LATTICE 2024 LIVERPOOL

Towards glueball scattering in lattice Yang-Mills theory

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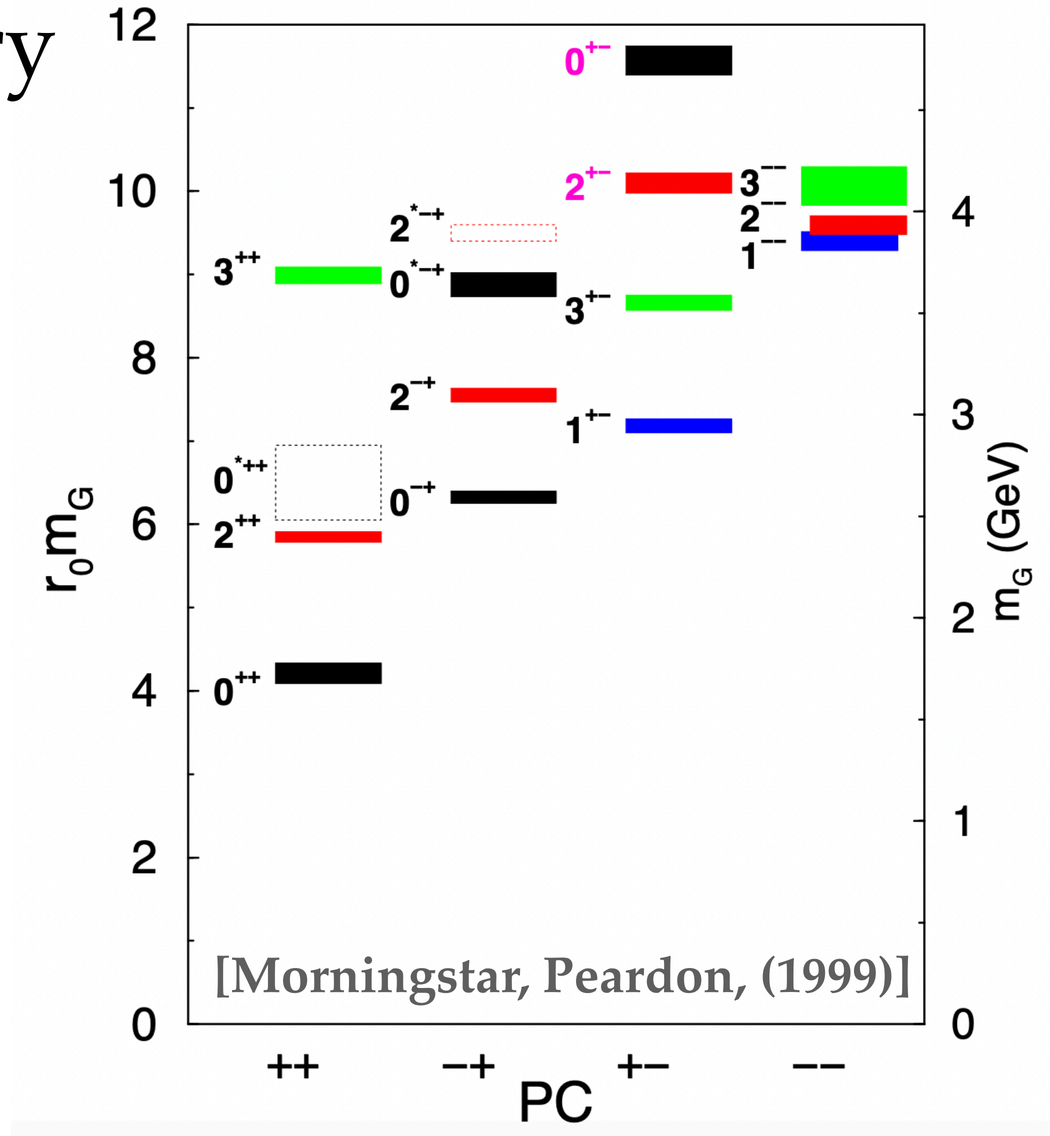
Scattering glue balls in Yang-Mills theory

- In Yang-Mills theory, low-lying spectrum consists of stable glue balls

$$\mathcal{L}_{\mathsf{YM}} = -\frac{1}{4} F^{a\mu\nu} F^{a}_{\mu\nu}$$

- Thinking in analogy to pions in QCD:
Can we use standard finite-volume methods to calculate glue-ball scattering amplitudes?



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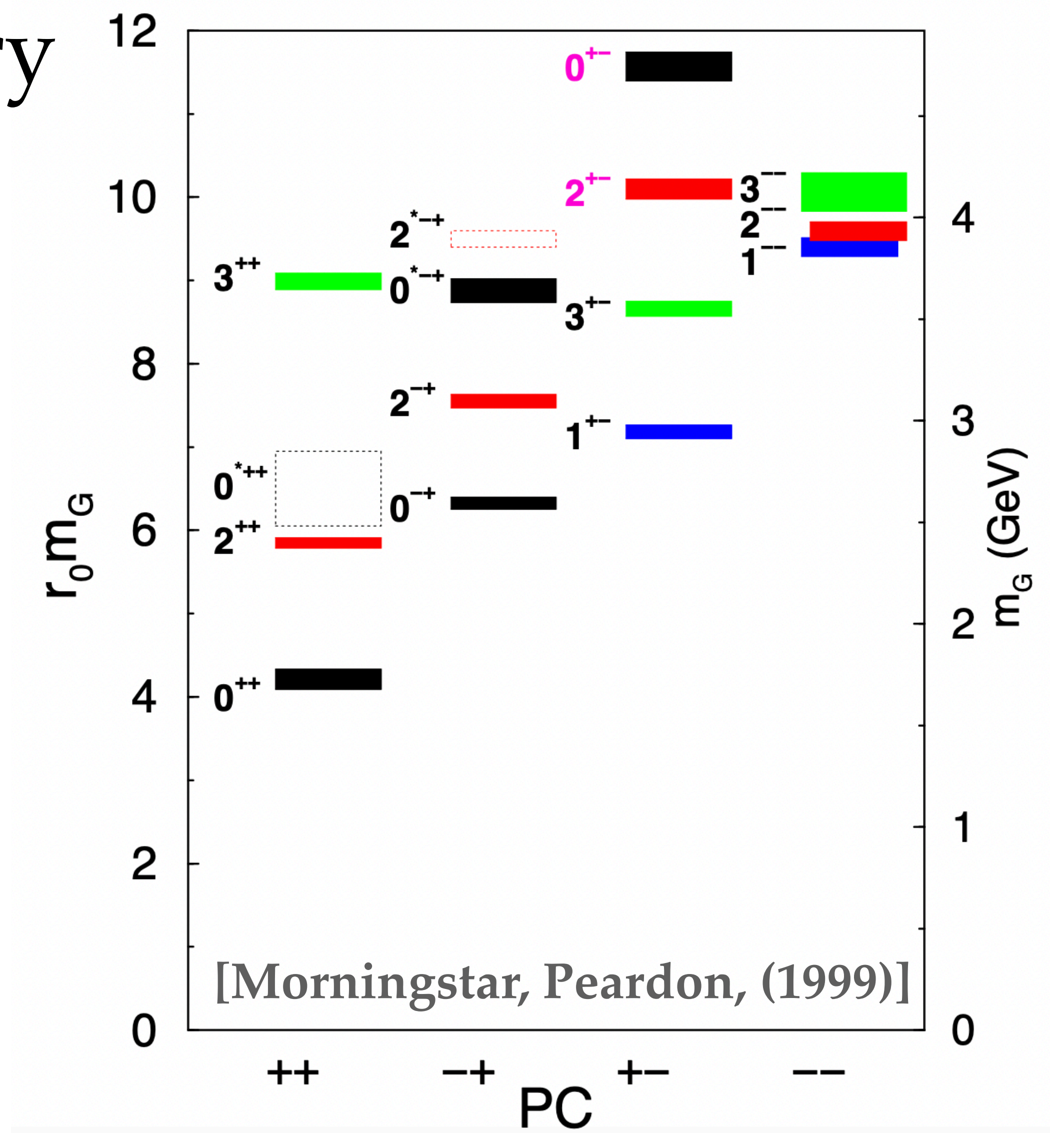

- Thinking in analogy to pions in QCD:
  Can we use standard finite-volume methods to calculate glue-ball scattering amplitudes?

- Motivations

  constrain strongly-interacting dark-matter models

  test S-matrix-bootstrap predictions

  better understand QCD glue balls in a *theoretically clean* environment

  experimental evidence that X(2370) has quantum numbers 0^{-+}
  (from $J/\psi \to \gamma\, K^0_S\, K^0_S\, \eta'$) [BES III (2024)]

Challenges for scattering glue balls

- Glue-ball correlators suffer from a signal-to-noise problem
- Require a large and varied set of operators to reliably extract excited states
- Window problem: finite-volume must be…

large enough to ignore unwanted effects small enough to resolve effects of interest

related to fact that YM = ***a single-scale theory***

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  large enough to ignore unwanted effects — small enough to resolve effects of interest

  related to fact that YM = ***a single-scale theory***

- Missing the analogous quark model → less obvious operator construction

- Finite-volume spectrum is more complicated then, say pions

  one-, two-, three-particle energies have the same quantum numbers

  di-torelon states could contaminate spectrum (more later)

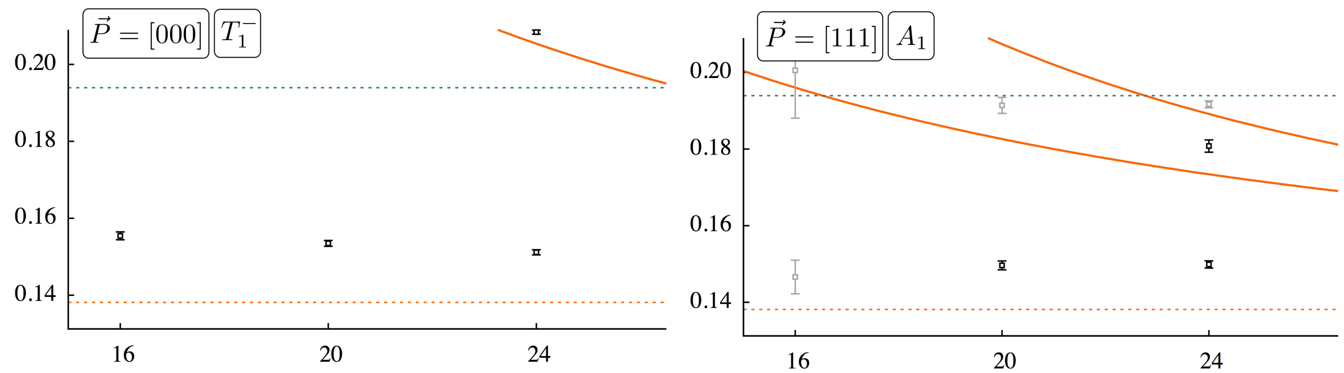
Finite-volume scattering (1/2)

- Standard in lattice QCD to use finite-volume energies to extract scattering amplitudes

scattering　　　　　　geometric function

$$\rho \cot \delta(s_n) = -F(E_n, \vec{P}, L)$$

equal at finite-volume energies



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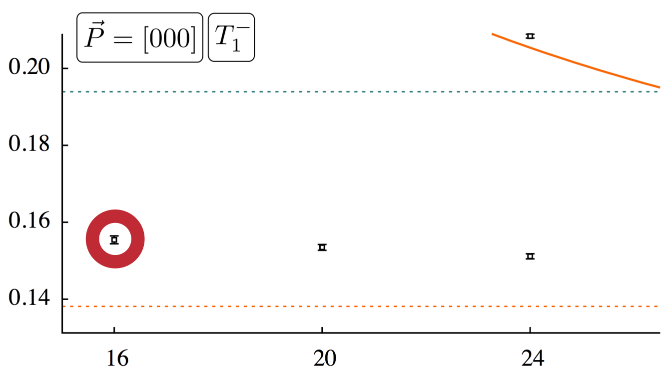

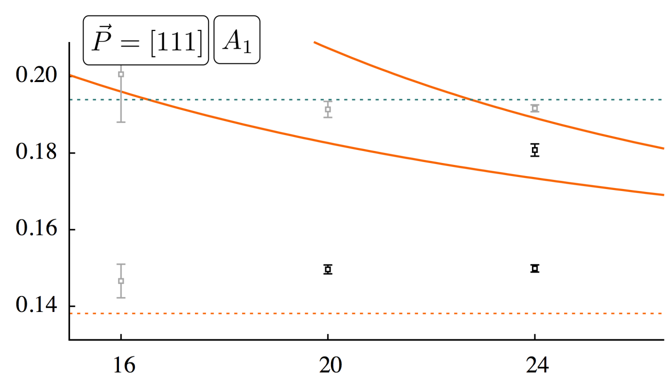

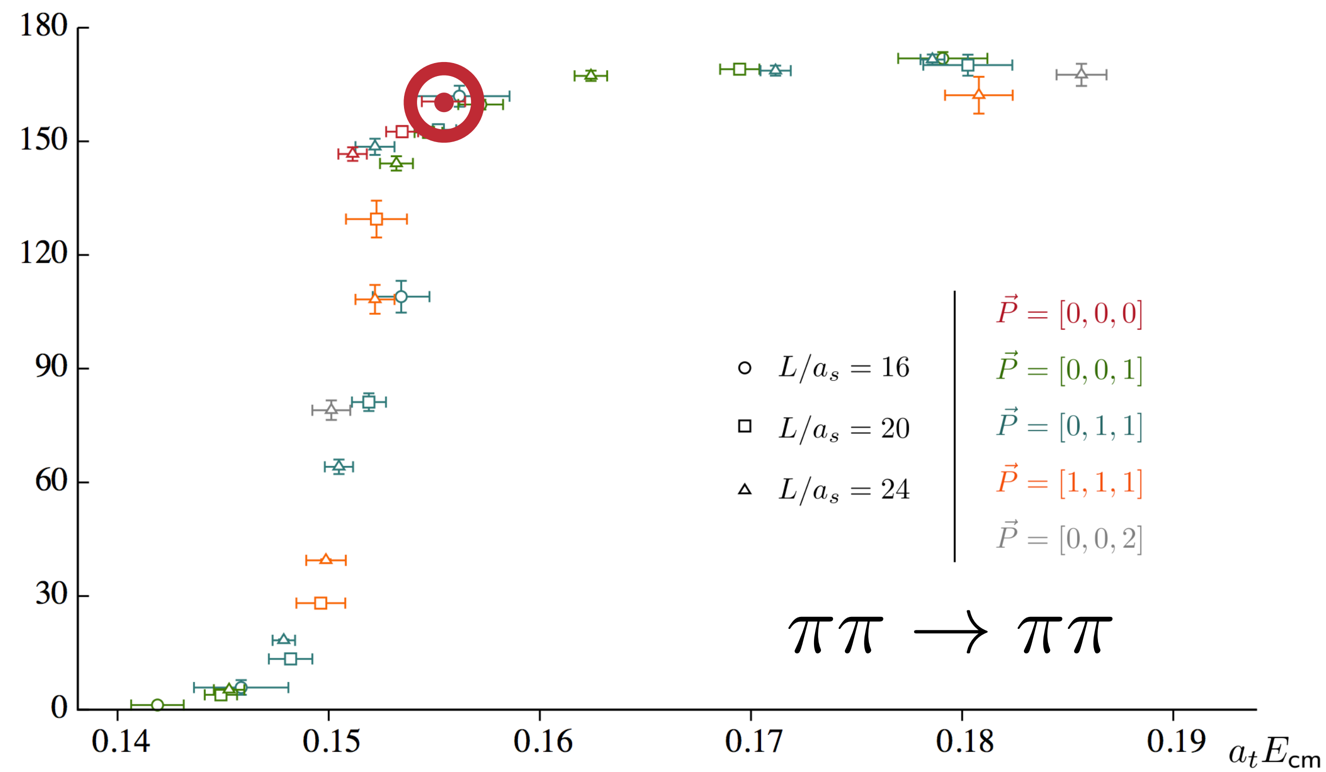

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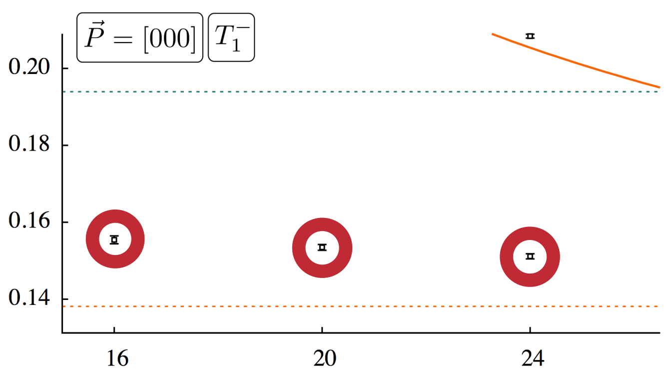

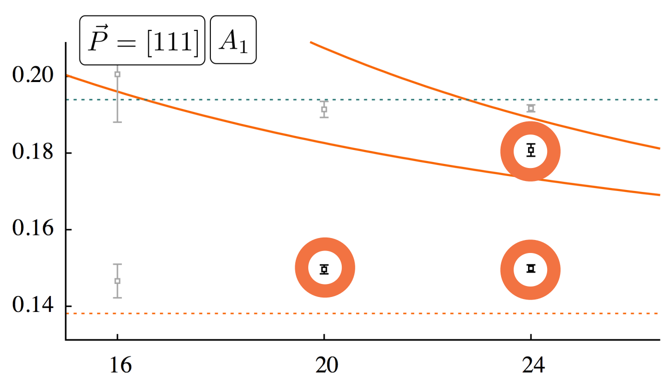

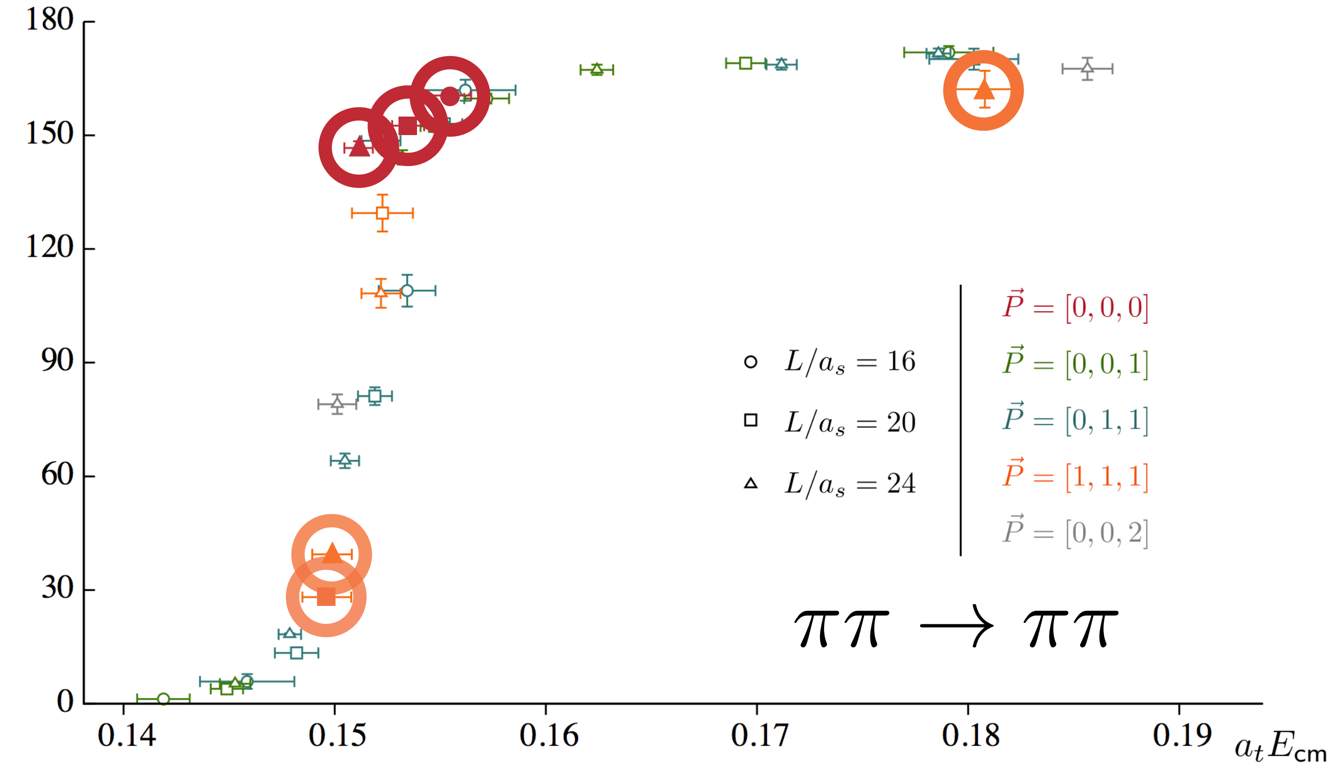

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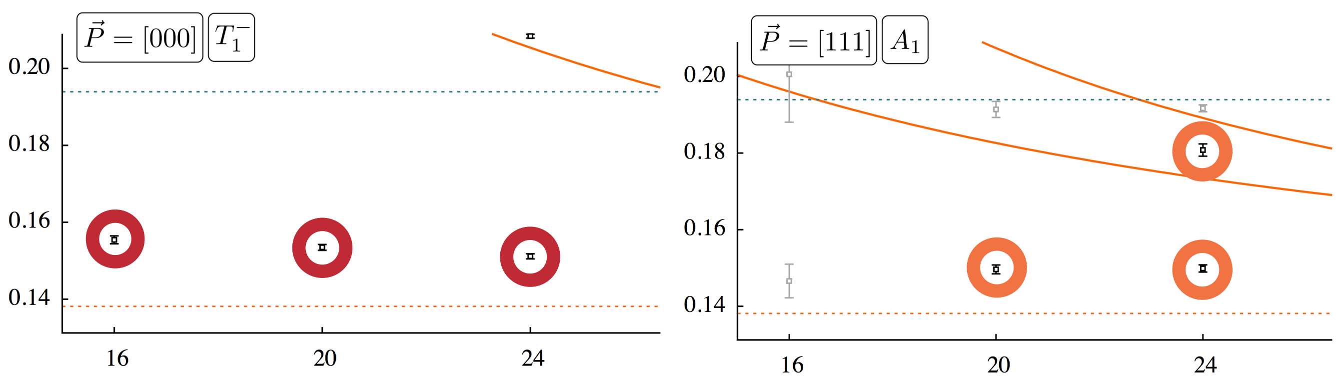

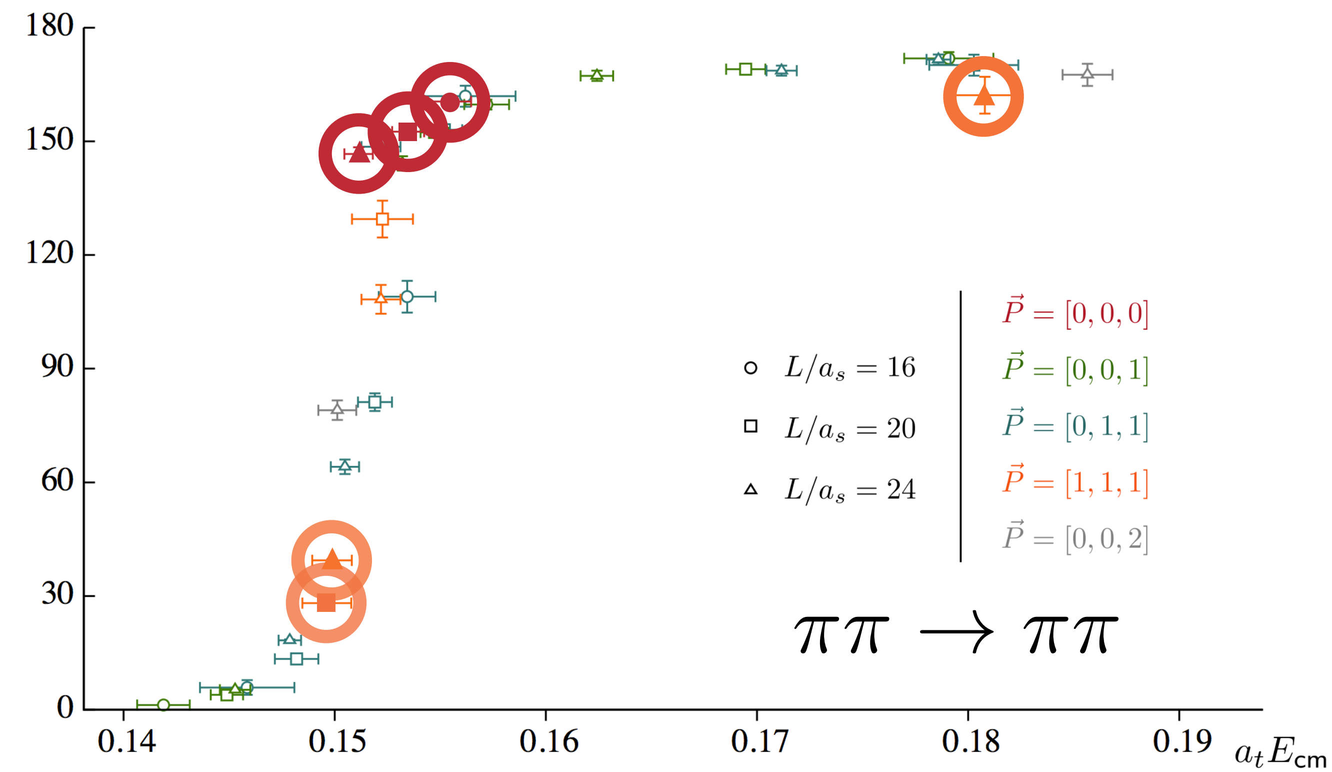

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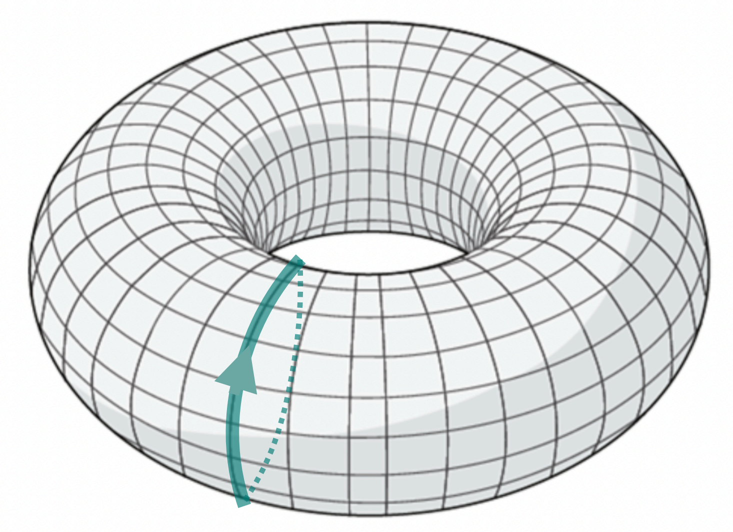
- Goal: Calculate finite-volume energies in Yang-Mills to extract

$$GG \to GG \quad \text{amplitudes} \qquad (G = 0^{++} \text{ ground state})$$

Expectations for the finite-volume energies

- ***torelon*** = closed flux tube, wrapping spatial direction

$$E_{\text{tor}}(L) = \sigma L + \mu + \mathcal{O}(1/L)$$



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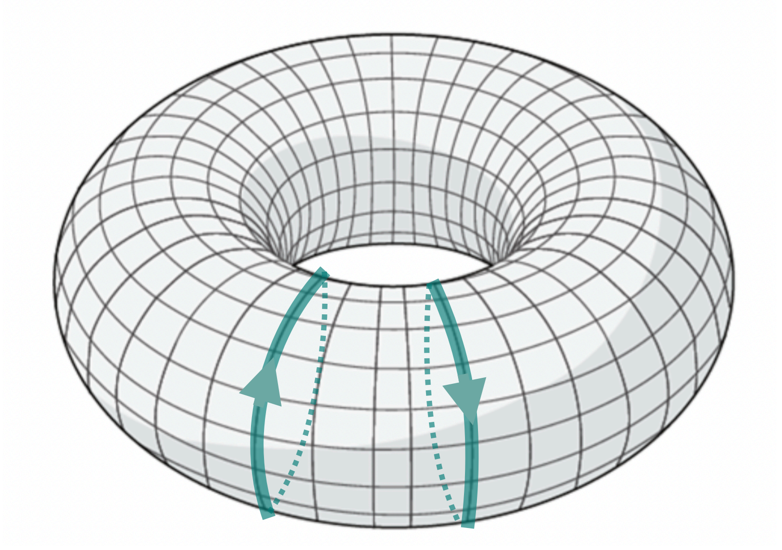
same quantum numbers as local 0^{++} states



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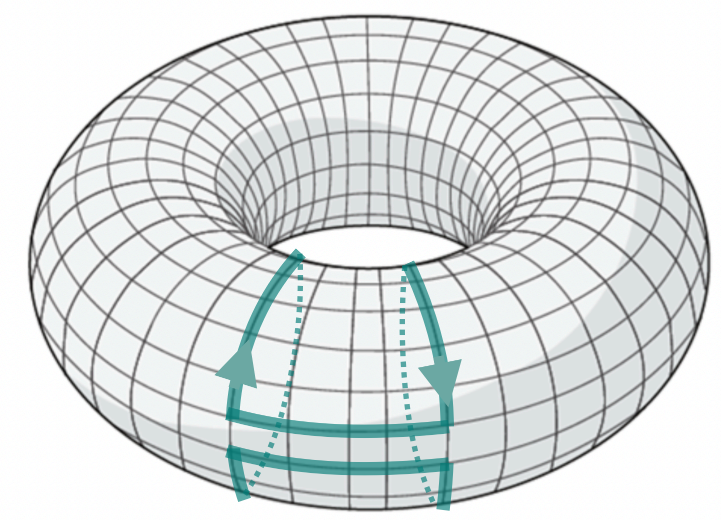

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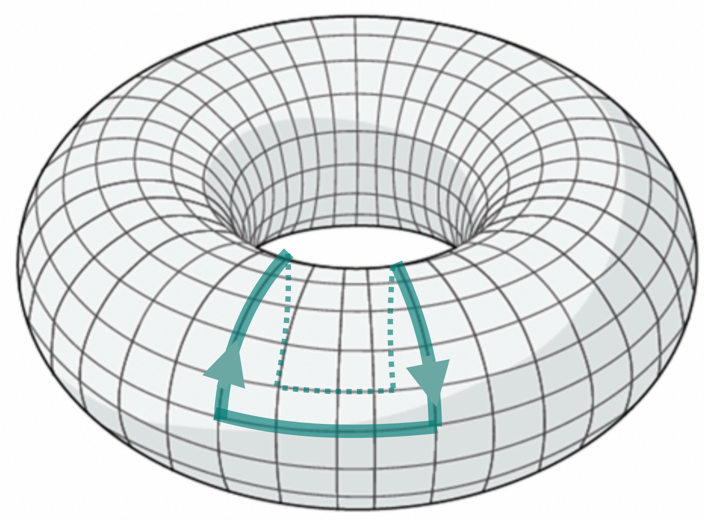

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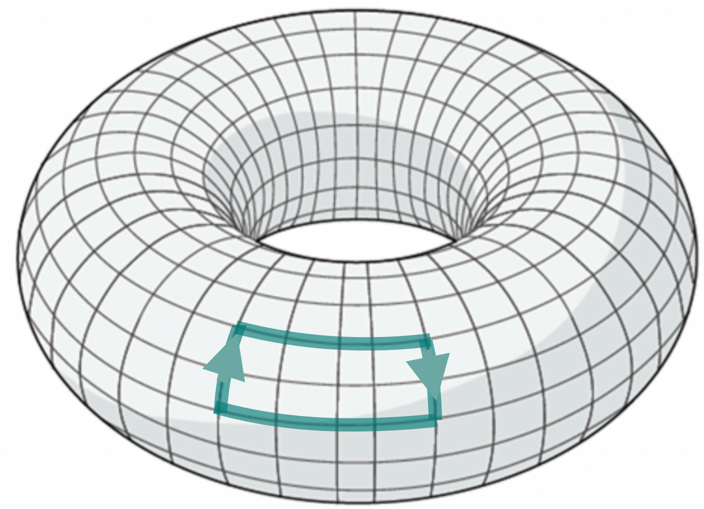

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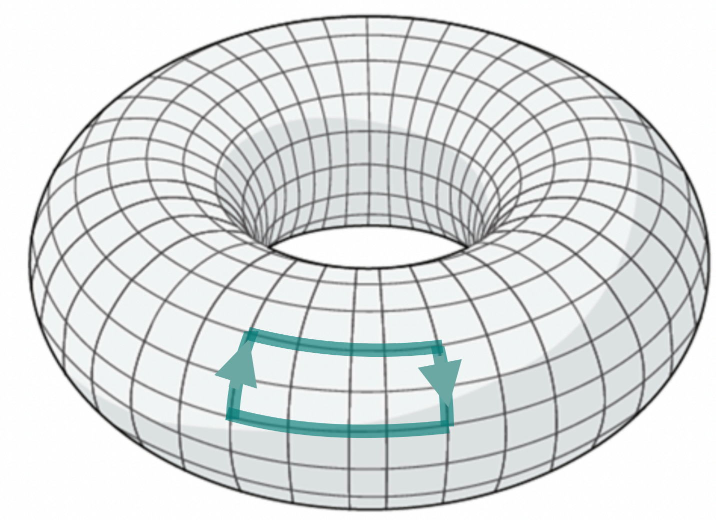
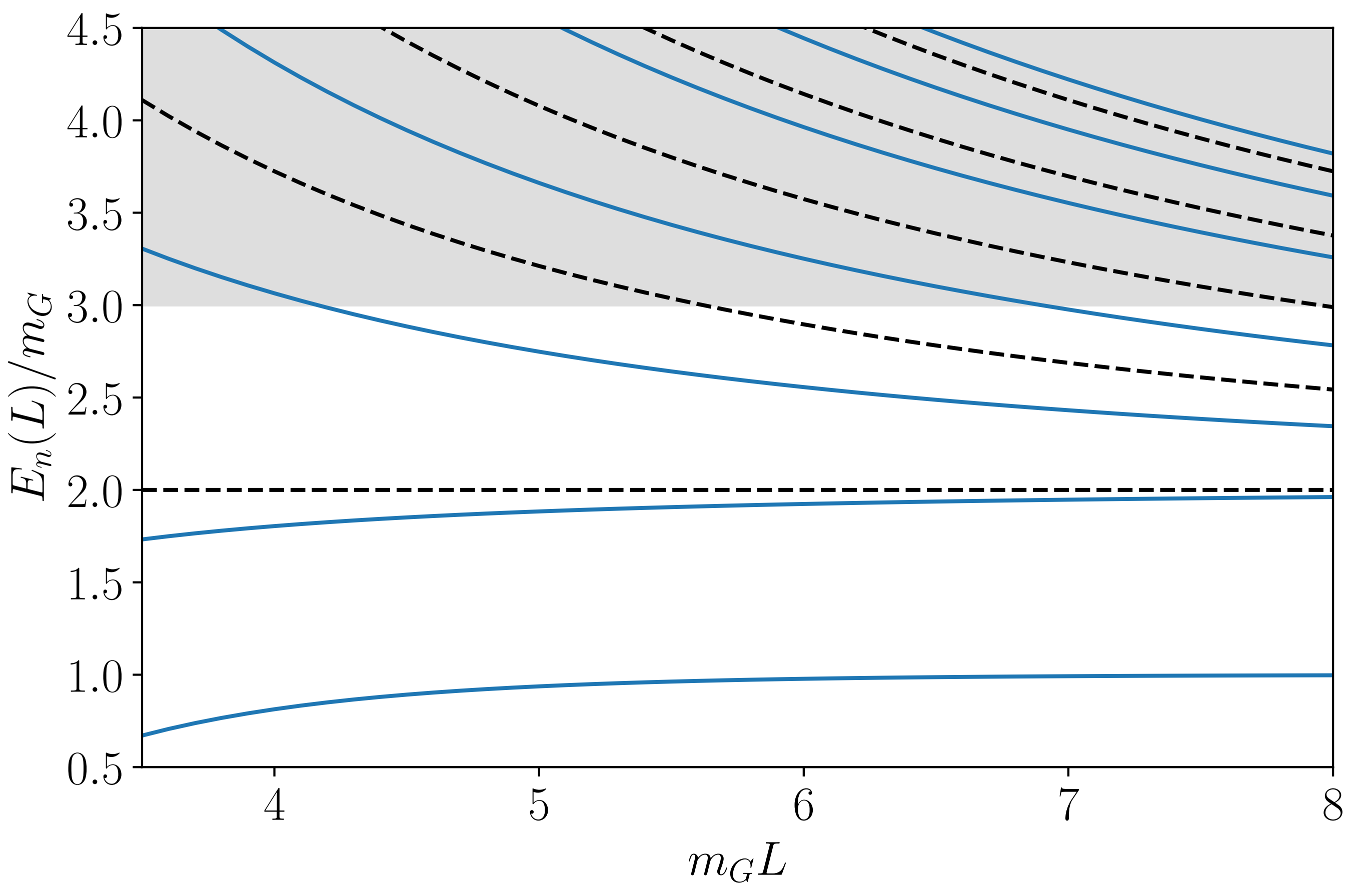
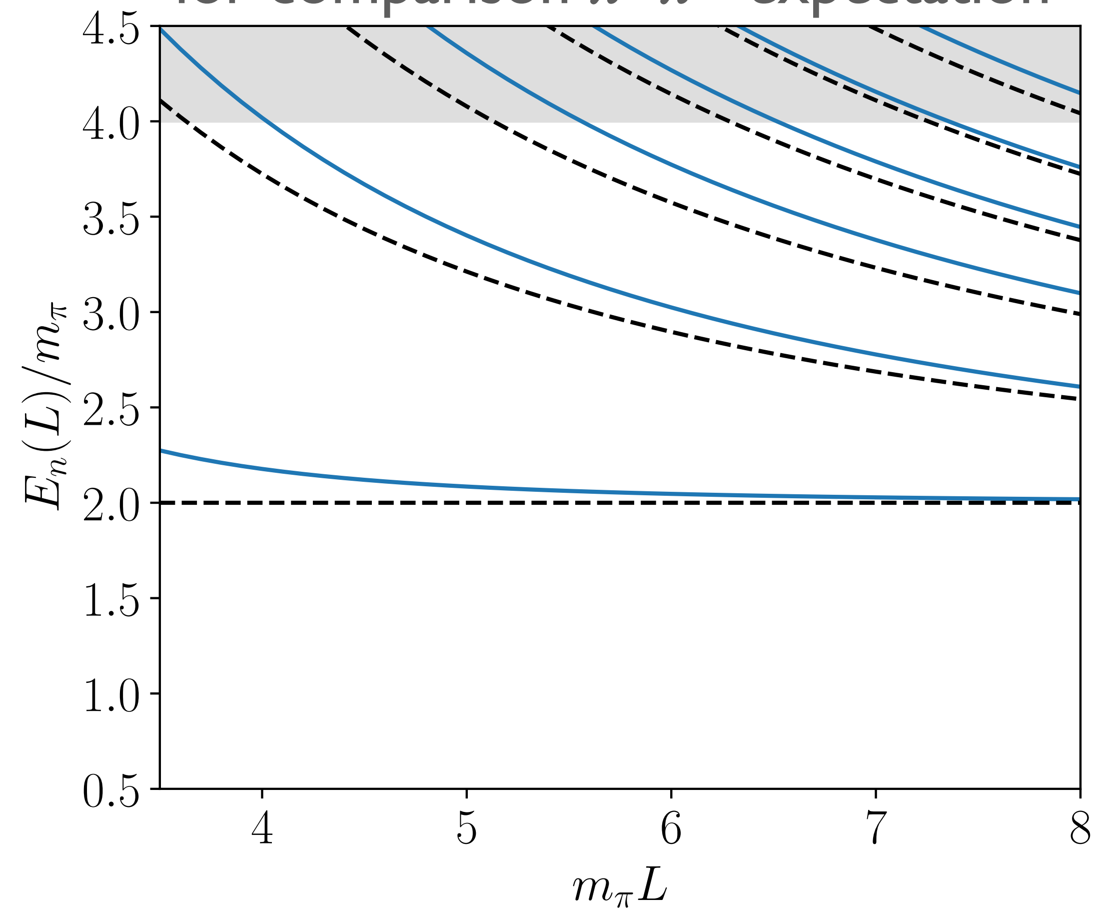


- What do we expect for the spectrum?…

0^{++} *qualitative* expectation



for comparison $\pi^+\pi^+$ expectation



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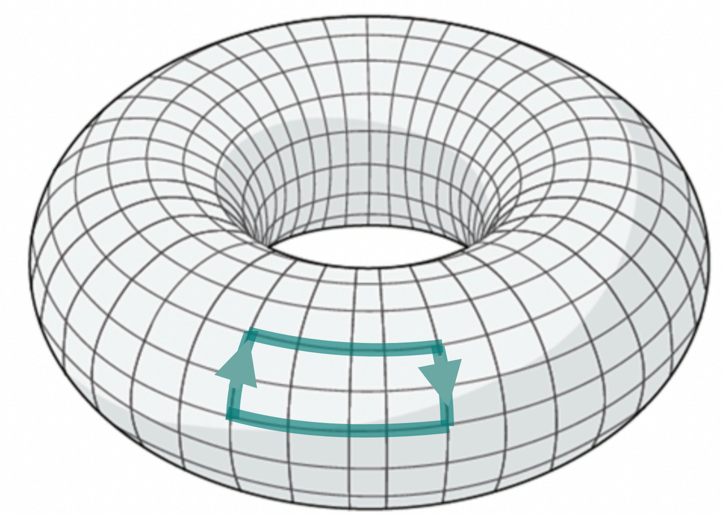

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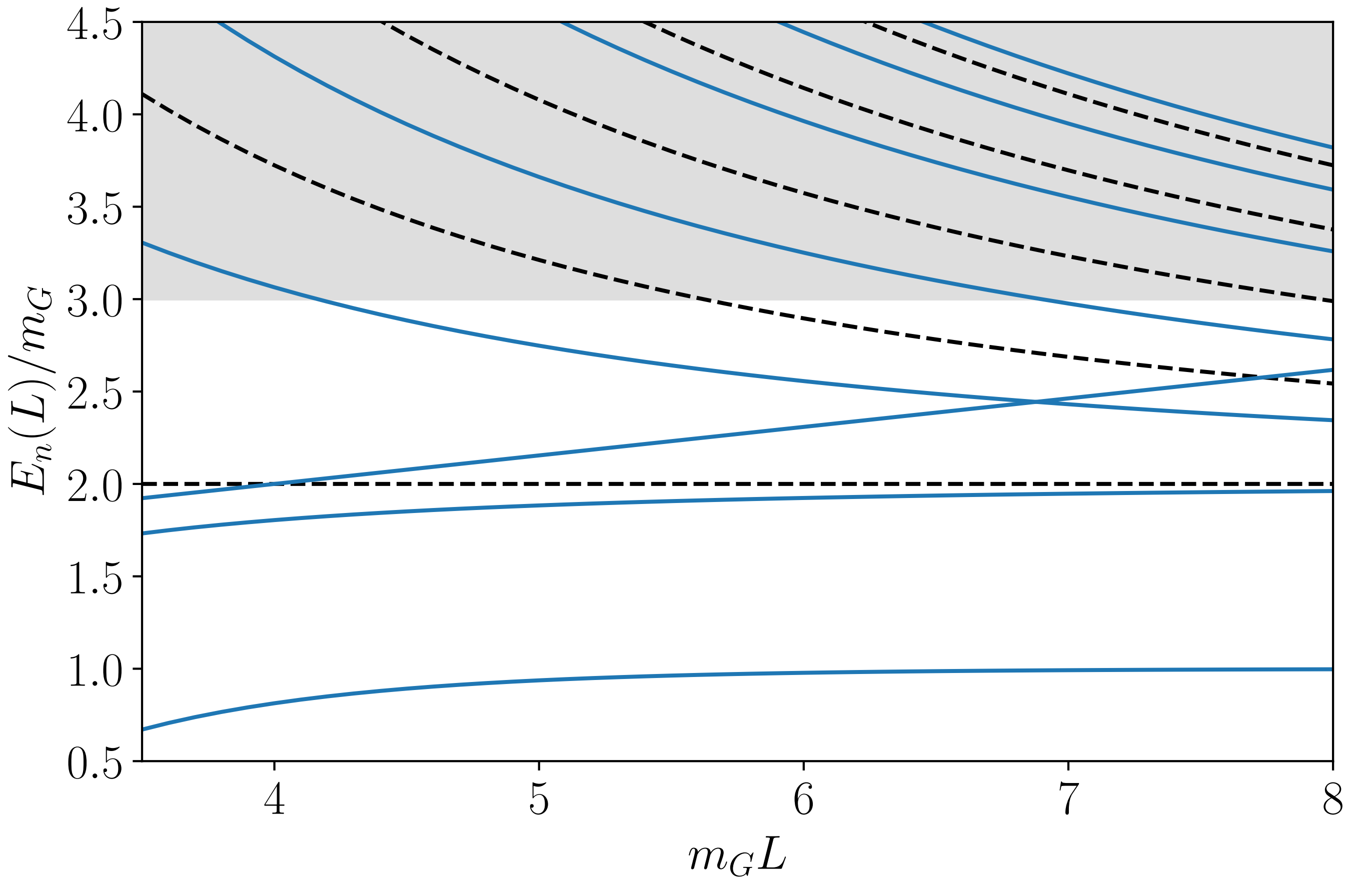

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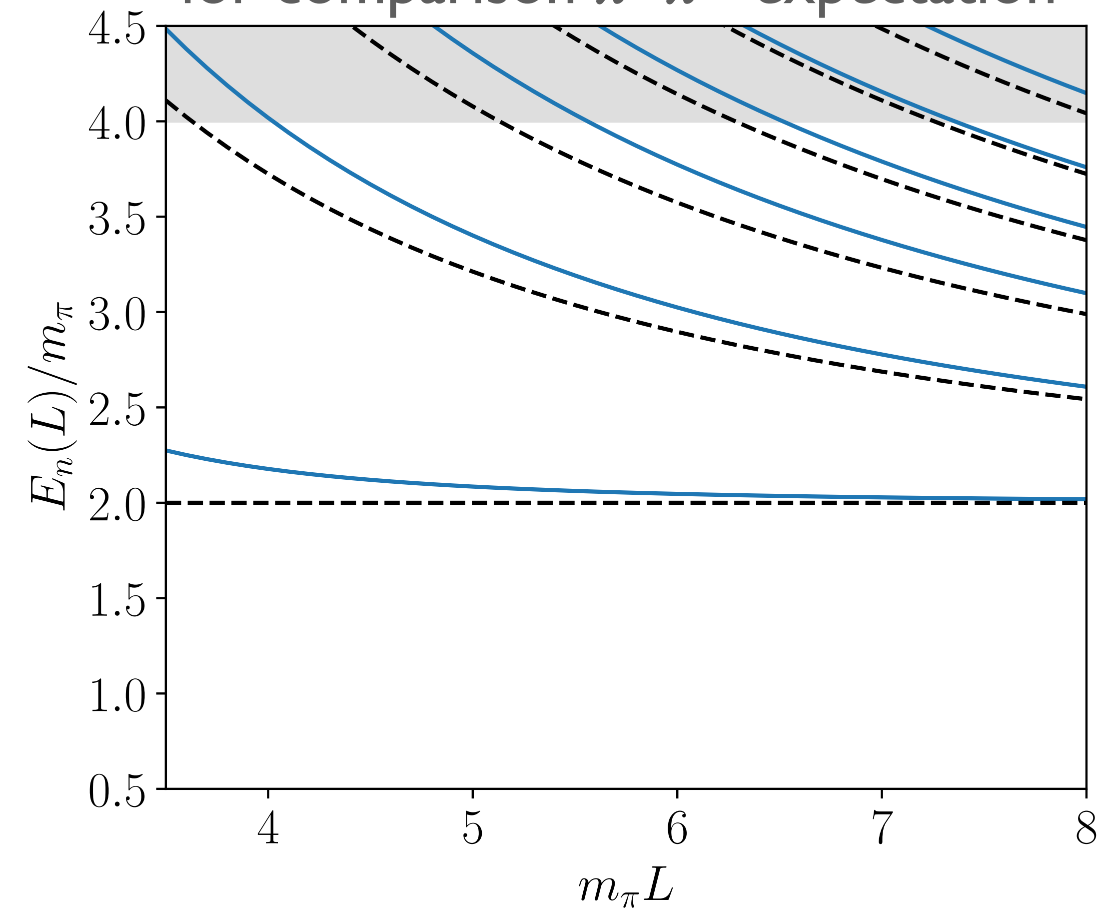

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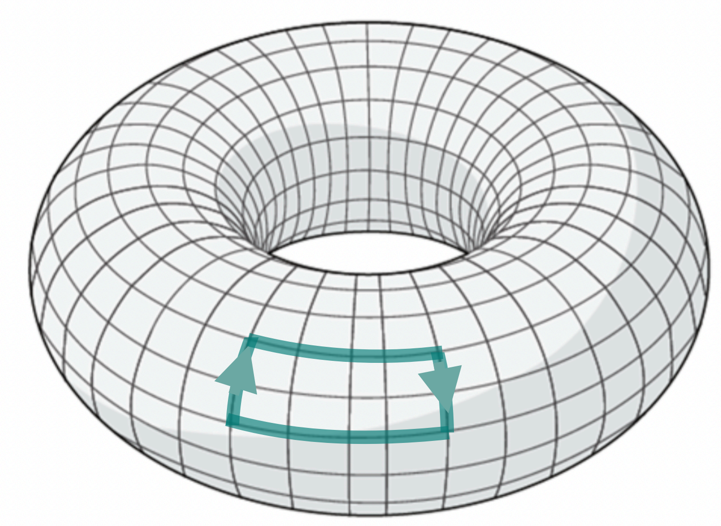

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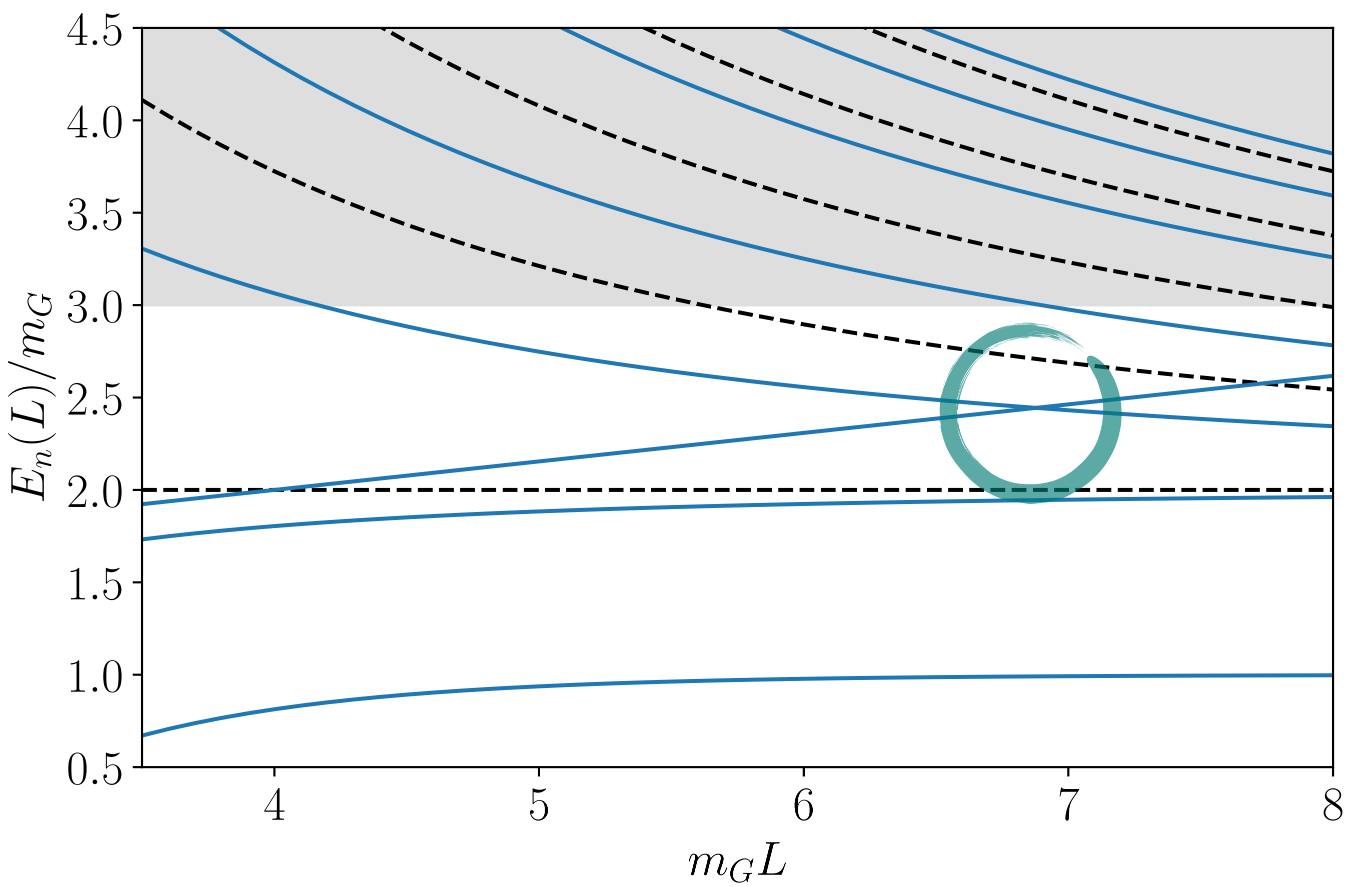

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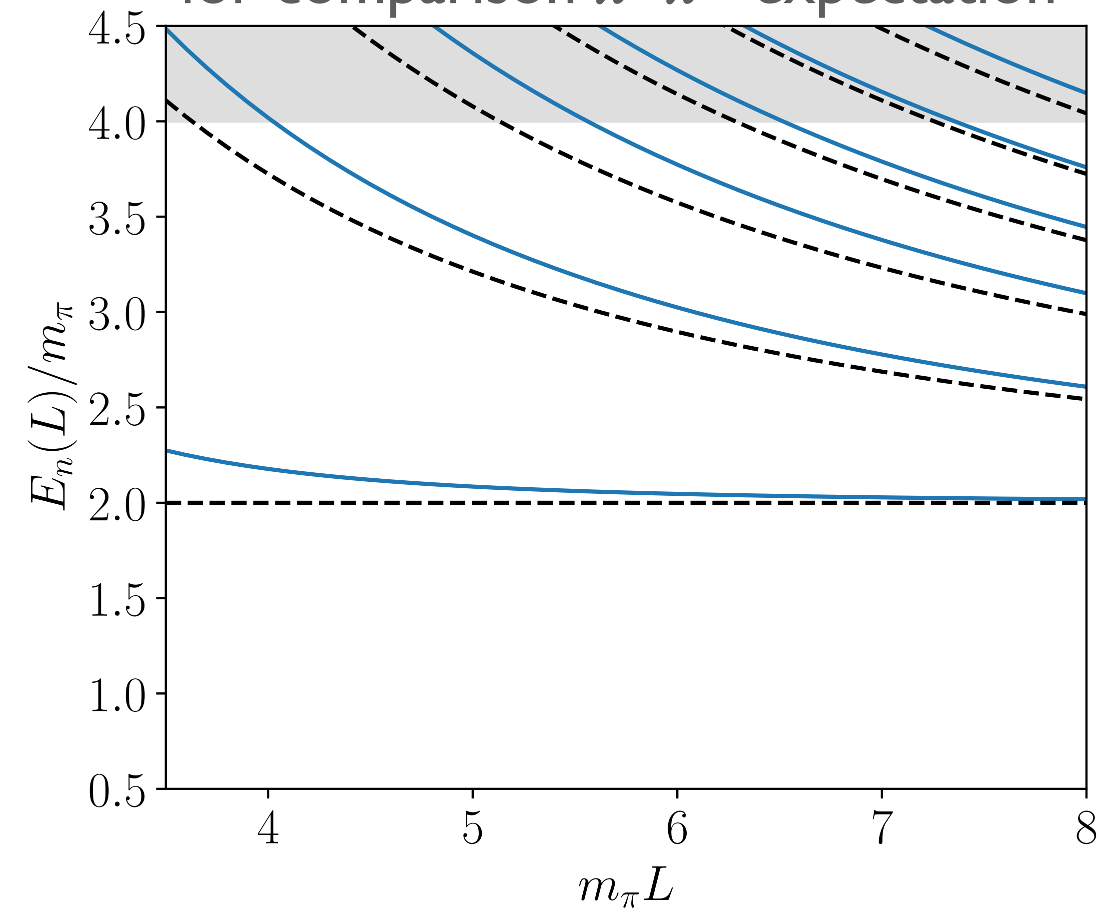

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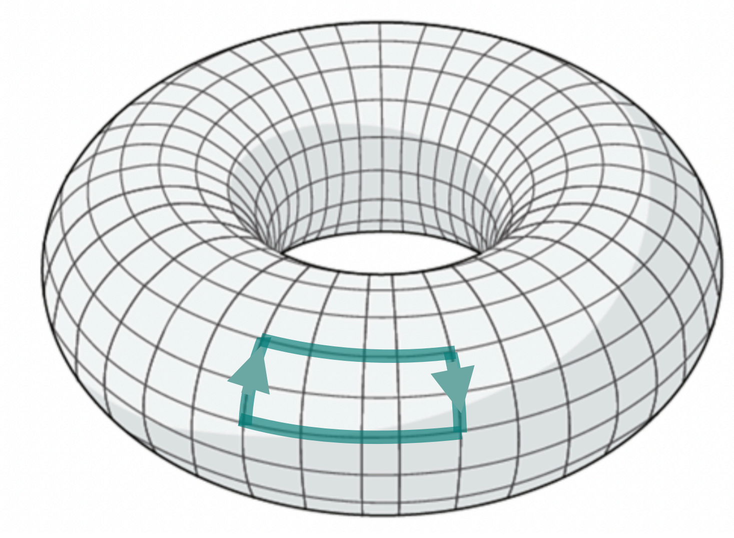

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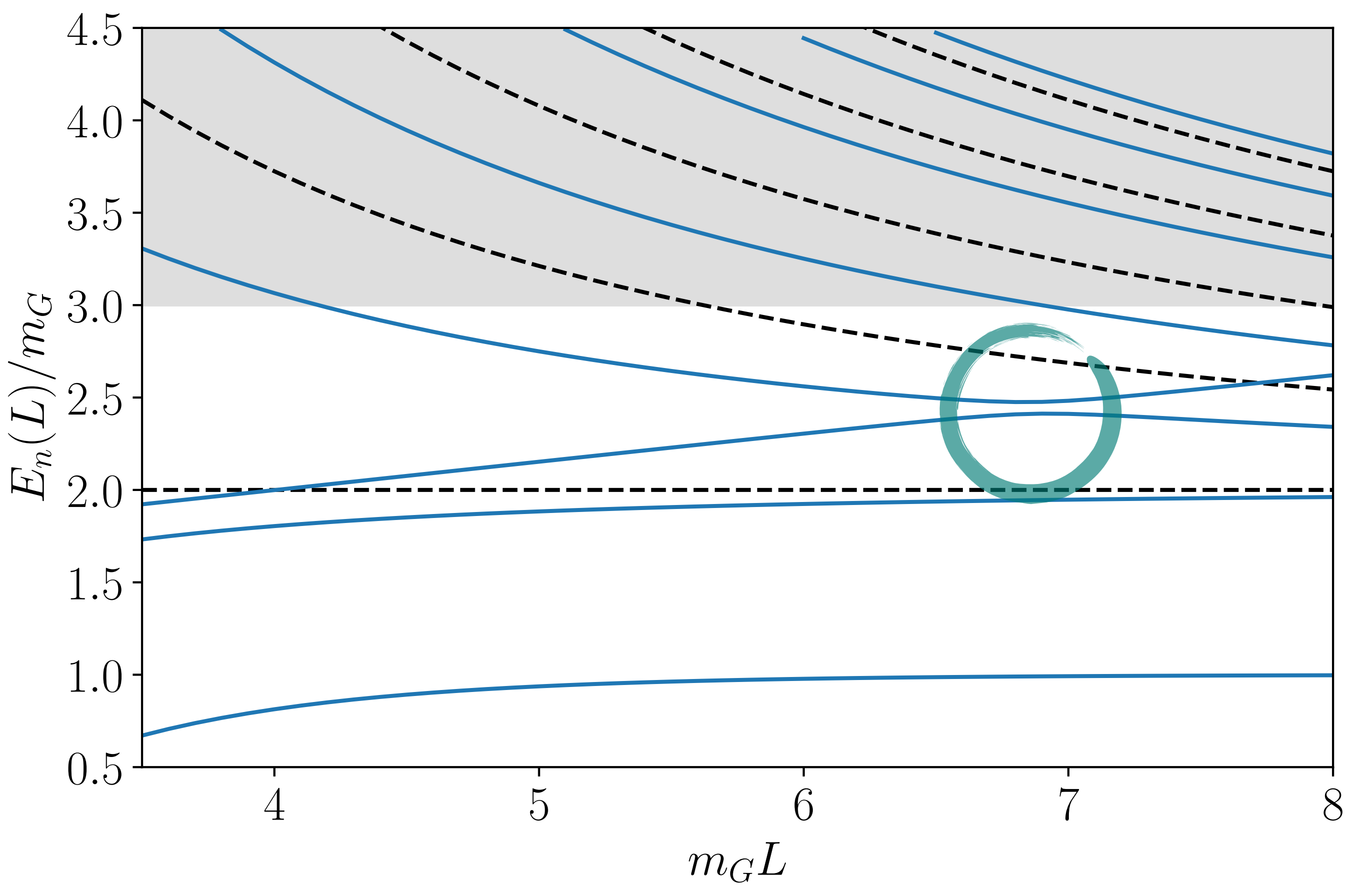

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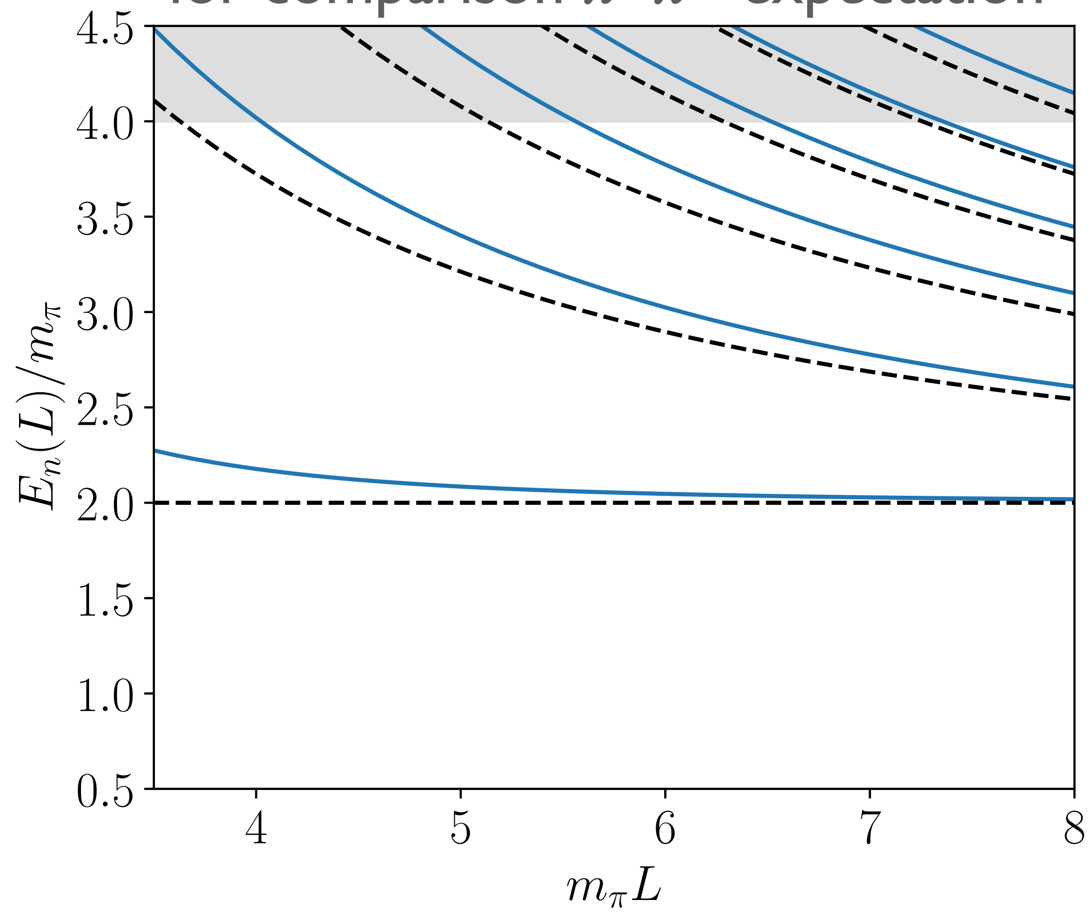

Set-up

- Anisotropic Wilson plaquette action ($a_\mathrm{t} \neq a_\mathrm{s}$)
- Renormalised anisotropy tuned via *spatial-spatial and spatial-temporal energy density*

$$\xi_R(\beta, \xi) = \frac{a_\mathrm{s}(\beta, \xi)}{a_\mathrm{t}(\beta, \xi)} = 4$$

[BMW]

- Multi-level algorithm *(described more on next slide)*

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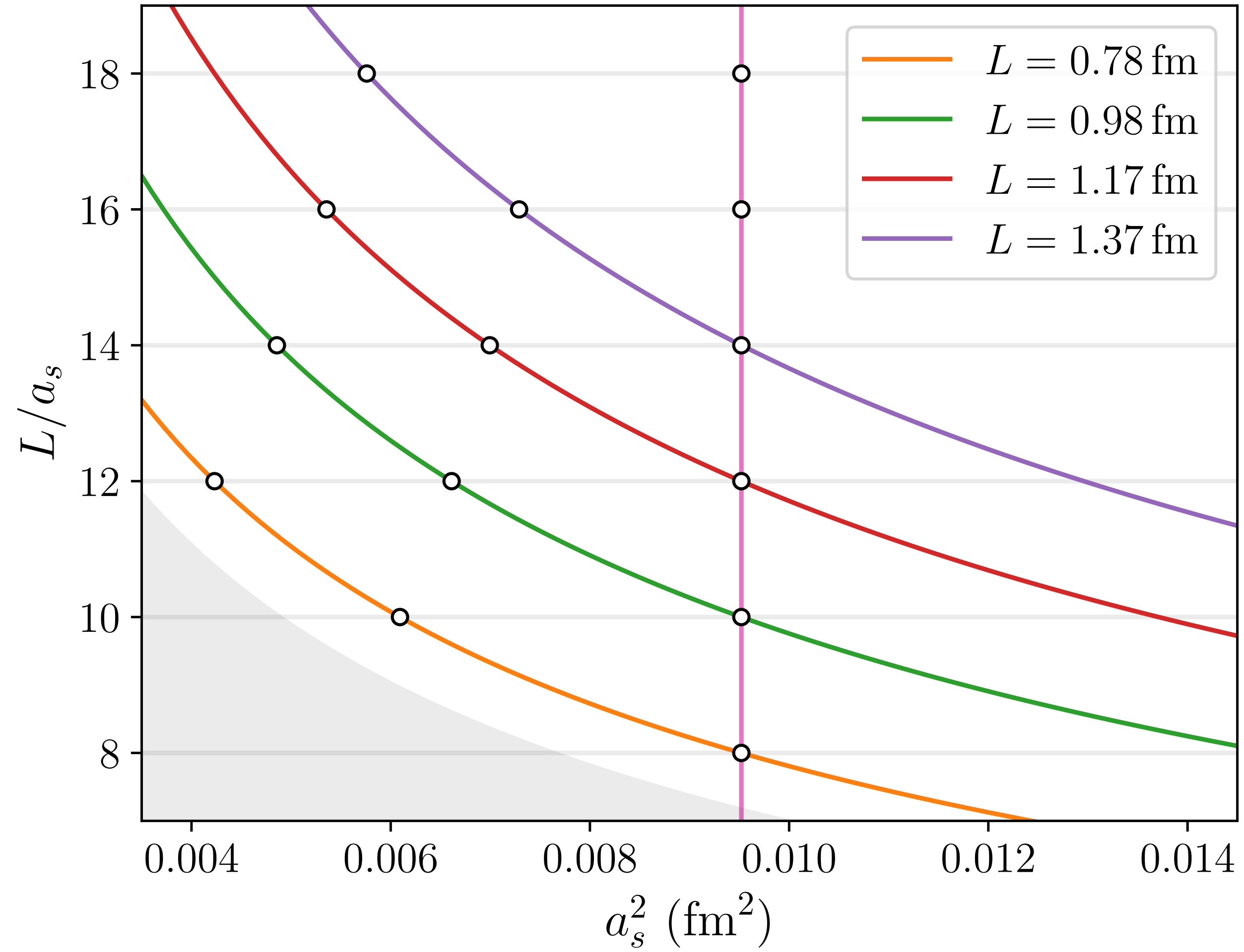
[BMW]

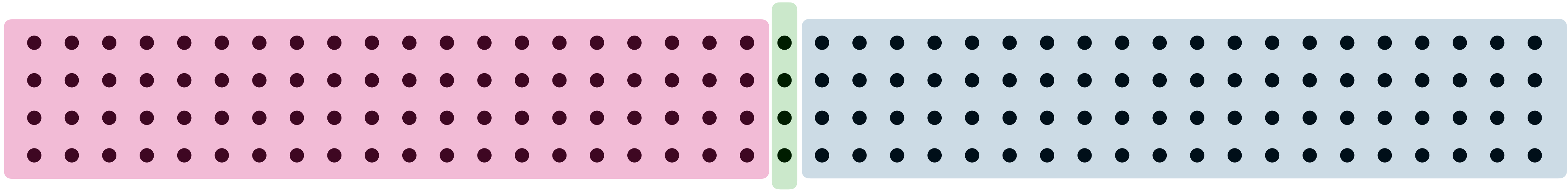
- Multi-level algorithm *(described more on next slide)*
- All ensembles tuned… so far only *vertical pink line* has been generated
- Scale-setting via t_0

$$t^2 \langle E(t) \rangle \Big|_{t=t_0} = 0.3$$



Multi-level algorithm

- Update parts of lattice to improve signal at fixed computational cost



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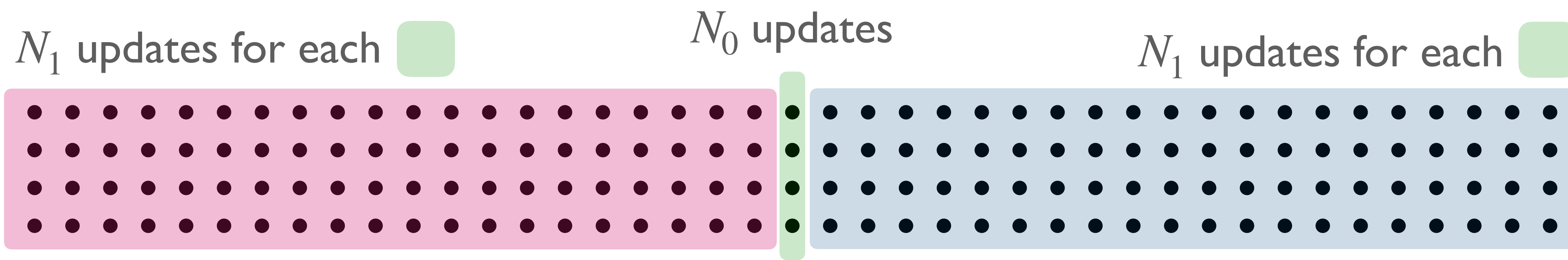


$$\langle \mathcal{O}_x^{(1)} \mathcal{O}_y^{(2)} \rangle = \sum_{\mathcal{A}\in\Omega} p(\mathcal{A}) \left(\sum_{X\in\Omega_{\text{left}}} p_{\mathcal{A}}(X) \mathcal{O}_x^{(1)}[\mathcal{A}, X] \right) \left(\sum_{Y\in\Omega_{\text{right}}} p_{\mathcal{A}}(Y) \mathcal{O}_y^{(2)}[\mathcal{C}, Y] \right)$$

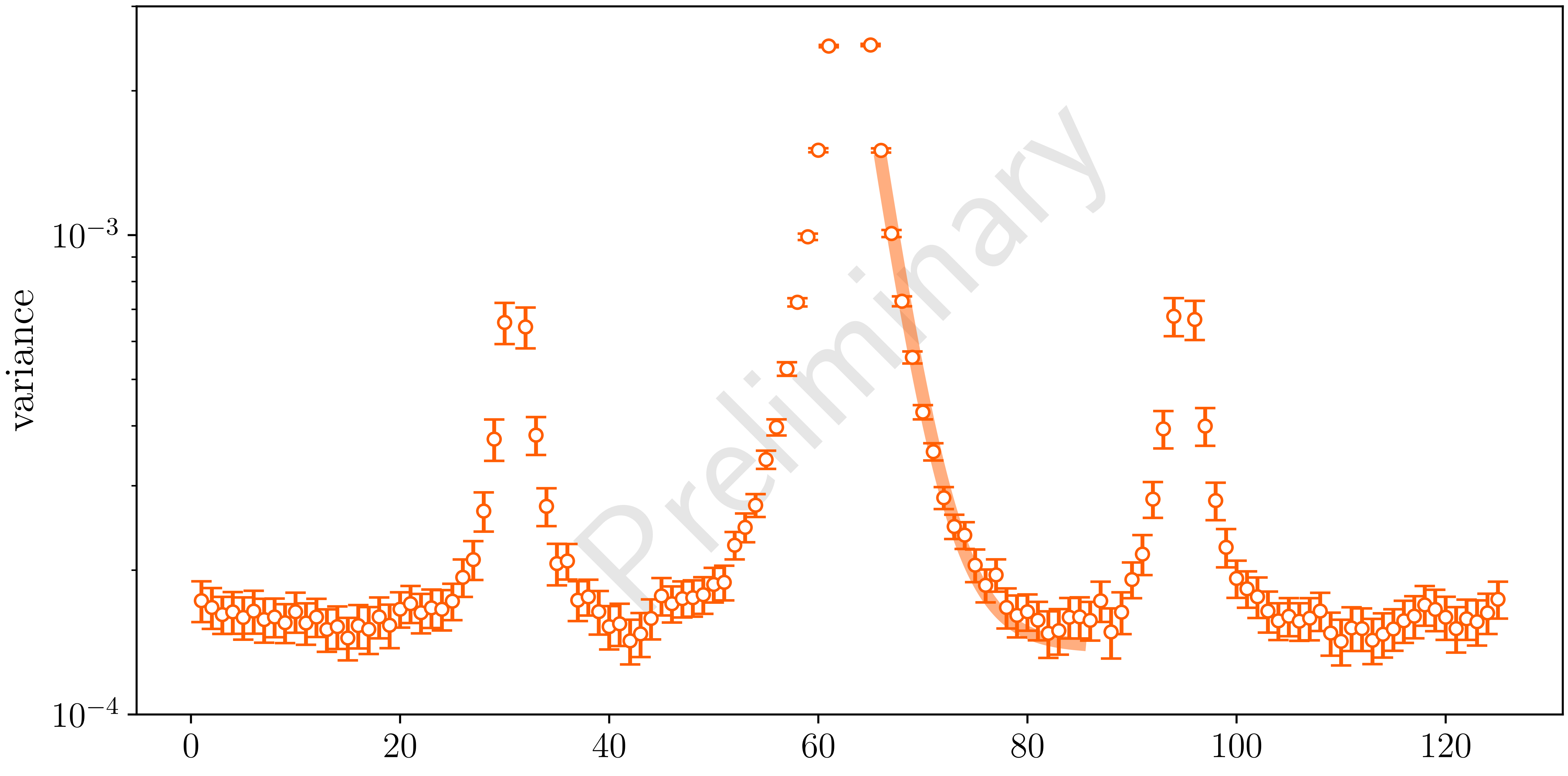
$$= \sum_{\mathcal{A}\in\Omega} p(\mathcal{A}) \left\langle \mathcal{O}_x^{(1)} \right\rangle_{\text{left},\mathcal{A}} \left\langle \mathcal{O}_x^{(2)} \right\rangle_{\text{right},\mathcal{A}}$$

- Ideally variance should scale as $N_1^2 N_0$ (and cost as $N_1 N_0$)

[Lüscher and Weisz, (2001)] - [Meyer (2003,2004)] - [Ce, Giusti, Schaefer (2017)] - [Barca et al. (2024)]

Theoretical understanding of scaling

- Variance (and uncertainty of variance) of the one point off the boundary



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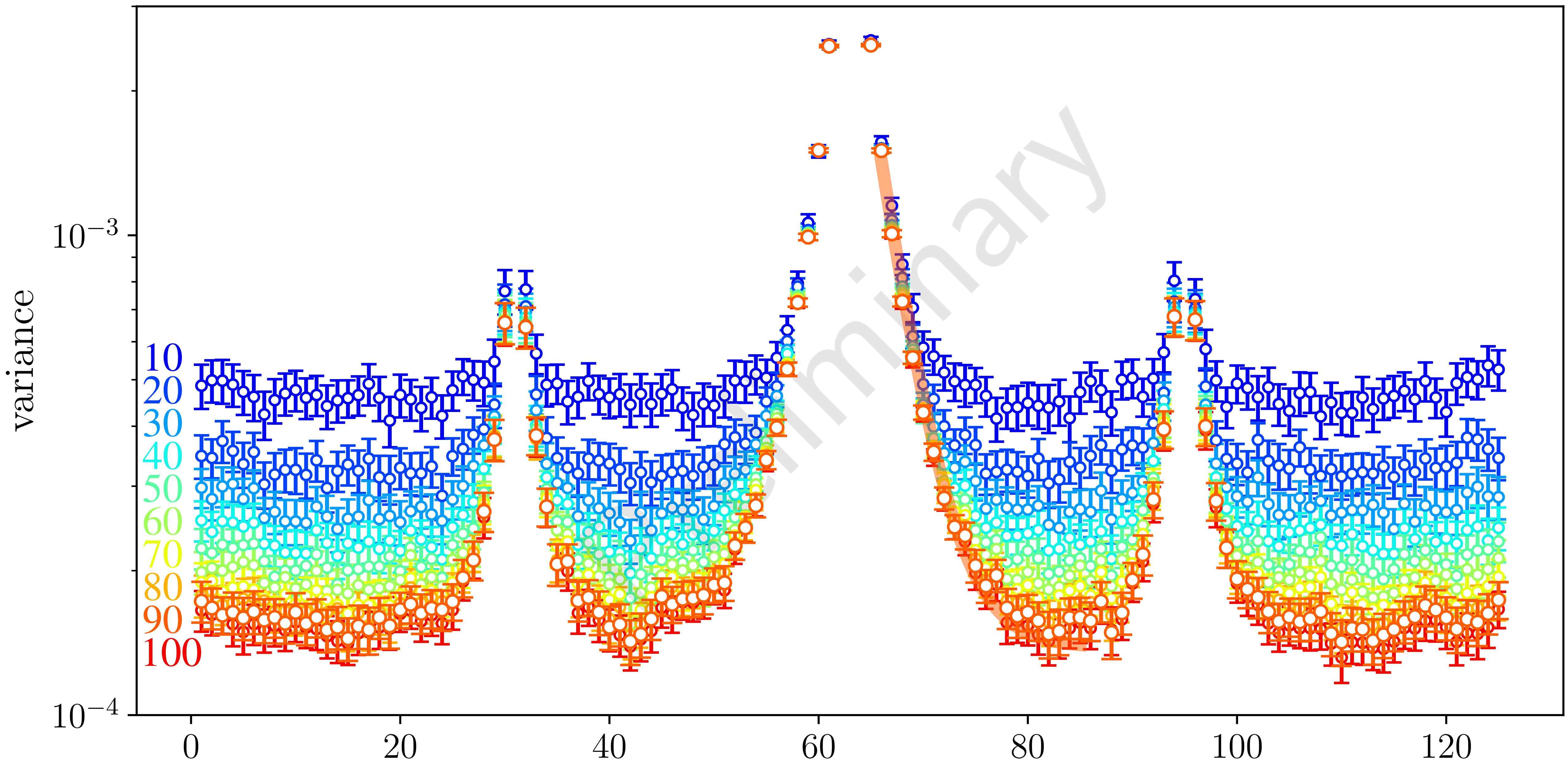

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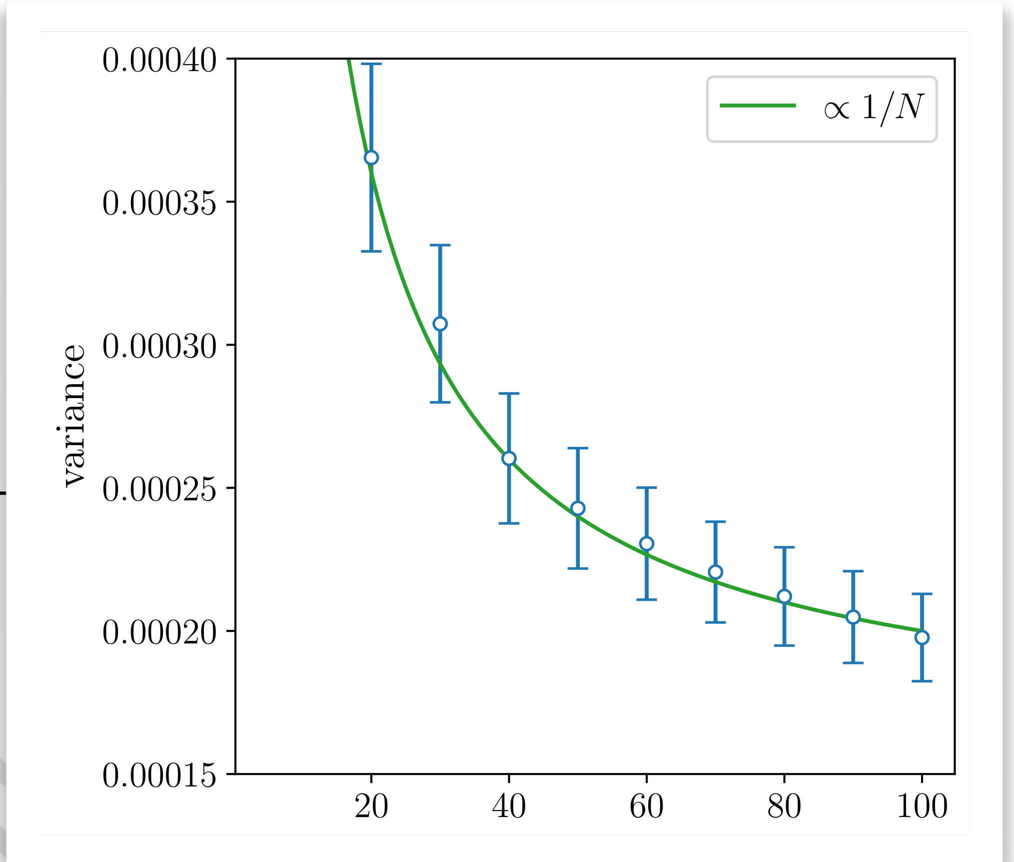
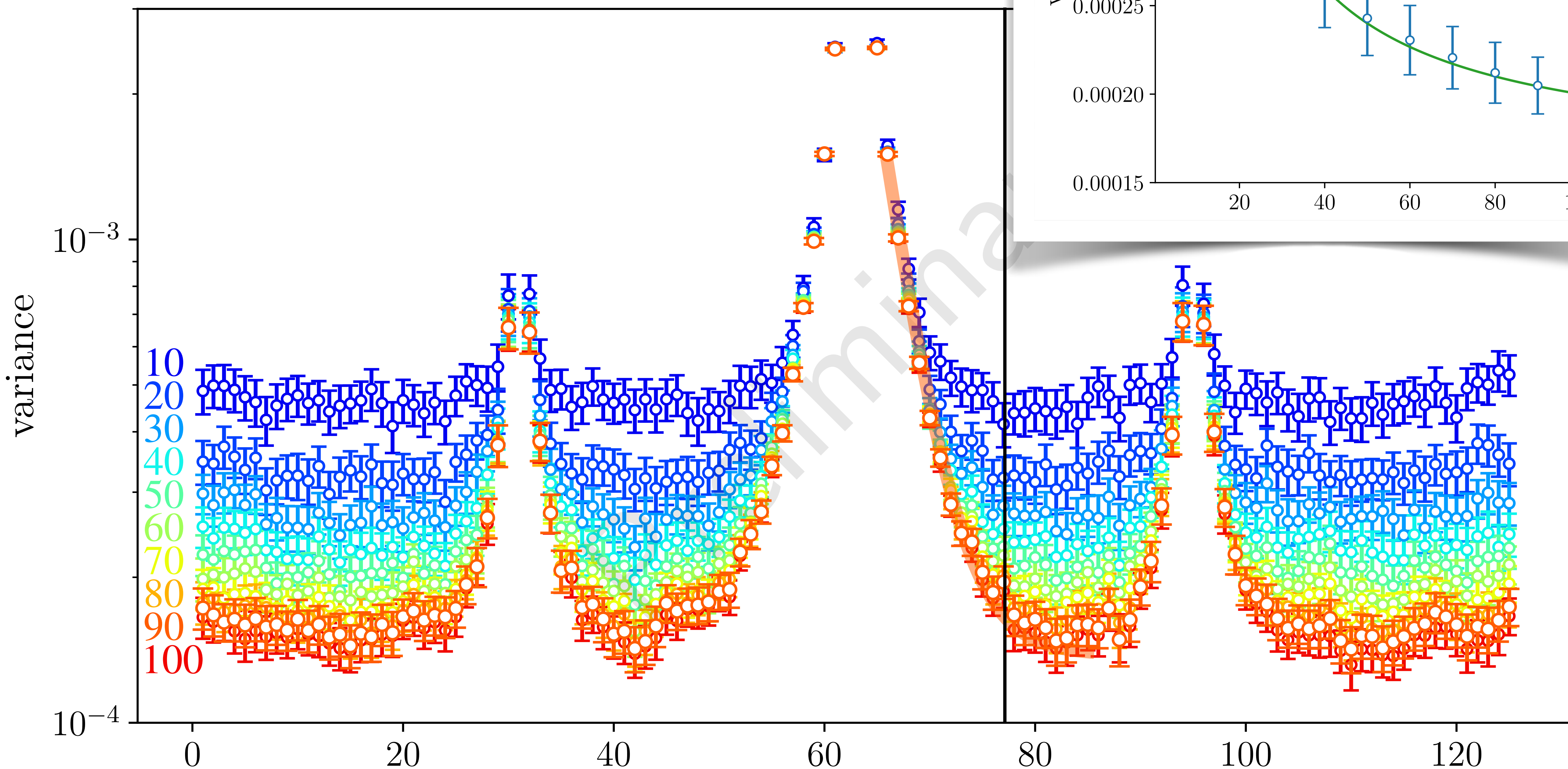
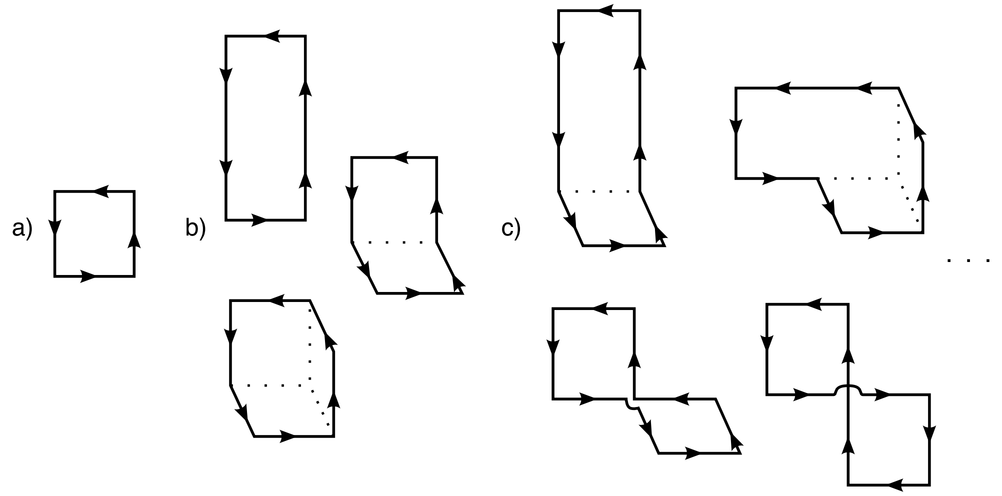




Operator construction

- General approach = many shapes of Wilson loops + smearing + blocking



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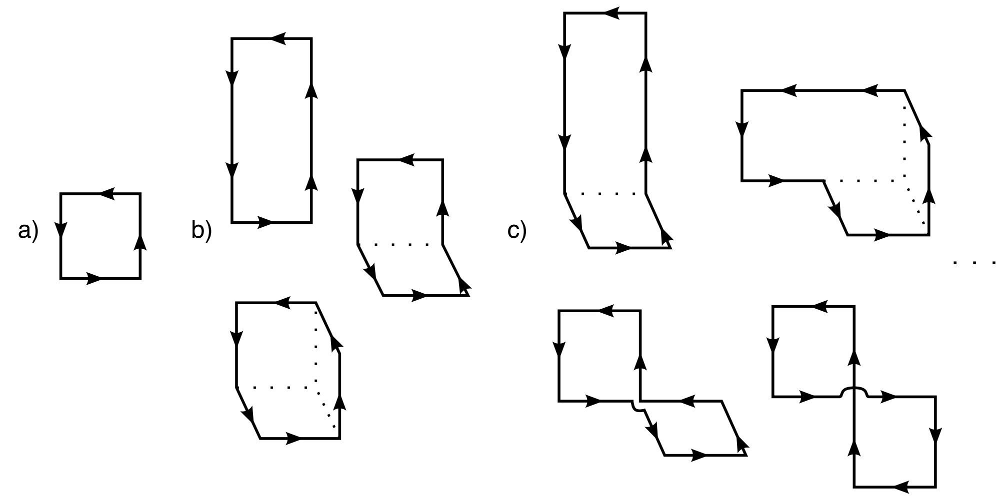


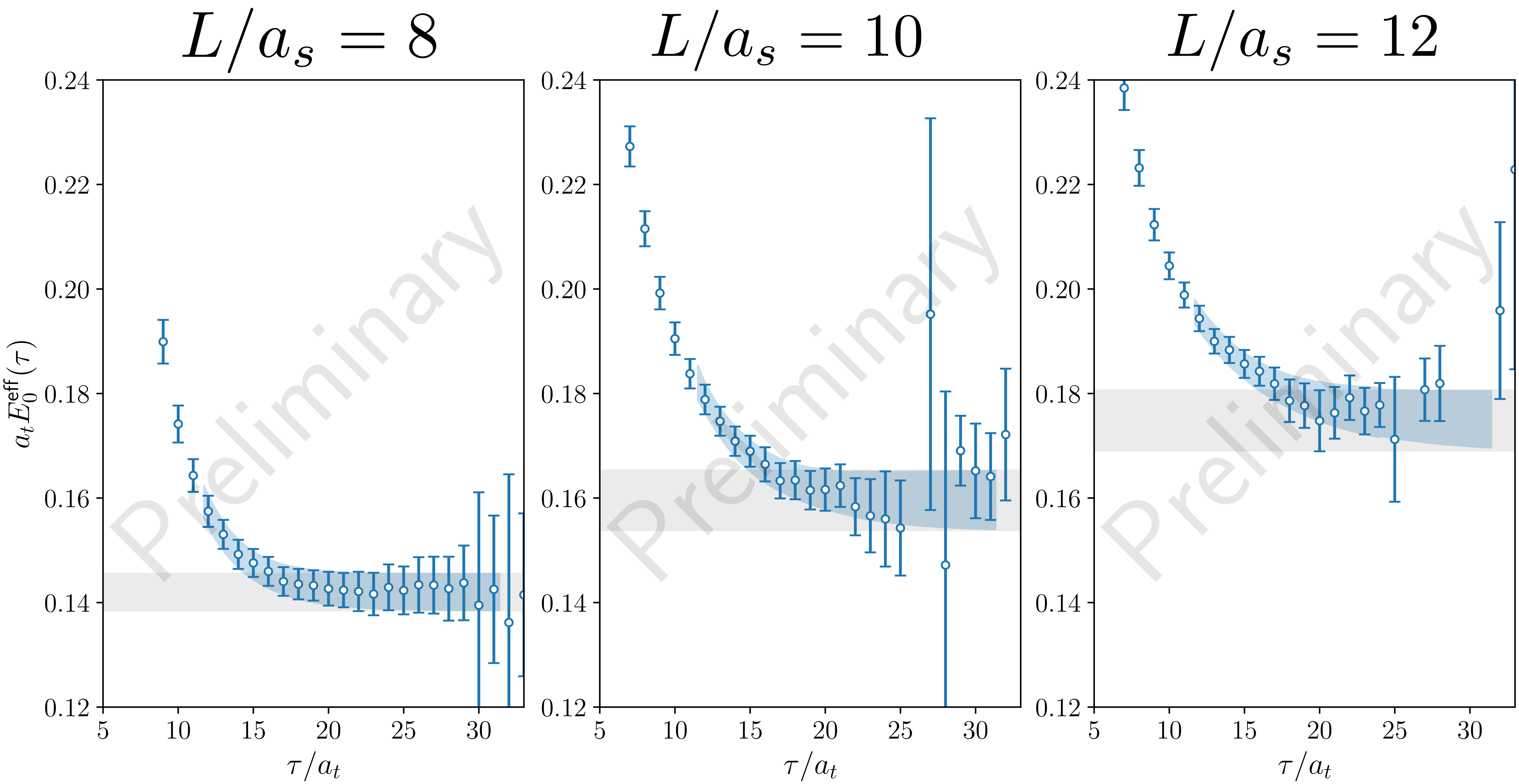
- Project to all good finite-volume quantum numbers as usual

- Also have two-trace operators to extract "two-glue-ball-like" excited states

$$\sum_{\boldsymbol{x},\boldsymbol{y}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}-i\boldsymbol{k}\cdot\boldsymbol{y}}\,\mathrm{tr}[W^{(1)}(\tau,\boldsymbol{x})]\mathrm{tr}[W^{(2)}(\tau,\boldsymbol{y})]$$

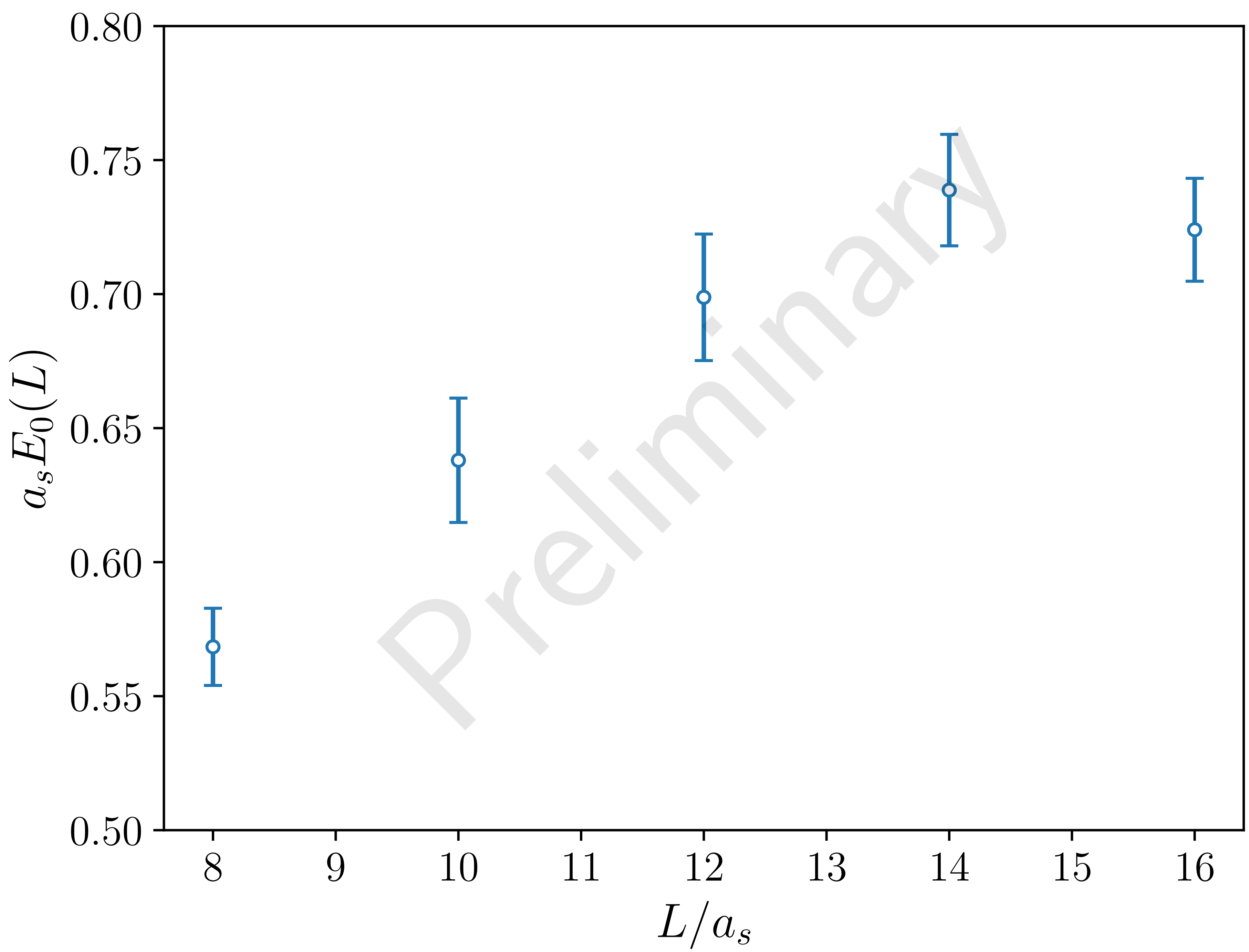
Extracting energies

- Use variational method to extract the ground state energy



$E_0^{[A_1^{++}]}(L)$ vs L

- Volume dependence of the ground state



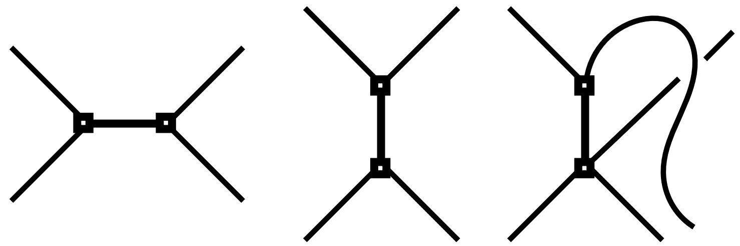
Interpreting the ground state L dependence

- Lüscher stable-states paper relates L dependence to trilinear coupling λ

$$\Delta m = -\frac{3}{16\pi m^2 L}\left\{\lambda^2 e^{-\frac{\sqrt{3}}{2}mL} + \frac{m}{\pi}\int_{-\infty}^{\infty} dy\, e^{-\sqrt{m^2+y^2}L} F(iy) + O(e^{-\bar{m}L})\right\}$$

[Lüscher (1986)]

- WARNING: If λ is too large, subleading exponentials could be important (also F contains terms proportional to λ^2)



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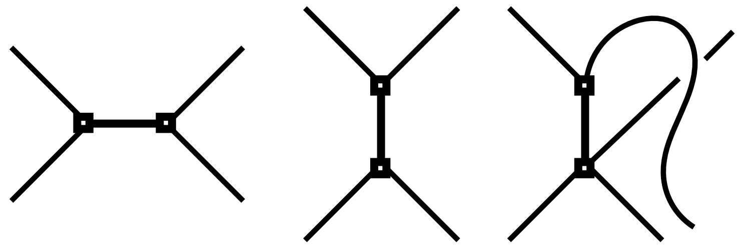

- Method was already used to estimate the coupling in 1984

$$\frac{\lambda}{m} = 50 \pm 8$$

[de Forcrand et al., (1984)]

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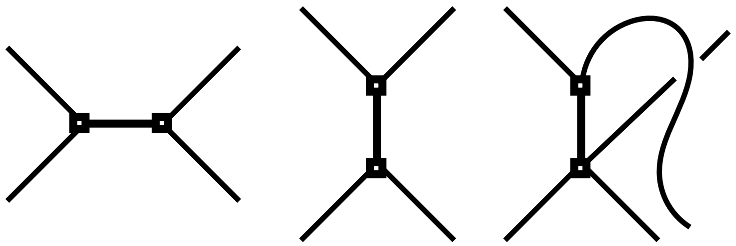

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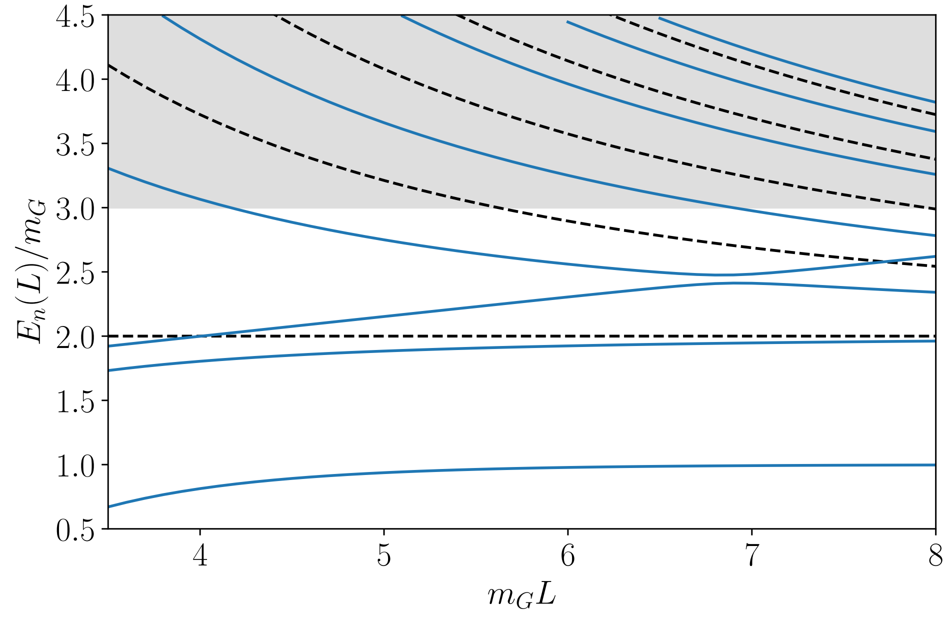
- Can also sum enhanced exponentials by continuing scattering-states formula below threshold

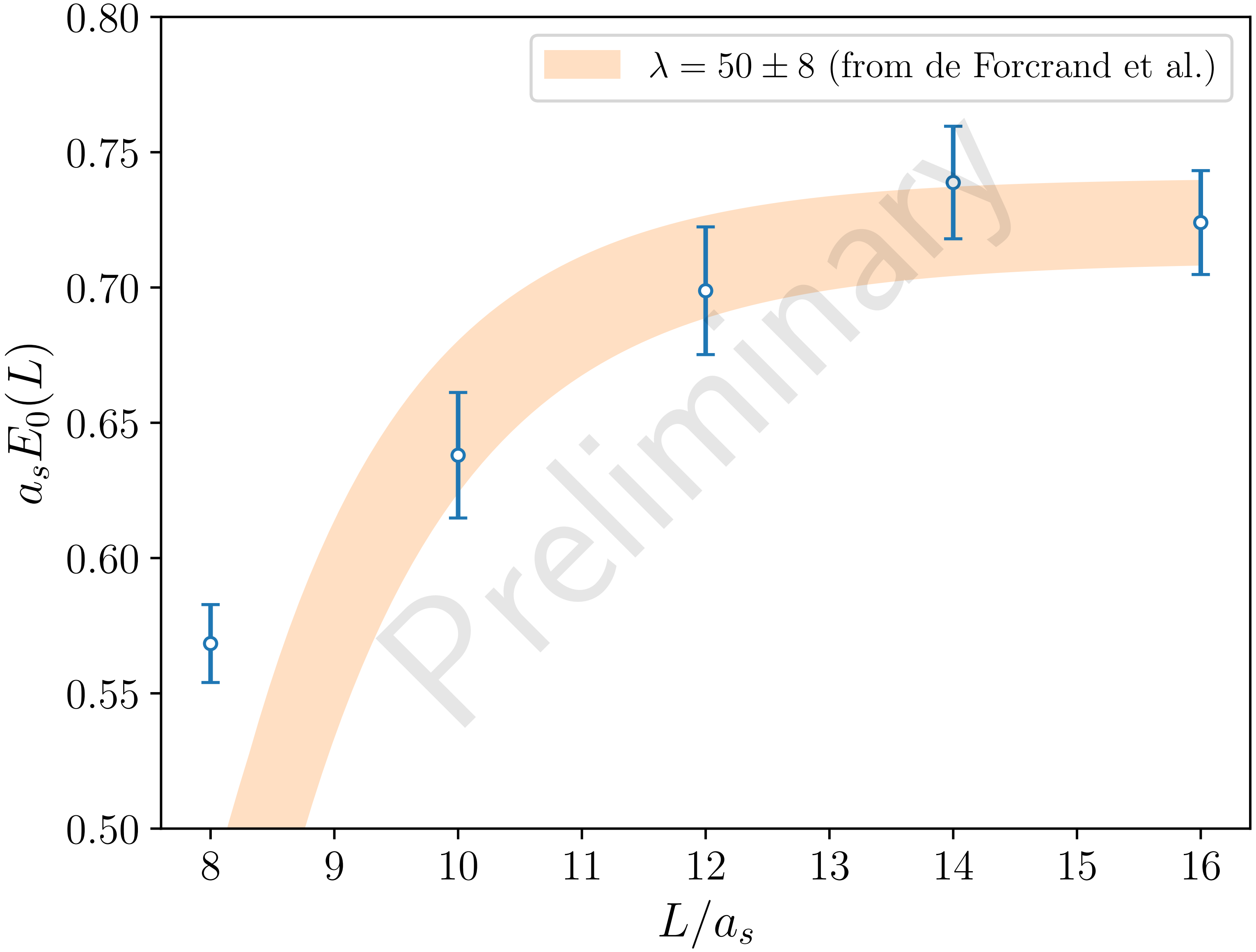
[Lüscher, (1986)]



- Requires left-hand cut formalism

[MTH and Raposo, (2024)]

$E_0^{[A_1^{++}]}(L)$ vs L



$$\Delta m = -\frac{3\lambda^2}{16\pi m^2 L} e^{-\sqrt{3}mL/2}$$

- Our interpretation = matching the *model and coupling* to de *Forcrand et al.* gives a good description

- For these parameters *sub-leading exponentials* could significantly change the story

Conclusions and next steps

- Presented results for 0^{++} ground state *vs L*: $E_0^{[A_1^{++}]}(L)$
- *L*-dependence gives a first probe of scattering: *tri-linear coupling*
- S-matrix bootstrap also constrains $\lambda/m < 200$

[Guerrieride Forcrand et al., (2023)]

- Next steps (most ambitious scenario)

extract excited-state energies… in moving frames… for various quantum numbers

determine and publish fully controlled, continuum-limit, finite-volume energies

analyse two- (and three-particle energies) and predict scattering amplitudes

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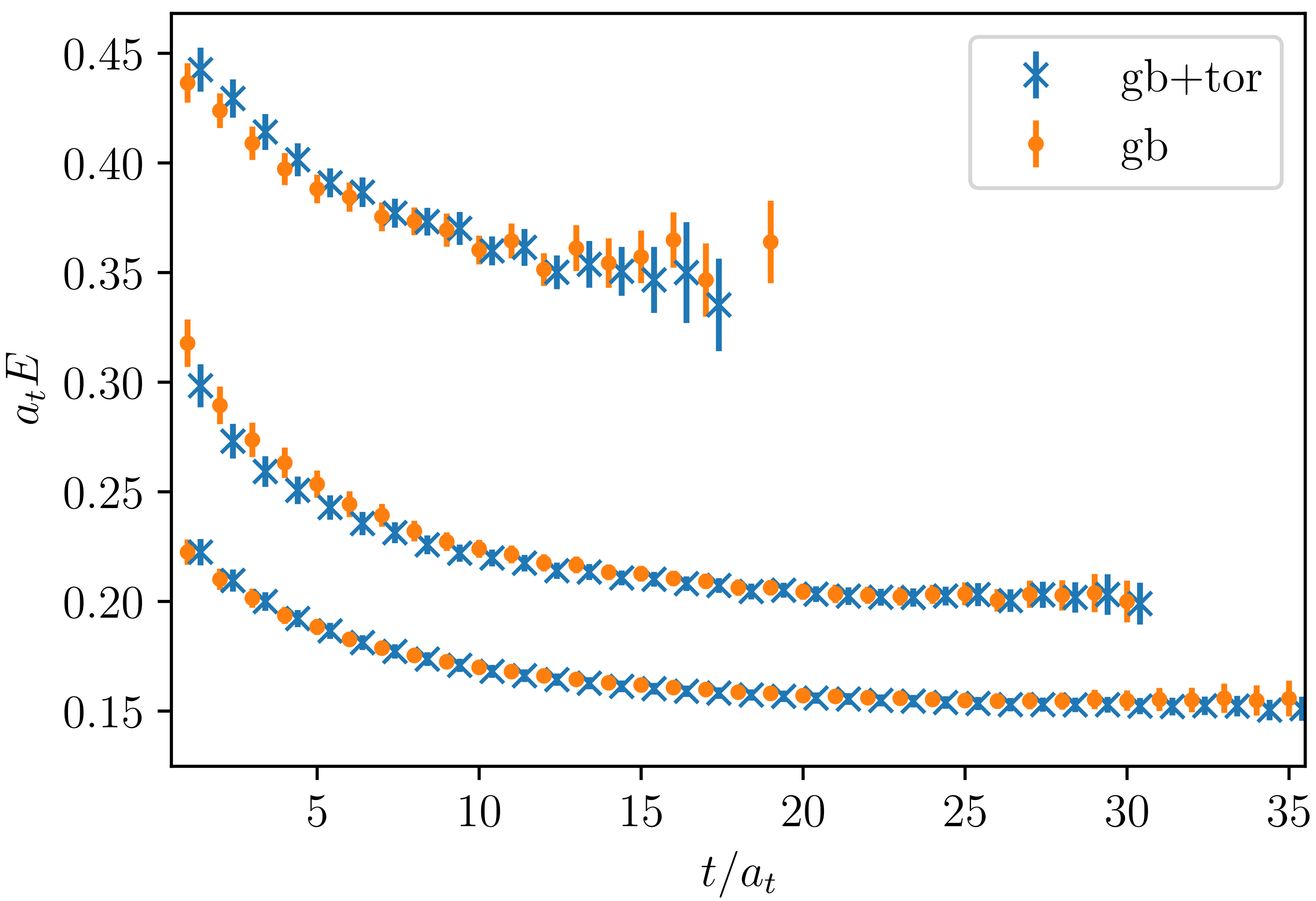
Backup slides

Di-torelon stability

- Stability with respect to inclusion/exclusion of di-torelon operator



GEVP improvement