Results on meson-meson scattering at large N_c

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The large N_c limit of QCD

QCD simplifies at the **large N_c or 't Hooft limit**

$N_c \to \infty \quad N_f = \text{const}$

- Keeps **non-perturbative features** (asymptotic freedom, confinement...)
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Long-term goal: Understand subleading N_c effects in the lattice:

- Pion mass and decay constant [Hernández et al. 2019]
- $K \to (\pi\pi)_{I=0,2}$ [Donini et al. 2016, 2020]
- Meson-meson scattering [JBB et al. 2022 and in preparation]

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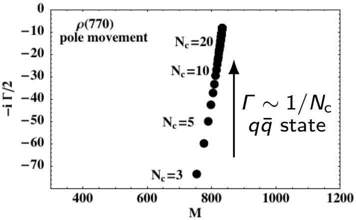
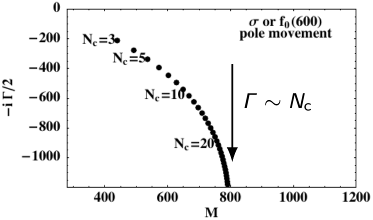
Large N_c in phenomenology

Large N_c + Unitarized ChPT ⟶ **N_c scaling of resonances** [Peláez 2004]





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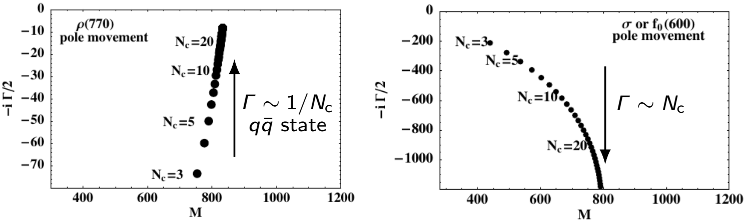

Neglects **subleading N_c effects**

Tetraquarks at large N_c

Recent **controversy about the existence of tetraquarks at large N_c**

- [Witten 1979, Coleman 1980]: Tetraquarks do not exist at large N_c
- [Weinberg 2013]: Tetraquarks can exist at large N_c, with $\Gamma \sim 1/N_c$ (as ordinary resonances)
- [Knetch, Peris 2013]: $\Gamma \sim 1/N_c$ or $\Gamma \sim 1/N_c^2$ depending on the flavor content
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Lattice QCD can allow us to directly answer this question

Meson-meson scattering at large N_c

This talk: N_c scaling of meson-meson scattering

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Large N_c scaling
of **scattering amplitudes**

↓

Constrain LECs from ChPT

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Large N_c scaling of **scattering amplitudes**

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Possible **tetraquark resonances**

↓

Nature of tetraquarks at large N_c

Meson-meson scattering at large N_c

$$N_f = 4 \ (m_u = m_d = m_s = m_c)$$

Used to study $K \to \pi\pi$

[Donini et al. 2020]

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$$15 \otimes 15 = \underset{\pi^+\pi^+}{\overset{\text{even } J}{\mathbf{84}\,(SS)}} \oplus \overset{\text{odd } J}{\mathbf{45}\,(SA)} \oplus \overset{\text{odd } J}{\mathbf{45}\,(AS)} \oplus \underset{D_s^+\pi^+ - D^+K^+}{\overset{\text{even } J}{\mathbf{20}\,(AA)}} \oplus 15 \oplus 15 \oplus 1$$

$$C_{SS} = D - C + (p_1 \leftrightarrow p_2)$$
$$C_{AA} = D + C + (p_1 \leftrightarrow p_2)$$
$$C_{SA} = D - C - (p_1 \leftrightarrow p_2)$$
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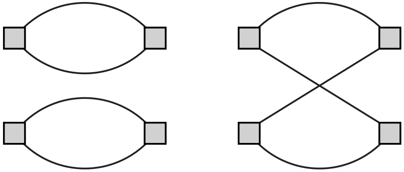

D **C**

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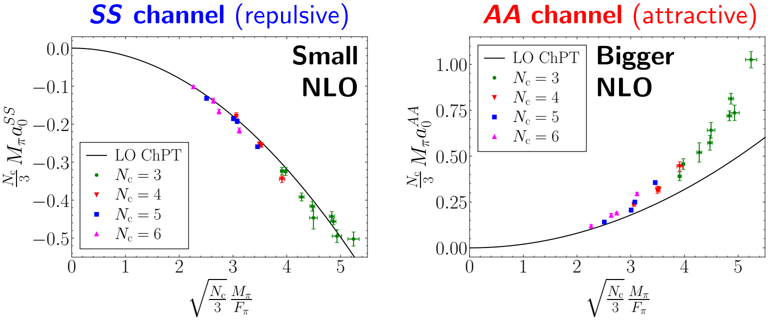
Large N_c counting

$$\mathcal{M}^{SS,AA} = \mp \frac{1}{N_c}\left(a + b\frac{N_f}{N_c} \pm c\frac{1}{N_c}\right) + \ldots$$

$a, b, c \sim \mathcal{O}(1)$ constants

$\pi\pi$ scattering lengths

Previous work [JBB et al. 2022]: Pion-pion scattering near threshold



Compare to LO ChPT: $M_\pi a_0^{SS,AA} = \mp \frac{M_\pi^2}{16\pi F_\pi^2}$ $\qquad k \cot \delta_0 = \frac{1}{a_0} + \ldots$

$\pi\pi$ scattering at large N_c

AA channel is **attractive** $\longrightarrow$ **Possible tetraquark**

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Recently found **exotic states at LHCb** [LHCb 2020, 2022]:

$J = 0$: $T^0_{cs0}(2900)$ in D^+K^-; $T^{++}_{c\bar{s}0}(2900)$ and $T^0_{c\bar{s}0}(2900)$ in $D_s^{\pm}\pi^+$ $\longrightarrow$ ***AA* channel**

$J = 1$: $T^0_{cs1}(2900)$ in D^+K^- $\longrightarrow$ $84 \oplus \mathbf{45}(\boldsymbol{SA}) \oplus \mathbf{45}(\boldsymbol{AS}) \oplus 20 \oplus \ldots$

Below $D_s^*\rho$ threshold $\longrightarrow$ Described as **meson-meson bound states**

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Goal: N_c scaling of meson-meson scattering + tetraquark

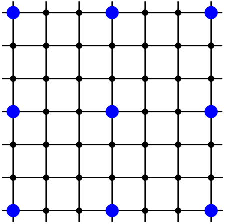
Lattice computations

$N_c = 3, 4, 5, 6$ ensembles with $a \sim 0.075$ fm and $M_\pi \sim 590$ MeV

Lattice simulations are performed using **HiRep** [Del Debbio et al. 2010]

Operator set: $\pi\pi + \rho\rho\,(M_\rho/M_\pi \approx 1.7 - 2)$ + local tetraquark

- Local tetraquark operators ⟶ **Point sources in a sparse lattice $\tilde{\Lambda}$** [NPLQCD 2019]



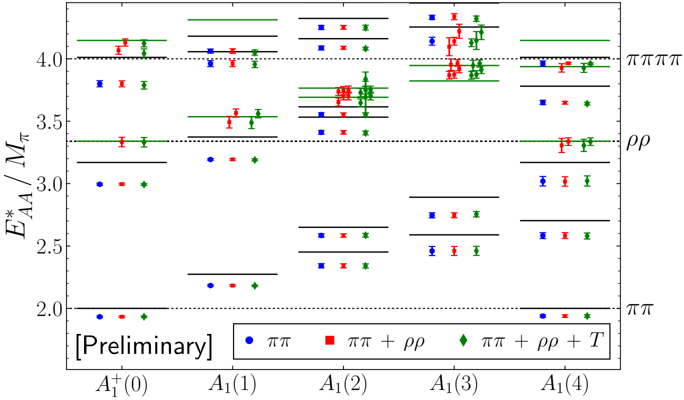
$$T(\boldsymbol{P}) \propto \sum_{\boldsymbol{x}\in\tilde{\Lambda}} e^{-i\boldsymbol{P}\boldsymbol{x}}\, T(\boldsymbol{x})$$

$$T(x) \sim \bar{d}\Gamma_1 u\, \bar{s}\Gamma_2 c - \bar{s}\Gamma_1 u\, \bar{d}\Gamma_2 c$$

Quantum numbers of AA channel

Finite-volume energies: *AA* channel

We study the **effect of different operators** for $N_c = 3$:



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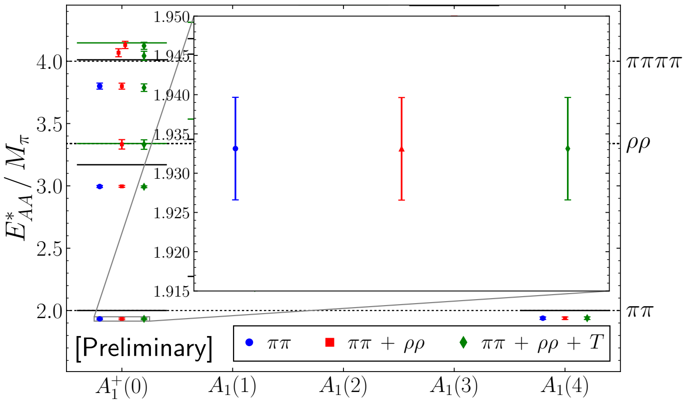

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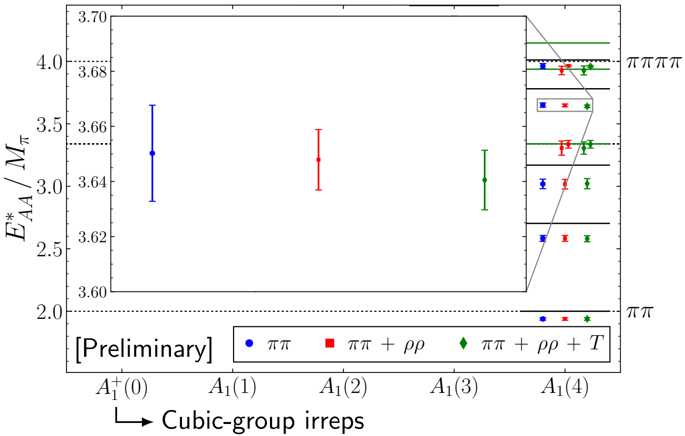
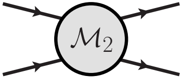


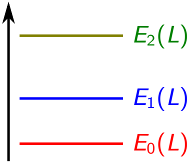
Finite-volume scattering formalism

Infinite-volume scattering amplitudes



Quantization condition (QC)

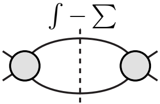
Finite-volume spectrum



Two-particle QC [Lüscher 1986, Rummukainen and Gotlieb 1995, He et al. 2005]:

$$\det[\mathcal{K}_2^{-1} + F(P, L)] = 0$$

$\rho(E)\cot\delta(E)$



$\sim \frac{1}{L^n}$

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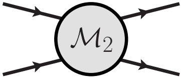

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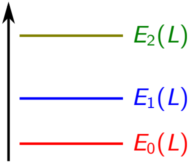

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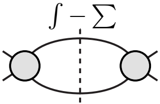
$\rho(E)\cot\delta(E)$ ← $\mathcal{K}_2^{-1}$; $F(P, L)$ →



$\sim \frac{1}{L^n}$

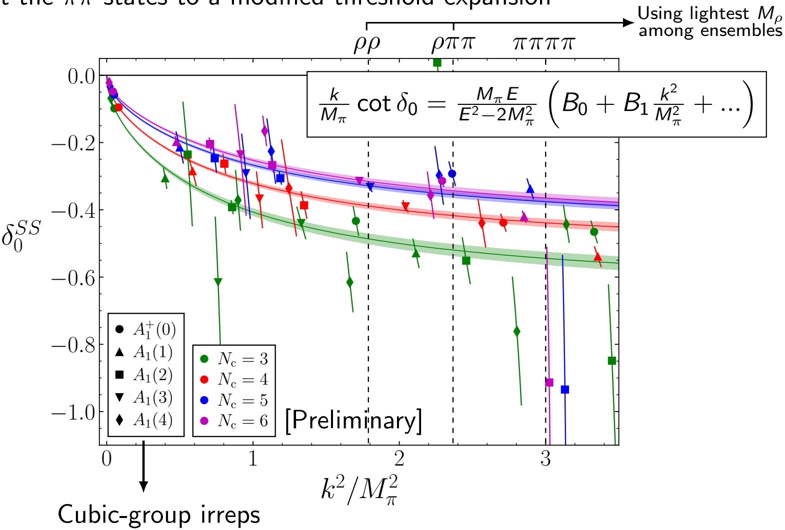
Single-channel, *s*-wave ⟶

(Similar for *p* wave)

$$k \cot\delta_0 = \frac{2}{\gamma L \pi^{1/2}} \mathcal{Z}_{00}^{\boldsymbol{P}}\left(\frac{kL}{2\pi}\right)$$

Scattering phase shift: *SS* channel

We fit the $\pi\pi$ states to a modified threshold expansion



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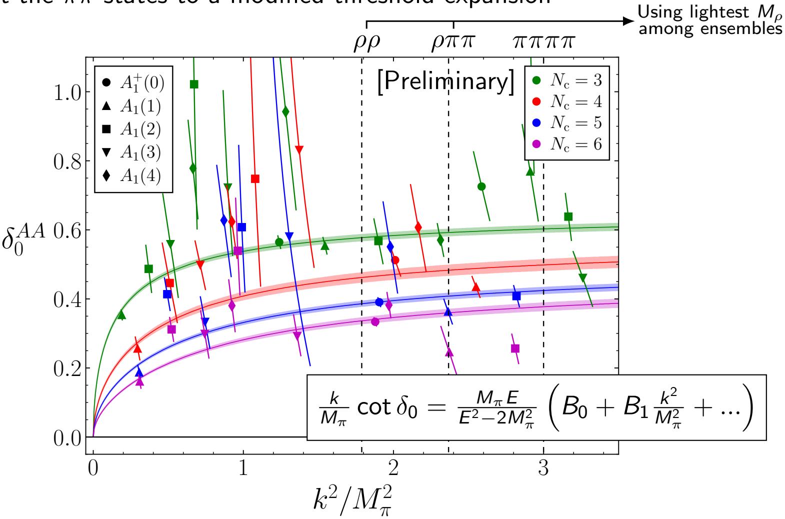

$$\frac{k}{M_\pi}\cot\delta_0 = \frac{M_\pi E}{E^2 - 2M_\pi^2}\left(B_0 + B_1\frac{k^2}{M_\pi^2} + \dots\right)$$

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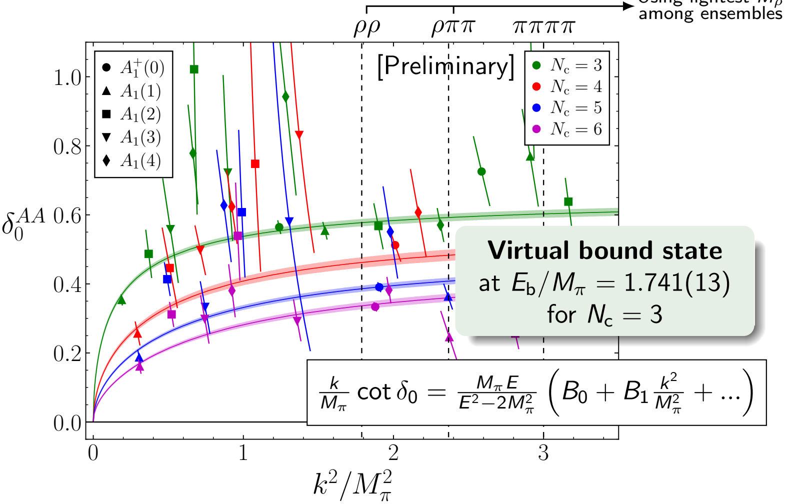

Virtual bound state
at $E_b/M_\pi = 1.741(13)$
for $N_c = 3$

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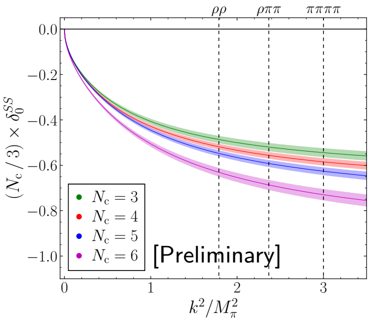
Scattering phase shift: *SS* and *AA* channels

Results present the **expected N_c scaling**: $\delta_0 \propto N_c^{-1}$

SS channel



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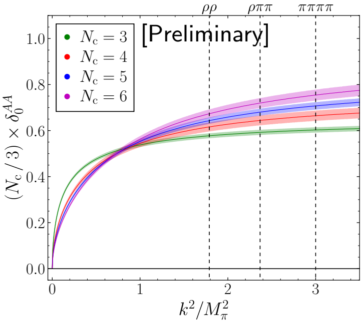
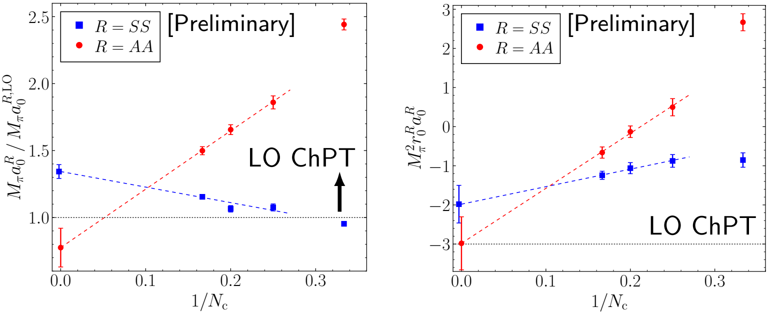


We are **sensitive to subleading N_c corrections**

Large N_c scaling of scattering parameters

We study the **N_c scaling of scattering observables**



LO ChPT: $M_\pi a_0^{SS,AA} = \mp \frac{M_\pi^2}{16\pi F_\pi^2}$ $\qquad M_\pi^2 a_0^{SS,AA} r_0^{SS,AA} = -3$

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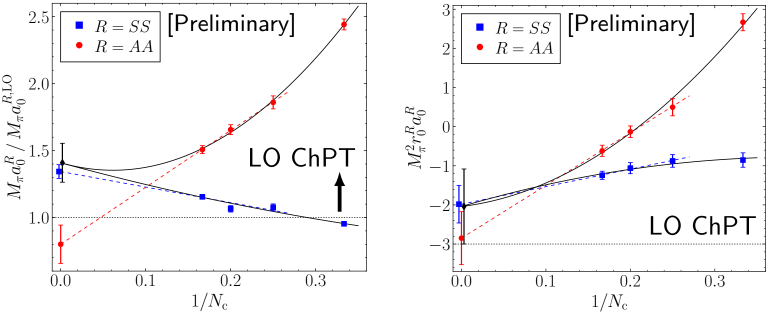

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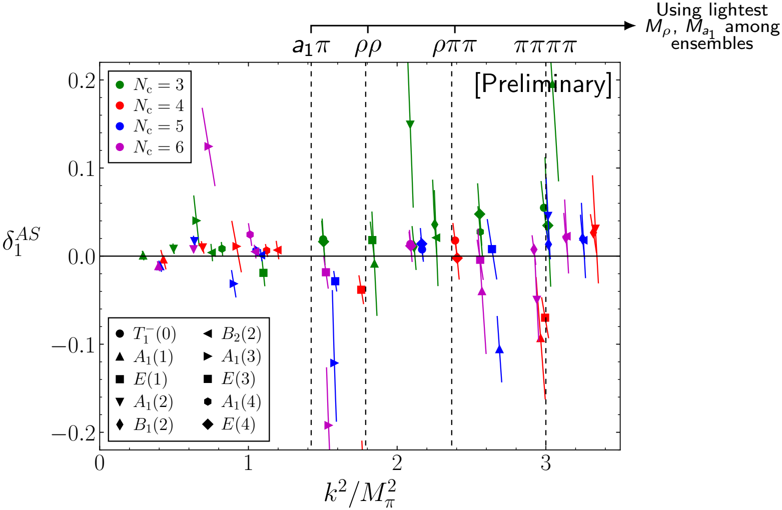
Scattering phase shift: *AS* channel



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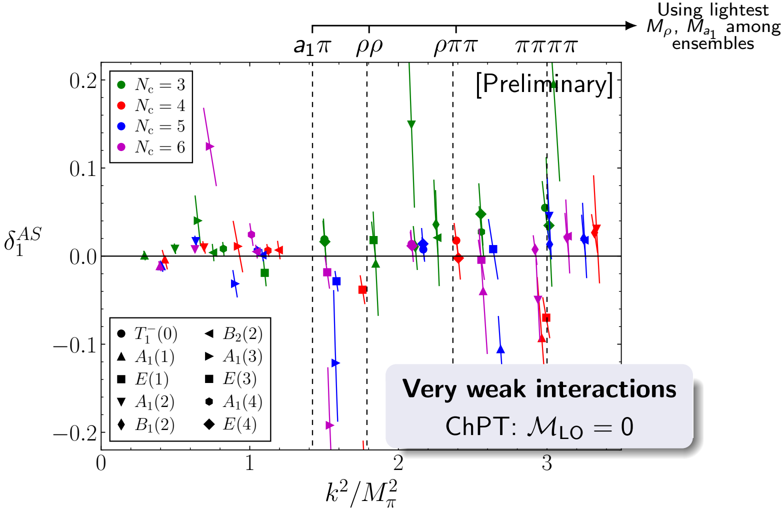

Very weak interactions

ChPT: $\mathcal{M}_{\rm LO} = 0$

Summary and outlook

Goal: Large N_c scaling of meson-meson interactions

- We have characterized the $\pi\pi$ scattering amplitudes in the *SS* and *AA* channel, and are **sensitive to subleading N_c effects**
- We find a **virtual bound state in the *AA* channel** for $N_c = 3$
- We find **very weak interactions in the *AS* channel**, as expected

Next steps: Match to ChPT, $\rho\rho - \pi\pi$, higher partial waves...

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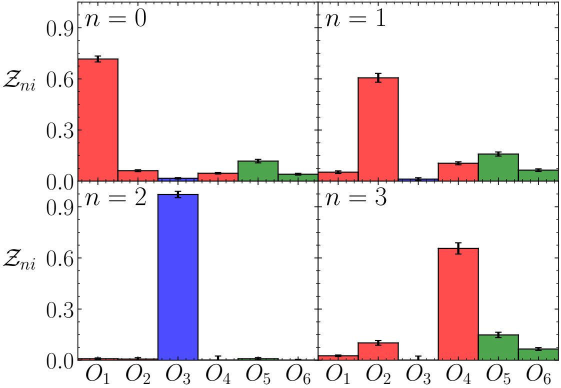
Thank you for your attention!

Operator overlap

Eigenvectors of the GEVP provide intuition on the effect of each operator

AA channel, A_1^+ irrep, rest frame

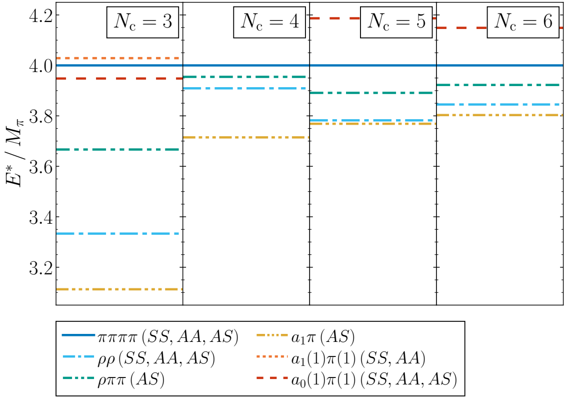


$$\mathcal{Z}_{ni} = \frac{\langle 0|O_i|n\rangle}{\sum_\chi \langle 0|O_\chi|n\rangle}$$

$$O_1 = \pi(0)\pi(0)$$
$$O_2 = \pi(1)\pi(1)$$
$$O_3 = \rho(0)\rho(0)$$
$$O_4 = \pi(2)\pi(2)$$
$$O_5 = T_{\gamma_5\gamma_5}$$
$$O_6 = \sum_i T_{\gamma_i\gamma_i}$$

Two-particle energy spectrum

We observe significant N_c dependence of meson masses (no $\pi\pi$ mixing)

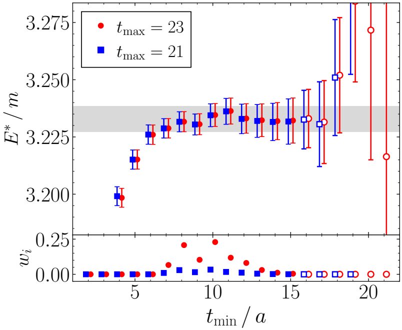


Meson mass spectrum

Average plateaux using **Akaike Information Criterion** [Jay, Neil 2020]

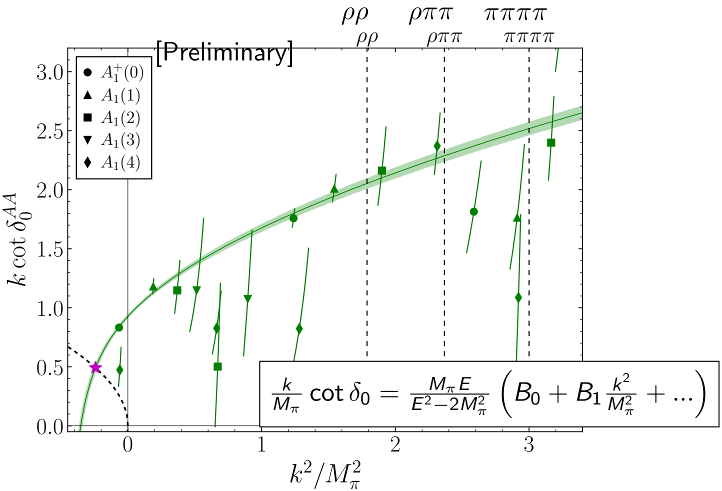
$$w_i \propto \exp\left[-\frac{1}{2}\left(\chi^2 - 2N + 2N_{\text{par}}\right)\right]$$

- Reduces human bias
- Allows to automatically find plateaux for accurate data



Virtual bound state for $N_c = 3$

We find a **virtual bound state** for $N_c = 3$



$$\frac{k}{M_\pi} \cot \delta_0 = \frac{M_\pi E}{E^2 - 2M_\pi^2} \left(B_0 + B_1 \frac{k^2}{M_\pi^2} + \ldots \right)$$