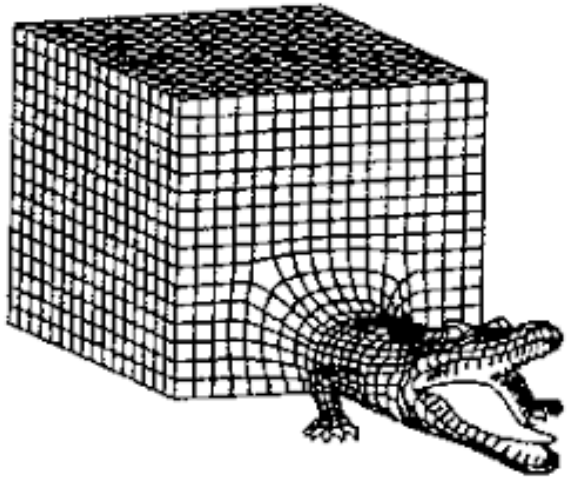


Study of pion-mass dependence of rho meson on the lattice

based on PhysRevD.109.034505,

K. Yu, J.J. Wu, Y. Li, D.B. Leinweber, A.W. Thomas

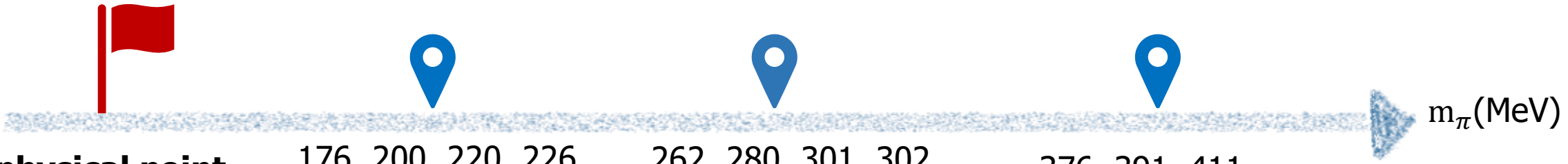
1.1 short review on rho meson

- $J^P = 1^-$
- ~100% \pi\pi mode

- single partial wave
- single channel

Lüscher formula is simple practically

Many work has been done ...



physical point (~140)

176, 200, 220, 226, 236, 247, 248, ...

262, 280, 301, 302, 315, 316,330, 346, ...

376, 391, 411, ...

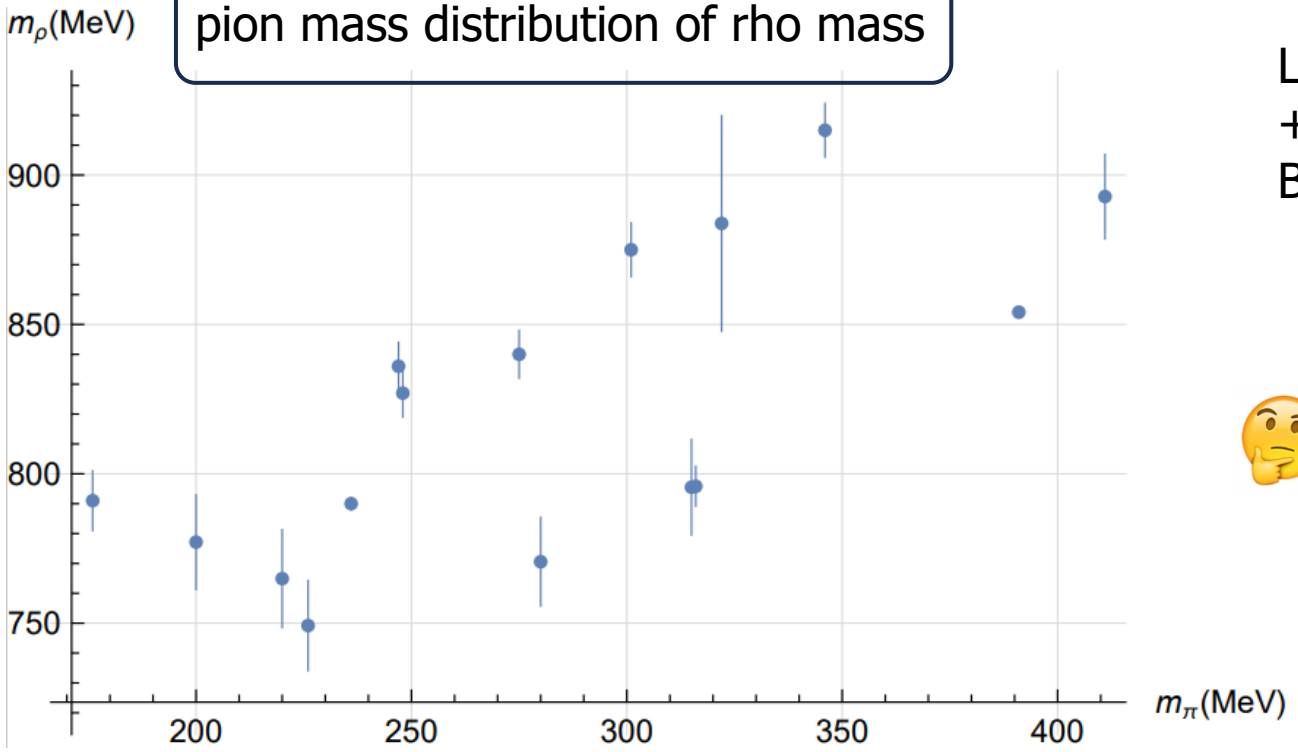
(most with N_f=2+1)

by HSC, ETMC, MILC, PACS-CS, J.Bulava et al. etc.

1.1 short review on rho meson

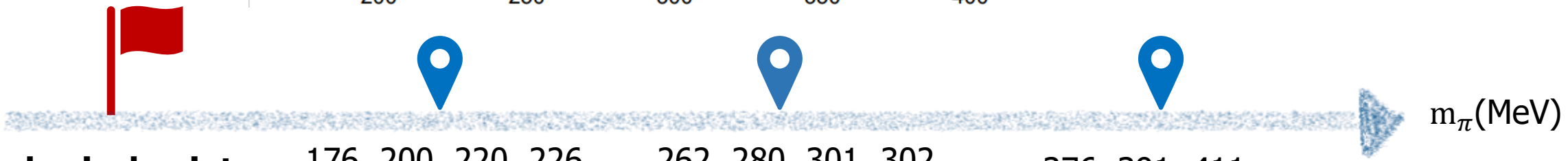
However ...

pion mass distribution of rho mass



Lüscher formula
+
BW parameterization

🤔 any physical reason ?



physical point (~140)

176, 200, 220, 226, 236, 247, 248, ...

262, 280, 301, 302, 315, 316,330, 346, ...

376, 391, 411, ...

(most with N_f=2+1)

1.2 rho meson in phenomenological theory

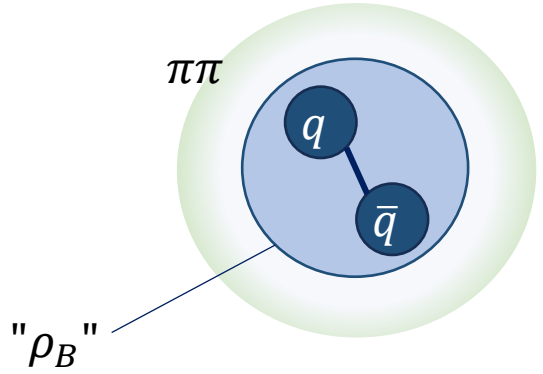
- Dynamically generated from $\pi\pi - K\bar{K}$ couple channel in (SU(3)) Unitarized ChPT

PhysRevLett.117.122001
PhysRevLett.126.102002
PhysRevD.96.034520
...

Picture here ...

- observed rho = **bare** rho core (at quark level) **dressed** by $\pi\pi$ meson pairs(hadronic level)
  (similar to CDD pole)

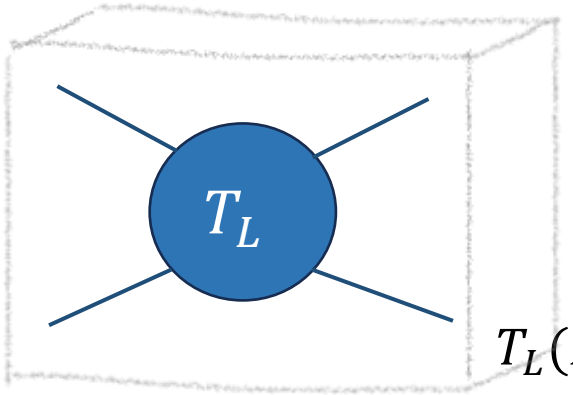
  - rho survives in the large N_C limit where QCD mesons are free
    hep-ph/0701038



more regular pion-mass dependence ?

2. brief introduction to Hamiltonian EFT

finite volume lattice spectra = poles of



$$T_L(E) = V + V\frac{1}{E - H_L}V$$

finite volume effective Hamiltonian

infinite volume

finite volume

H

share some parameters

H_L

- ☑ rest frame
- ☑ moving frame
- ☑ elongated frame

for more details, talk on Friday by Wu

solve scattering equation
(ex. LS Equation)

unitarity,
analyticity...

O(3) breaking,
irrep decomposition...

calculate eigenvalues

phase shift,
pole position, ...

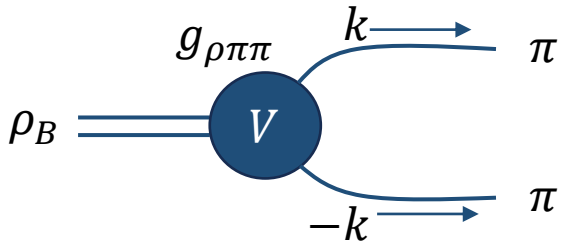
Lüscher formula

lattice spectra

PhysRevLett.116.082004
PhysRevD.95.014506
PhysRevD.97.094509
PhysRevD.103.094518
...

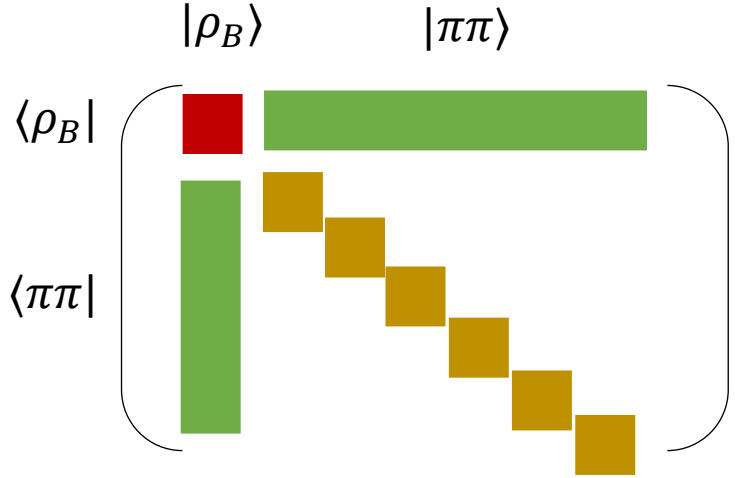
3. rho meson on the lattice within Hamiltonian EFT

- Hilbert space spanned by $|\rho_B\rangle, |\pi\pi\rangle$
- interaction: $\langle\rho_B|V|\pi(k)\pi(-k)\rangle \propto g_{\rho\pi\pi}\, k \left(\frac{\Lambda_{\rho\pi\pi}^2}{k^2+\Lambda_{\rho\pi\pi}^2}\right)^2$



  - $g_{\rho\pi\pi}$: bare coupling constant
  - $\Lambda_{\rho\pi\pi}$: energy cutoff of effective Hamiltonian
  - m_ρ^B : mass of bare rho core

determined by fitting the lattice spectra



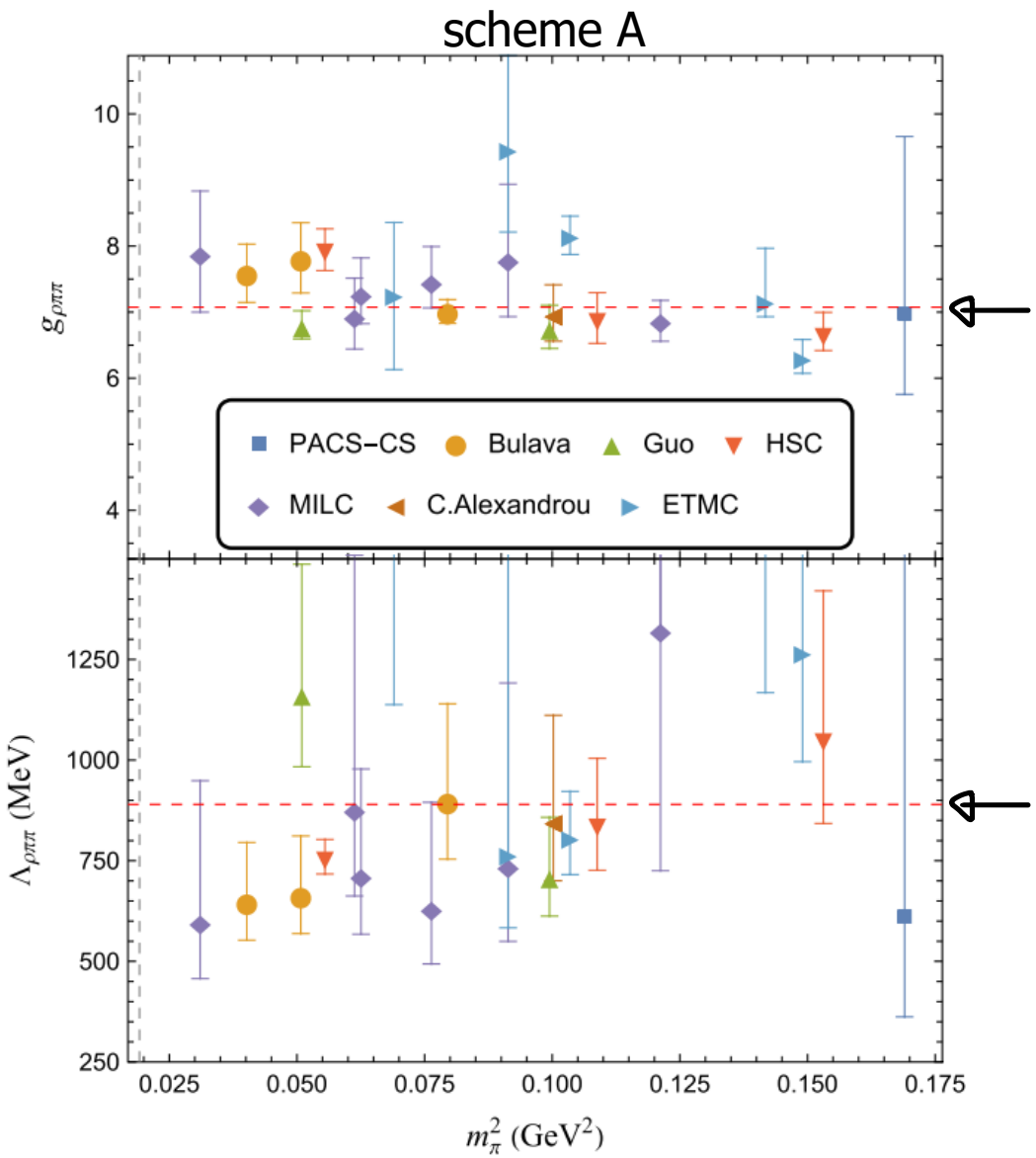
As a first try, all these three parameters are free (scheme A)

3. rho meson on the lattice within Hamiltonian EFT

- $g_{\rho\pi\pi}$ and $\Lambda_{\rho\pi\pi}$ are not sensitive to m_π
- $g_{\rho\pi\pi}$ is dimensionless
- the insensitivity for $\Lambda_{\rho\pi\pi}$ is in line with the spirit of effective theory
- $g_{\rho\pi\pi} \approx 7.0$ ($g^{BW}_{\rho\pi\pi} \approx 6.0$) and $\Lambda_{\rho\pi\pi} \approx 900$ MeV fixed.

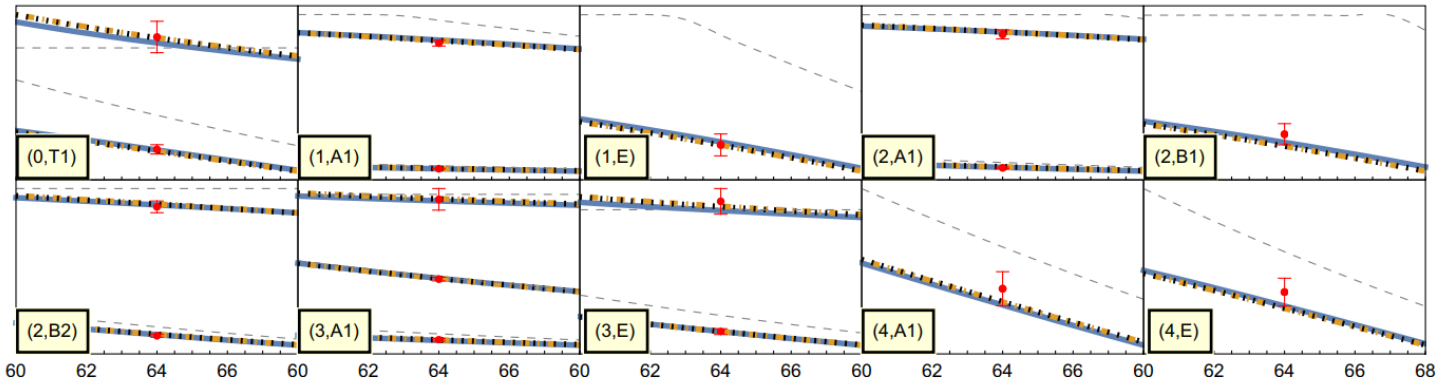
only **one** parameter m_ρ^B to fit lattice spectra (scheme B)

- only m_ρ^B will be dependent on m_π

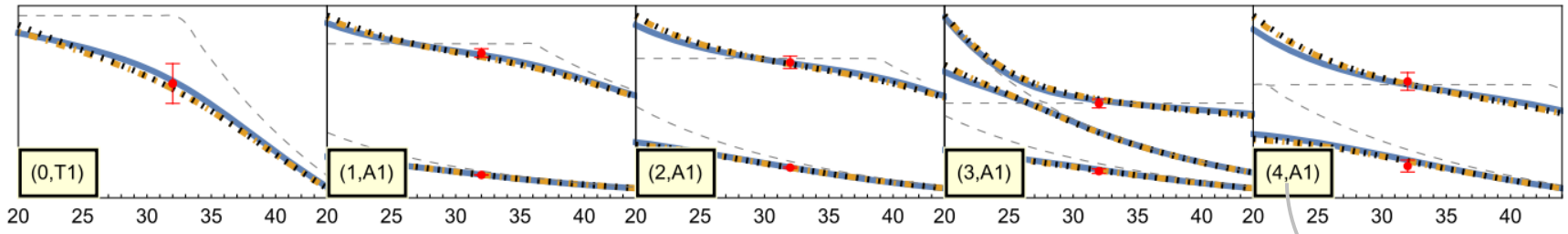


3. rho meson on the lattice within Hamiltonian EFT

220 MeV
J.Bulava

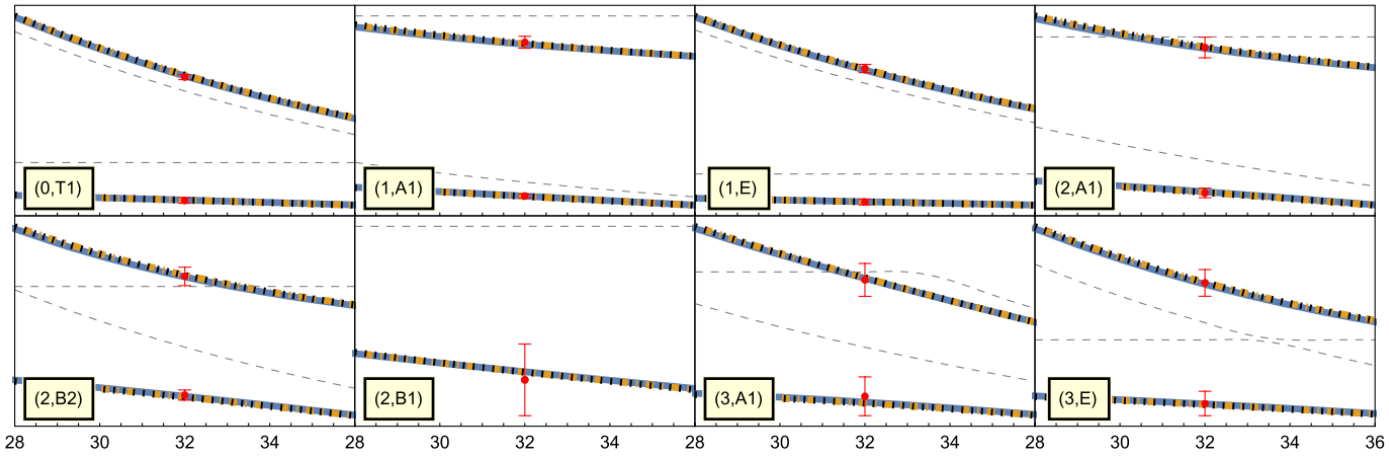


247 MeV
MILC



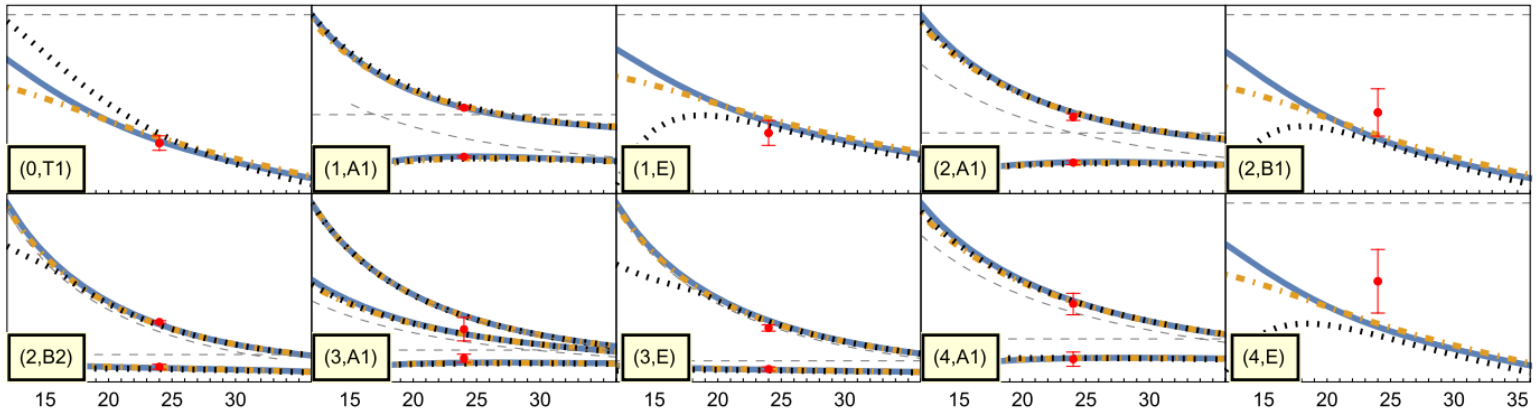
total momentum & irrep

316 MeV
C.Alexrandru

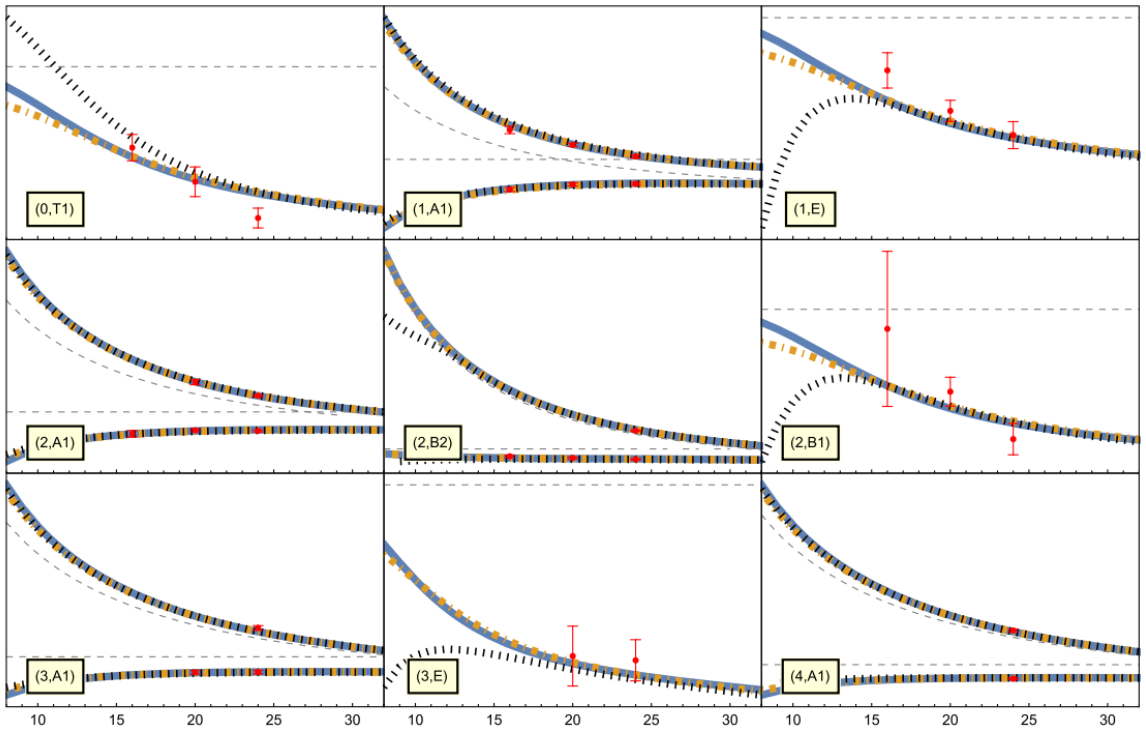


3. rho meson on the lattice within Hamiltonian EFT

386 MeV
ETMC



391 MeV
HSC



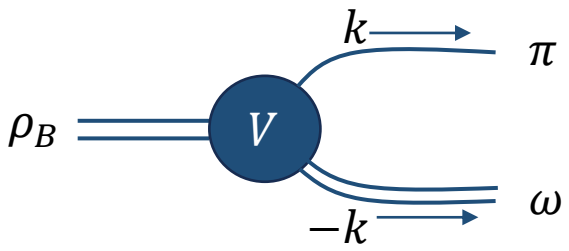
However ...

3. rho meson on the lattice within Hamiltonian EFT

pion mass dependence of m_ρ^B is also messy ☹

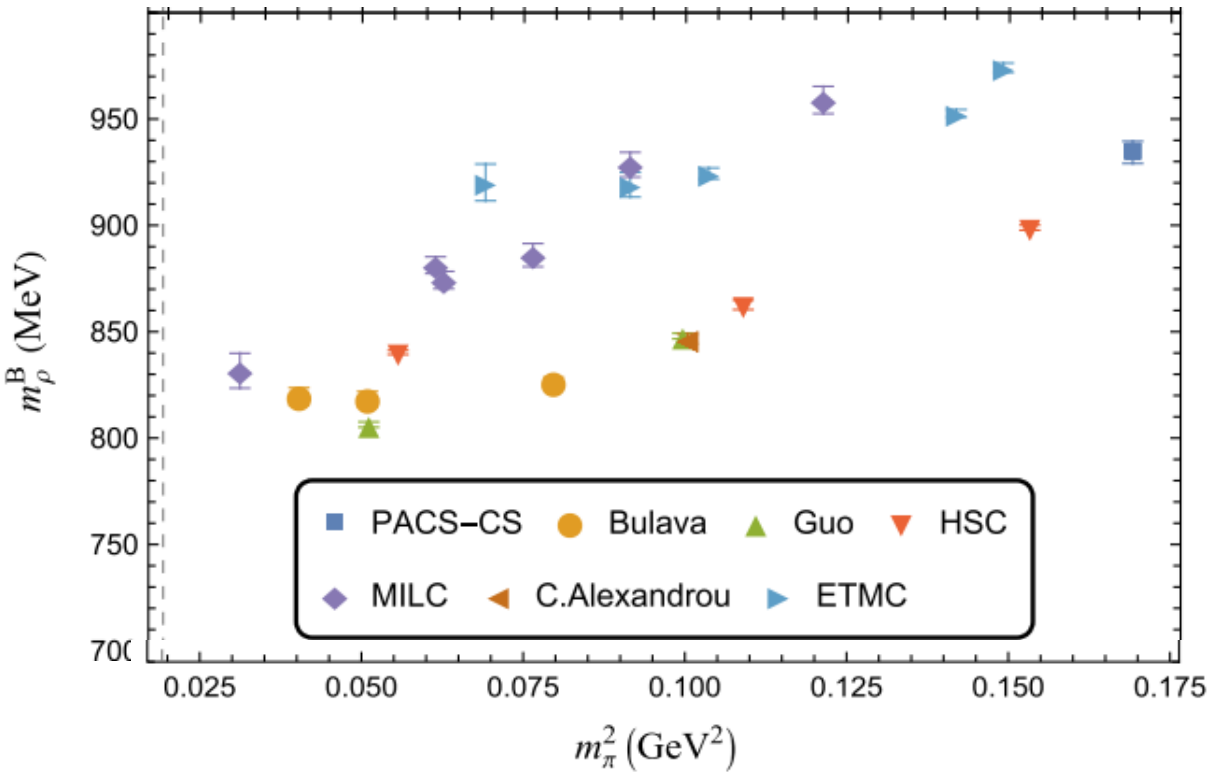
$E_{\text{thr}}(\omega\pi) \sim 920$ MeV
$E_{\text{thr}}(K\bar{K}) \sim 1000$ MeV contribution from $|\omega\pi\rangle$?

$|\omega\pi\rangle$ without introducing additional free parameters



$$\propto g_{\omega\rho\pi} k \left(\frac{\Lambda_{\omega\rho\pi} - m_\pi^{\text{phys},2}}{k^2 + \Lambda_{\omega\rho\pi}^2} \right)^2$$

scheme B



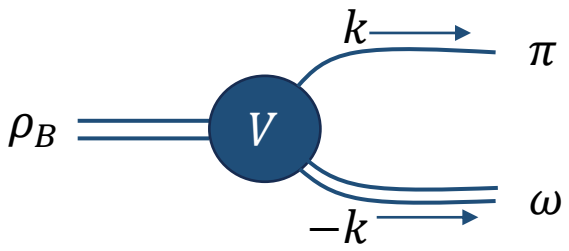
updated Hamiltonian (scheme C), but still not good ☹

3. rho meson on the lattice within Hamiltonian EFT

pion mass dependence of m_ρ^B is also messy ☹

$E_{\text{thr}}(\omega\pi) \sim 920$ MeV
$E_{\text{thr}}(K\bar{K}) \sim 1000$ MeV contribution from $|\omega\pi\rangle$?

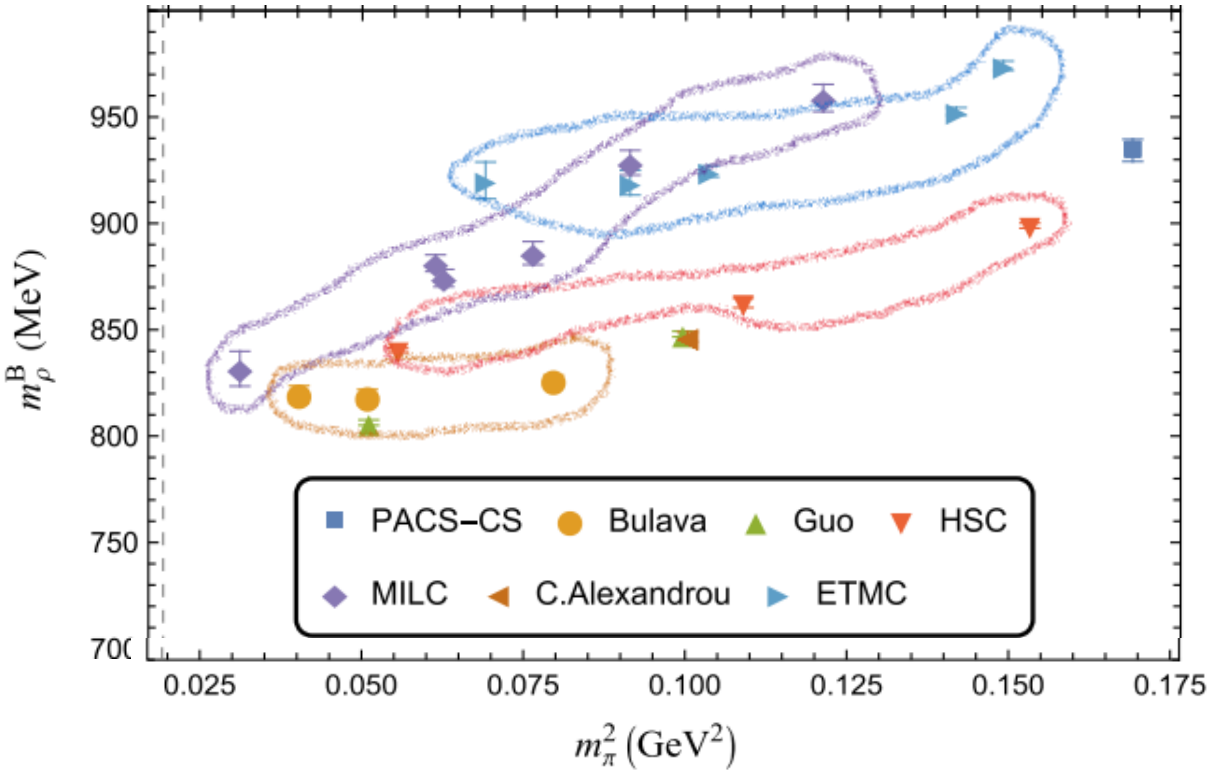
$|\omega\pi\rangle$ without introducing additional free parameters



$$\propto g_{\omega\rho\pi} k \left(\frac{\Lambda_{\omega\rho\pi} - m_\pi^{\text{phys},2}}{k^2 + \Lambda_{\omega\rho\pi}^2} \right)^2$$

updated Hamiltonian (scheme C), but still not good ☹

scheme B



intrinsic discrepancy may from ...

lattice spacing effect? different action? scale setting?...

4. Extrapolation within Hamiltonian EFT

m_ρ^B exhibit a linear relation to m_π^2 ?

From some previous work...

epjc/s2005-02118-0

$$M_\rho = M_\rho^0 + c_1 M_\pi^2 + c_2 M_\pi^3 + c_3 M_\pi^4 \ln\left(\frac{M_\pi^2}{M_\rho^2}\right) + \mathcal{O}(M_\pi^4)$$

identified as bare rho mass

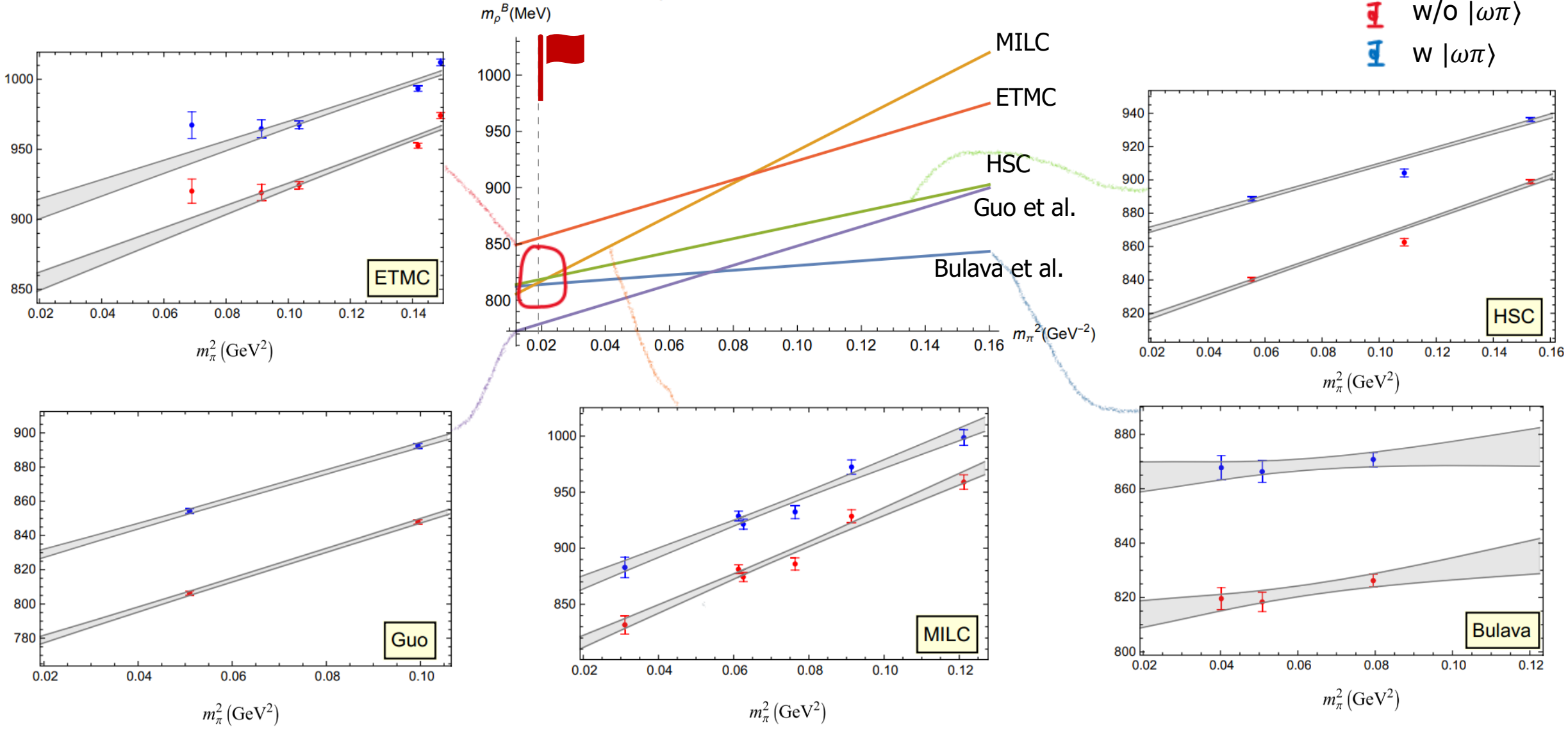
contributed by self-energy

In our model, at LO $m_\rho^{\mathrm{B}}(m_\pi) = c_0 + c_1 m_\pi^2.$ $m_\rho^{\mathrm{B}}(m_\pi; a) = c_0 + c_1 m_\pi^2 + \xi a^2,$

work flow $c_0, c_1 \longrightarrow m_\rho^{B,\mathrm{phys}} = m_\rho^B\left(m_\pi^{\mathrm{phys}}\right) \longrightarrow F = F(m_\rho^{B,\mathrm{phys}};\ g_{\rho\pi\pi}, \Lambda_{\rho\pi\pi}, g_{\omega\rho\pi}, \Lambda_{\omega\rho\pi})$

4. Extrapolation within Hamiltonian EFT

$$m_\rho^{\mathrm{B}}(m_\pi) = c_0 + c_1 m_\pi^2.$$

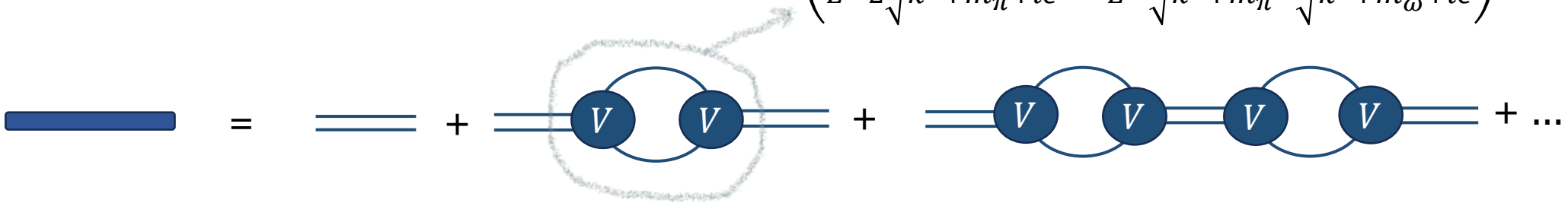


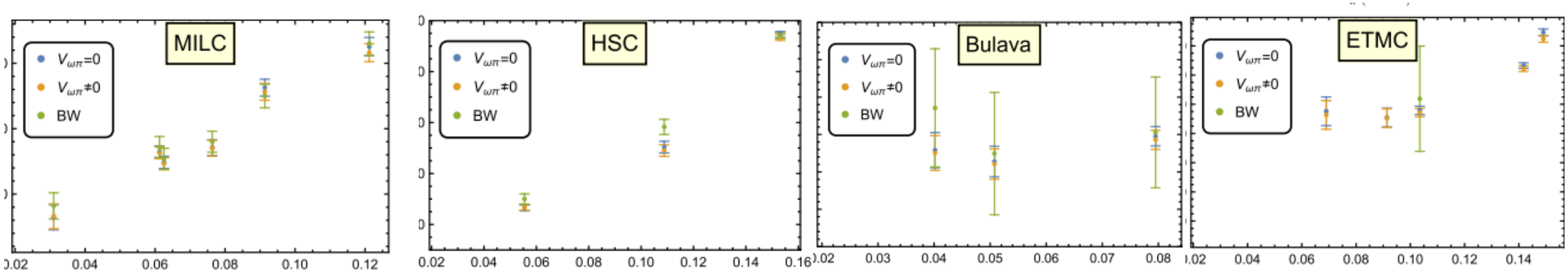
4. Extrapolation within Hamiltonian EFT

to get observables ...

pole mass of rho: $m_\rho^p = \Re(z_0),\ G^{-1}(z_0) = 0$ where z_0 is at unphysical Riemann sheet

- $G^{-1}(E) = E - m_\rho^B - \Sigma(E)$ with $\Sigma(E) = \int k^2 dk \left(\frac{|V_{\rho\pi\pi}(k)|^2}{E - 2\sqrt{k^2+m_\pi^2}+i\epsilon} + \frac{|V_{\omega\rho\pi}(k)|^2}{E - \sqrt{k^2+m_\pi^2} - \sqrt{k^2+m_\omega^2}+i\epsilon} \right)$





consistent with Breit-Wigner parametrization within error

4. Extrapolation within Hamiltonian EFT

to get observables ...

- $G^{-1}(E) = E - m_\rho^B - \Sigma(E)$ with $\Sigma(E) = \int k^2 dk \left(\frac{|V_{\rho\pi\pi}(k)|^2}{E - 2\sqrt{k^2+m_\pi^2}+i\epsilon} + \frac{|V_{\omega\rho\pi}(k)|^2}{E - \sqrt{k^2+m_\pi^2} - \sqrt{k^2+m_\omega^2}+i\epsilon} \right)$

  pole mass of rho: $m_\rho^p = \Re(z_0),\ G^{-1}(z_0) = 0$

pole mass extrapolated to physical point

Collaboration	c_0(MeV)	c_1(GeV^{-1})	$m_\rho^{\mathrm{B}}(\mu_\pi)$	$m_\rho^{\mathrm{p}}(\mu_\pi)$	
Bulava	809.8(7.0)	0.21(0.11)	814.0(5.0)	765.0(6.0)	w/o $\|\omega\pi\rangle$
	862.3(7.6)	0.11(0.12)	864.0(6.0)	765.0(6.0)	w $\|\omega\pi\rangle$
MILC	788.0(7.3)	1.45(0.10)	816.0(6.0)	768.0(6.0)	
	843.3(7.95)	1.32(0.11)	869.0(6.0)	769.0(6.0)	
HSC	806.7(1.71)	0.60(0.02)	818.2(1.4)	770.6(1.7)	
	861.3(1.9)	0.49(0.02)	870.7(1.6)	771.3(1.7)	
ETMC	838.9(7.7)	0.85(0.06)	855.0(7.0)	814.0(8.0)	
	892.5(8.3)	0.75(0.06)	907.0(7.0)	809.0(7.0)	
Guo	762.2(2.9)	0.86(0.04)	778.8(2.3)	719.3(2.6)	
	813.8(3.2)	0.79(0.04)	829.0(2.5)	719.0(2.6)	

PDG ...

$\rho(770)$ T-MATRIX POLE $\sqrt{s}$

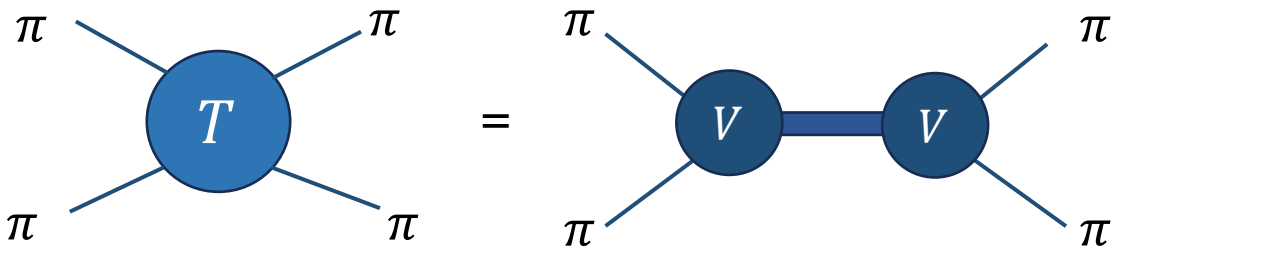
VALUE (MeV)

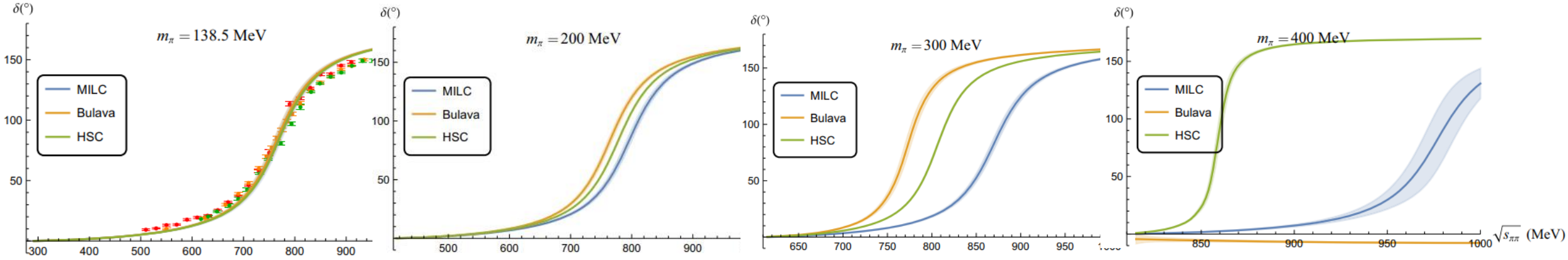
(761–765) $-$ i (71–74)

4. Extrapolation within Hamiltonian EFT

to get observables ...

- phase shift / t-matrix $e^{i\delta(E)} - 1 \propto t_{\pi\pi\to\pi\pi}(E) = \frac{|V_{\rho\pi\pi}(\bar{k})|^2}{E - m_\rho^B - \Sigma(E)}$ with $\bar{k} = \sqrt{\frac{E^2}{4} - m_\pi^2}$





$m_\pi = 139$ MeV $\quad m_\pi = 200$ MeV $\quad m_\pi = 300$ MeV $\quad m_\pi = 400$ MeV

extrapolation (not fit)

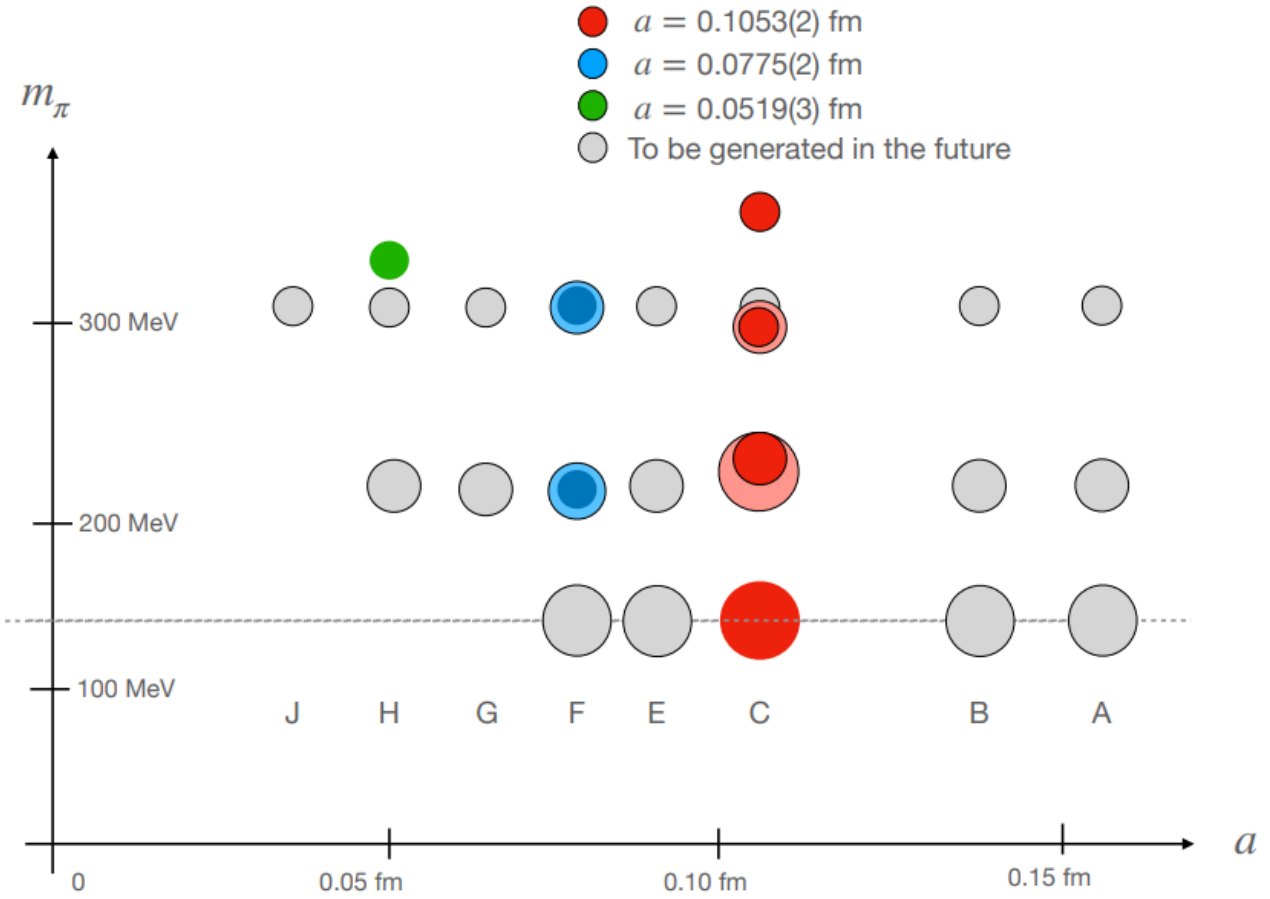
prediction

5. Summary

- many lattice spectra from different collabs are well fitted in our model with 1 parameter at each m_π .
- From fitting results, there are intrinsic discrepancies among different collabs.
  - lattice spacing, different action, scale-setting ...
- For each collab, m_ρ^B is linear to m_π^2
- Extrapolation is done individually for who gives at least two m_π .

6. Advertisement

- **rho meson on CLQCD is in preparation ...**

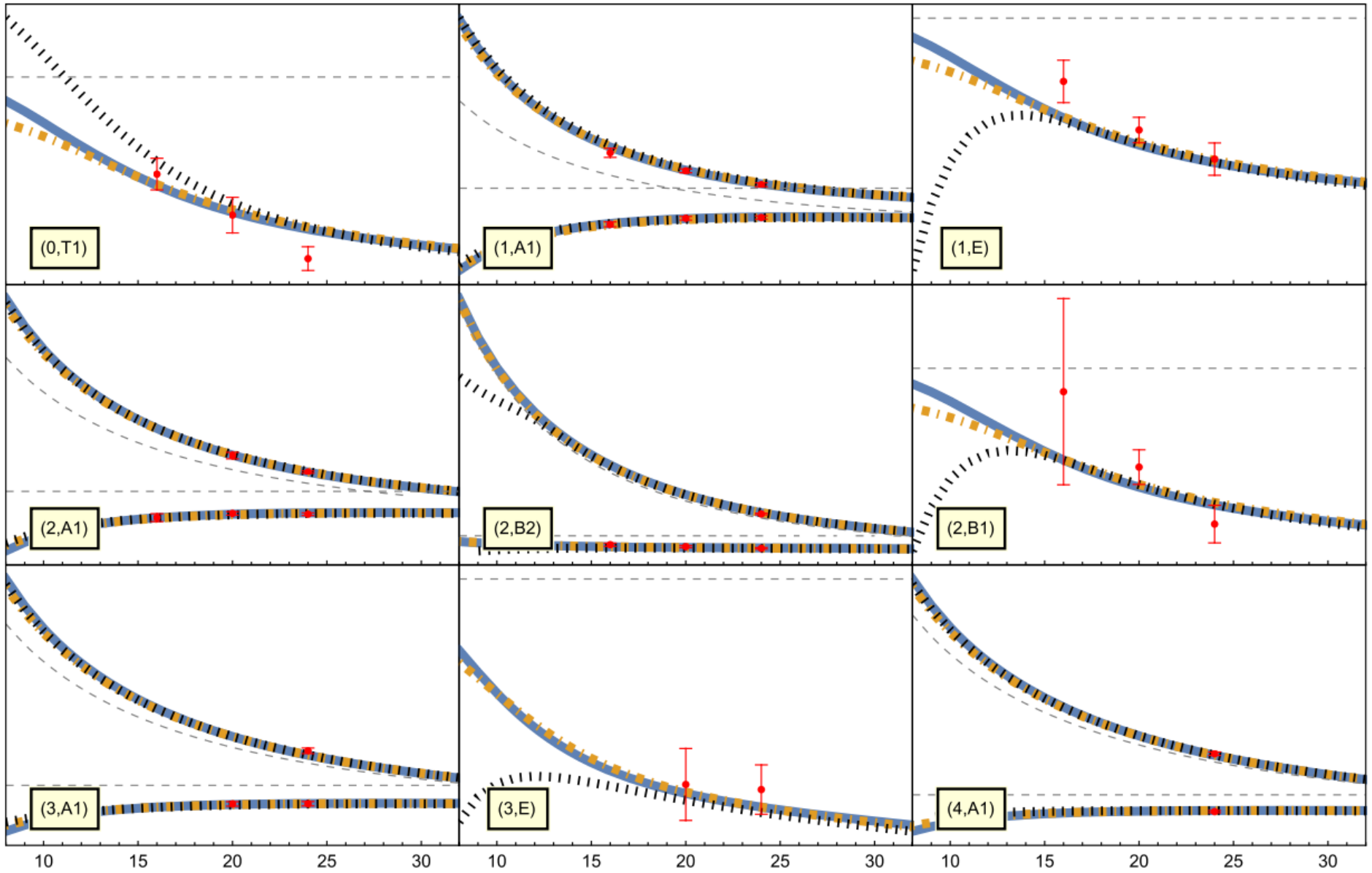


- **Hamiltonian EFT is developing to apply to three particles system ...**

Backup

lattice data fit

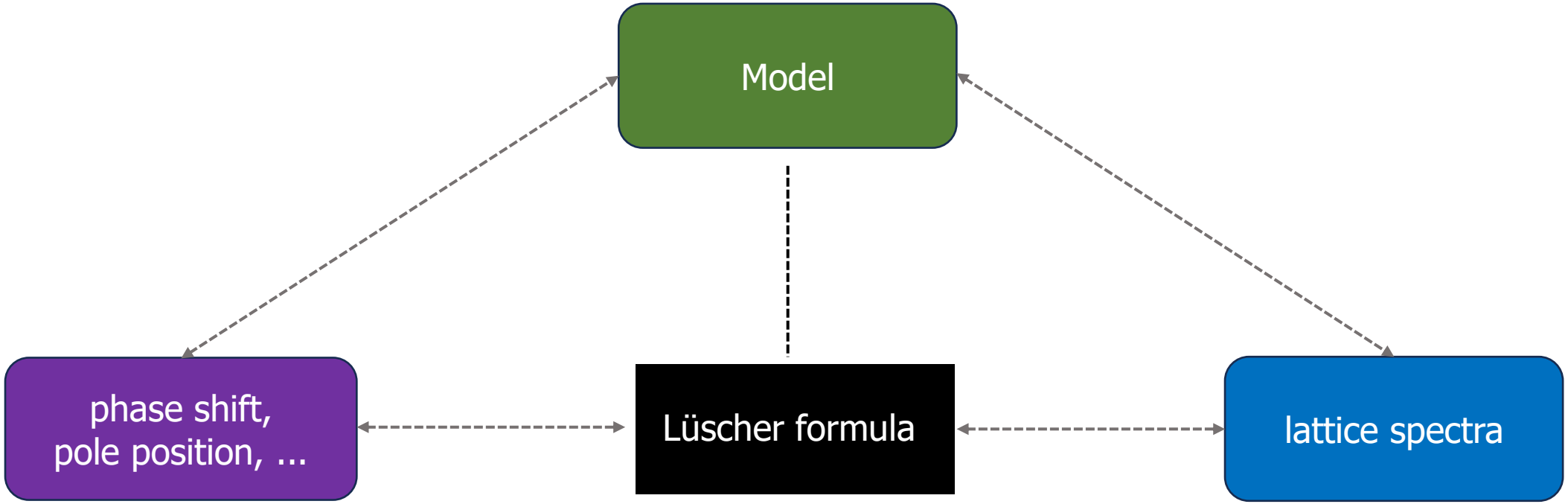
(HSC, 391)



(c) $m_\pi = 391$ MeV

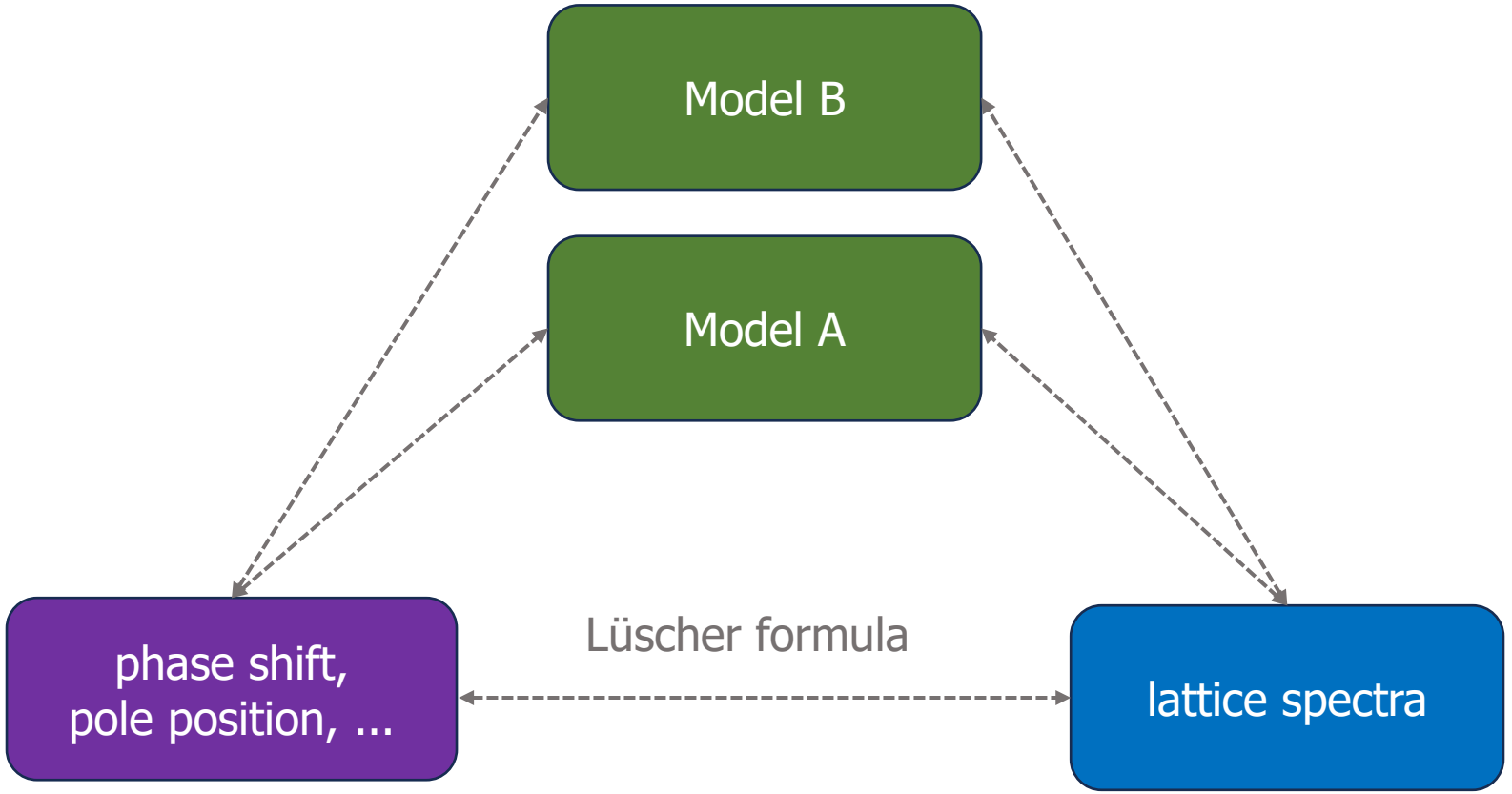
Backup

Relation to Lüscher Formula



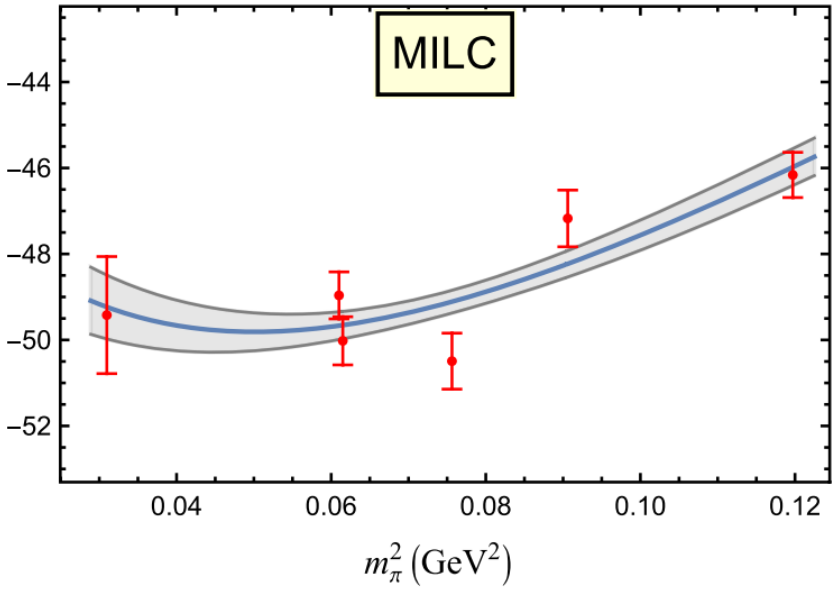
Backup

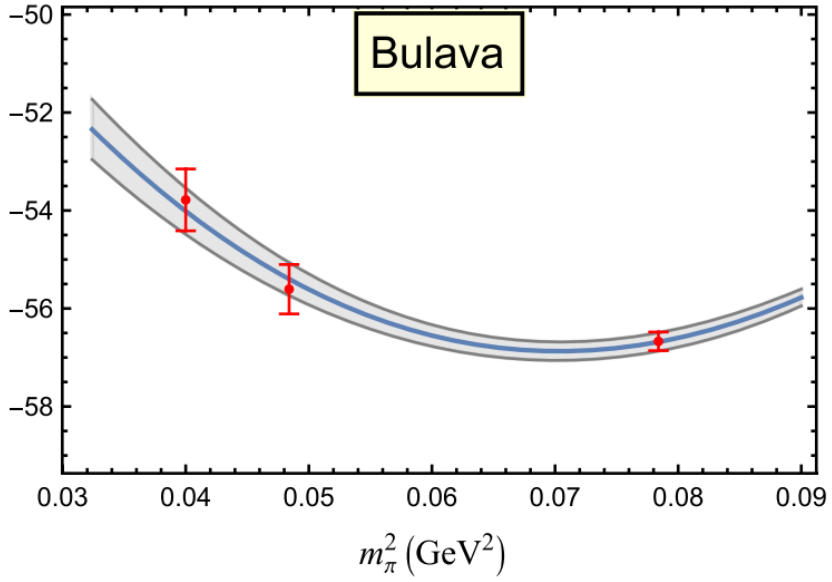
Model (in)dependence of Hamiltonian EFT

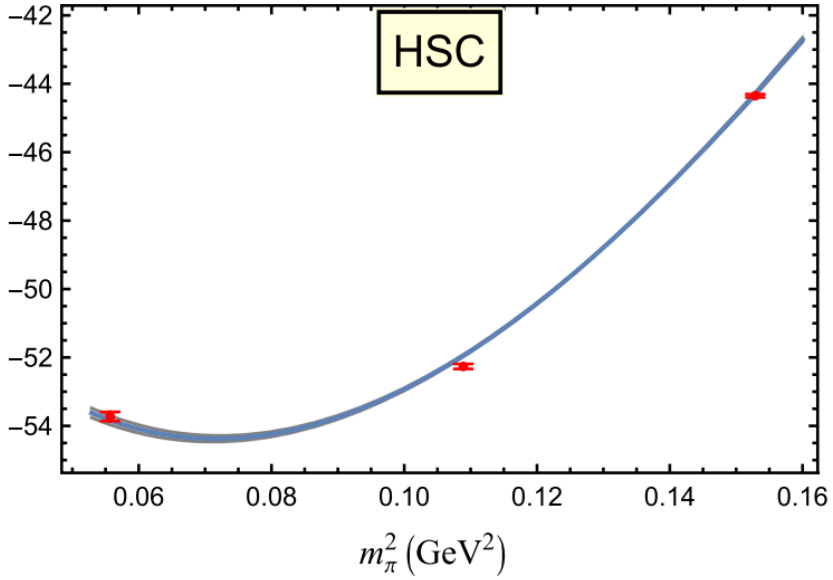


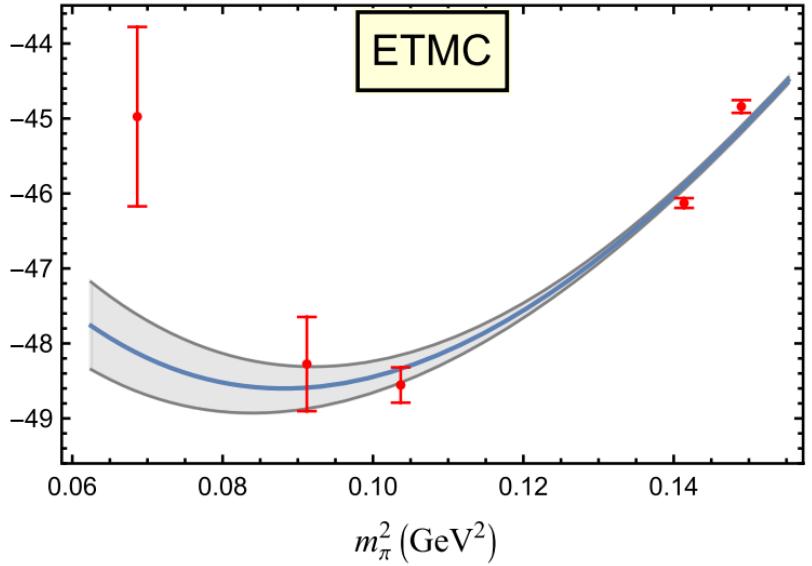
Backup

Self-energy correction





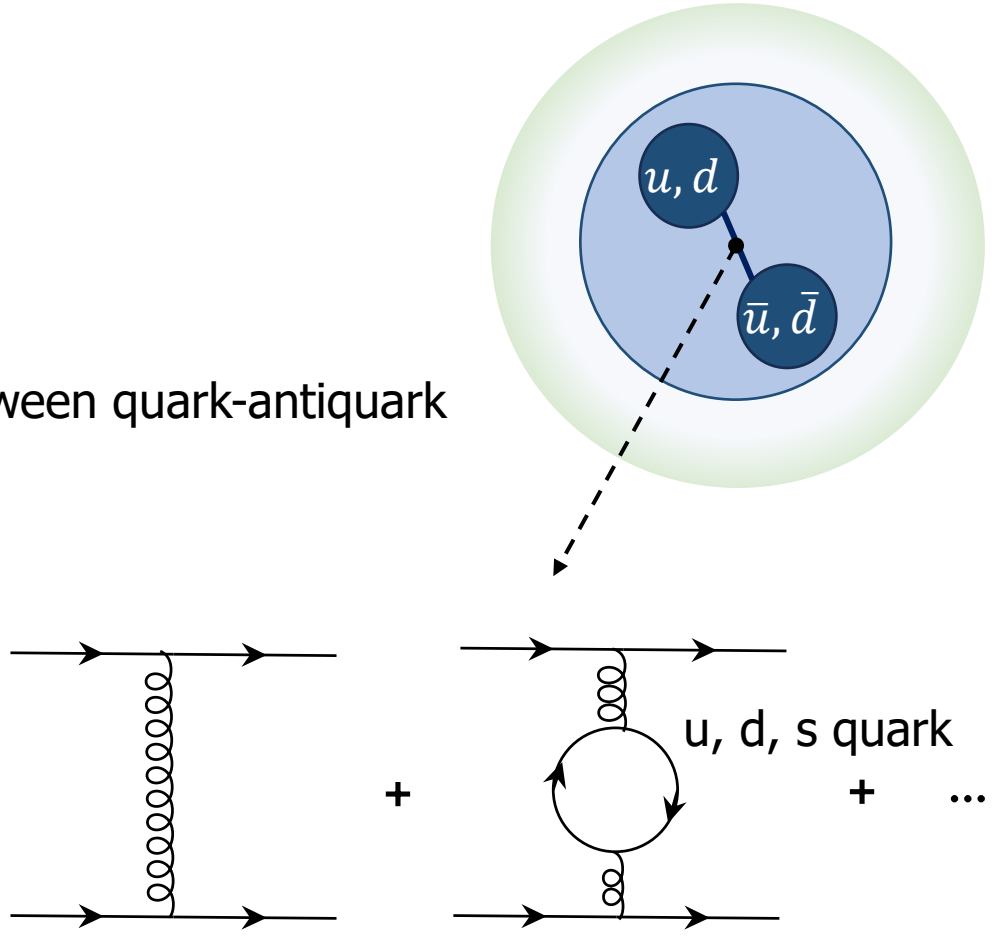




Backup

Role of strange quark

strange sea quark can affect the potential between quark-antiquark



Backup

lattice setups

Collaboration (year)	m_π(MeV)	N_f	L(fm)	a(fm)	Action	N_{lvl}	Method
Bulava (2018)	200	$2+1$	4.1	0.06	Improved Lüscher-Weisz gauge improved Wilson fermion	17	GEVP
	220		4.1	0.09		21	
	280		3.1	0.06		15	
HSC (2013)	391	$2+1$	1.9	0.12	Symanzik-improved gauge anisotropic Clover fermion	7	GEVP
			2.4	0.12		10	
			2.9	0.12		14	
HSC (2015)	236		3.8	0.12		23	
HSC (2023)	330		2.8	0.12		17	
MILC (2016)	176	$2+1$	5.4	0.09	Improved Lüscher-Weiss gauge staggered fermion	9	GEVP
	247		3.4	0.09		9	
	248		3.4	0.09		9	
	301		2.7	0.09		9	
	346		2.4	0.09		7	
	276		3.7	0.12		9	
ETMC (2020)	322	$2+1+1$	2.8	0.09	Iwasaki gauge twisted-mass Wilson fermion	18	GEVP
	386		2.1	0.09		16	
	262		2.6	0.08		13	
	302		3.9	0.08		23	
	376		2.6	0.08		14	
Alexandru (2017)	316	$2+1$	3.6	0.11	Symanzik-improved gauge clover Wilson fermion	15	GEVP
PACS-CS (2011)	411	$2+1$	2.9	0.09	Iwasaki gauge improved Wilson fermion	6	Exp Fit
Guo (2016)	226	2	2.9	0.12	Lüscher-Weiss gauge nHYP-smeared Clover fermion	8	GEVP
	315					20	