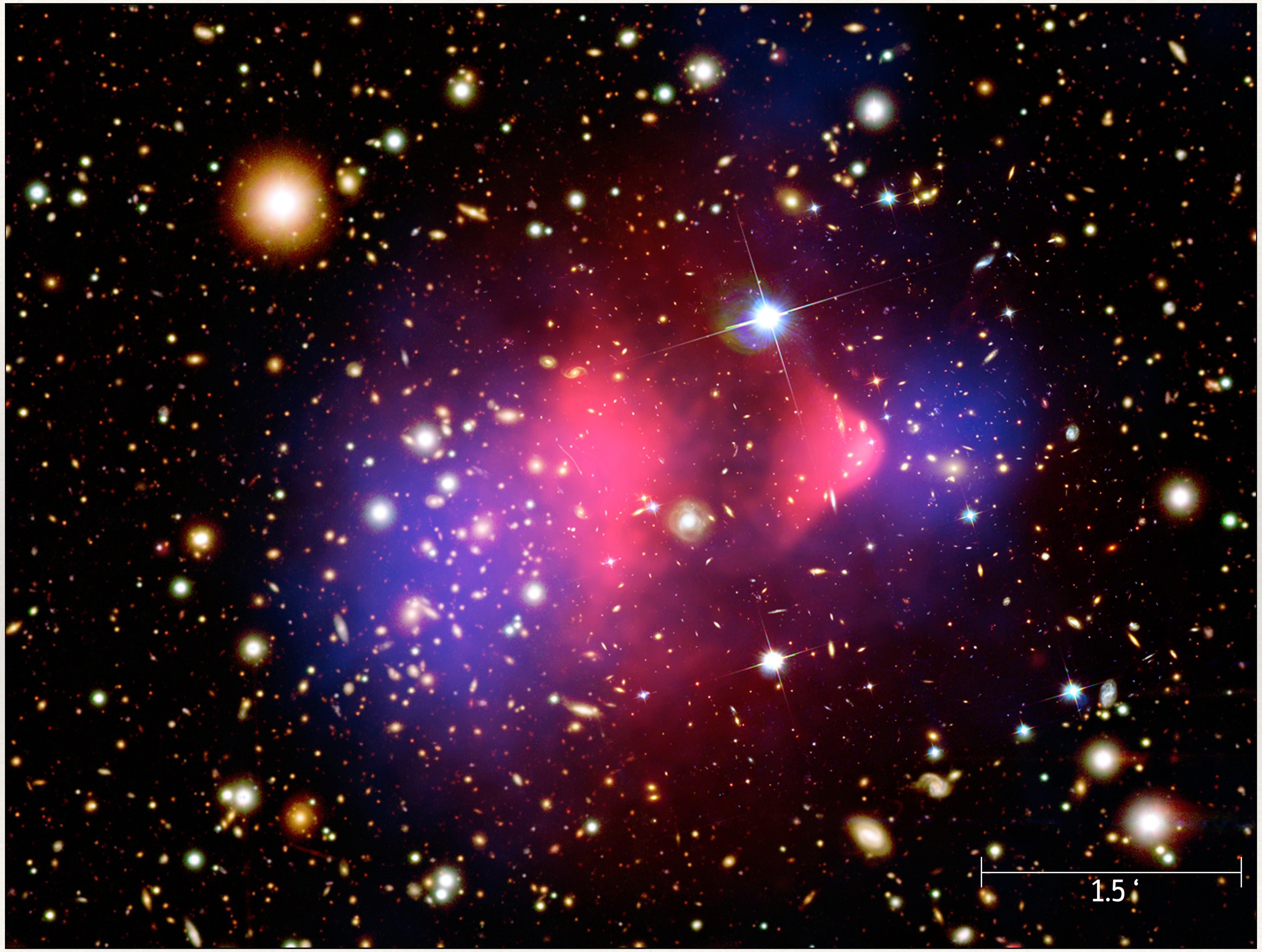


Chandra X-ray Observatory

Dark Matter

What can lattice do?

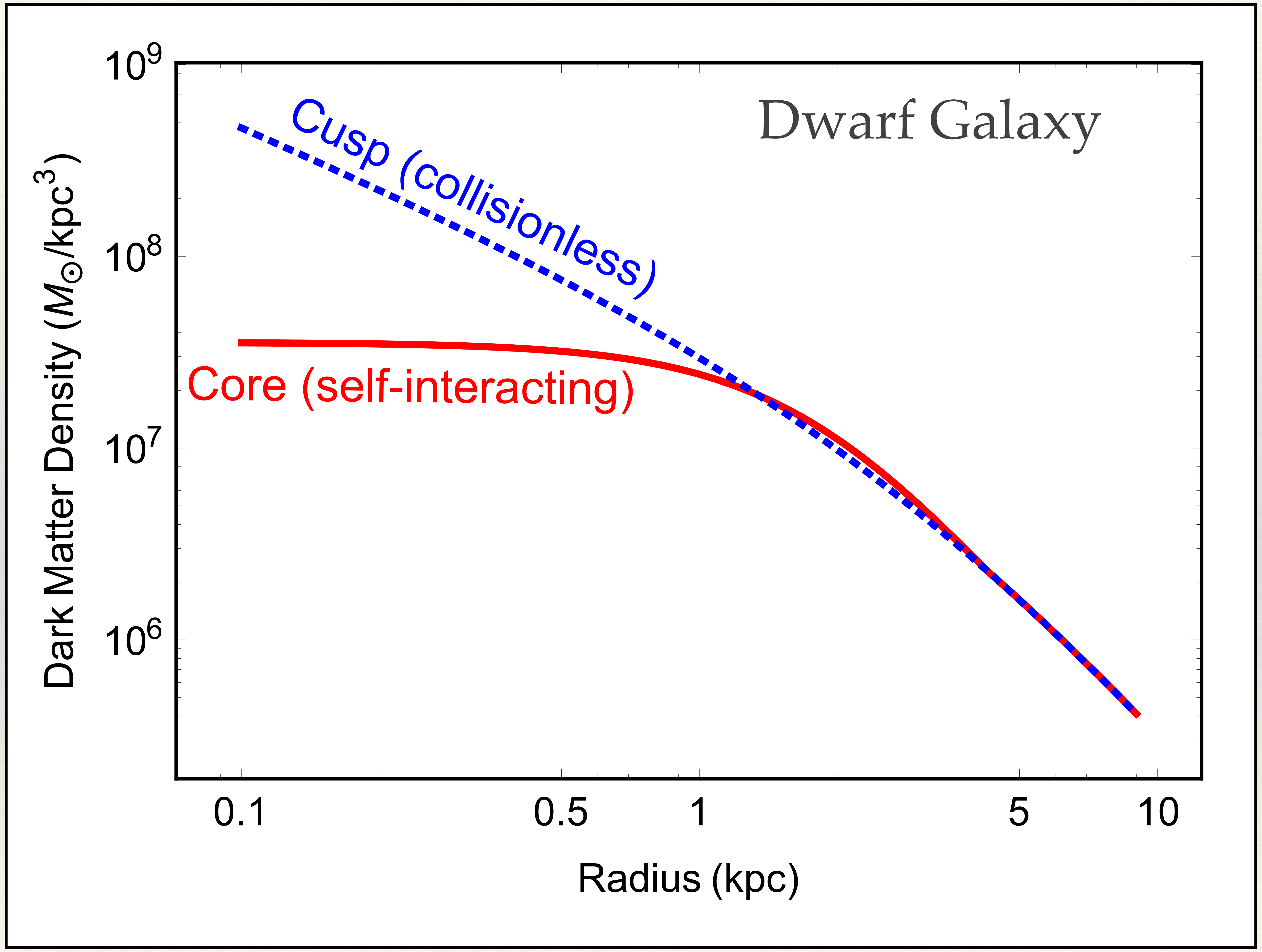
- Test / limit effective theories
- Provide first-principles verification of dark matter models
- Compare directly to i.e. astro data



Chandra X-ray Observatory

Self-interaction

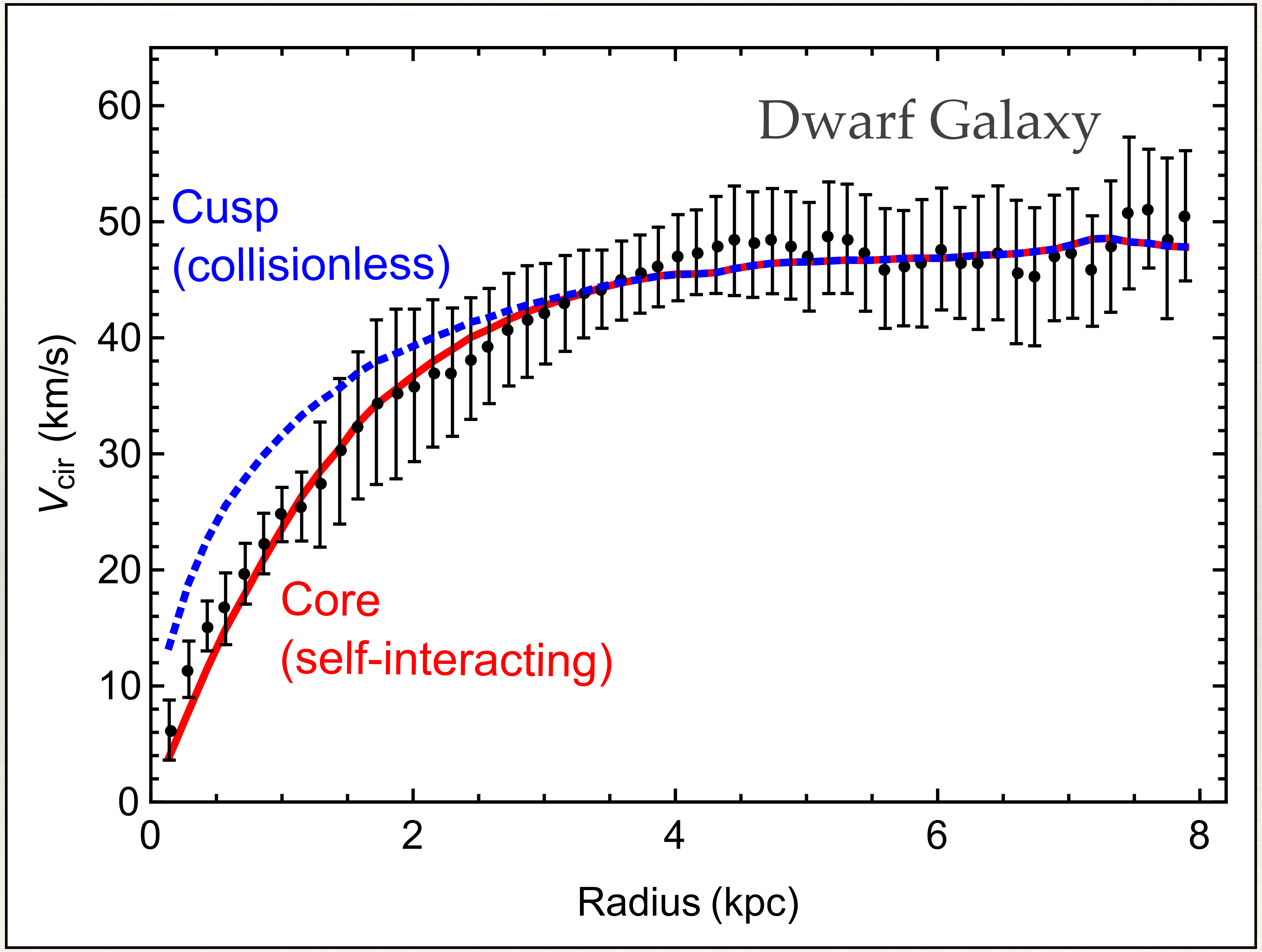
- "Small structure problems"
- Core-like shape preferred
  - Hints towards self-interaction
- Upper bounds on cross-section



Tulin, Yu: arXiv:1705.02358 (2017)

Self-interaction

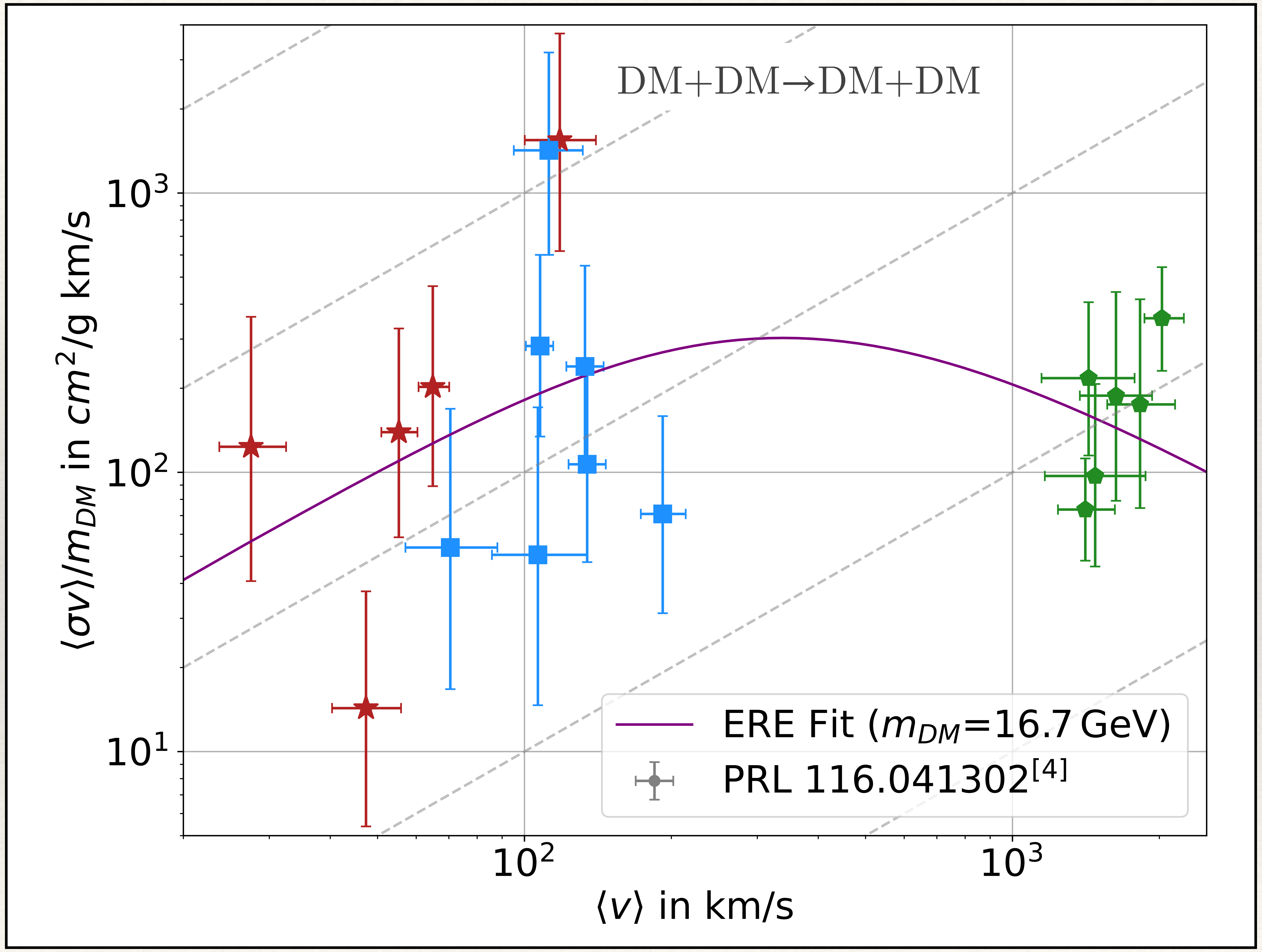
- "Small structure problems"
- Core-like shape preferred
  - Hints towards self-interaction
- Upper bounds on cross-section



Tulin, Yu: arXiv:1705.02358 (2017)

Velocity-dependent cross-section

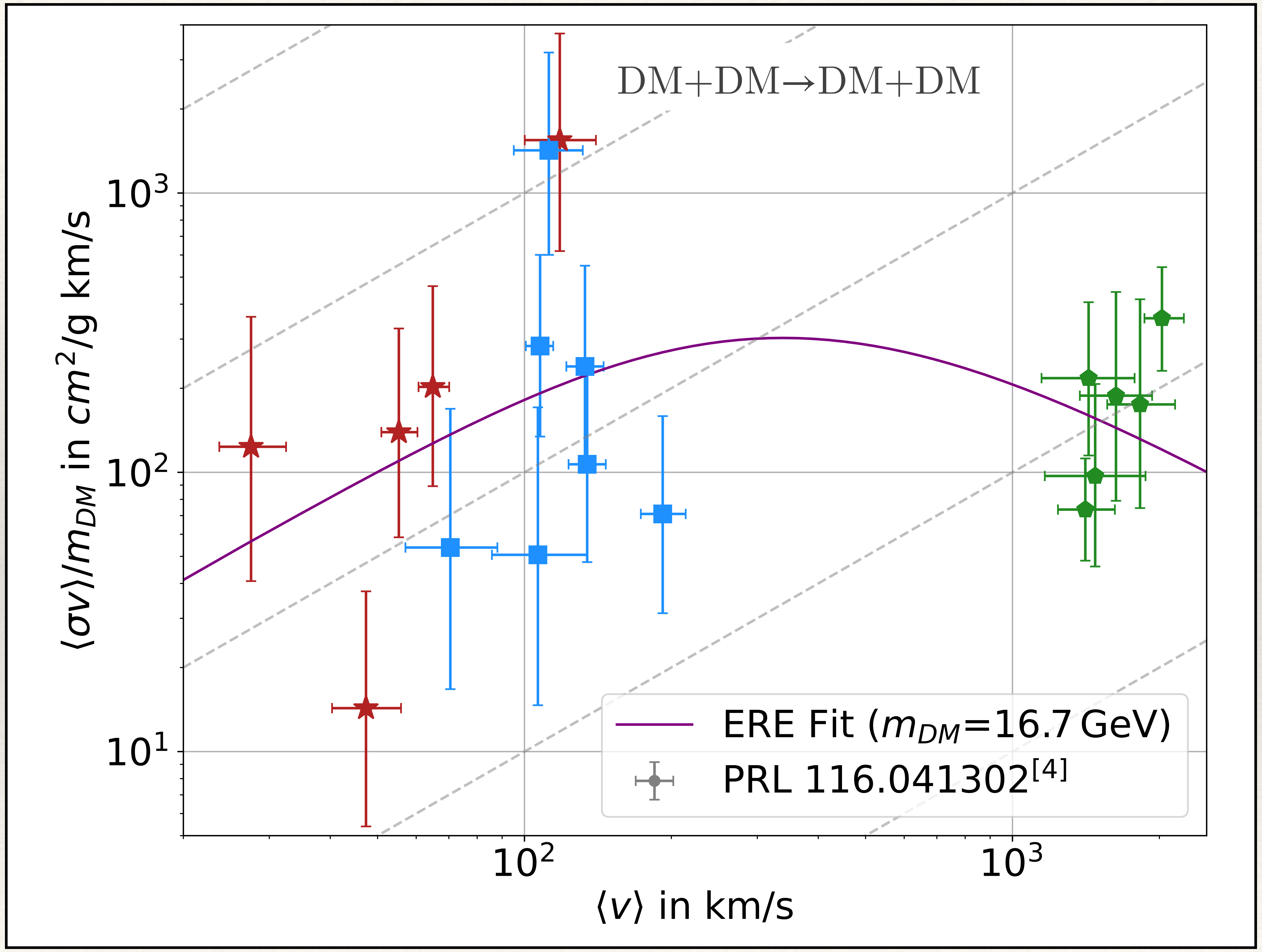
- "Dark matter halos as particle colliders"
- Mild velocity dependence @ non-relativistic velocities
- Relies on simulations of dark halos
  - model-dependent



Kaplinghat et al: Phys. Rev. Lett. 116 (2016)

Velocity-dependent cross-section

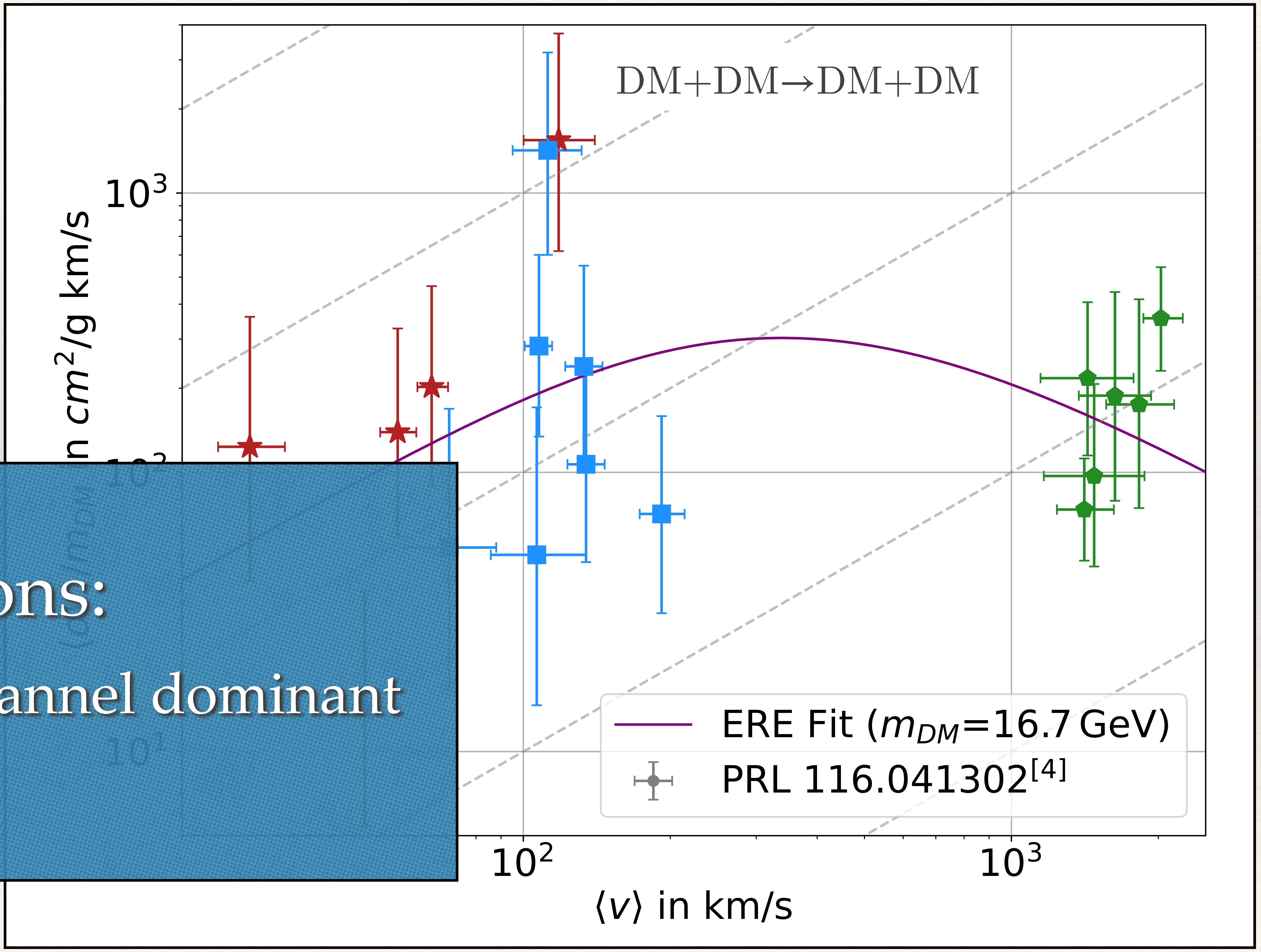
- DM in halo thermalized
- $\langle \sigma v \rangle = \int_0^{v_{esc}} dv\, \sigma(v)\, v\, f(v)$
  - v - rel. velocity, $f(v)$ - Maxwellian
- Can be done on the lattice
  - $\sigma(v)$ needed



Kaplinghat et al: Phys. Rev. Lett. 116 (2016)

Velocity-dependent cross-section

- DM in halo thermalized
- $\langle \sigma v \rangle = \int_0^{v_{esc}} dv\, \sigma(v)\, v\, f(v)$
  - v - rel. velocity, $f(v)$ - Maxwellian
- Can be done on the lattice
  - $\sigma(v)$ needed



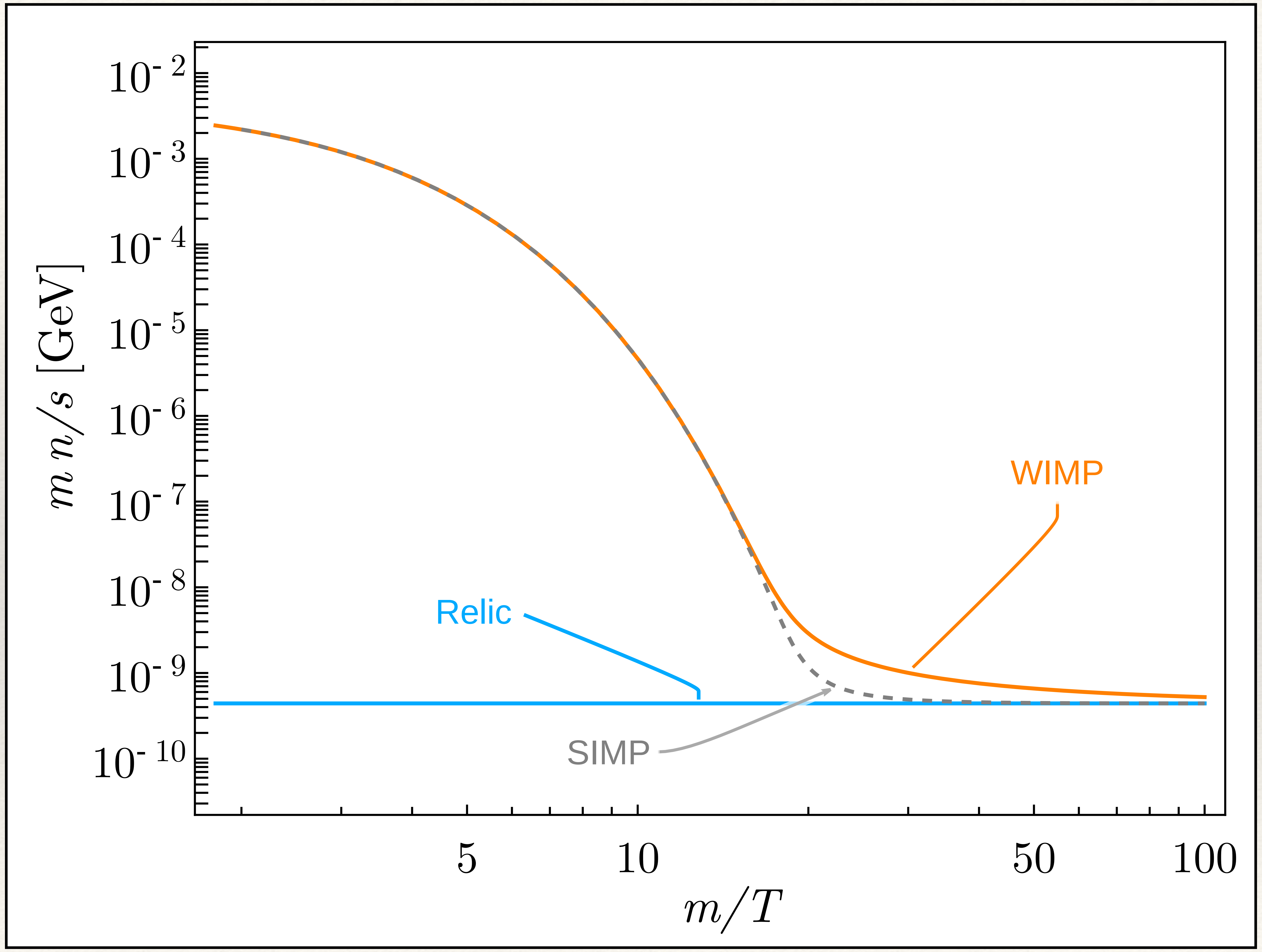
Assumptions:

1. Maximal scattering channel dominant
2. s-wave scattering

Kaplinghat et al: Phys. Rev. Lett. 116 (2016)

Relic density

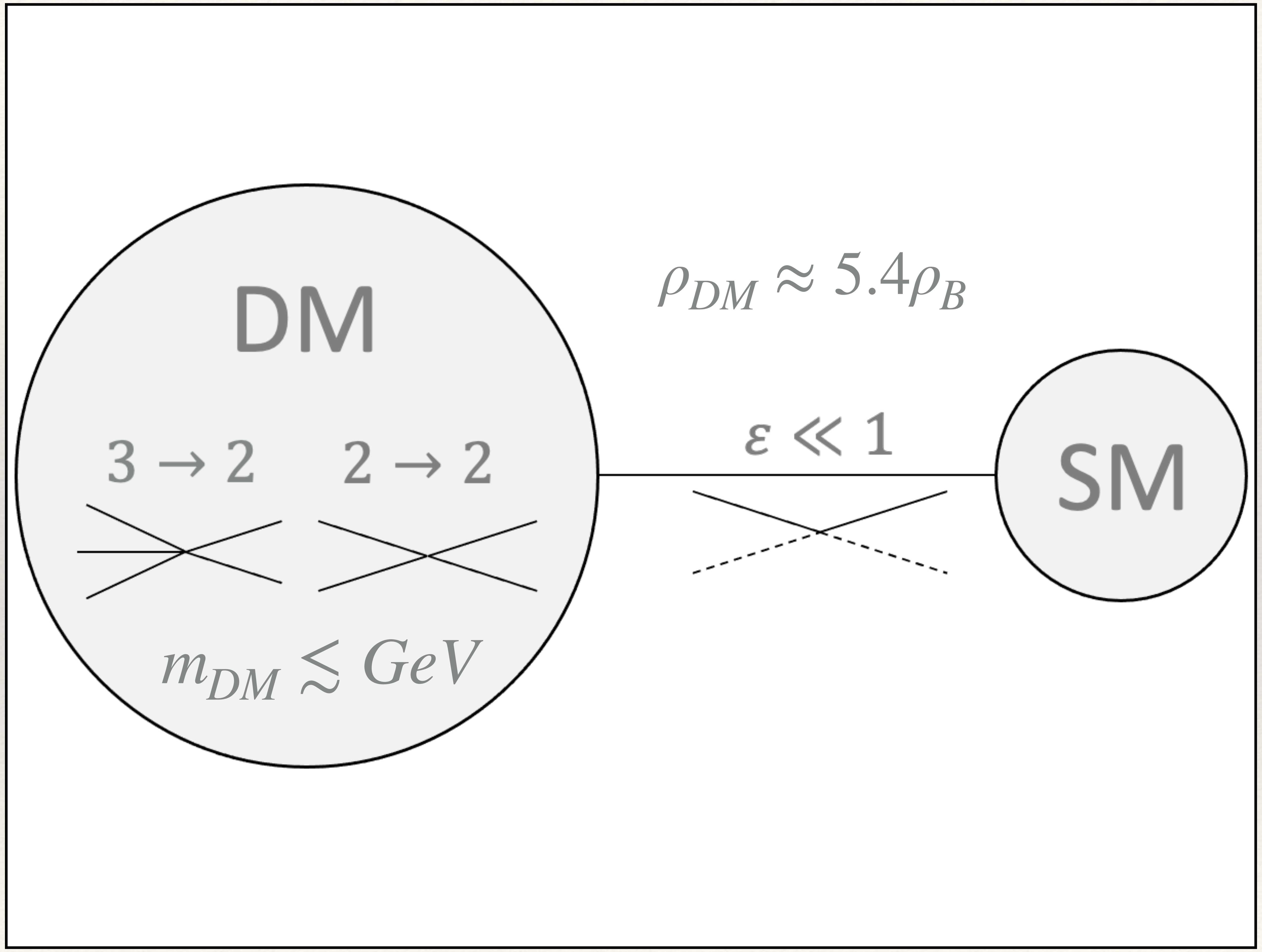
- Possibility: Dark matter as as a thermal relic from the early universe
- Handle on the dark matter abundance
- Temperature decreases → interaction "freezes out"
- Example:
  - WIMP: DM + DM → SM + SM



Frumkin et al: Phys.Rev.Lett. 130 (2023)

Strongly Interacting Massive Particles

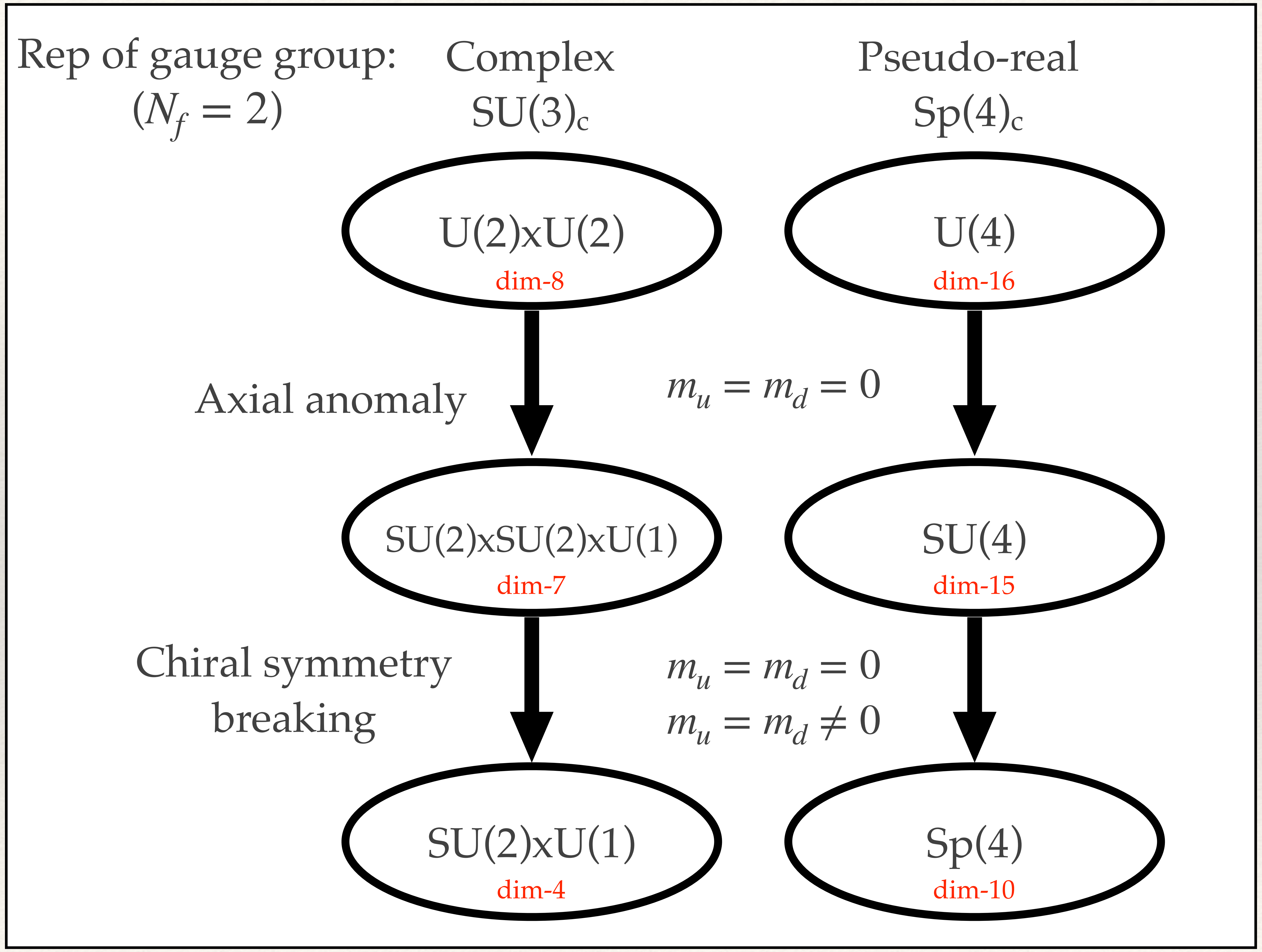
- Alternative freeze-out paradigm
- Number lowering process in the dark sector
  - Addresses self-interaction
- Coupling to the SM sector needed to prevent heat-up
  - Mediator enables direct detection
- Underlying UV theory?



Hochberg et al.: Phys. Rev. Lett. 113 (2014)

Minimal realisation

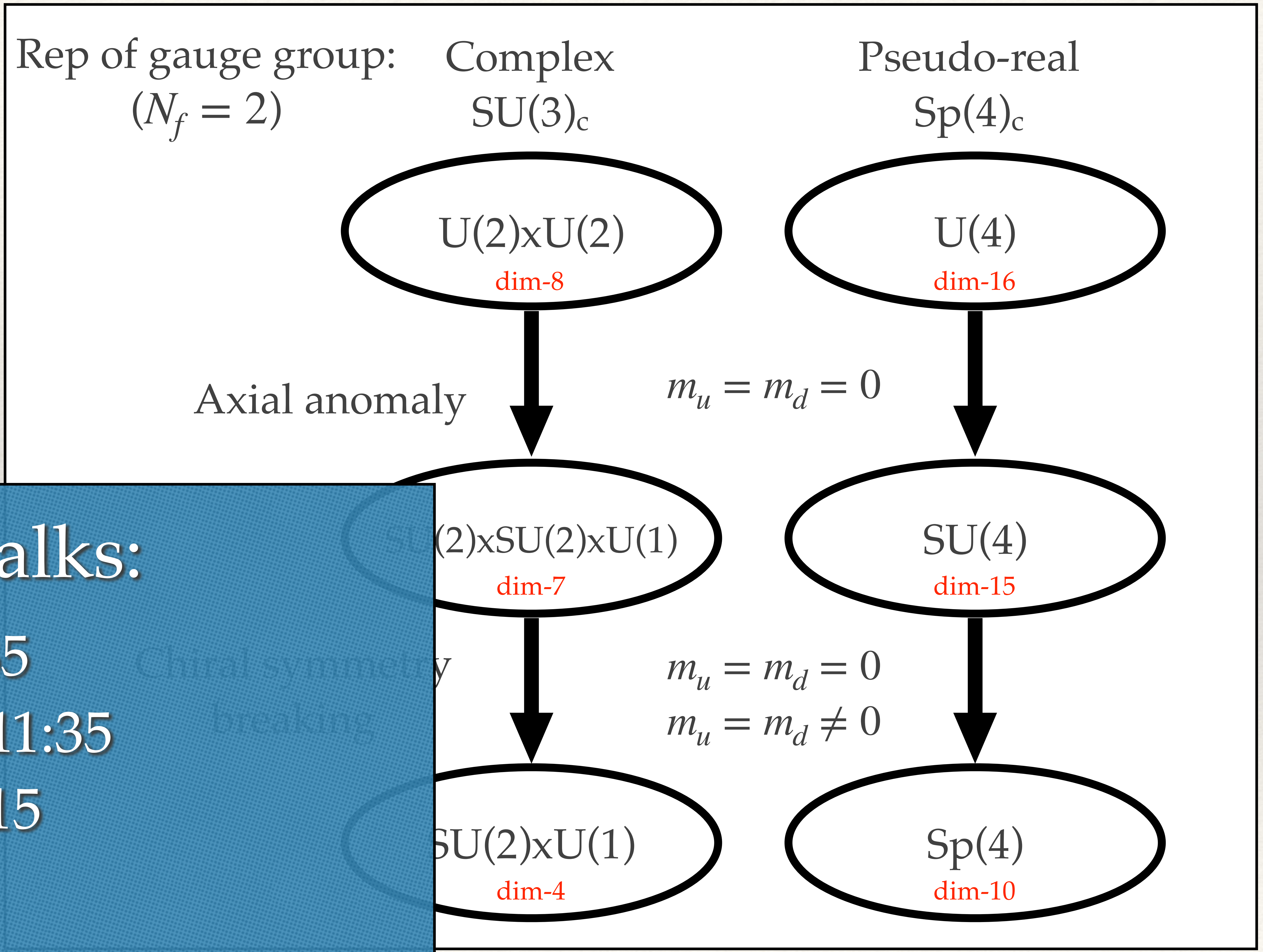
- Sp(4) flavour symmetry
- Pseudo-real rep of gauge group with $N_f = 2$
- Mixing of left- and right handed components (Weyl-fermions)
    - Symmetry is enlarged
- Result: 5 pNGBs
    - $3 \rightarrow 2$ process possible
    - WZW description in ChPT



Kulkarni et al.: SciPost Phys. 14 (2023)

Minimal realisation

- Sp(4) flavour symmetry
- Pseudo-real rep of gauge group with $N_f = 2$
- Mixing of left- and right handed components (Weyl-fermions)
  - Symmetry is
- Result: 5 pNGB
  - $3 \rightarrow 2$ proce
  - WZW descri

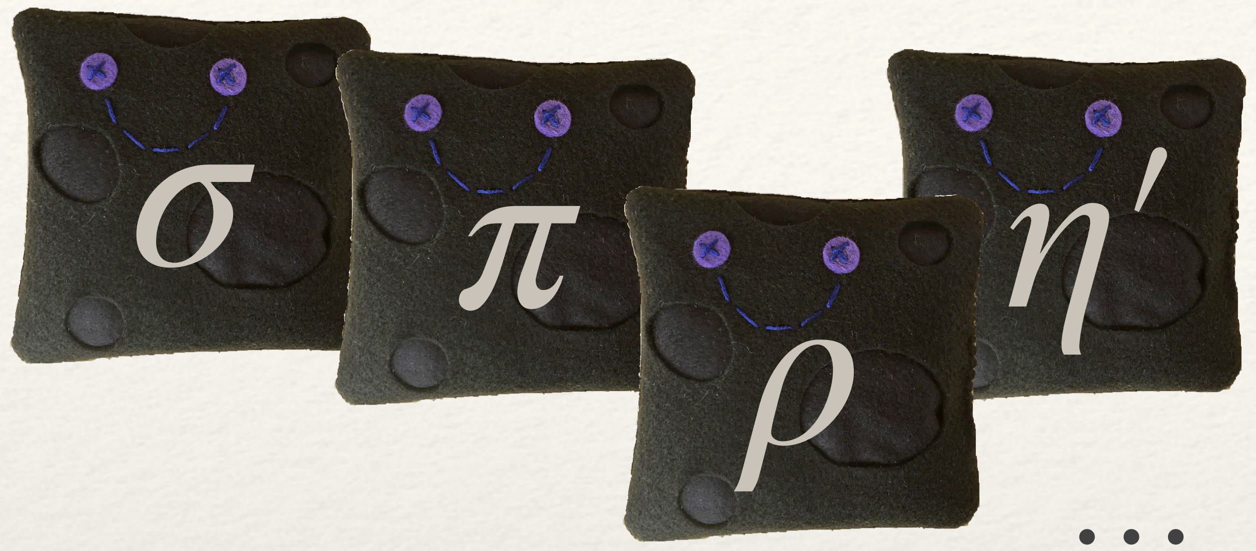


Other Sp(4) talks:

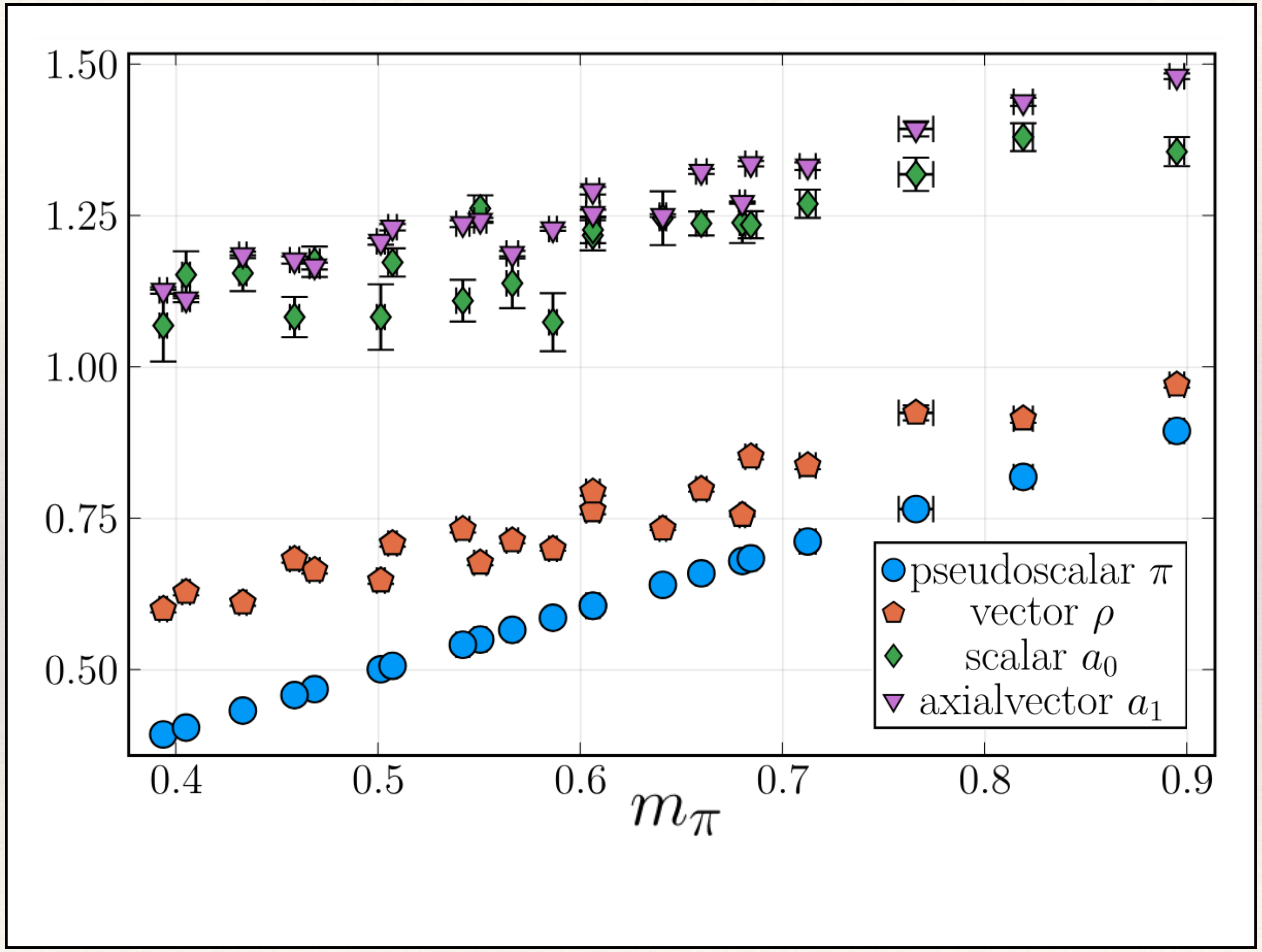
1. Fabian Zierler: Tue 11:35
2. Niccolo Forzano: Wed 11:35
3. David Mason: Wed 12:15
4. Ho Hsiao: Fri 11:15

Kulkarni et al.: SciPost Phys. 14 (2023)

Particle phenomenology

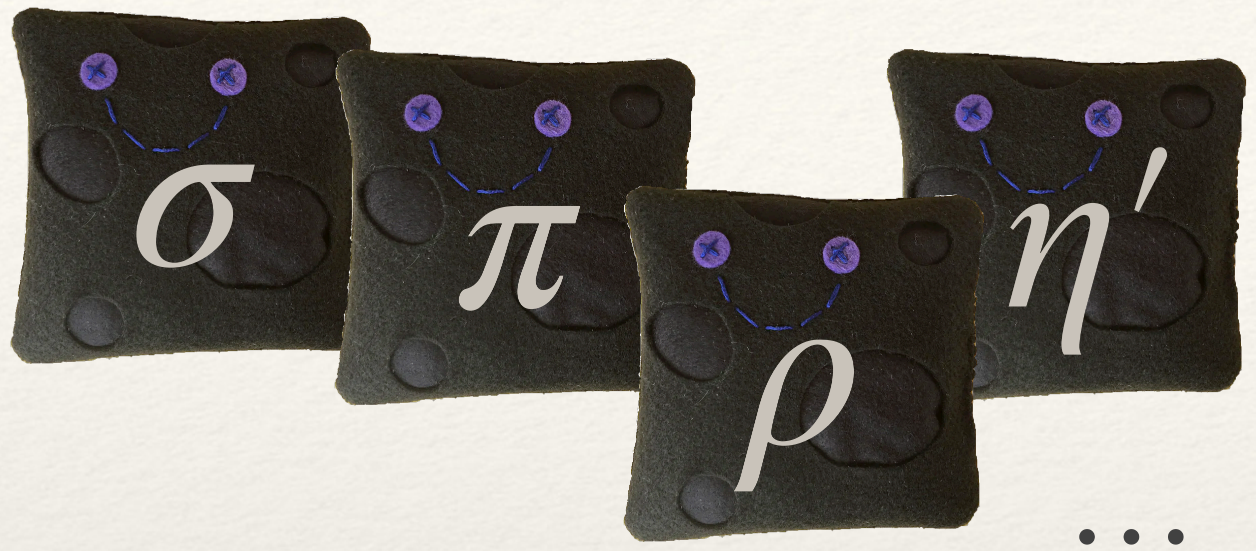


- Zoo of dark hadrons
- 5 Pions & 10 Rhos lightest non-singlets
- No fermionic bound states
- Light η' relevant for $\pi\pi$ scattering
  - Limits ChPT validity



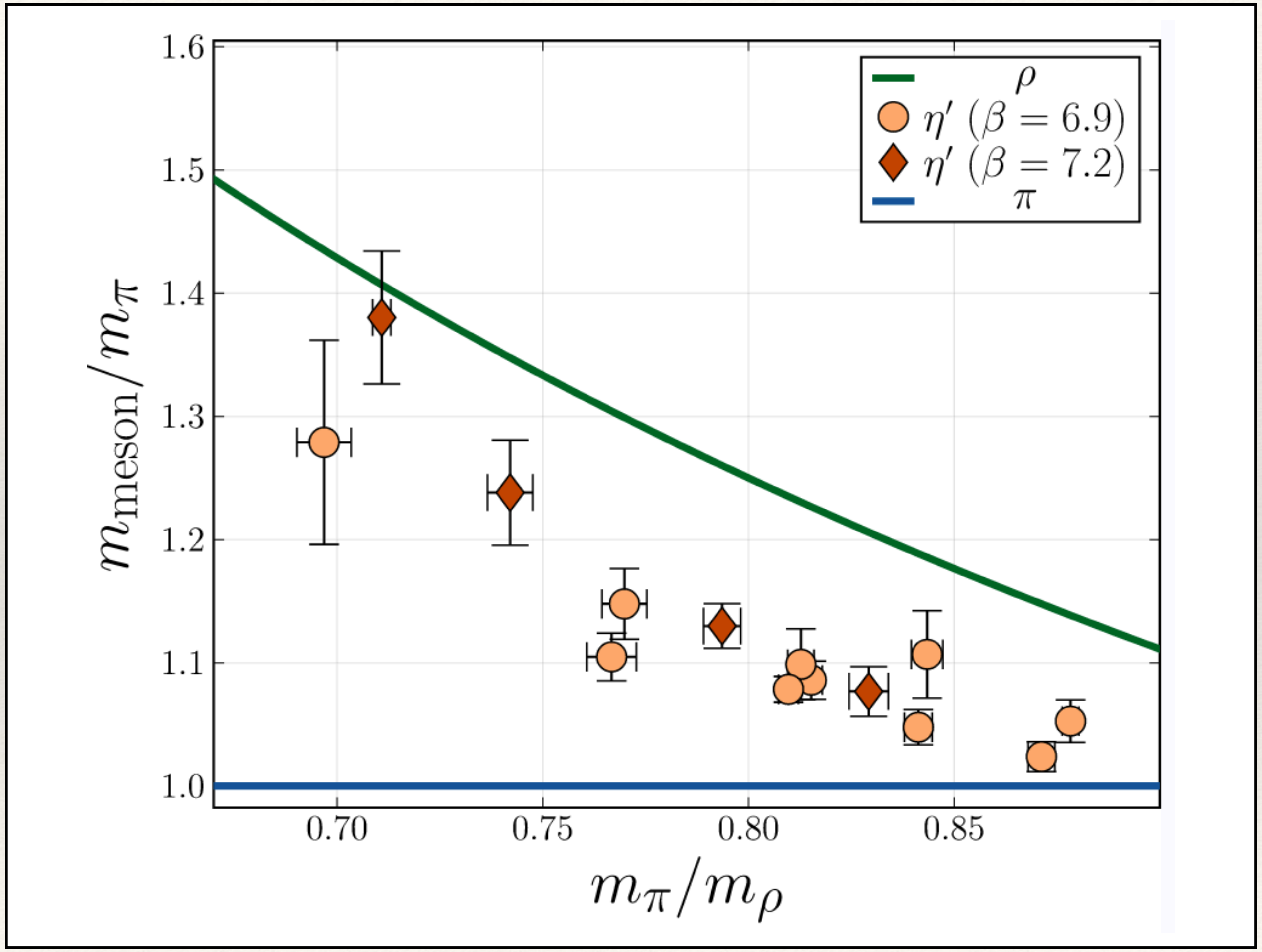
Bennett et al.: arXiv:1909.12662 (2019)
F. Zierler: PHD thesis

Particle phenomenology



…

- Zoo of dark hadrons
- 5 Pions & 10 Rhos lightest non-singlets
- No fermionic bound states
- Light η' relevant for $\pi\pi$ scattering
  - Limits ChPT validity



Bennett et al.: arXiv:1909.12662 (2019)
F. Zierler: PHD thesis

Scattering phenomenology

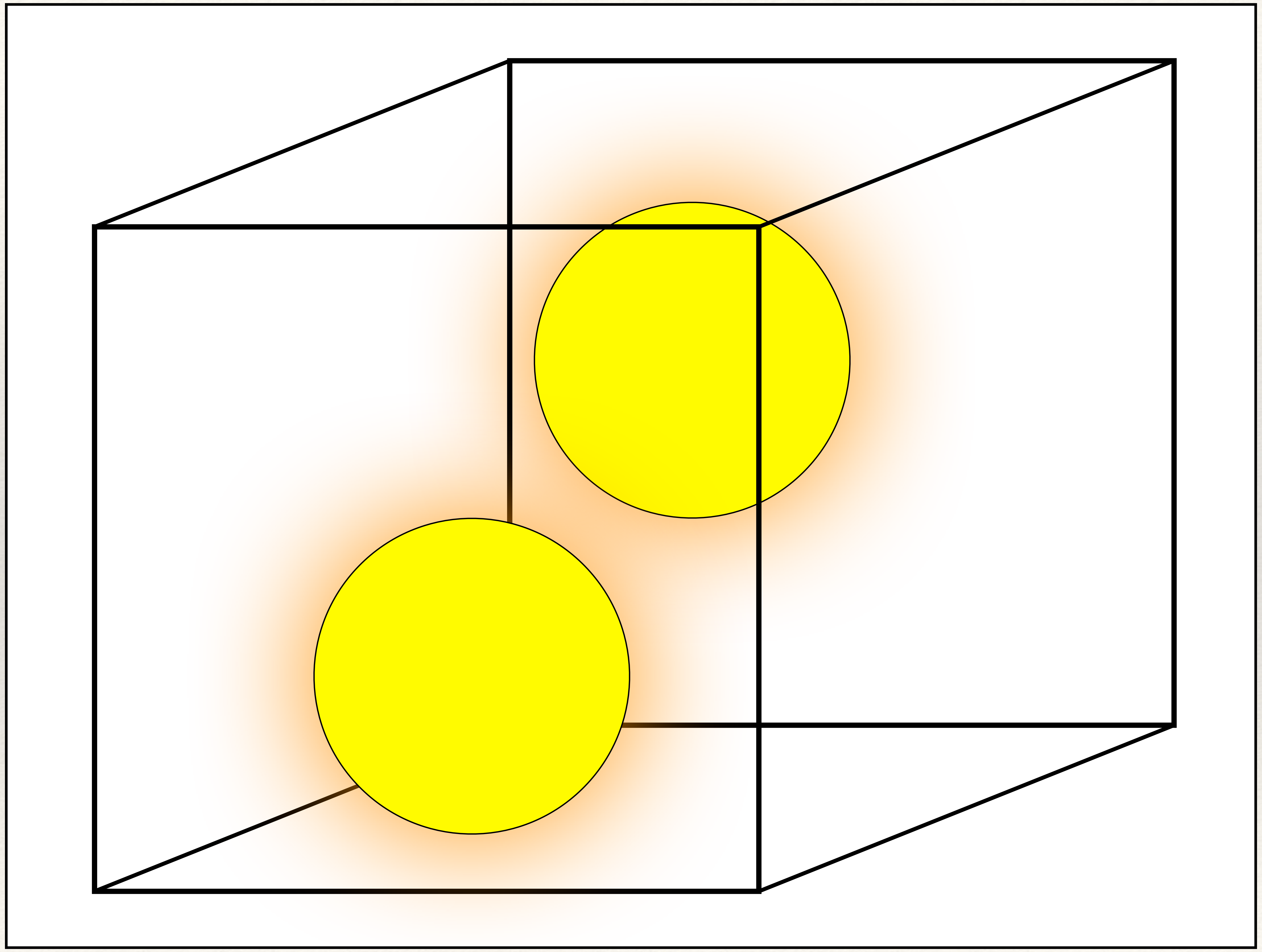
- 3 $\pi\pi$ scattering channels:
  - **1**-dim: Mixing with flavour singlets
  - **10**-dim: $\rho \to \pi\pi$, $3\pi \to 2\pi$
  - **14**-dim: "Maximal" - $\pi^+\pi^+ \to \pi^+\pi^+$
- Contribution to $\pi\pi \to \pi\pi$ scattering naively scales with dimension

$Sp(4)_f$
$5 \otimes 5 = 1 \oplus 10 \oplus 14$
$\pi\pi \to \pi\pi$ (dim $= 1, 10, 14$)
$\pi\pi \to \rho$ (dim $= 10$)
$5 \otimes 5 \otimes 5 = 3(5) \oplus 10 \oplus 30 \oplus 35$
$\pi\pi\pi \to \pi\pi$ (dim $= 10$)
etc.

Feger et al.: Comp.Phys.Com 257 (2020)

Scattering on the lattice

- Standard Lüscher
  - Relate finite volume energy levels with infinite volume scattering properties
- Zero total momentum:
  - $\tan(\delta) = \frac{\pi^{\frac{3}{2}} q}{\mathcal{Z}_{00}^{\vec{0}}(1, q^2)}, \; q = \frac{L}{2\pi} p$
- Result: Energy-dependent phase-shift



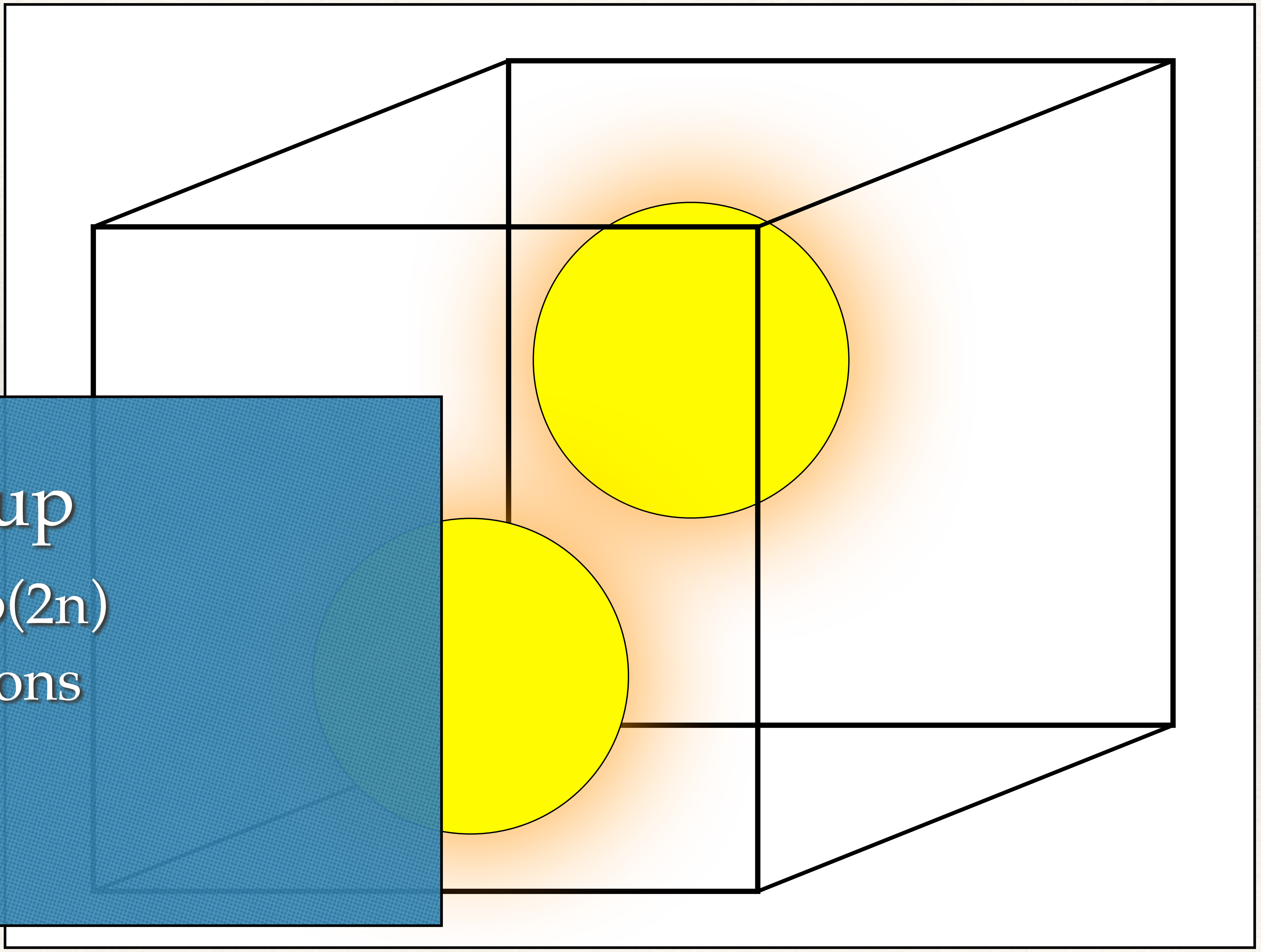
Lüscher et al.: Commun. Math. Phys. 104/105 (1986)
Briceño et al.: Rev.Mod.Phys. 90 (2018) 2, 025001

Scattering on the lattice

- Standard Lüscher
  - Relate finite volume energy levels with infinite volume scattering properties
- Zero total momentum:
  - $\tan(\delta) = \frac{\cdots}{\mathcal{Z}^{\vec{0}}_{00}(\cdots)}$
- Result: Energy-dependent phase shift

Lattice setup

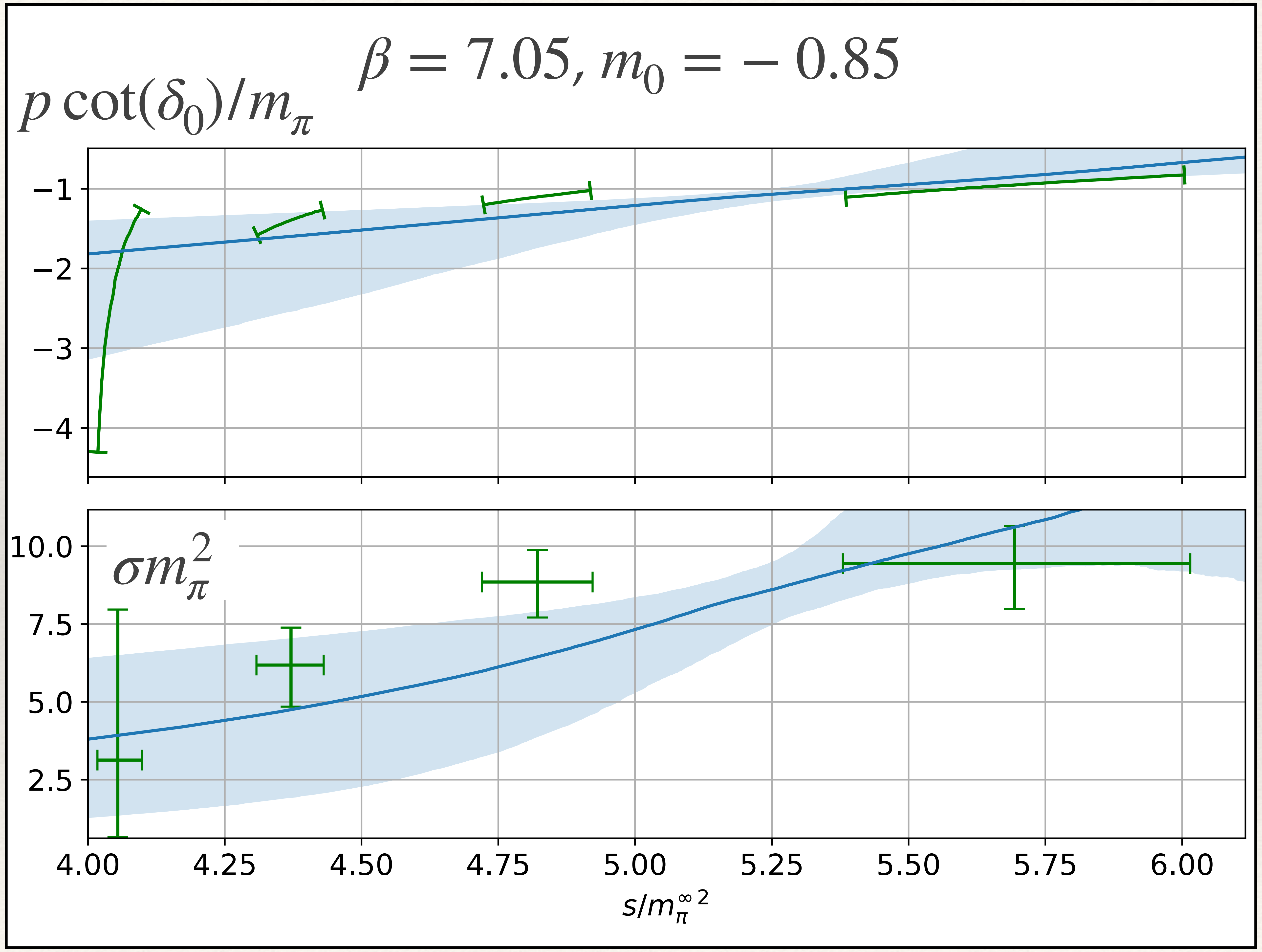
1. HiRep extension for Sp(2n)
2. Standard Wilson fermions
3. $\frac{m_\pi}{m_\rho} = 0.70 - 0.87$



Lüscher et al.: Commun. Math. Phys. 104/105 (1986)
Briceño et al.: Rev.Mod.Phys. 90 (2018) 2, 025001

Phase shift

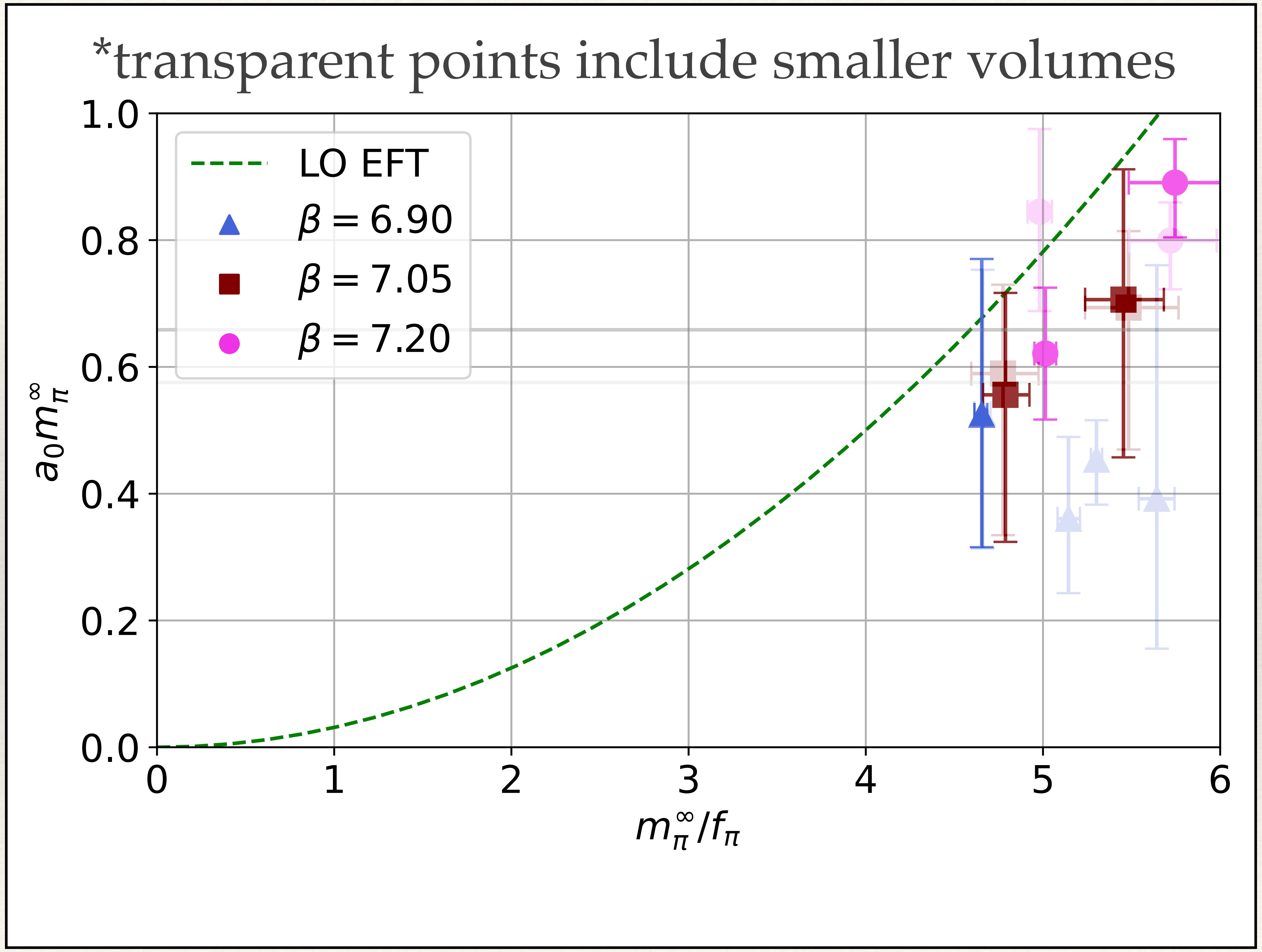
- Effective range expansion:
  - Expand phase shift in $\mathcal{O}(p^2)$
- Access to $\sigma(s)$
- No quantitative effect from using different models for the fit



YD, F. Zierler and A. Maas: arXiv:2405.06506

χ-pT comparison

- Prediction: $a_0 m_\pi = \frac{1}{32}\left(\frac{m_\pi}{f_\pi}\right)^2$
- Potential systematics
- Promising for ChPT
  - NLO?

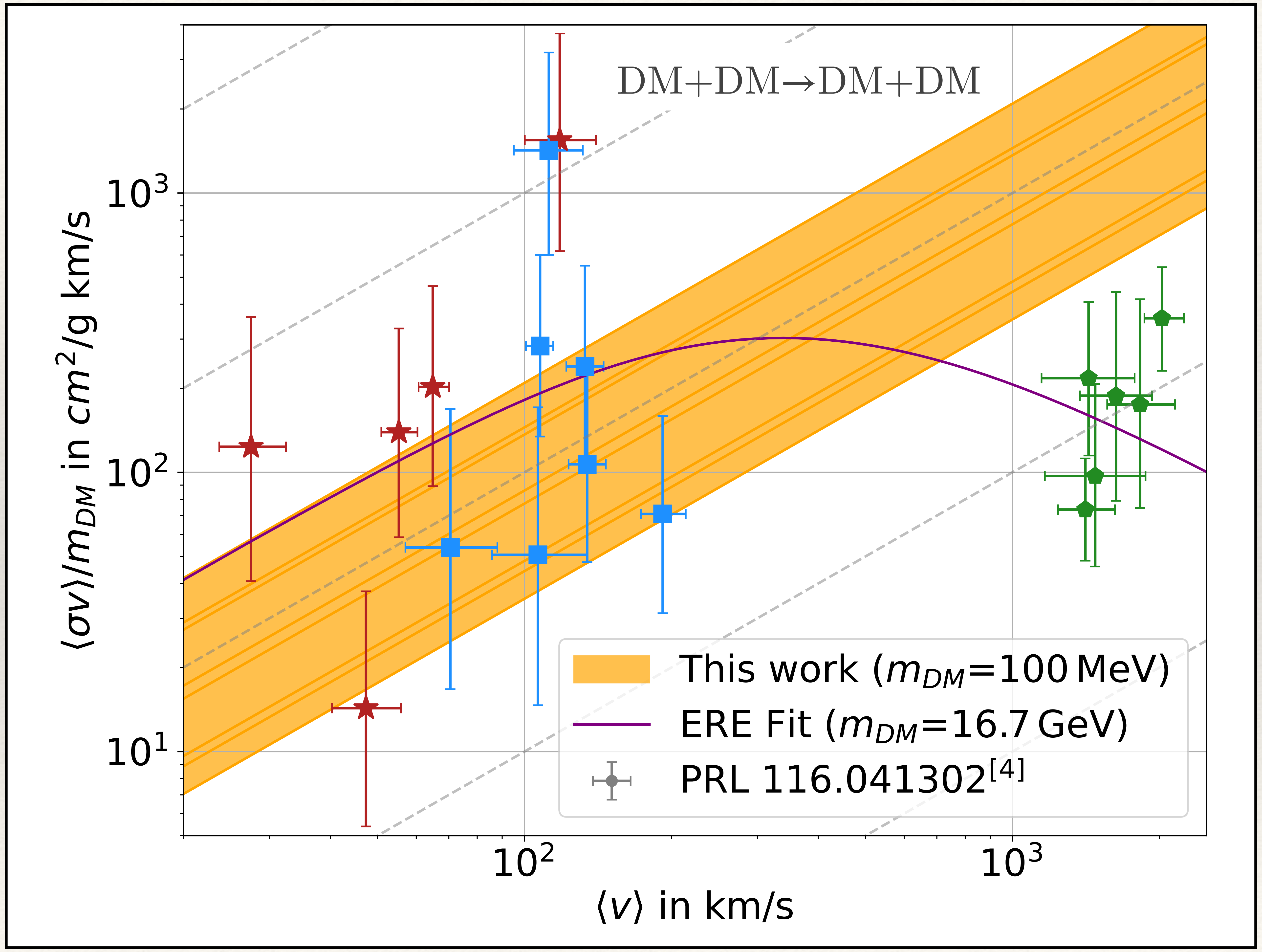


YD, F. Zierler and A. Maas: arXiv:2405.06506
Bijnens and Lu: JHEP, 03:028, (2011)

Velocity-weighted cross-section

$$\langle \sigma v \rangle = \int_0^{v_{esc}} dv\, \sigma(v)\, v\, f(v)$$

- No sign for a velocity dependence
  - Discrepancy in $a_0 m_{DM}$
- $m_{DM} \sim 100\,\mathrm{MeV}$ predicted by SIMP
- Sp(4) not ruled out



YD, F. Zierler and A. Maas: arXiv:2405.06506
Kaplinghat et al: Phys. Rev. Lett. 116 (2016)

Summary & Outlook

- $\pi\pi$-scattering in the most common channel
- Decent agreement with astro-data with $m_{DM} = 100\,\mathrm{MeV}$

- $\rho \to \pi\pi$ scattering (with Bennett et. al.)
- 3→2: Parametrizing $\mathcal{K}_{3\to 2}$ from ChPT (WZW) + finite volume levels (with Max Hansen)

Thank you!

Result tables

β	am_0	N_L	N_T	n_{config}	m_π/m_ρ	am_π	$aE_{\pi\pi}$	af_π	$\langle P\rangle$
6.9	-0.87	10	20	976	0.8744(43)	0.7425(12)	1.4961(22)	0.1313(25)	0.550680(46)
6.9	-0.87	12	24	400	0.8754(41)	0.7414(15)	1.4891(27)	0.13195(91)	0.550441(54)
6.9	-0.87	16	32	100	0.8762(28)	0.74060(96)	1.4820(23)	0.1324(35)	0.550525(61)
6.9	-0.9	8	16	651	0.795(11)	0.6241(25)	1.2799(48)	0.1014(44)	0.557959(99)
6.9	-0.9	8	24	402	0.811(12)	0.6267(26)	1.2806(53)	0.0969(20)	0.55796(10)
6.9	-0.9	10	20	1273	0.7998(38)	0.5738(12)	1.1602(26)	0.1044(21)	0.557172(40)
6.9	-0.9	12	24	2904	0.8110(22)	0.56409(54)	1.1339(14)	0.10484(87)	0.557009(18)
6.9	-0.9	14	24	942	0.8115(27)	0.56222(63)	1.1280(16)	0.10599(58)	0.556981(26)
6.9	-0.9	16	32	546	0.8156(28)	0.56275(57)	1.1283(12)	0.1064(13)	0.556921(25)
6.9	-0.9	18	36	356	0.8135(24)	0.56121(58)	1.1245(12)	0.10576(91)	0.556987(24)
6.9	-0.91	12	24	1268	0.7698(77)	0.4920(10)	0.9950(27)	0.0949(13)	0.559351(28)
6.9	-0.91	14	24	513	0.7756(81)	0.4857(12)	0.9781(29)	0.0945(23)	0.559409(34)
6.9	-0.91	16	32	435	0.7658(65)	0.48610(86)	0.9765(19)	0.0948(23)	0.559353(27)
6.9	-0.92	12	24	63	0.738(61)	0.416(19)	0.885(14)	0.0853(68)	0.56145(14)
6.9	-0.92	14	24	550	0.699(10)	0.3926(14)	0.7914(40)	0.0724(19)	0.562096(34)
6.9	-0.92	16	32	176	0.670(11)	0.3894(14)	0.7848(35)	0.0821(15)	0.562116(42)
6.9	-0.92	24	32	467	0.7035(31)	0.38649(51)	0.7734(12)	0.08260(35)	0.562077(14)
7.05	-0.835	8	24	402	0.777(13)	0.6585(60)	1.345(15)	0.0481(47)	0.577085(74)
7.05	-0.835	12	24	313	0.790(11)	0.4616(15)	0.9424(32)	0.0792(14)	0.575237(39)
7.05	-0.835	14	24	619	0.7877(91)	0.4417(17)	0.9085(30)	0.0793(20)	0.575368(25)
7.05	-0.835	20	36	100	0.7945(61)	0.4380(10)	0.8792(27)	0.0796(31)	0.575269(29)
7.05	-0.85	12	24	84	0.611(33)	0.3778(57)	0.786(22)	0.0582(39)	0.577835(69)
7.05	-0.85	14	24	167	0.716(26)	0.3496(25)	0.7236(67)	0.0675(17)	0.577429(44)
7.05	-0.85	16	32	101	0.660(17)	0.3375(17)	0.6892(41)	0.0669(11)	0.577413(41)
7.05	-0.85	24	36	100	0.7118(64)	0.33076(97)	0.6638(23)	0.0684(19)	0.577371(24)
7.2	-0.78	8	24	401	0.8770(81)	0.8089(42)	1.617(11)	0.0402(50)	0.590527(59)
7.2	-0.78	10	20	195	0.648(16)	0.5508(48)	1.1345(88)	0.0497(20)	0.589788(65)
7.2	-0.78	12	24	150	0.835(19)	0.4382(34)	0.9024(84)	0.0569(14)	0.589547(56)
7.2	-0.78	14	24	425	0.7762(83)	0.3857(14)	0.7951(35)	0.06569(73)	0.589362(26)
7.2	-0.78	16	32	265	0.7930(90)	0.3809(11)	0.7703(31)	0.0645(11)	0.589253(22)
7.2	-0.78	24	36	508	0.7852(30)	0.36963(39)	0.74360(79)	0.0646(26)	0.5892779(85)
7.2	-0.794	12	24	101	0.732(26)	0.3932(63)	0.823(13)	0.0389(24)	0.590837(54)
7.2	-0.794	14	24	234	0.691(31)	0.3234(26)	0.6888(66)	0.0533(14)	0.590422(39)
7.2	-0.794	16	32	101	0.796(27)	0.3097(17)	0.6463(50)	0.0570(13)	0.590330(40)
7.2	-0.794	28	36	504	0.7163(57)	0.28524(35)	0.57582(97)	0.05689(71)	0.5904516(67)

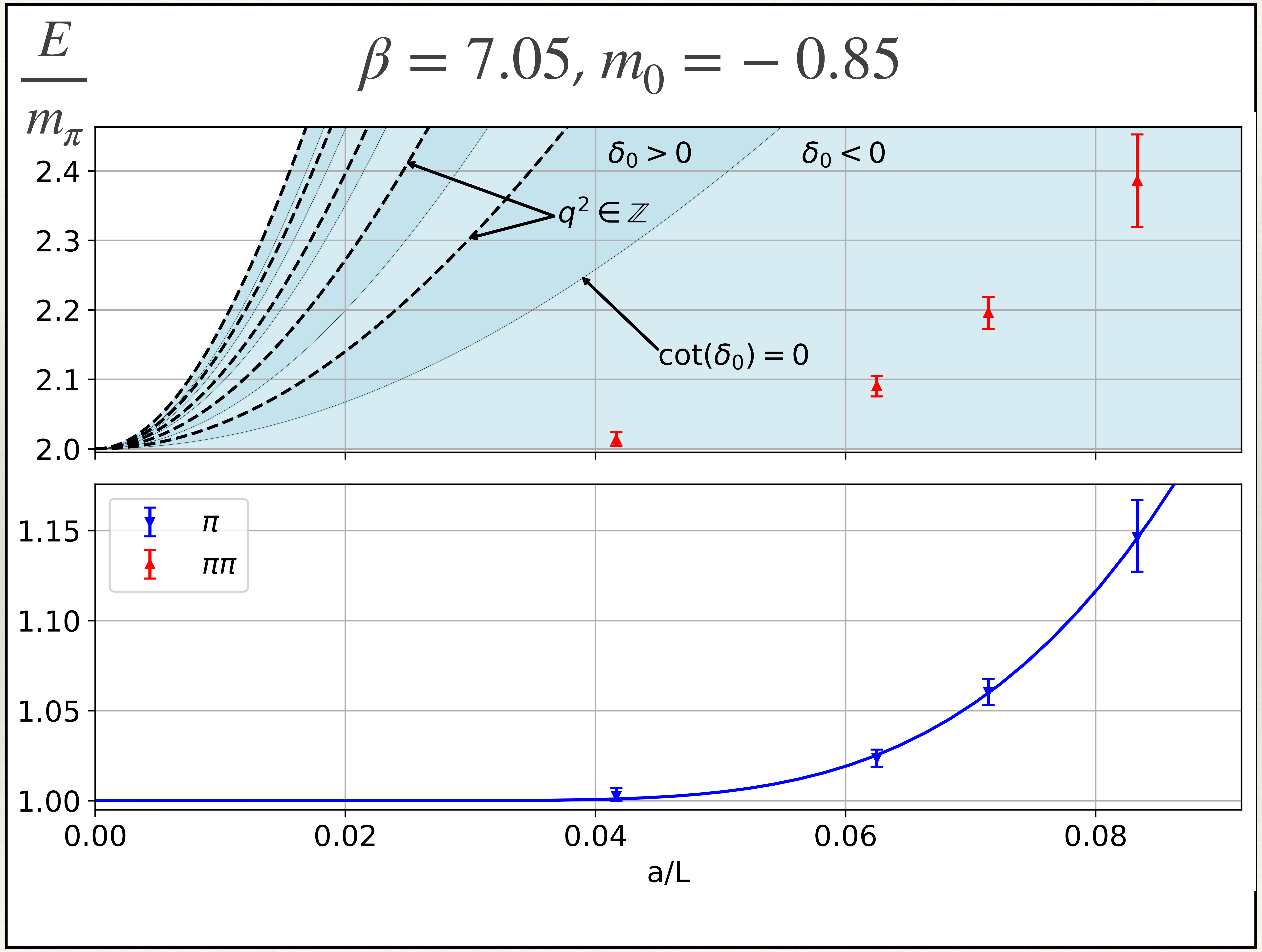
Energy levels ←

Effective range expansion →

β	am_0	$m_\pi^\infty \times 10^4$	$a_0 m_\pi$	$r_0 m_\pi$
6.9	-0.87	7401^{+9}_{-9}	$0.39^{+0.35}_{-0.25}$	59^{+329}_{-110}
6.9	-0.9	5608^{+4}_{-4}	$0.45^{+0.06}_{-0.07}$	$9.3^{+3.5}_{-2.2}$
6.9	-0.91	4845^{+9}_{-9}	$0.36^{+0.12}_{-0.12}$	42^{+57}_{-27}
6.9	-0.92	3845^{+18}_{-31}	$0.52^{+0.23}_{-0.21}$	$6.7^{+9.5}_{-3.8}$
7.05	-0.835	4373^{+9}_{-9}	$0.70^{+0.12}_{-0.21}$	$1.9^{+1.1}_{-0.4}$
7.05	-0.85	3297^{+11}_{-13}	$0.60^{+0.14}_{-0.25}$	$3.7^{+7.4}_{-1.7}$
7.2	-0.78	3696^{+4}_{-4}	$0.80^{+0.06}_{-0.08}$	$2.0^{+0.3}_{-0.2}$
7.2	-0.794	2837^{+12}_{-14}	$0.84^{+0.13}_{-0.16}$	$1.3^{+0.7}_{-0.4}$

Energy levels

- Infinite volume pion mass
- $q = \frac{L}{2\pi} P, \tan(\delta) = \frac{\pi^{\frac{3}{2}} q}{\mathcal{Z}_{00}^{\vec{0}}(1, q^2)}$
  - Non-interacting levels: $q^2 \in \{1,2,..\}$
  - Resonances: $\mathcal{Z}(1, q^2) = 0$
- One to one mapping of $E_{\pi\pi}(L)$ to sign of phase shift

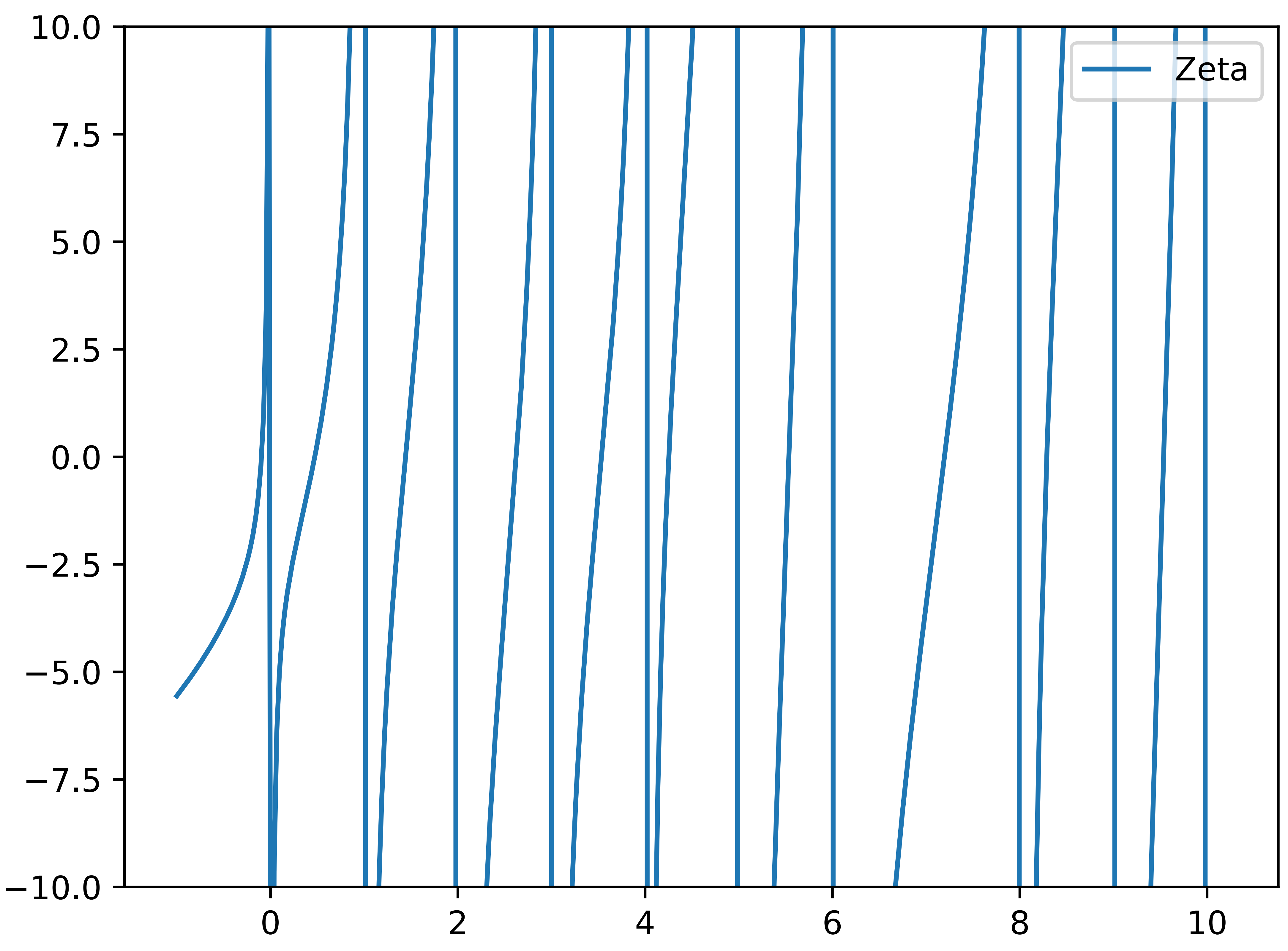


YD, F. Zierler and A. Maas: arXiv:2405.06506
Jenny et al.: Phys. Rev. D 105 (2022)

The Zeta function

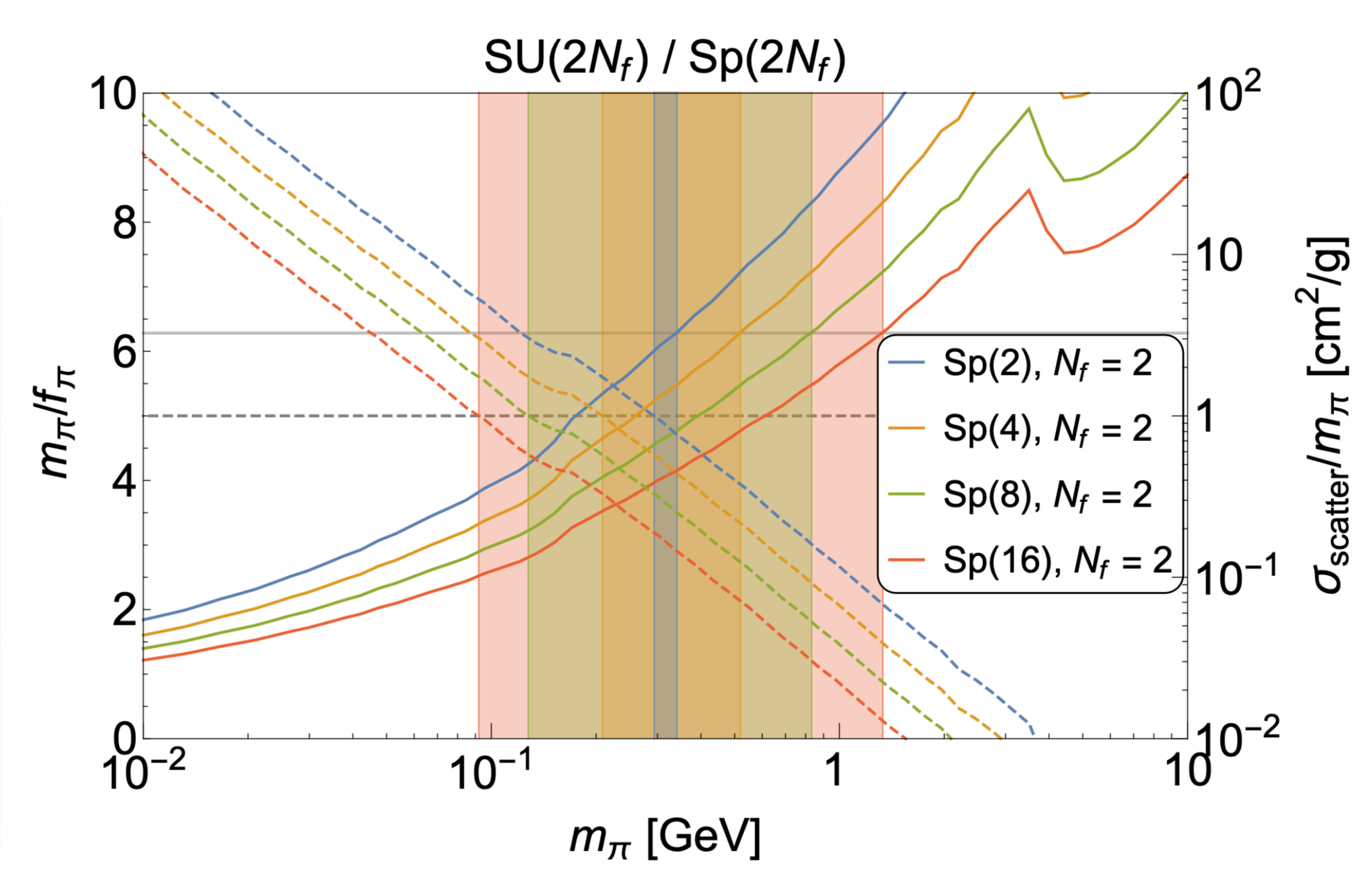
$$\mathcal{Z}^{\vec{\mathbf{d}}}_{Jm}(r, q^2) = \sum_{\vec{\mathbf{x}} \in P_{\vec{\mathbf{d}}}} \frac{|\vec{\mathbf{x}}|^J Y_{Jm}(\vec{\mathbf{x}})}{(\vec{\mathbf{x}}^2 - q^2)^r}$$

$$P_{\vec{\mathbf{d}}} = \left\{ \vec{\mathbf{x}} \in \mathbb{R}^3 \,\middle|\, \vec{\mathbf{x}} = \vec{\mathbf{y}} + \frac{\vec{\mathbf{d}}}{2}, \vec{\mathbf{y}} \in \mathbb{Z}^3 \right\}$$



Why not SU(2)

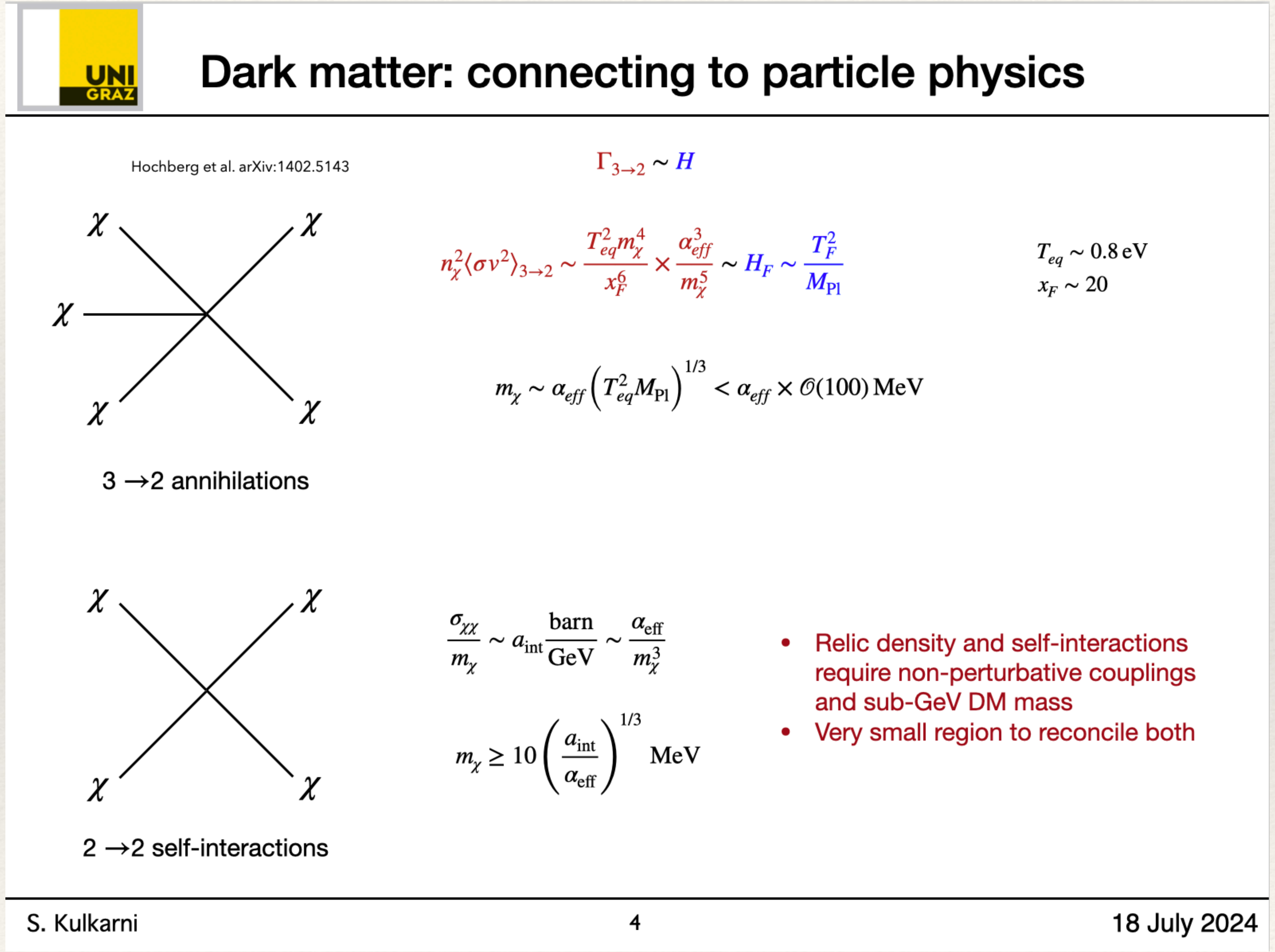
- Sp(2) = SU(2)
- "Large N$_C$" further away from the conformal window for fixed N_f
- Studies of SU(2) exist but are less compatible



"m_π less constraint for larger N_c"

Why confining gauge theory

- Large coupling needed
  - Arises naturally in confining theories
  - Hard to make it work with elementary particles



Small-scale structure problems

Core-cusp problem: High-resolution simulations show that the mass density profile for CDM halos increases toward the center, scaling approximately as $\rho_{\rm dm} \propto r^{-1}$ in the central region [47, 48, 49]. However, many observed rotation curves of disk galaxies prefer a constant "cored" density profile $\rho_{\rm dm} \propto r^0$ [50, 51, 52], indicated by linearly rising circular velocity in the inner regions. The issue is most prevalent for dwarf and low surface brightness (LSB) galaxies [53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65], which, being highly DM-dominated, are appealing environments to test CDM predictions.

Diversity problem: Cosmological structure formation is predicted to be a self-similar process with a remarkably little scatter in density profiles for halos of a given mass [49, 66]. However, disk galaxies with the same maximal circular velocity exhibit a much larger scatter in their interiors [67] and inferred core densities vary by a factor of $\mathcal{O}(10)$ [68].

Missing satellites problem: CDM halos are rich with substructure, since they grow via hierarchical mergers of smaller halos that survive the merger process [69]. Observationally, however, the number of small galaxies in the Local Group are far fewer than the number of predicted subhalos. In the MW, simulations predict $\mathcal{O}(100-1000)$ subhalos large enough to host galaxies, while only 10 dwarf spheroidal galaxies had been discovered when this issue was first raised [70, 71]. Nearby galaxies in the field exhibit a similar underabundance of small galaxies compared to the velocity function inferred through simulations [36, 72, 73].

Too-big-to-fail problem (TBTF): In recent years, much attention has been paid to the most luminous satellites in the MW, which are expected to inhabit the most massive suhalos in CDM simulations. However, it has been shown that these subhalos are too dense in the central regions to be consistent with stellar dynamics of the brightest dwarf spheroidals [74, 75]. The origin of the name stems from the expectation that such massive subhalos are too big to fail in forming stars and should host observable galaxies. Studies of dwarf galaxies in Andromeda [76] and the Local Group field [77] have found similar discrepancies.

Sp(4) particle spectrum

Label (M)	Interpolating operator $(\mathcal{O}_M)$	Meson	J^P	$Sp(4)$
PS	$\overline{Q^i}\gamma_5 Q^j$	π	0^-	5
S	$\overline{Q^i} Q^j$	a_0	0^+	5
V	$\overline{Q^i}\gamma_\mu Q^j$	ρ	1^-	10
T	$\overline{Q^i}\gamma_0\gamma_\mu Q^j$	ρ	1^-	10(+5)
AV	$\overline{Q^i}\gamma_5\gamma_\mu Q^j$	a_1	1^+	5
AT	$\overline{Q^i}\gamma_5\gamma_0\gamma_\mu Q^j$	b_1	1^+	10(+5)

Bennett et al.: arXiv:1909.12662 (2019)

Energy levels on the lattice

- Each operator in a specified quantum number channel contains the full energy spectrum with some non-trivial (not possible to tell a priori) overlap
  - Solution: Try/use a lot of operators and perform variational analysis

$$C(t) = \left\langle \mathcal{O}(t)\mathcal{O}^{\dagger}(0)\right\rangle = \sum_k \langle 0|\mathcal{O}|k\rangle \left\langle k|\mathcal{O}^{\dagger}|0\right\rangle \exp^{-tE_k}$$

$$\lim_{t\to\infty} C(t) = e^{-tm}$$

- Correlation functions can expressed as diagrams

Variational Analysis

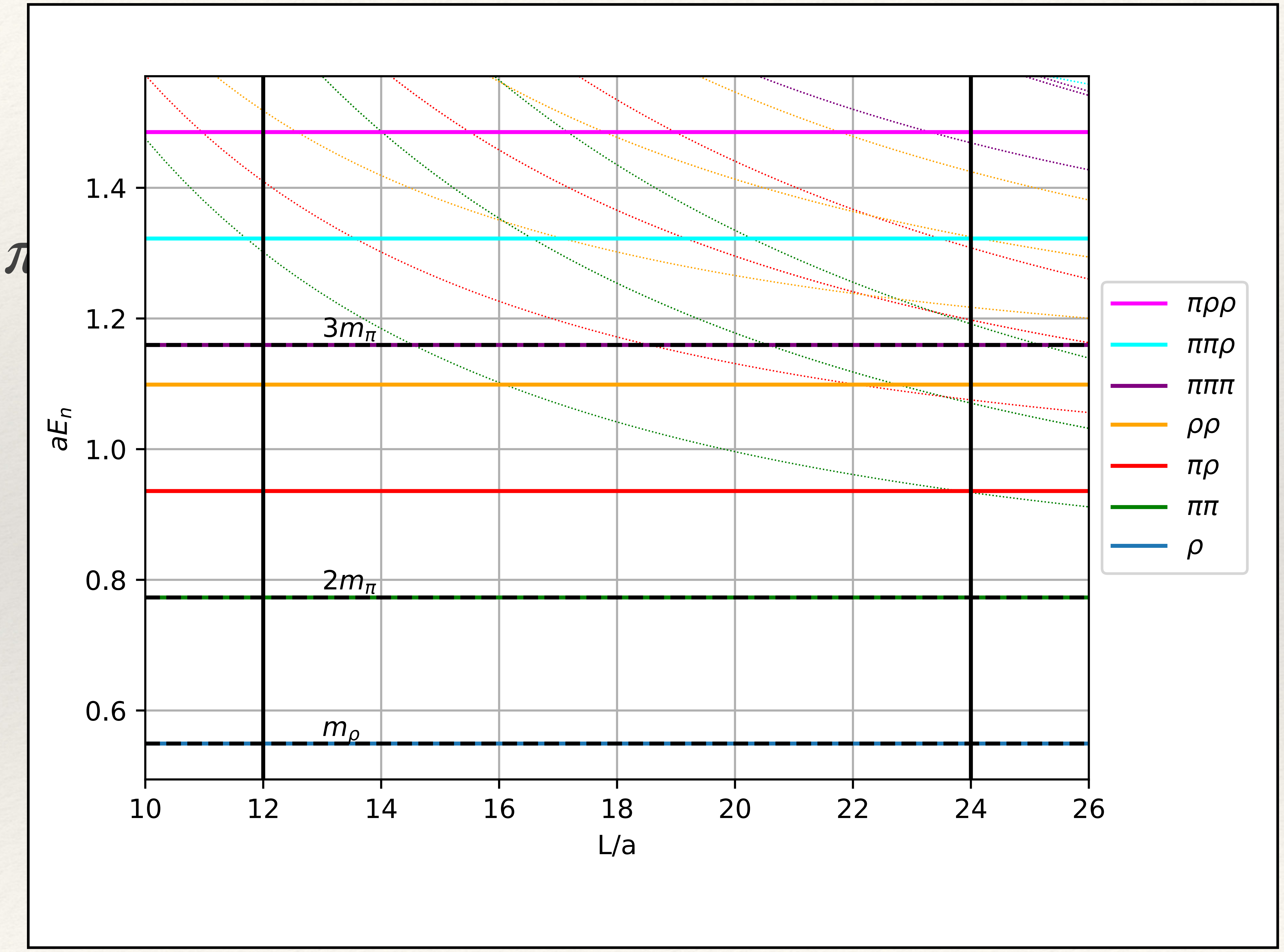
$$C_{ij}(t) = \left\langle \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0) \right\rangle$$

- Build cross-correlation matrix
- The Eigenvalues of this matrix disentangle the energy levels
- $\lambda_k(t) \propto e^{tE_k}$

- Works best with large operator basis

Trivial energy levels

- Lot of scattering states possible
- Possible operators: ρ, $\pi\pi$, $\pi\rho$, $\rho\rho$, $\pi\pi\pi$, π
- $E = \sum_i \sqrt{m_i^2 + p_i^2}$
- Trivial momenta in finite volume:
  - $p = \frac{2\pi|\vec{n}|}{L}, \vec{n} \in \mathbb{Z}^3$



Lüscher method

Zero momentum $\mathbf{P} = (\mathbf{0}, \mathbf{0}, \mathbf{0})$
(for irrep T_1^- in O_h) [3]:

$$\tan \delta(q) = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1; q^2)}$$

Nonzero momentum $\mathbf{P} = (\mathbf{0}, \mathbf{0}, \mathbf{1})\frac{\mathbf{2\pi}}{\mathbf{L}}$
(for irrep A_2^- in D_{4h}) [7]:

$$\tan \delta(q) = \frac{\gamma \pi^{3/2} q^3}{q^2 \mathcal{Z}^{\mathbf{d}}_{00}(1; q^2) + \sqrt{\frac{4}{5}}\, \mathcal{Z}^{\mathbf{d}}_{20}(1; q^2)}$$

P=(1,1,0)

(for irrep B_1^- in D_{2h})

$$\tan \delta(q) = \frac{\gamma \pi^{3/2} q^3}{q^2 \mathcal{Z}^{\mathbf{d}}_{00}(1; q^2) - \sqrt{\frac{1}{5}}\, \mathcal{Z}^{\mathbf{d}}_{20}(1; q^2) + \mathrm{i}\sqrt{\frac{3}{10}}\, (\mathcal{Z}^{\mathbf{d}}_{22}(1; q^2) - \mathcal{Z}^{\mathbf{d}}_{2\bar{2}}(1; q^2))}$$

Lang et. al.: arXiv:1105.5636v3

Phenomenology of scattering channels

- **14**-dim:
  - (Probably) contributes most to $\pi\pi$-scattering
  - 14 out of 25 possible combinations of Pions

$Sp(4)_f$

$5 \otimes 5 = 1 \oplus 10 \oplus 14$

$10 \otimes 5 = 5 \oplus 10 \oplus 35$

$5 \otimes 5 \otimes 5 = 3(5) \oplus 10 \oplus 30 \oplus 35$

$\pi\pi \rightarrow \pi\pi$ (I=0,1,2)

$\pi\pi \rightarrow \rho$ (I=1)

$\pi\pi \rightarrow \pi\pi\pi$ (I=1)

$\pi\pi \rightarrow \pi\pi\rho$ (I=0,1,2)

etc.

Feger et al.: Comp.Phys.Com 257 (2020)

Phenomenology of scattering channels

- **1**-dim:
  - (Probably) no large contribution to $\pi\pi$-scattering
  - Mixes in other scattering channel
  - Numerically challenging

$Sp(4)_f$

$5 \otimes 5 = 1 \oplus 10 \oplus 14$

$10 \otimes 5 = 5 \oplus 10 \oplus 35$

$5 \otimes 5 \otimes 5 = 3(5) \oplus 10 \oplus 30 \oplus 35$

$\pi\pi \rightarrow \pi\pi$ (I=0,1,2)

$\pi\pi \rightarrow \rho$ (I=1)

$\pi\pi \rightarrow \pi\pi\pi$ (I=1)

$\pi\pi \rightarrow \pi\pi\rho$ (I=0,1,2)

etc.

Feger et al.: Comp.Phys.Com 257 (2020)

Phenomenology of scattering channels

- **10**-dim:
  - Mixing with the Rho
  - $\pi\pi\pi \to \pi\pi$
- Work in progress

$Sp(4)_f$

$5 \otimes 5 = 1 \oplus 10 \oplus 14$

$10 \otimes 5 = 5 \oplus 10 \oplus 35$

$5 \otimes 5 \otimes 5 = 3(5) \oplus 10 \oplus 30 \oplus 35$

$\pi\pi \to \pi\pi$ (I=0,1,2)

$\pi\pi \to \rho$ (I=1)

$\pi\pi \to \pi\pi\pi$ (I=1)

$\pi\pi \to \pi\pi\rho$ (I=0,1,2)

etc.

Feger et al.: Comp.Phys.Com 257 (2020)

Phenomenology of scattering channels

- **14**-dim:
  - Makes up most $\pi\pi$ scattering (14/25)
  - Easiest on the lattice
- **10**-dim:
  - Mixing with dark ρ
  - $\pi\pi\pi \to \pi\pi$
- **1**-dim:
  - Mixing with other states

$$Sp(4)_f$$

$$5 \otimes 5 = 1 \oplus 10 \oplus 14$$

$$10 \otimes 5 = 5 \oplus 10 \oplus 35$$

$$5 \otimes 5 \otimes 5 = 3(5) \oplus 10 \oplus 30 \oplus 35$$

$\pi\pi \to \pi\pi$ (I=0,1,2)

$\pi\pi \to \rho$ (I=1)

$\pi\pi \to \pi\pi\pi$ (I=1)

$\pi\pi \to \pi\pi\rho$ (I=0,1,2)

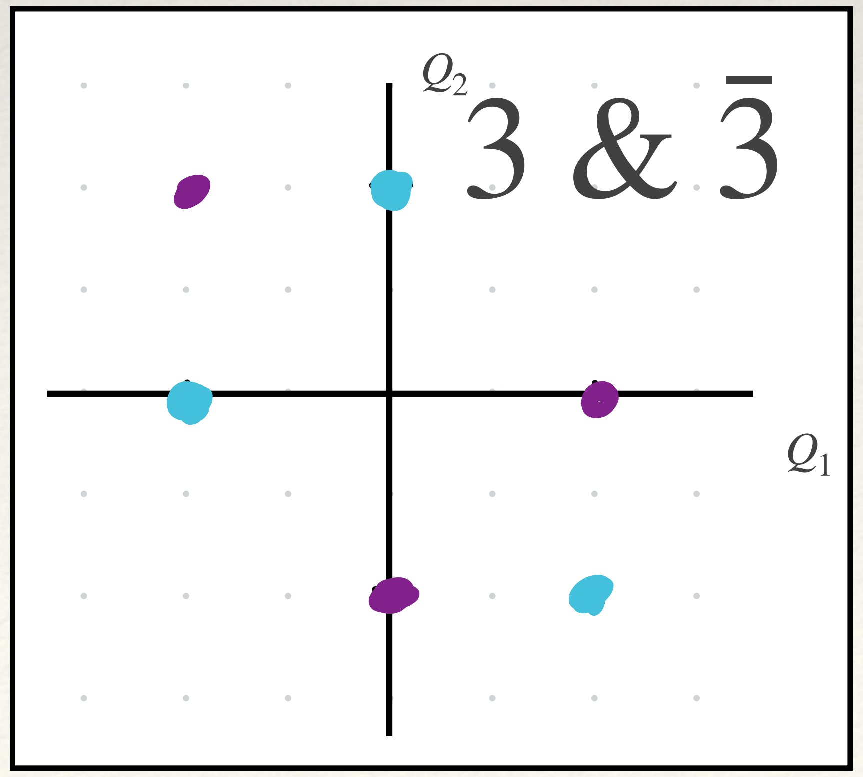
etc.

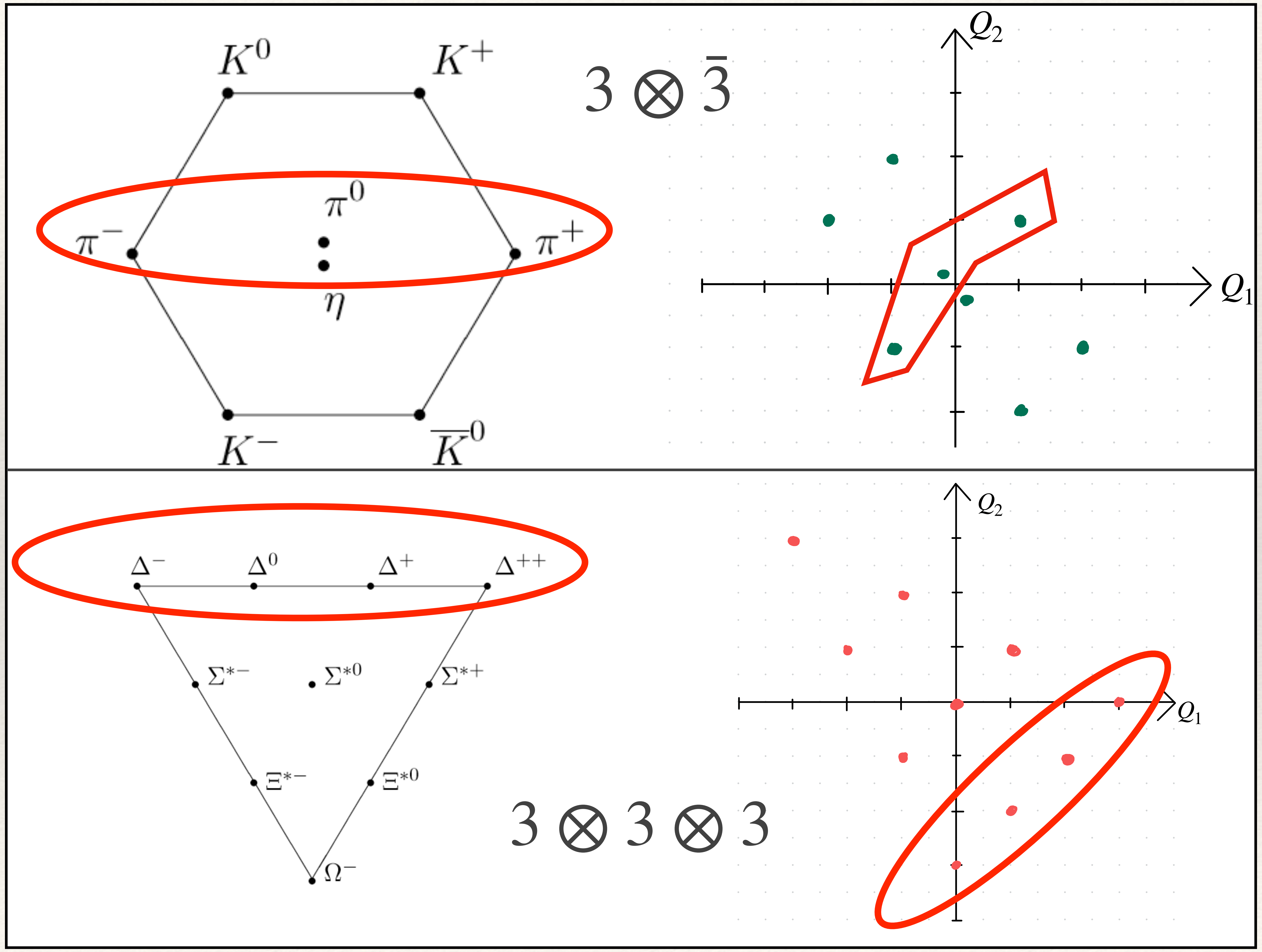
Feger et al.: Comp.Phys.Com 257 (2020)

Flavour quantum numbers

- Composite states live in irreps of the flavour symmetry
- Can be represented in diagrams given by the weight system
  - "Meson-octet" and "Baryon-Decuplet" in $SU(3)_F$ (mass-degenerate)
  - Mass-degenerate → perfect symmetry

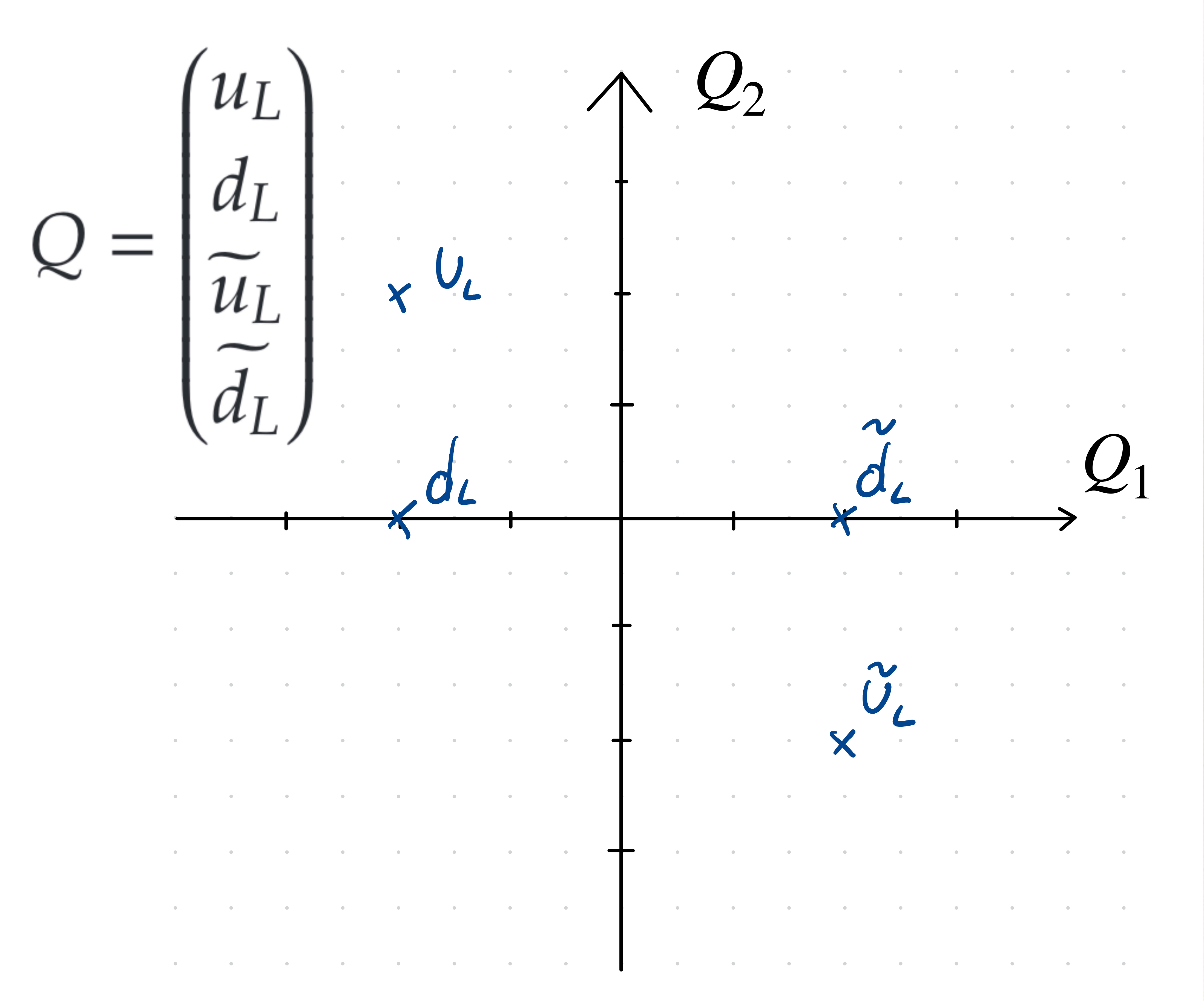
Weight system of the fundamental of SU(3):





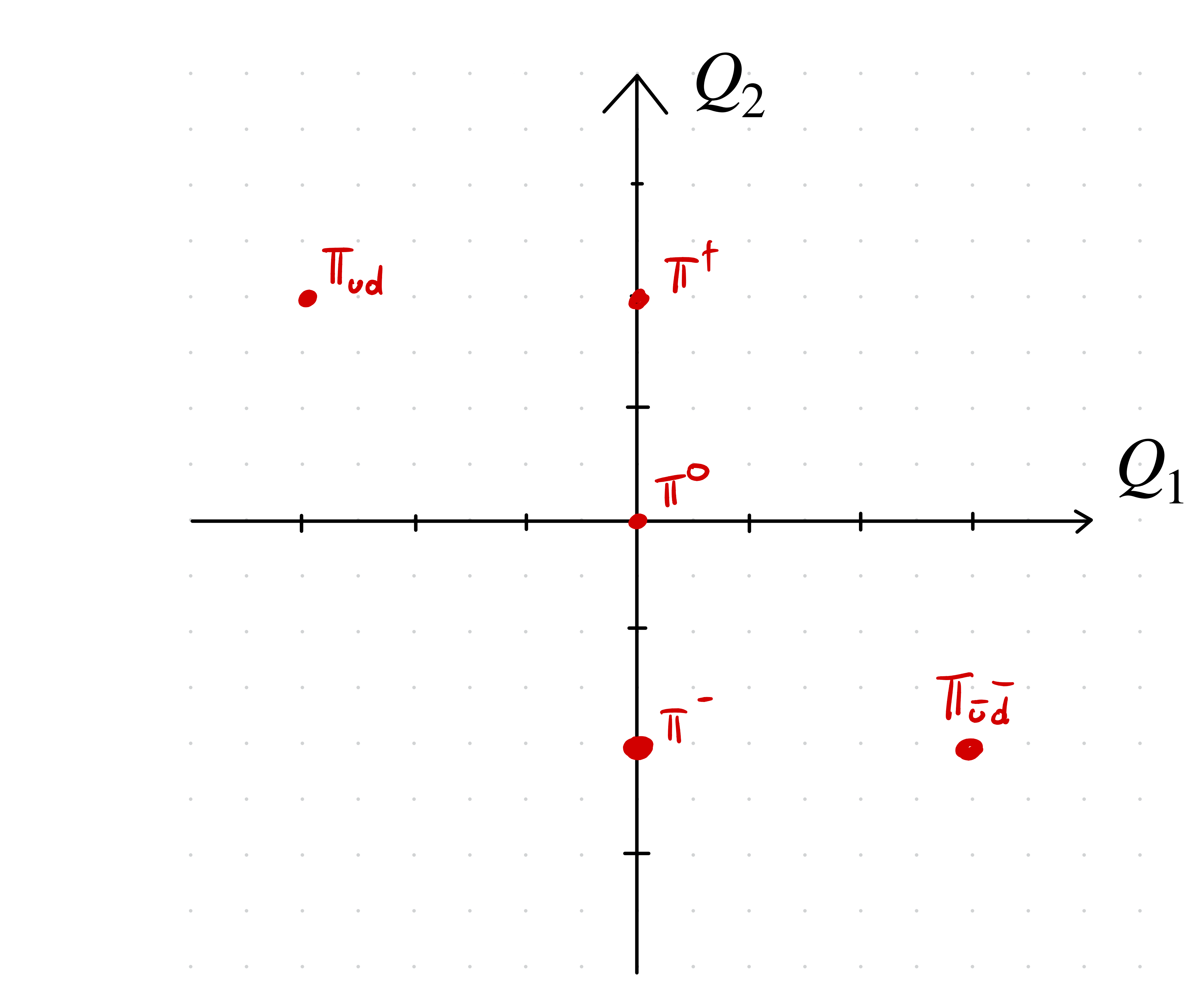
Flavour quantum numbers in Sp(4)

- Similar in Sp(4) for visualising scattering states
- Quarks in fundamental of Sp(4) (**4**-plet)
- $4 \otimes 4 = 1 \oplus 5 \oplus 10$
  - Pions in **5**
  - Rhos in **10**



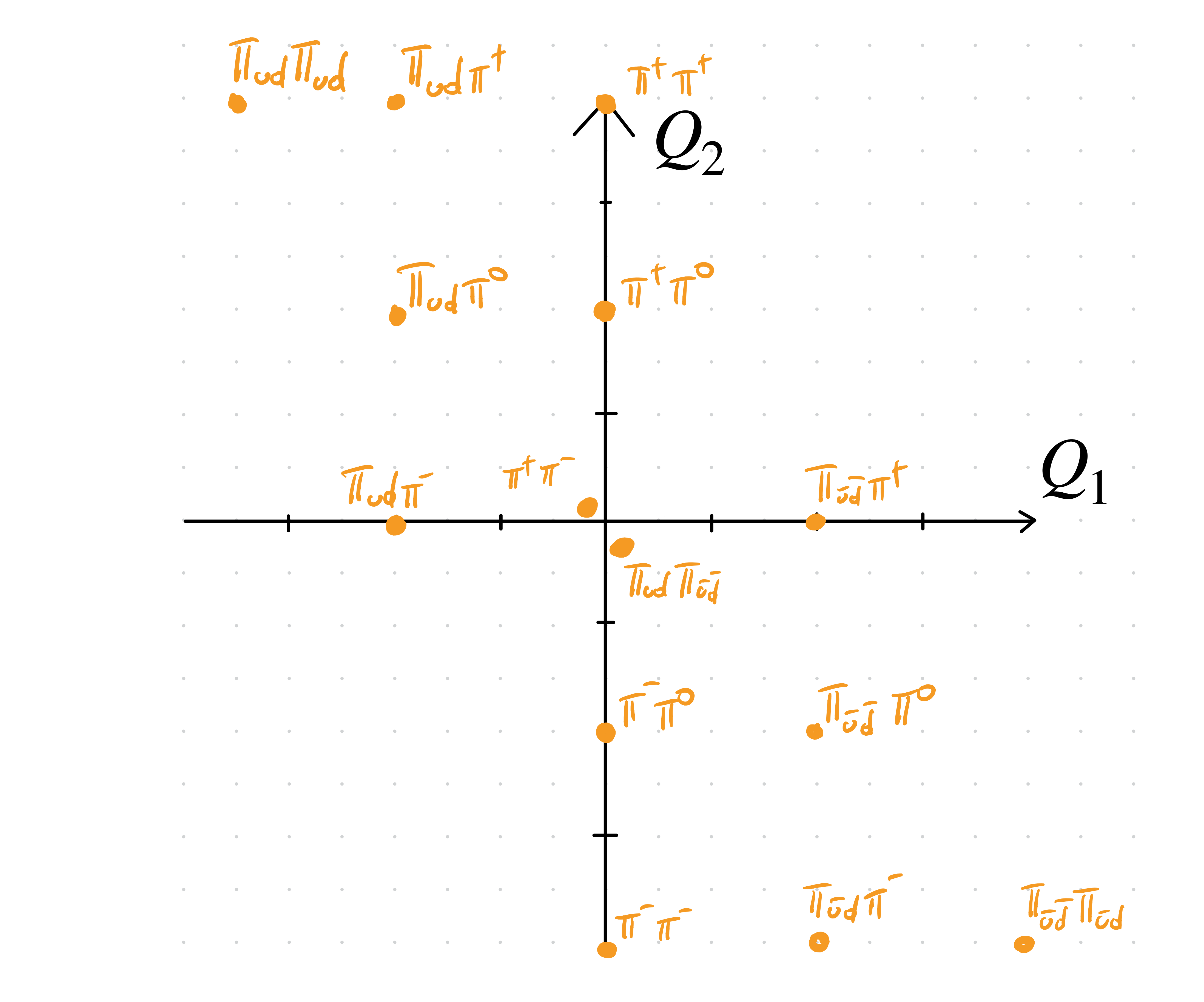
Pions form a **5**-plet

- Isomorphism: SO(5) = Sp(4)
- Quark content can be read off
  - $\pi^+ = u\gamma_5\bar{d}$
- Scattering states:
  - $5 \otimes 5 = 1 \oplus 10 \oplus 14$



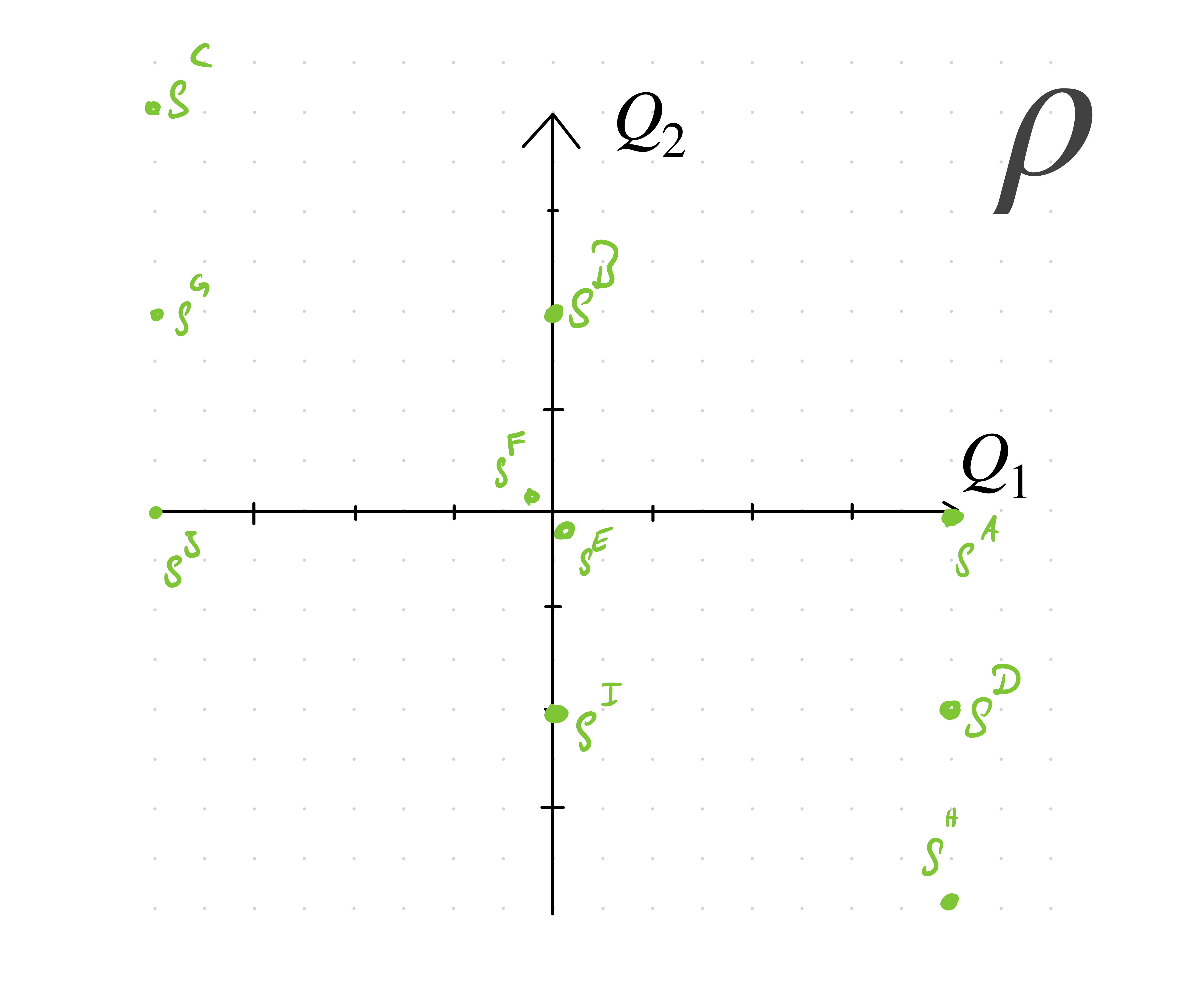
The **14**-plet

- Reminder: $5 \otimes 5 = 1 \oplus 10 \oplus 14$
- $\pi^+\pi^+$ is unique to the **14**
- $\mathcal{O}^{14}_{\pi\pi} = \pi^+\pi^+ = \pi^-\pi^- = \Pi_{ud}\Pi_{ud} = \Pi_{\bar{u}\bar{d}}\Pi_{\bar{u}\bar{d}}$



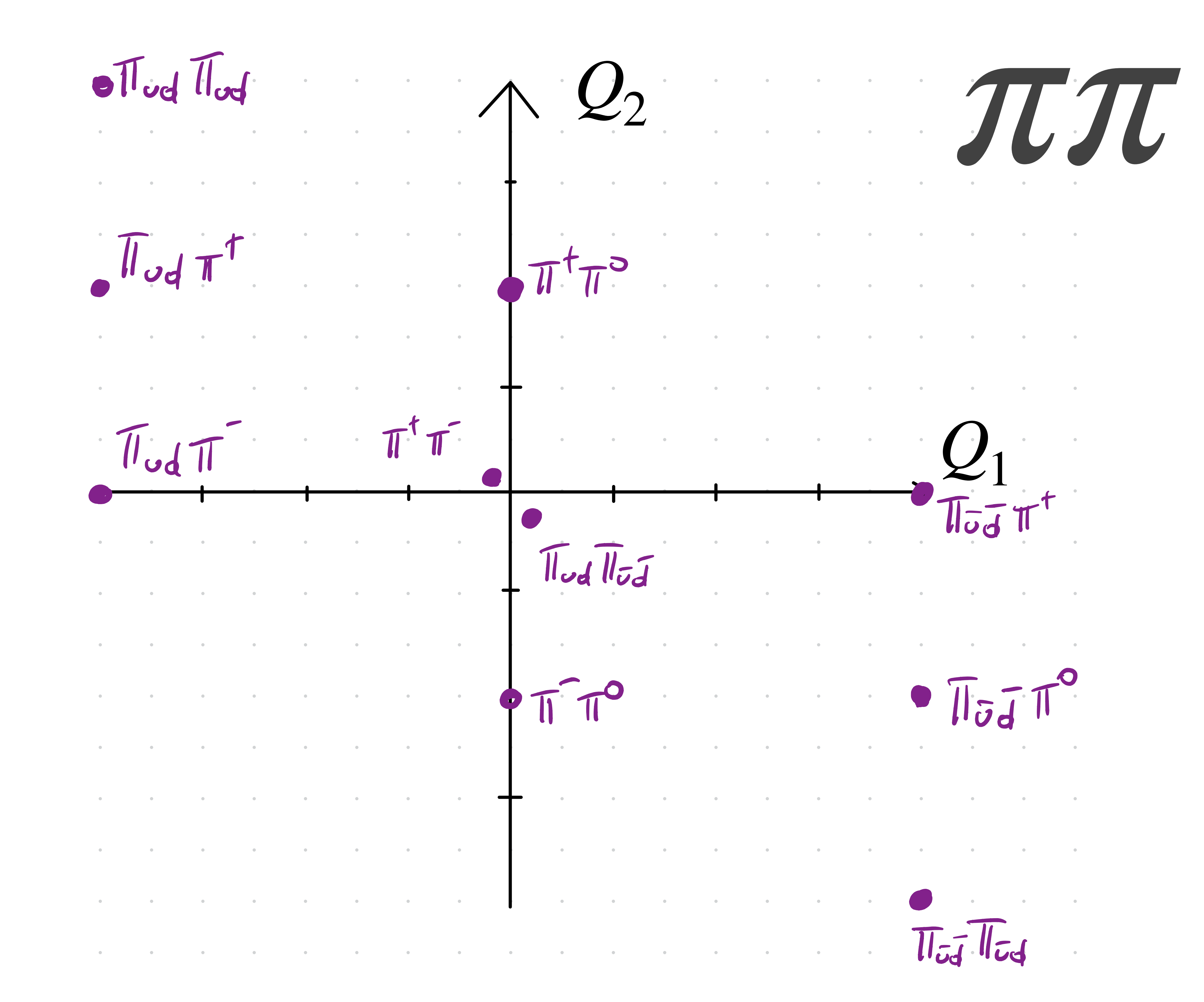
The **10**-plet

- Contains ρ, $\pi\pi$, $\pi\pi\pi$
- Trick from before does not work
- Use young-diagrams



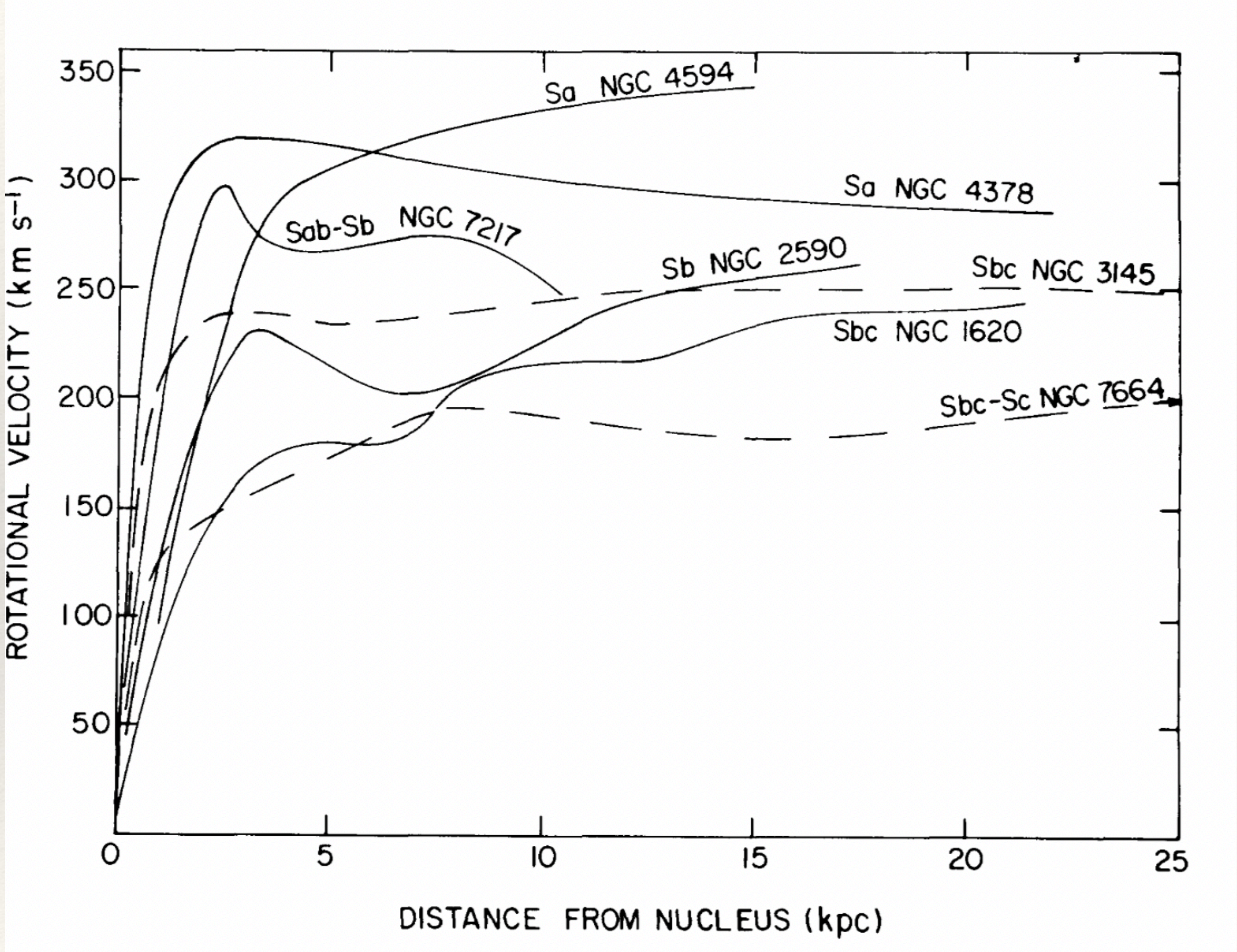
The **10**-plet

- Contains ρ, $\pi\pi$, $\pi\pi\pi$
- Trick from before does not work
- Use young-diagrams



Dark Matter

- Collection of phenomena with no explanation in the standard model (SM)
  - Rotation curves, structure formation, etc.
- Possible explanations:
  - Modified gravity
  - Non observable form of matter
  - Particle beyond the SM



Rubin et al.: Ap.J.L. 225 (1978)