Three-meson scattering amplitudes with physical quark masses

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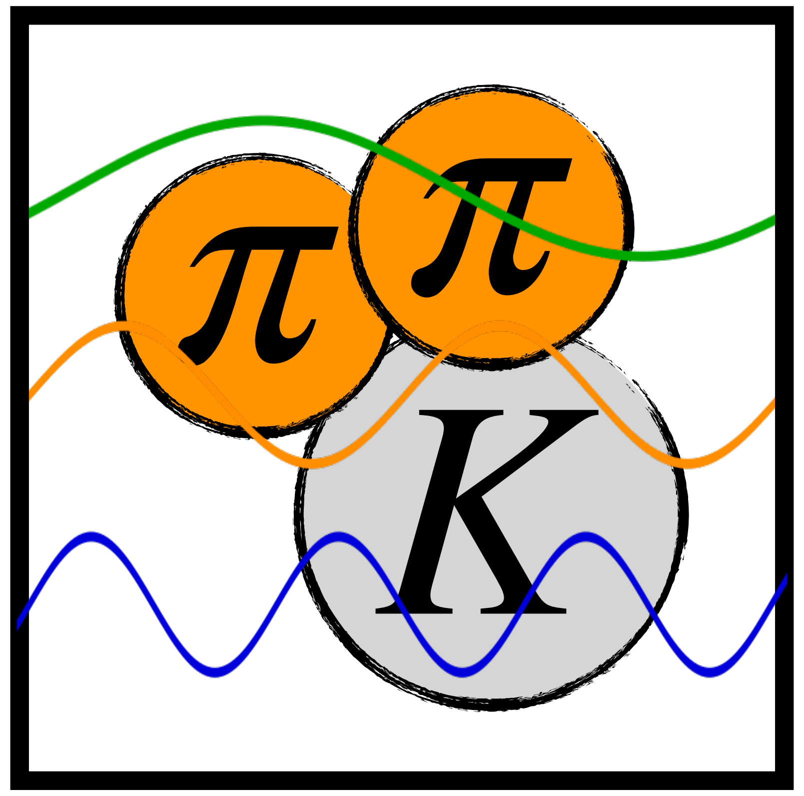

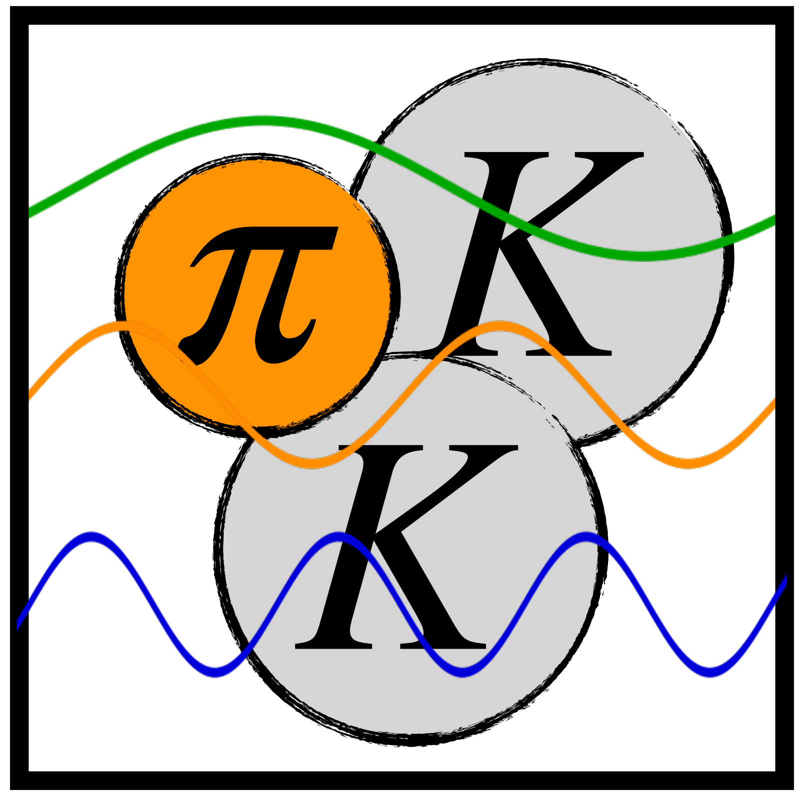

Work in progress with

Zack Draper (UW)

Sebastian Dawid (UW)

Andrew Hanlon (CMU)

Colin Morningstar (CMU)

Ben Hörz (Intel)

Sarah Skinner (CMU)

Steve Sharpe (UW)

The three-hadron frontier

Many reasons to study the three-hadron problem from lattice QCD

- Resonances that decay to three (or more) hadrons
  - **Exotics:** $T_{cc} \to DD^*, DD\pi$
  - 3π **resonances:** $\omega(782)$, $h_1(1170)$, $a_1(1260)\ldots$
  - **Roper:** $N(1440) \to \Delta\pi \to N\pi\pi$
- Interest in three-baryon forces: NNN, NNY
- Electroweak processes $K \to 3\pi, \quad K^0 \leftrightarrow 3\pi \leftrightarrow \overline{K}^0, \quad \gamma \to 3\pi$

- ☑ Major developments in the three-particle finite-volume formalism
  [Hansen, Sharpe, PRD 2014 & 2015], [Hammer, Pang, Rusetsky, JHEP 2017] x 2
  [Mai, Döring, EPJA 2017] [...]

- ☑ See other related talks at this conference:
  [A. Alotaibi, S. Dawid, W. Schaaf, M. Sjö, S. Sharpe, H. Yan, ...]

Three-meson systems

Important benchmark system: three pseudoscalar mesons at maximal isospin

- Implement formalism and explore its features
- Test fitting strategies to extract three-body K matrix
- Interpret results in combination with EFTs
- Investigate features of scattering amplitudes

$$3\pi^+, \quad 3K^+, \quad \pi^+\pi^+K^+, \quad K^+K^+\pi^+$$

[Blanton ... FRL ... et al., PRL 2020 & JHEP 2021]
[Draper ... FRL ... et al., JHEP 2023],
[Fischer ... FRL ... et al (ETMC), EPJC 2021]
[Alexandrou et al, Brett et al, Culver et al, Mai et al. (GWQCD)]
[Hansen et al (HadSpec)]

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This work:

- Extend previous calculations to the physical point
  [Blanton ... FRL ... et al., PRL 2020 & JHEP 2021]
  [Draper ... FRL ... et al., JHEP 2023]
- Compute physical three-meson scattering amplitudes
  [Hansen et al (HadSpec)]

	$(L/a)^3 \times (T/a)$	M_π[MeV]	M_K[MeV]	N_{cfg}
N203	$48^3 \times 128$	340	440	771
N200	$48^3 \times 128$	280	460	1712
D200	$64^3 \times 128$	200	480	2000
E250	$96^3 \times 192$	130	500	505

$$a \simeq 0.063 \text{ fm}$$

$$\text{tr } m_q = 2m_{ud} + m_s \simeq \text{const}$$

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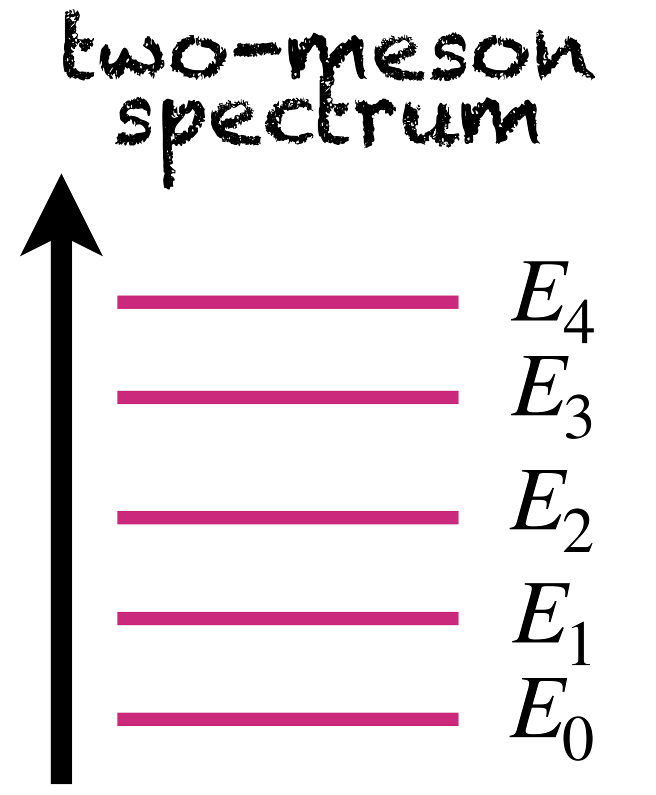
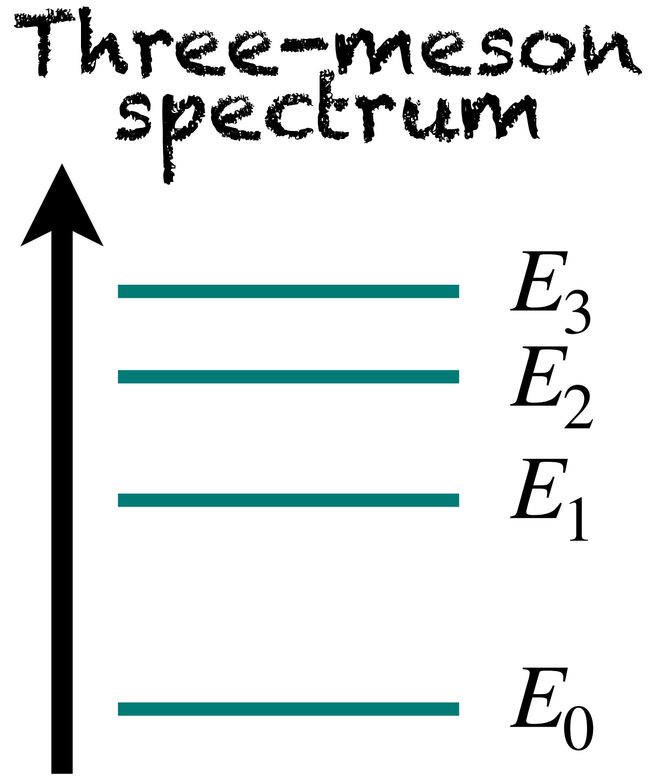
Formalism



two-meson spectrum
E_4
E_3
E_2
E_1
E_0



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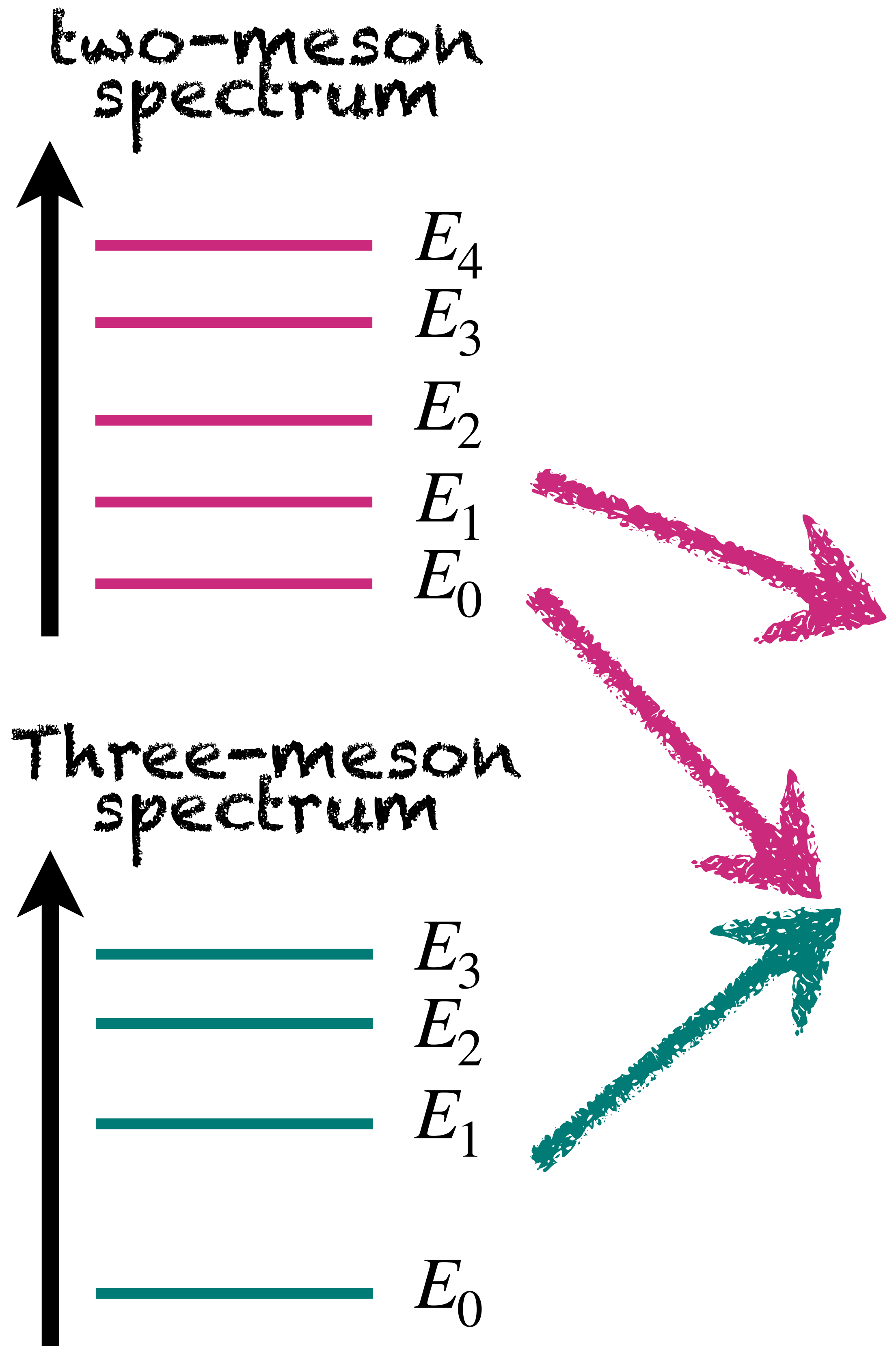
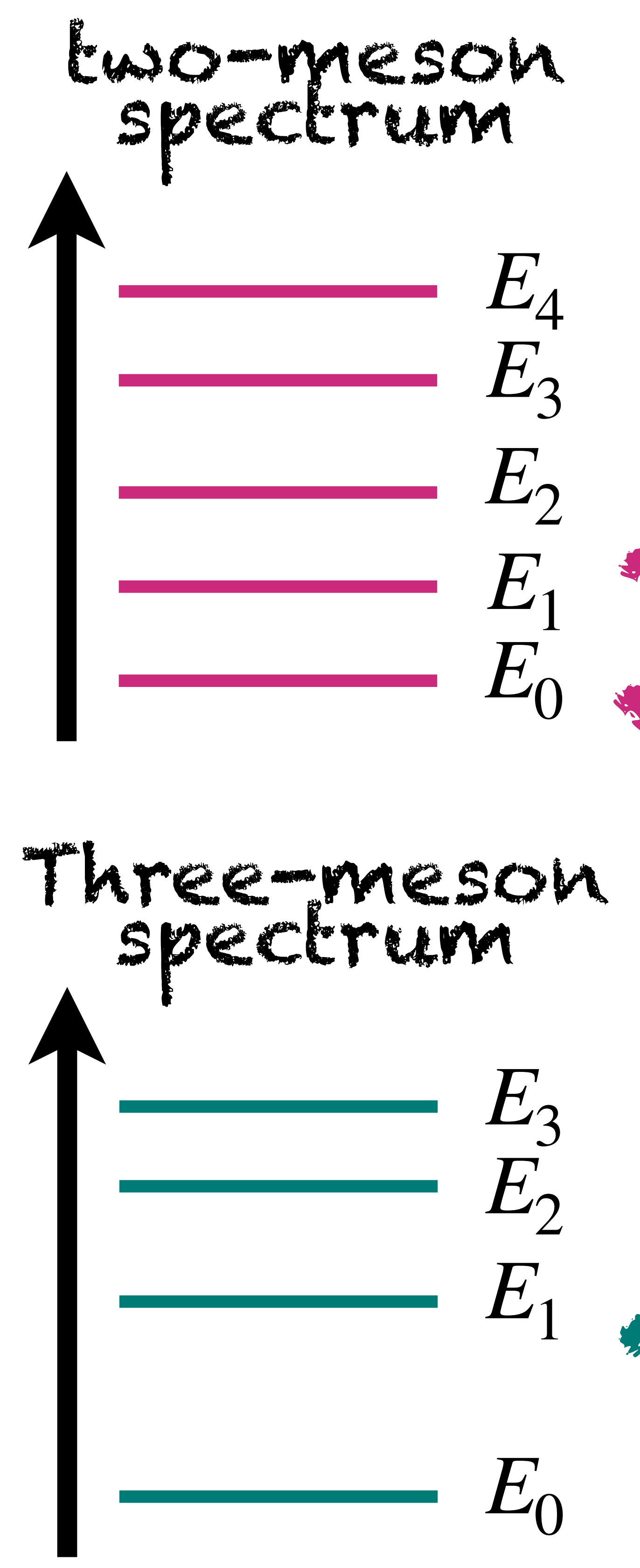


Quantization conditions

$$\det_{\ell m}\left[\mathcal{K}_2 + F_2^{-1}\right] = 0$$

$$\det_{k\ell m}\left[\mathcal{K}_{\mathrm{df},3} + F_3^{-1}\right] = 0$$

Formalism



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Fit →

K-matrices

$$\mathcal{K}_2$$

$$\mathcal{K}_{\mathrm{df},3}$$

Parametrize:

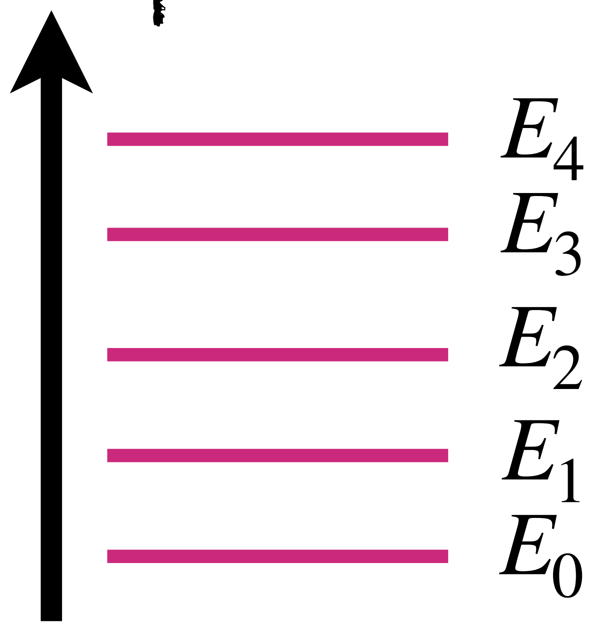
$$\mathcal{K}_2 = c_0 + c_1 k^2 + \ldots$$

$$\mathcal{K}_{\mathrm{df},3} = \mathcal{K}_{\mathrm{df},3}^{\mathrm{iso},0} + \mathcal{K}_{\mathrm{df},3}^{\mathrm{iso},1}\left(\frac{s-9m^2}{9m^2}\right) + \ldots$$

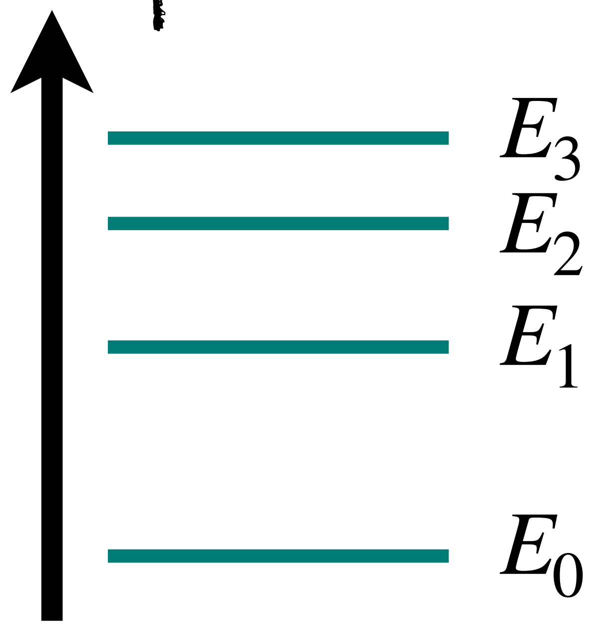
[Blanton, FRL, Sharpe, JHEP 2019]

Formalism

two-meson spectrum



Three-meson spectrum



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[Blanton, FRL, Sharpe, JHEP 2019]

Unitarity relations

Integral equations

[Briceño et al., PRD 2018]
[Hansen et al., PRL 2021]
[Jackura et al., PRD 2021]
[Dawid et al., 2303.04394]
[See talk by S. Dawid]

Scattering amplitudes

$$\mathcal{M}_2$$

$$\mathcal{M}_3$$

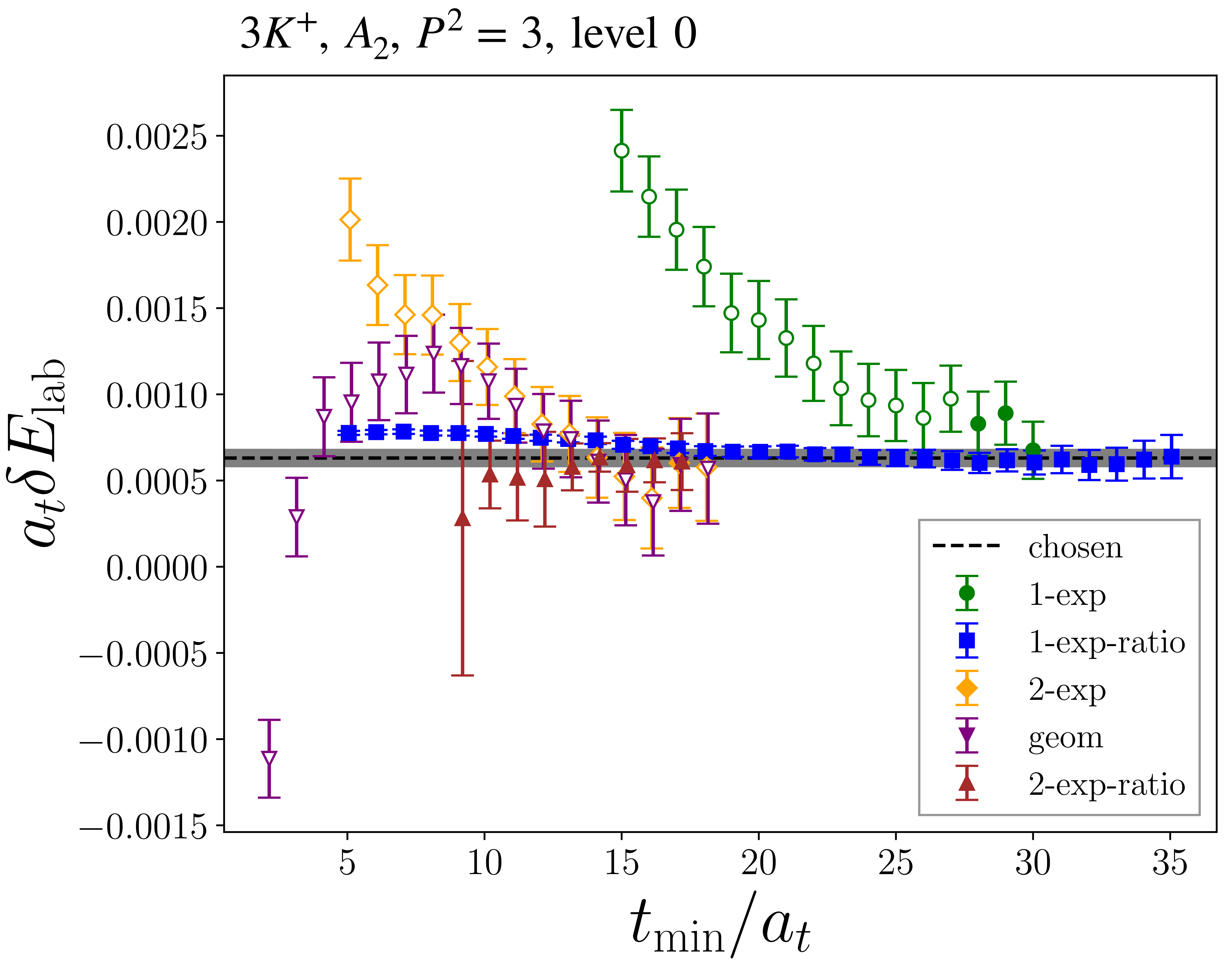
Extracting energies

- Stochastic LapH method, multi-hadron operators
  [Morningstar et al, 1104.3870]

- GEVP and look for consistency between methods.
  - Single and double exponential
  - Ratio fit with single and double exponential

- Use ratio fit to benefit from correlated cancellations

$$R_n(t) = \frac{C_{\text{three-meson}}(t)}{C_{\text{meson}}(t) C_{\text{meson}}(t) C_{\text{meson}}(t)}$$



Fitting the spectrum

- Requires a correlated fit for several systems at one. For instance: $\pi\pi K + \pi\pi + \pi K$
- Fit energy shifts in the lab frame ("spectrum method")

$$\chi^2(\vec{p}) = \sum_{ij} \left(\Delta E_{\mathrm{lab},i} - \Delta E^{\mathrm{QC}}_{\mathrm{lab},i}(\vec{p}) \right) \left(C^{-1} \right)_{ij} \left(\Delta E_{\mathrm{lab},j} - \Delta E^{\mathrm{QC}}_{\mathrm{lab},j}(\vec{p}) \right)$$

parameters in K-matrices

covariance matrix of lab-shifts

"predicted minus measured" lab-frame energy shifts

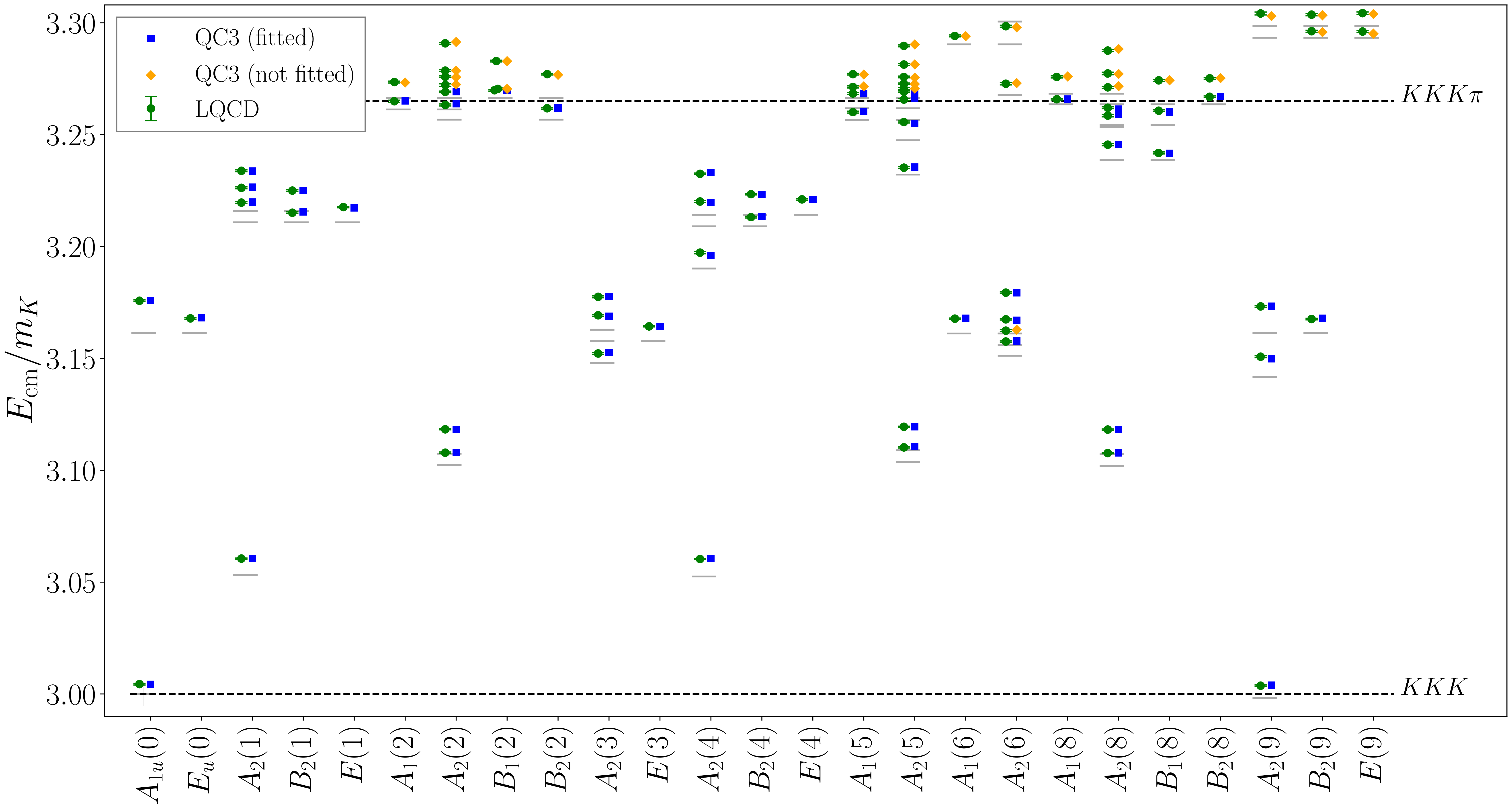
- Include several two-meson partial waves:
  - s and d waves for $\pi\pi$ and KK
  - s and p waves for πK
- Threshold expansion in three-body K matrix

$$\mathcal{K}_{\mathrm{df},3} = \mathcal{K}_0 + \mathcal{K}_1 \Delta + \mathcal{K}_2 \Delta^2 + \mathcal{K}_A \Delta_A + \mathcal{K}_B \Delta_B,$$

$$\Delta \equiv \frac{s - 9m^2}{9m^2}$$

Functions of Mandelstam variables

3K spectrum



$\chi^2/\mathrm{dof} = 1.49$

$\mathrm{dof} = 87, \;\; n_{\rm params} = 6$

$s + d$ waves

$\mathcal{K}_{\rm df,3} \neq 0, \;$ with $\; > 10\sigma$

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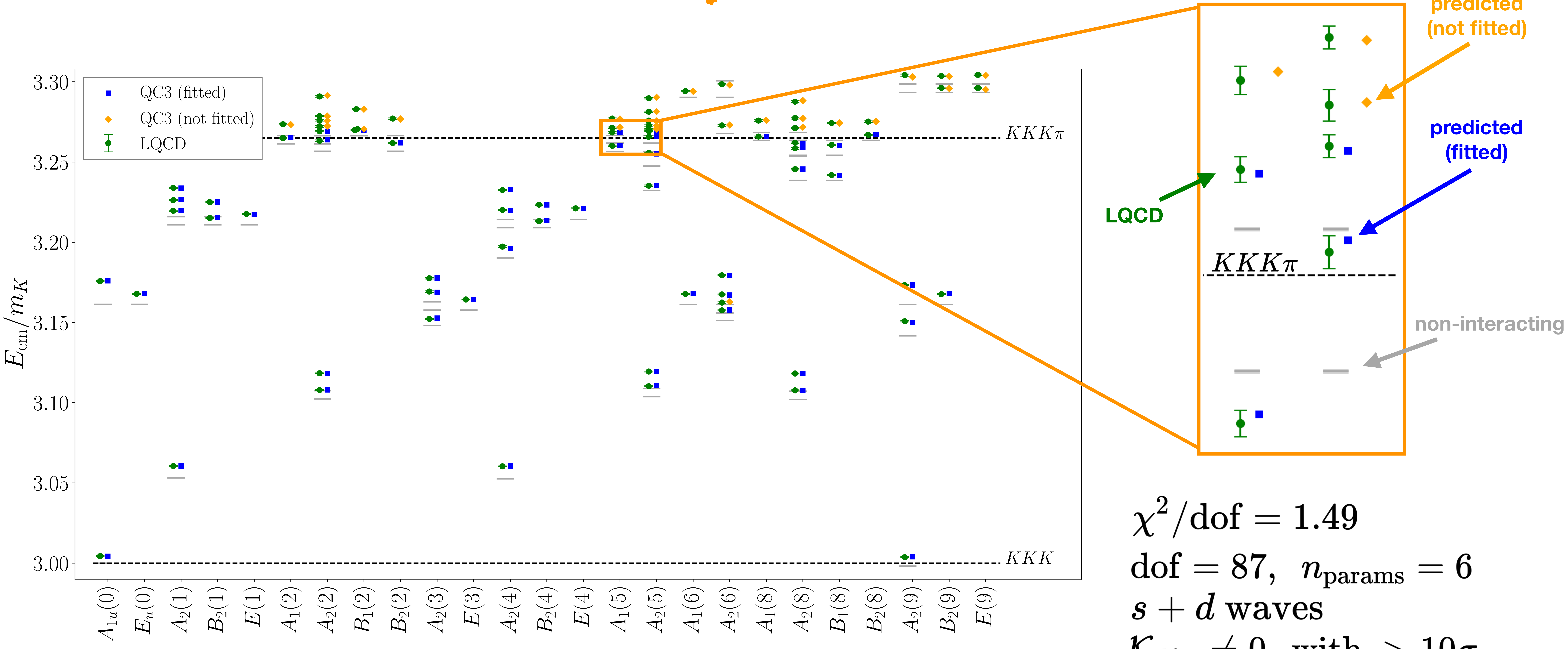
3K spectrum



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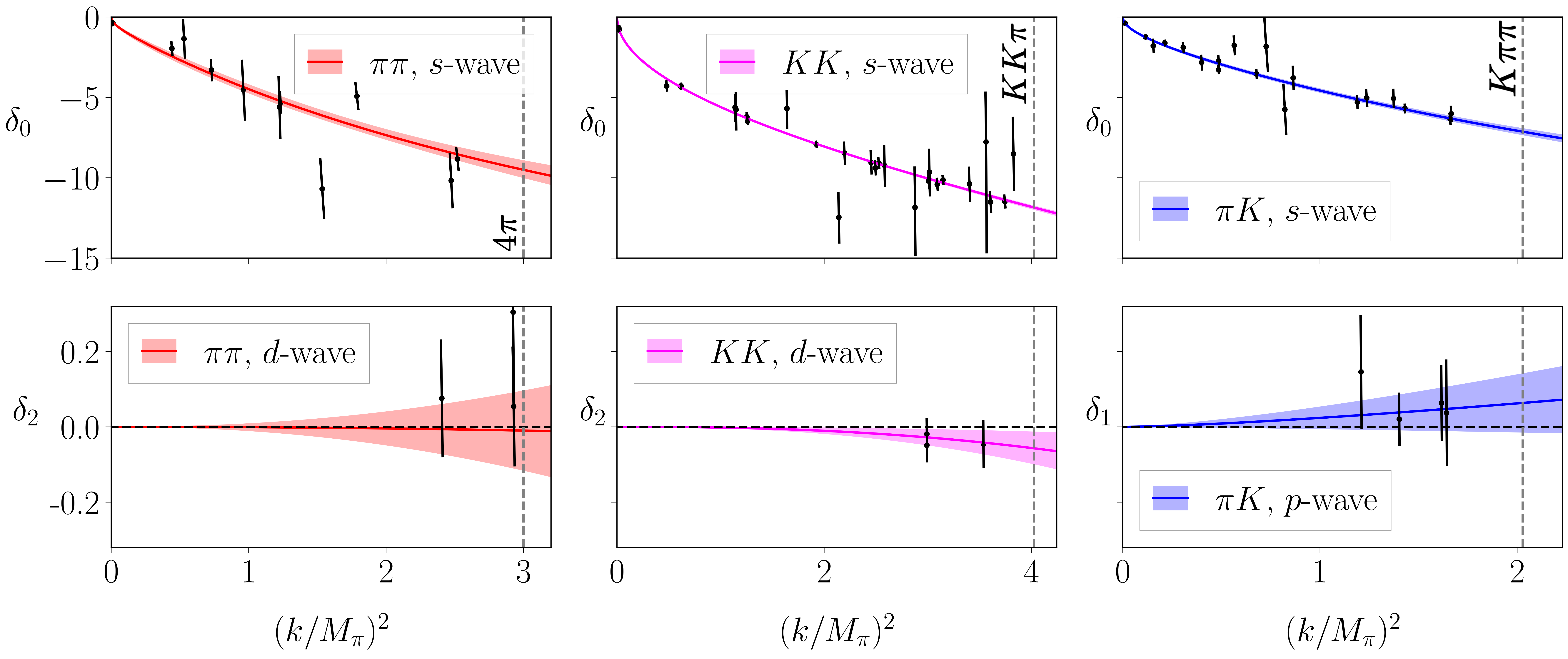
Two-body phase shift

- Required input for three-meson calculations



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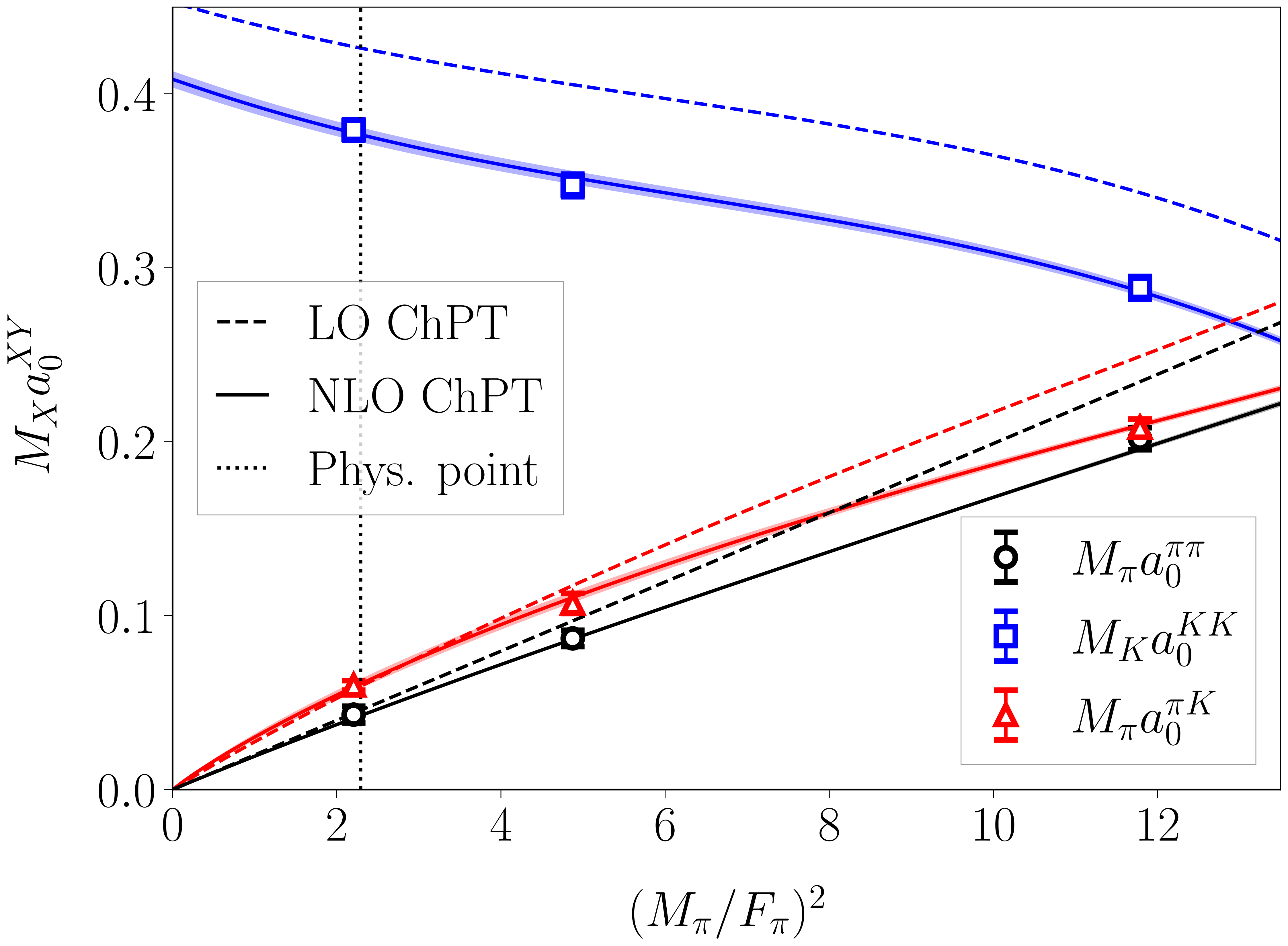
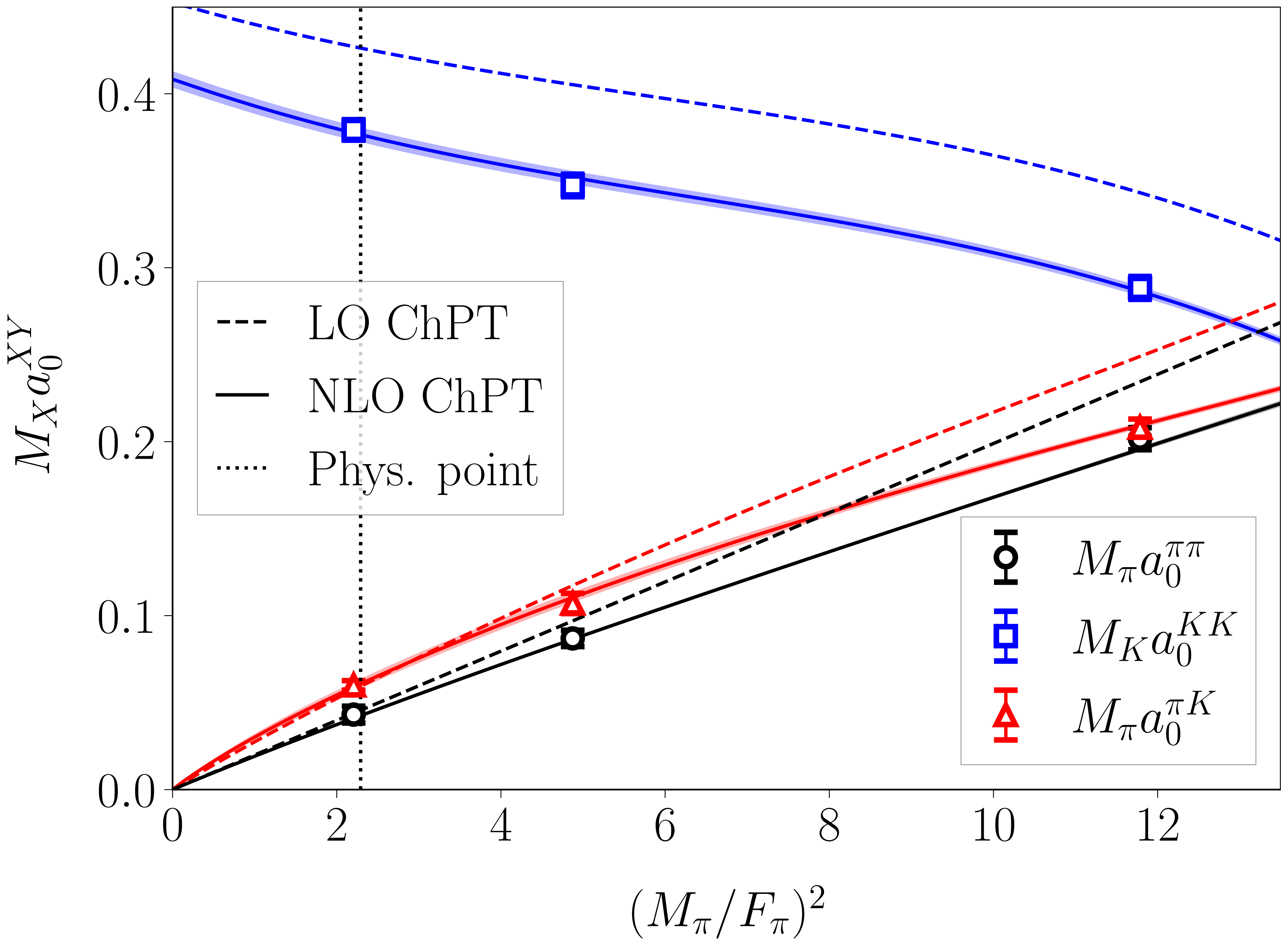
Scattering lengths

s-wave scattering lengths

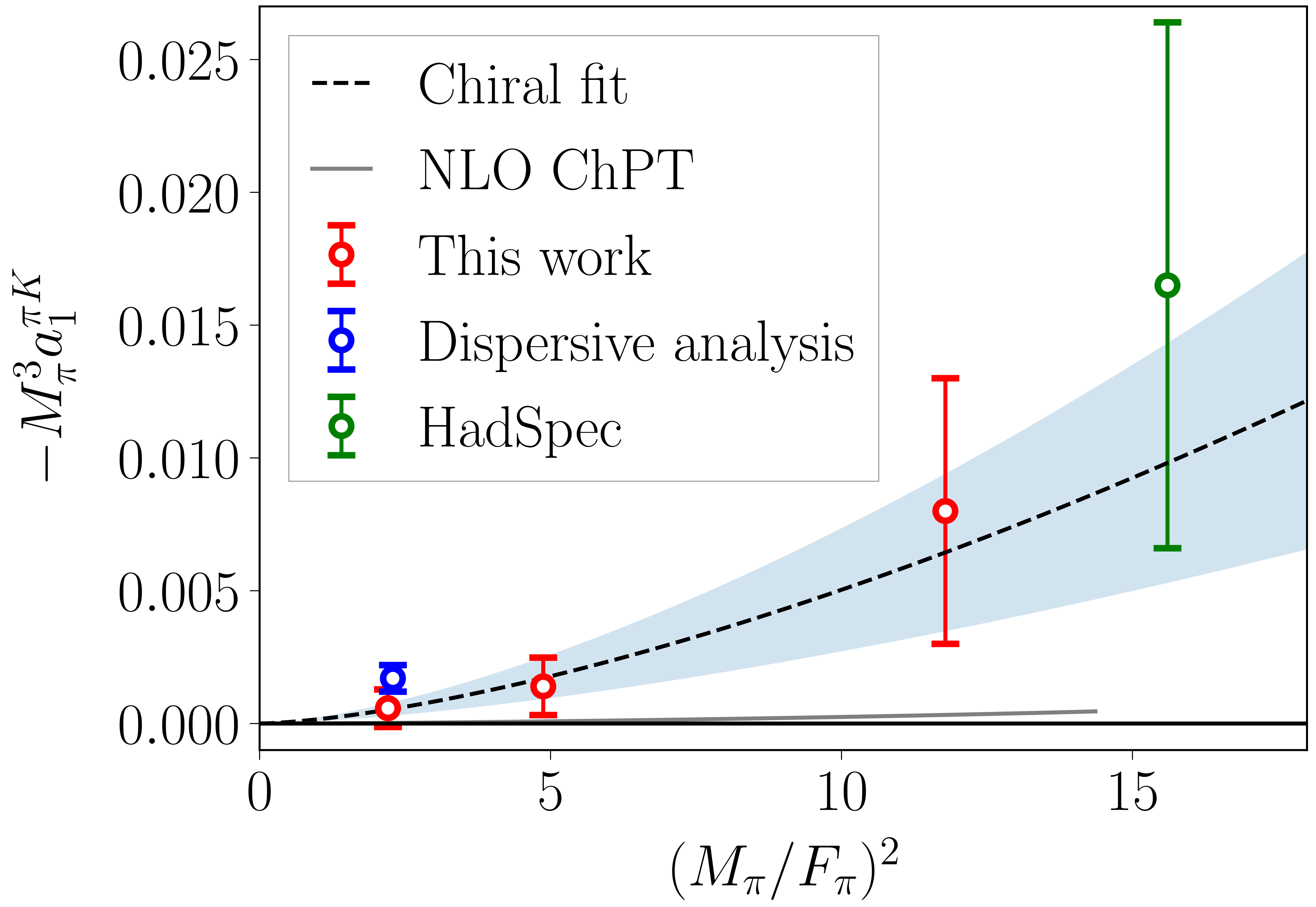


Scattering lengths

s-wave scattering lengths



p-wave πK scattering lengths



Dispersive results: [Pelaez, Rodas, 2010.11222]

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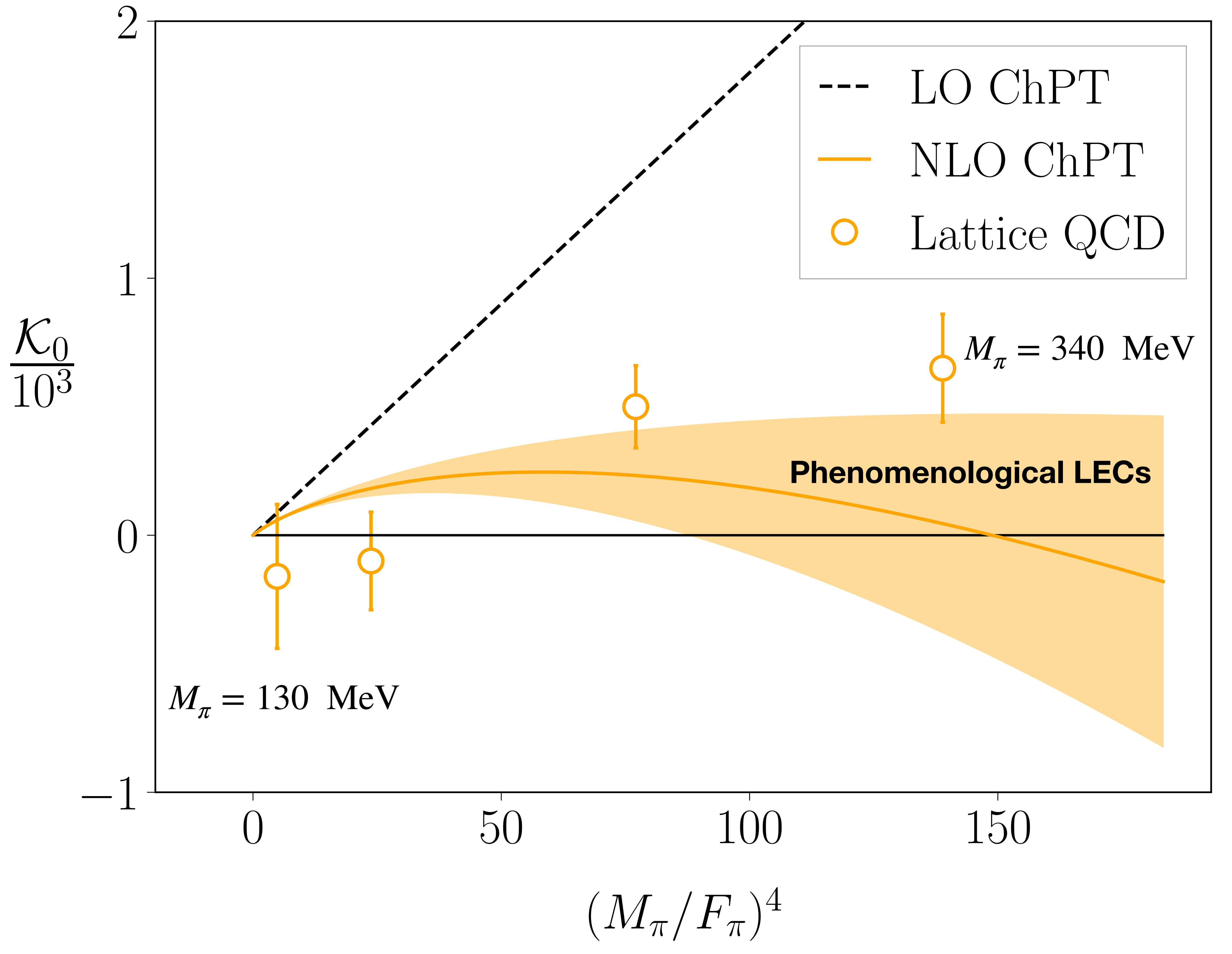
Three-pion K matrix

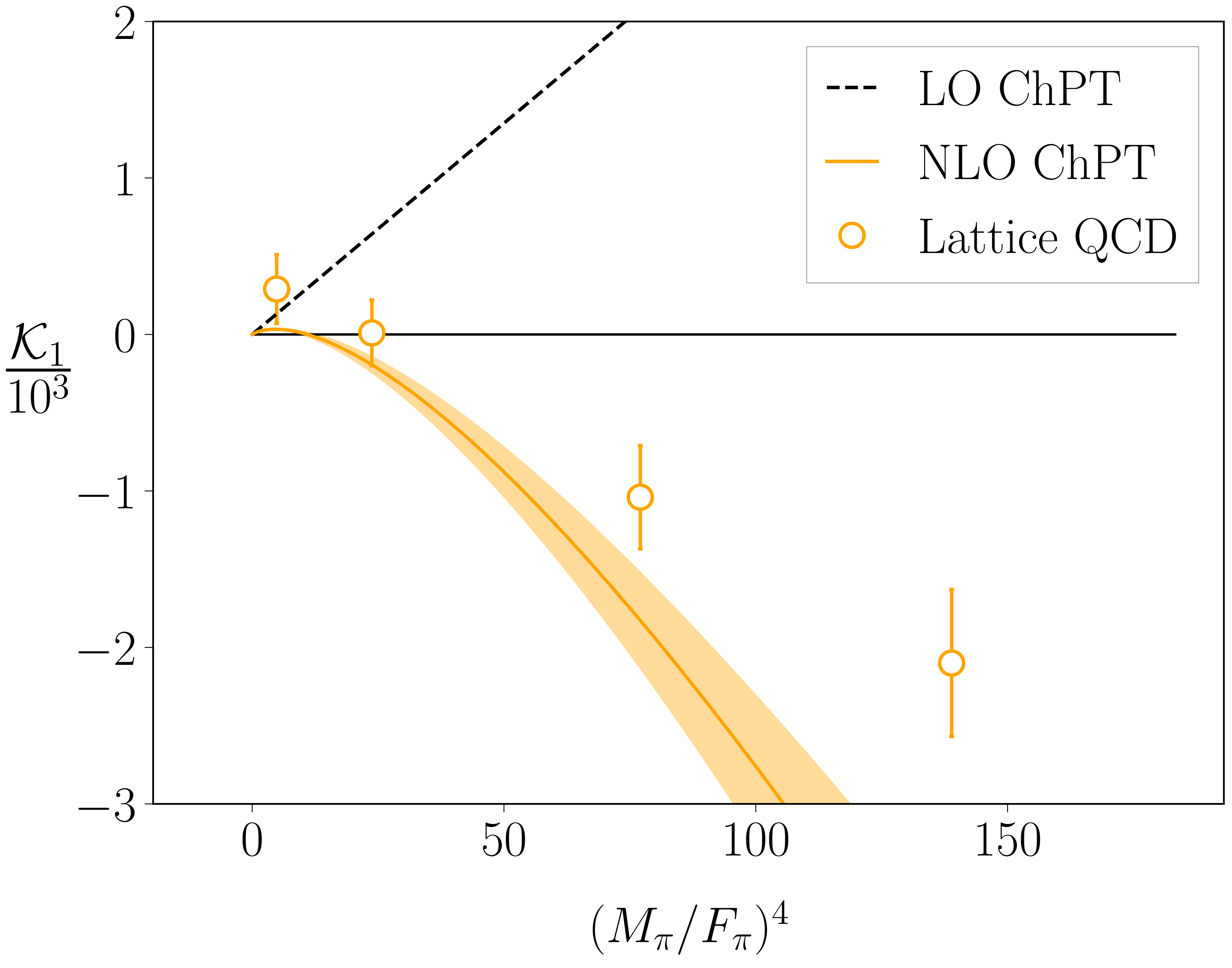
- Compare to chiral perturbation theory

NLO ChPT: [Baeza-Ballesteros, Bijnens, Husek, FRL, Sharpe, Sjö, JHEP 2023]

[See talk by M. Sjö]

$$\mathcal{K}_{\mathrm{df},3} = \mathcal{K}_0 + \mathcal{K}_1\left(\frac{s - 9M_\pi^2}{9M_\pi^2}\right) + \cdots$$





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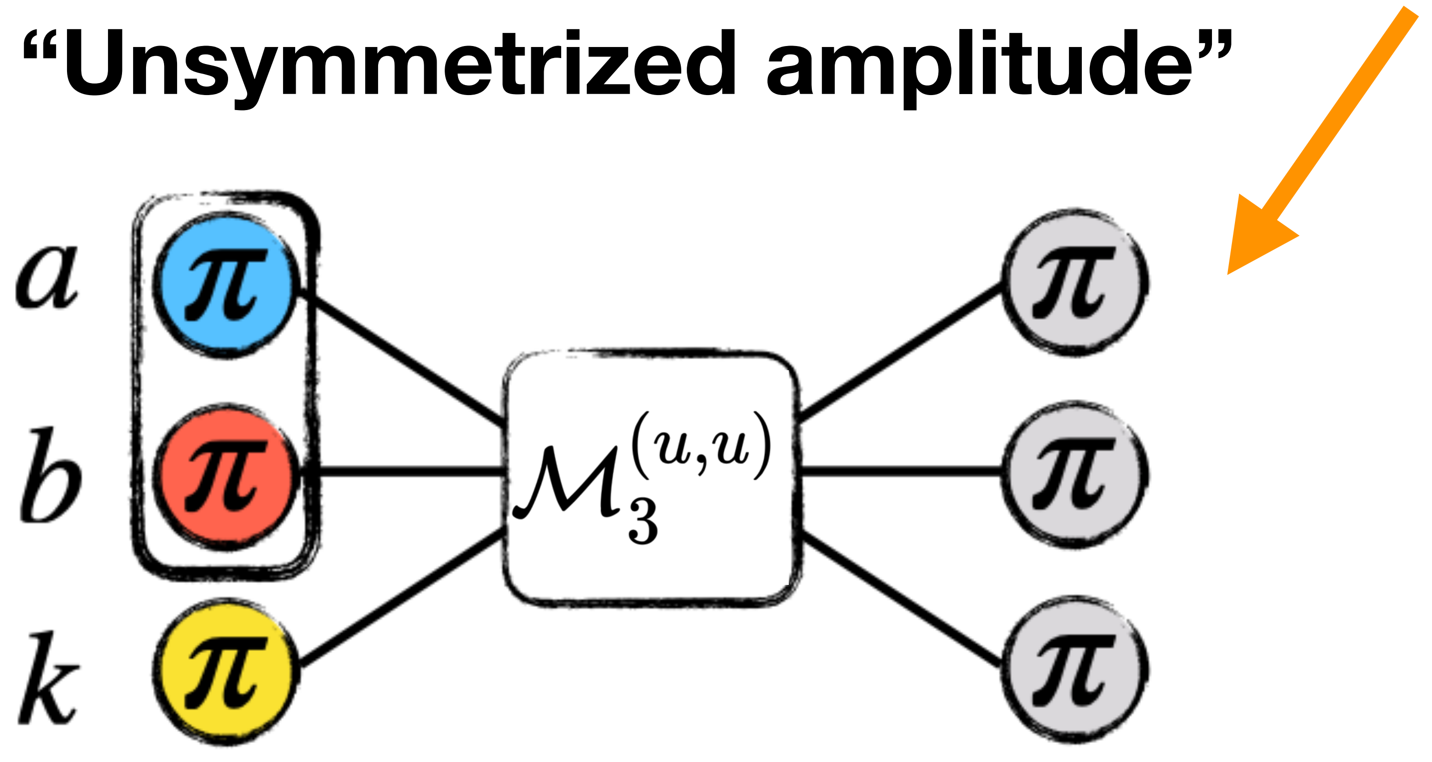
Scattering amplitudes

- Physical amplitudes can be obtained after solving integral equations:

$$\mathcal{M}_3^{(u,u)}$$

"Unsymmetrized amplitude"



- Choice of spectator is fixed
- Needs "symmetrization"

$$\mathcal{M}_3 = \mathcal{S}\left[\mathcal{M}_3^{(u,u)}\right]$$

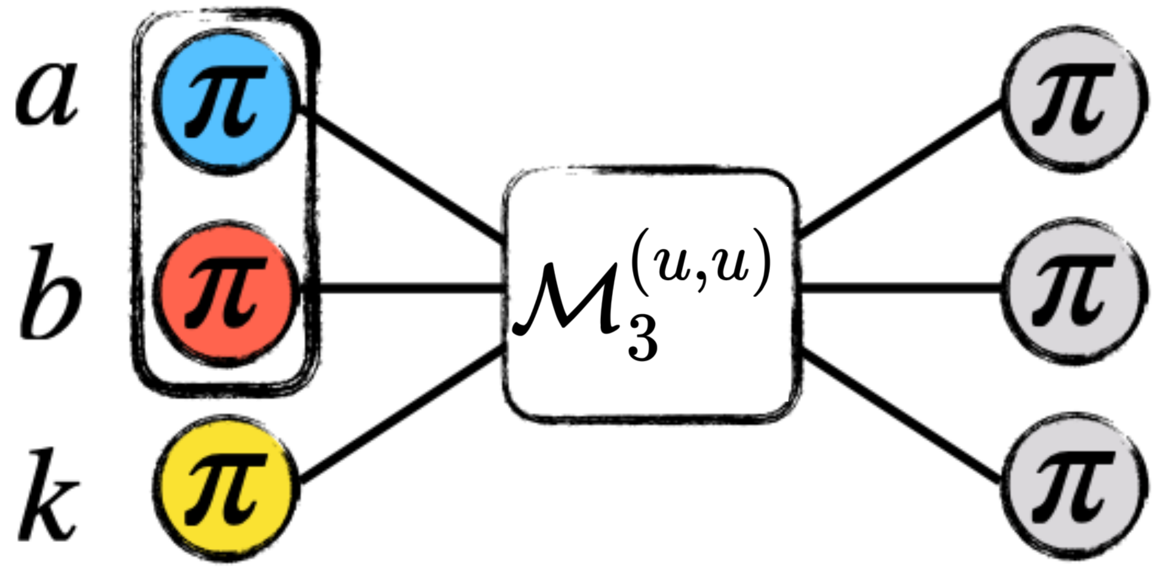
Scattering amplitudes

- Physical amplitudes can be obtained after solving integral equations:

Partial-wave projection
[Jackura, Briceño, 2312.00625]

$$\mathcal{M}_3^{(u,u)} = \mathcal{D}^{(u,u)} + \mathcal{M}_{\text{df},3}^{(u,u)}$$

"Unsymmetrized amplitude"



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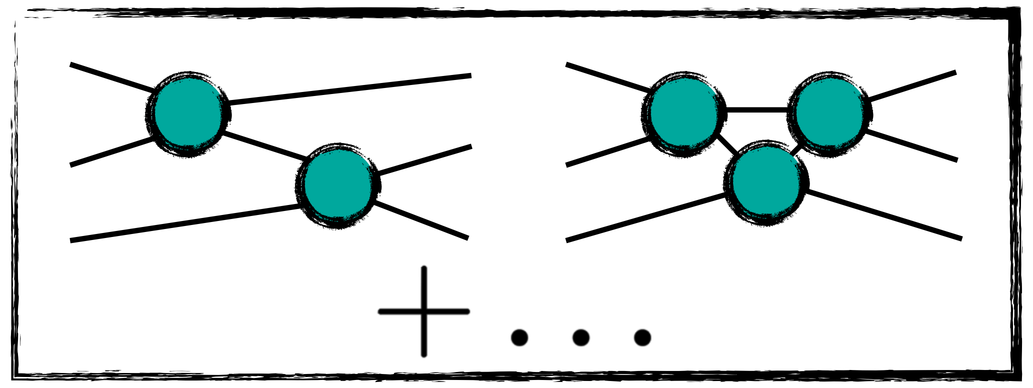
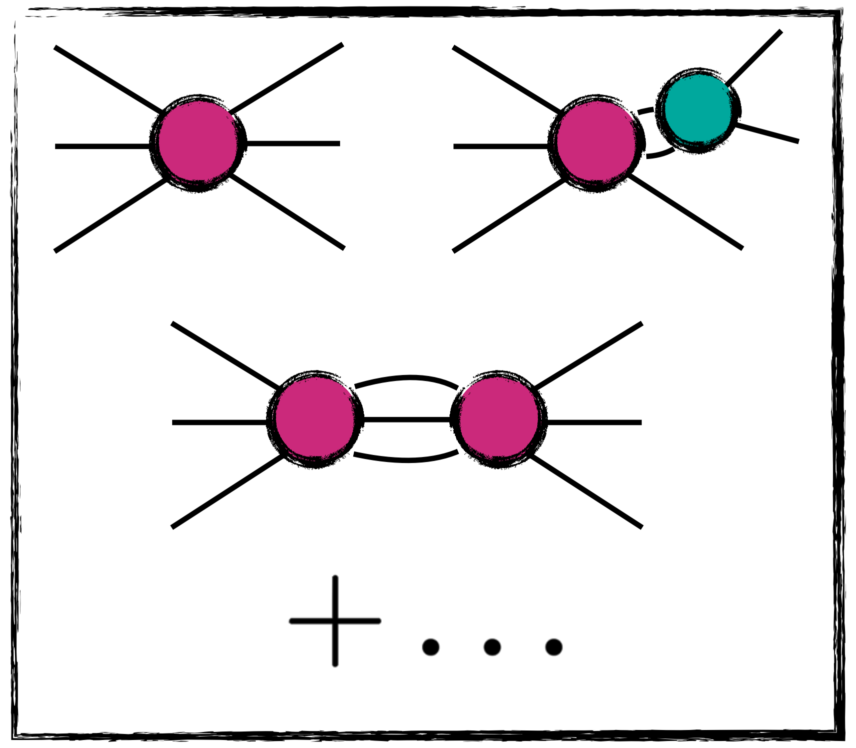
$$\mathcal{M}_3 = \mathcal{S}\left[\mathcal{M}_3^{(u,u)}\right]$$

"ladder amplitude"



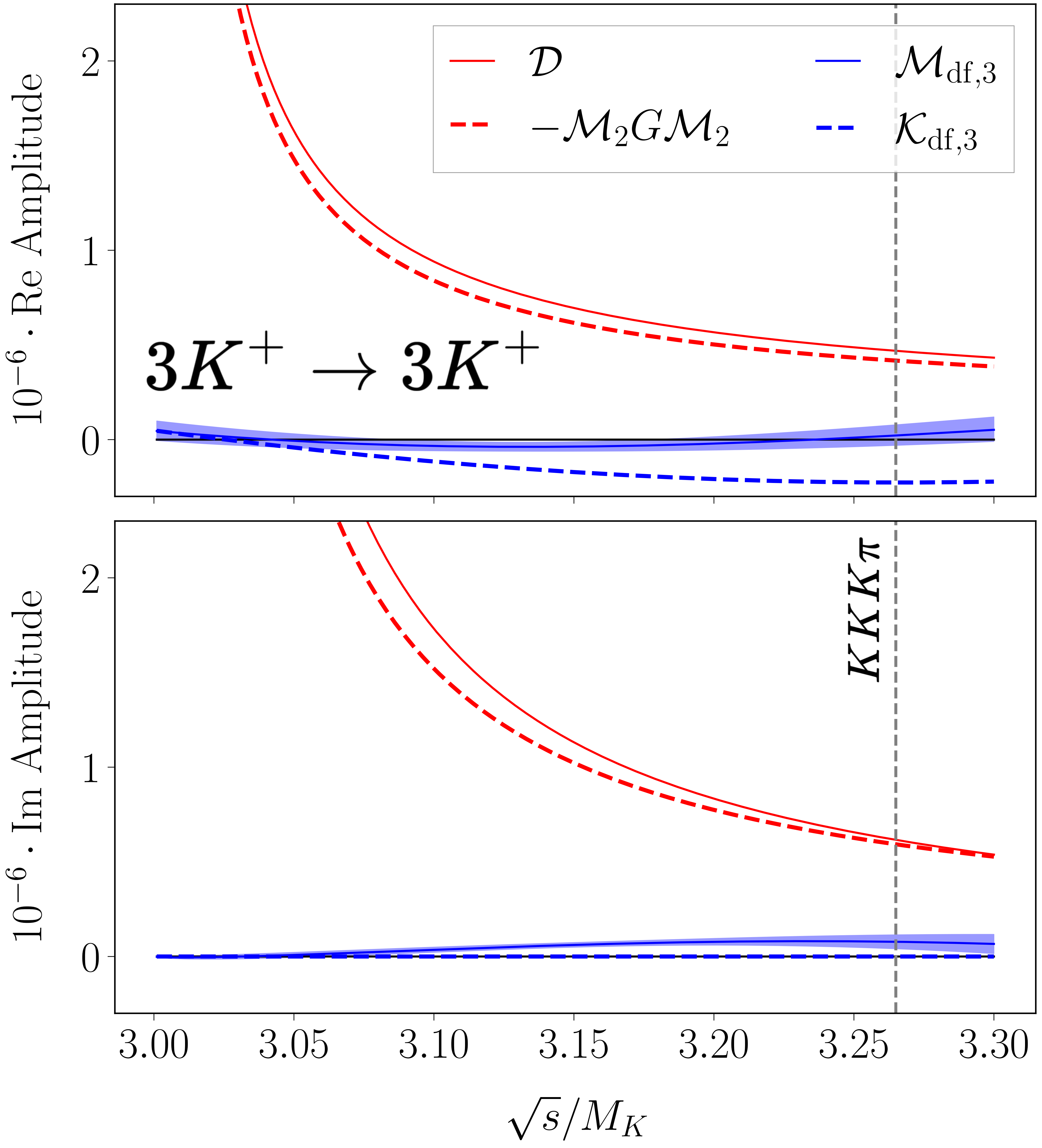
$$\mathcal{D}^{(u,u)} = -\mathcal{M}_2 G \mathcal{M}_2 - \int \mathcal{M}_2 G \mathcal{D}^{(u,u)}$$

"divergence-free amplitude"



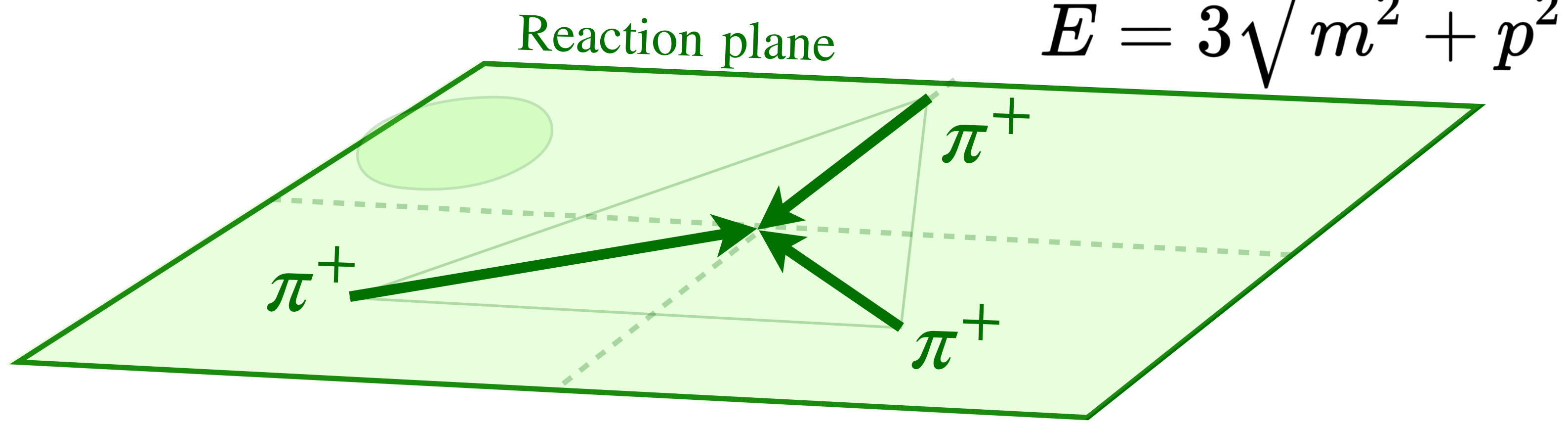
At least one three-body interaction

Three-kaon $J^P=0^-$ amplitude



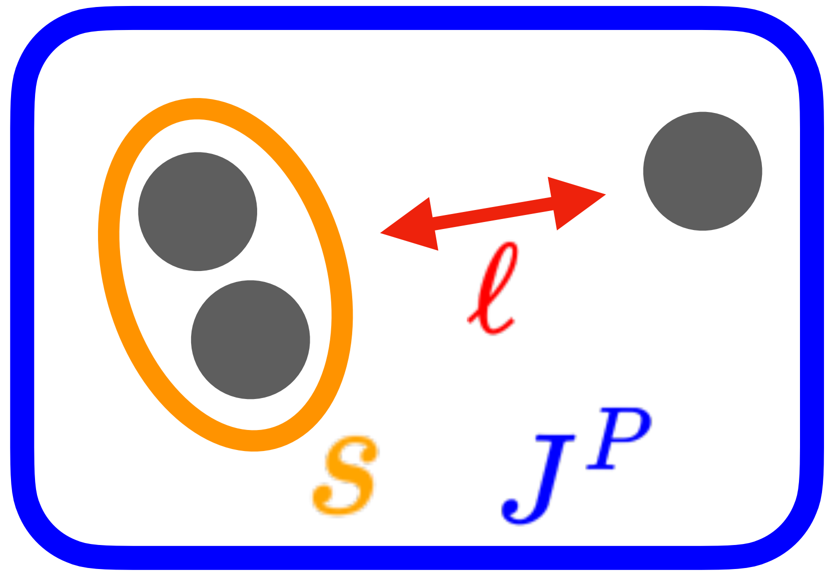
$$\mathcal{M}_3 = \mathcal{D} + \mathcal{M}_{\mathrm{df},3}$$

Equilateral kinematic configuration



- **Partial-wave projected to $J^P = 0^-$**

  [Jackura, Briceño, 2312.00625]



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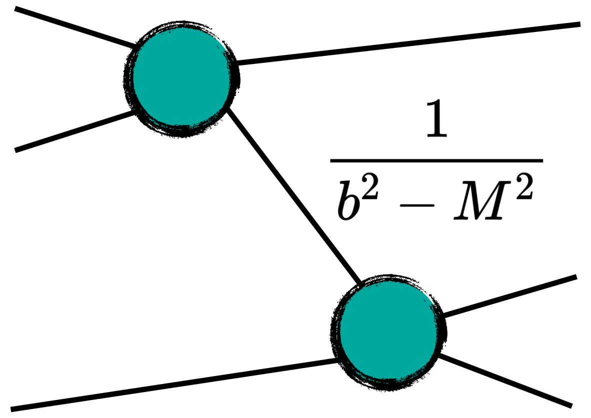
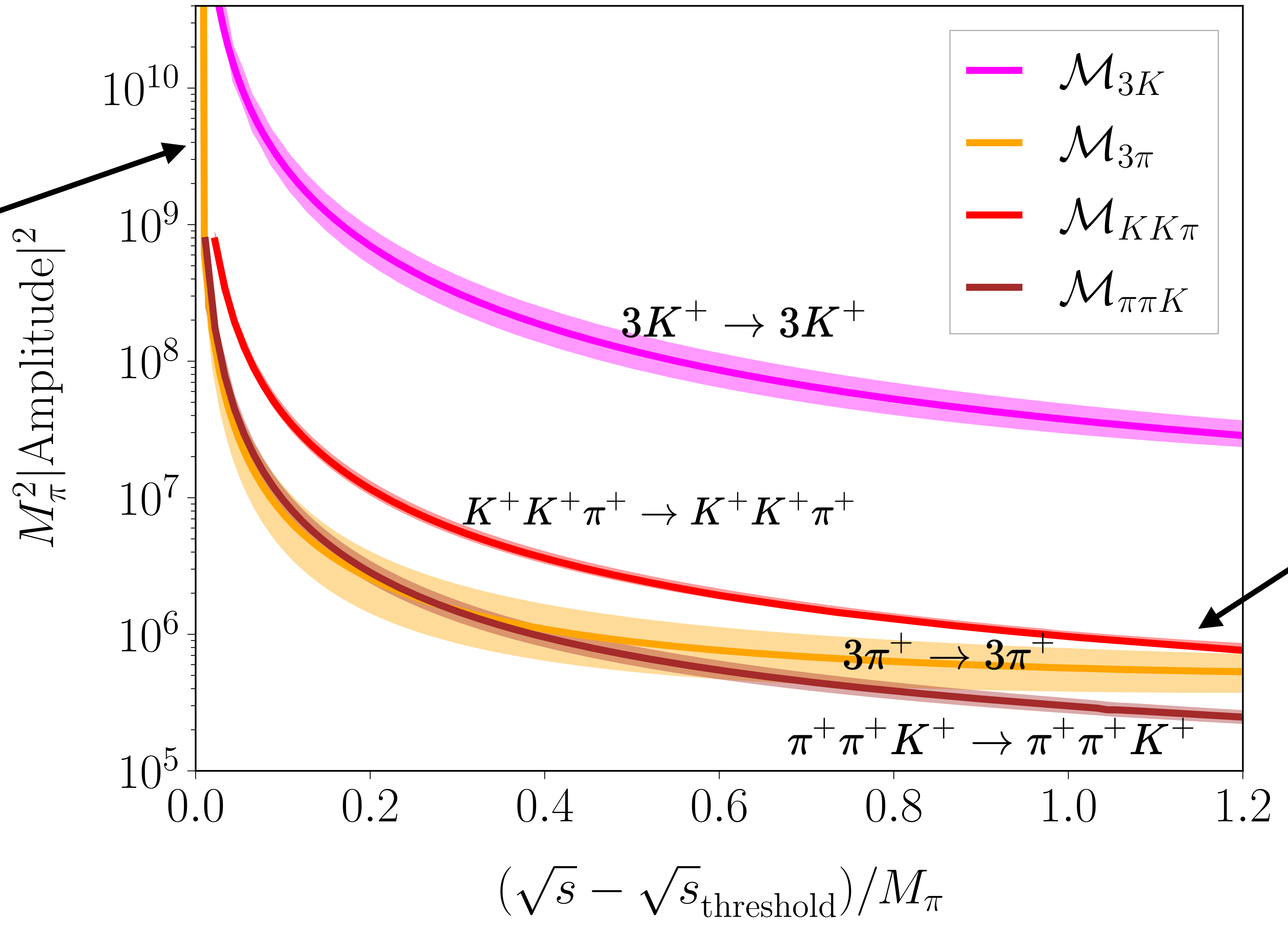
Three-meson amplitudes

Lattice QCD predictions for physical three-meson scattering amplitudes

Divergent at threshold





Pion interactions are chirally suppressed

$$M_\pi^2 \mathcal{M}_3 = O(M_\pi^4/F_\pi^4)$$

Conclusion & Outlook

- We have studied systems of two and three-mesons at maximal isospin at the physical point
  - Important benchmark system for three-hadron spectroscopy
- Constraints on two-meson scattering amplitudes at the physical point
  - Some evidence for d waves (KK) and p waves (πK)
  - Chiral dependence of scattering lengths
- Solving the integral equations, we have determined physical three-meson scattering amplitudes
  - Dominated by two-meson rescattering
  - Evidence for “contact” three-hadron force ($3K$)
- Next steps involve other three-pion isospin channels, DDπ systems and systems with baryons

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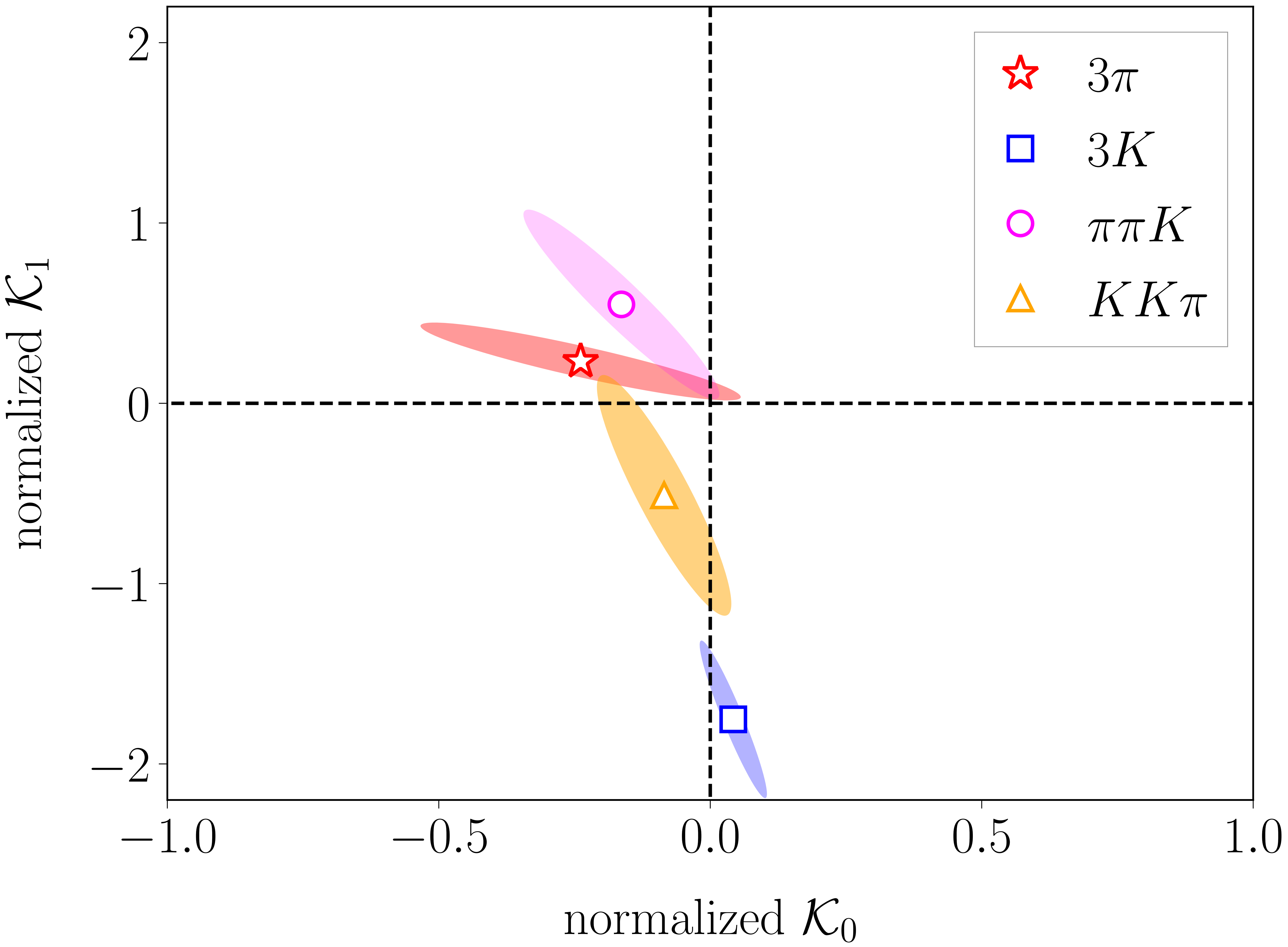
Thanks!

Back-up

Significance of Kdf3



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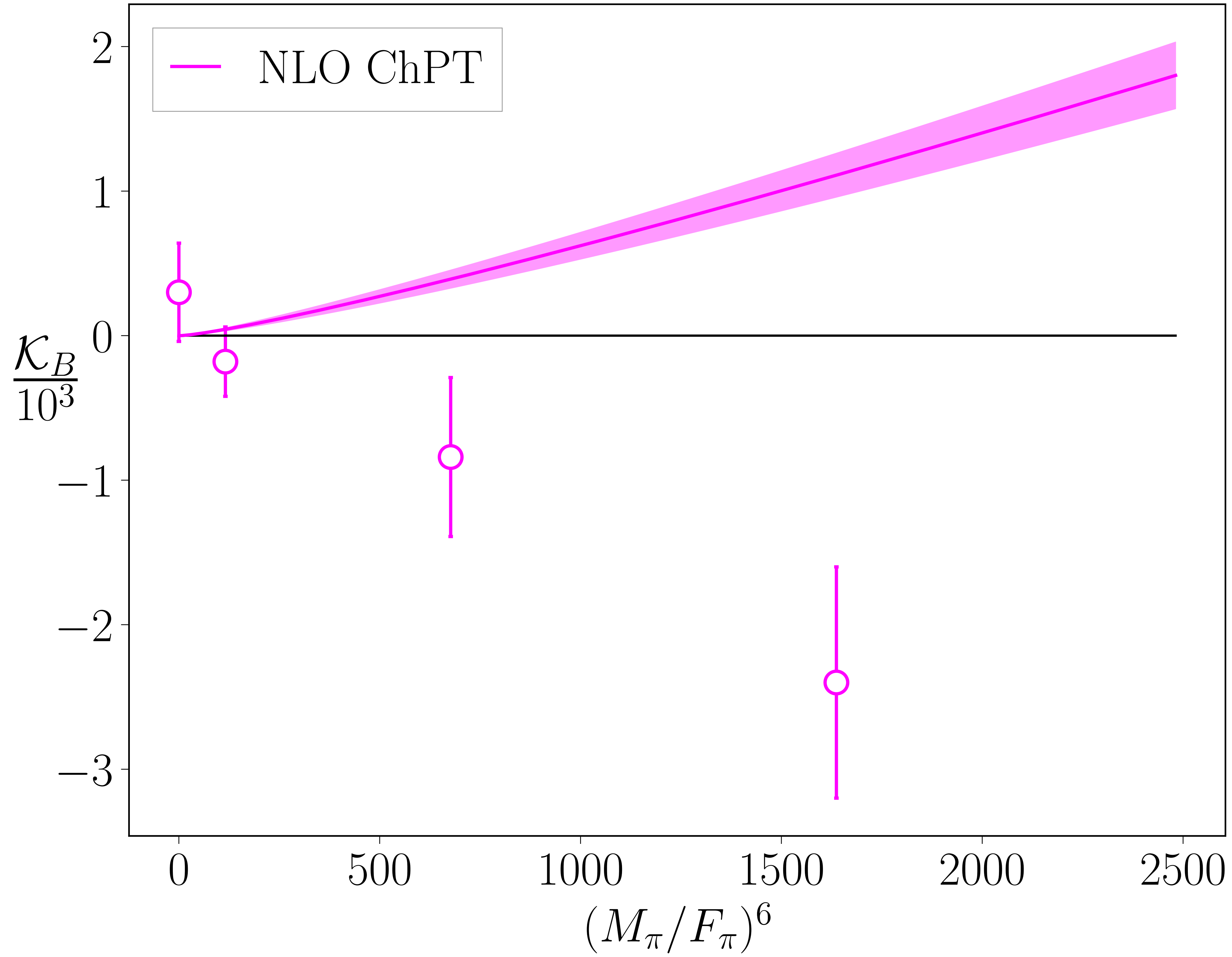
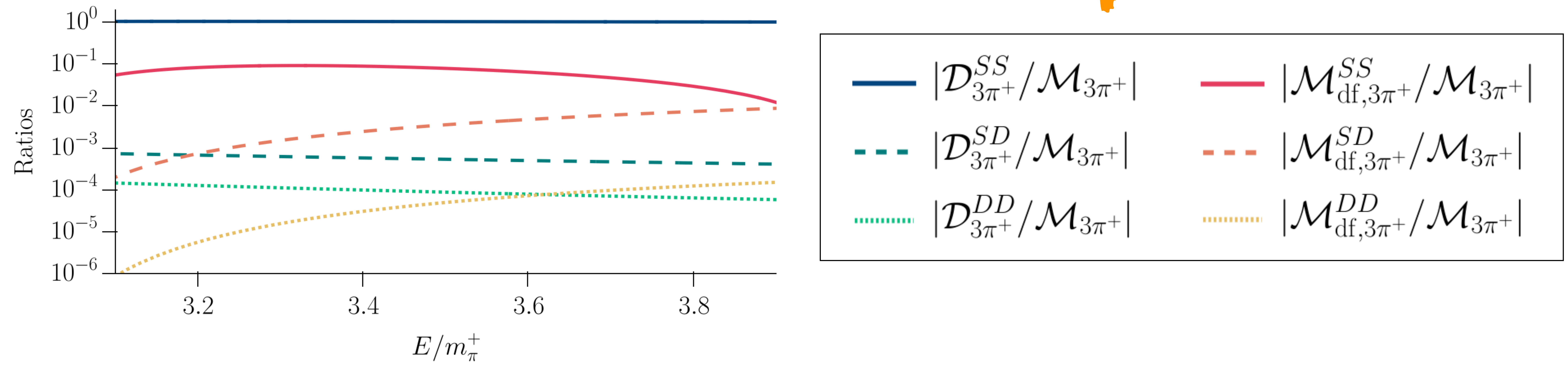
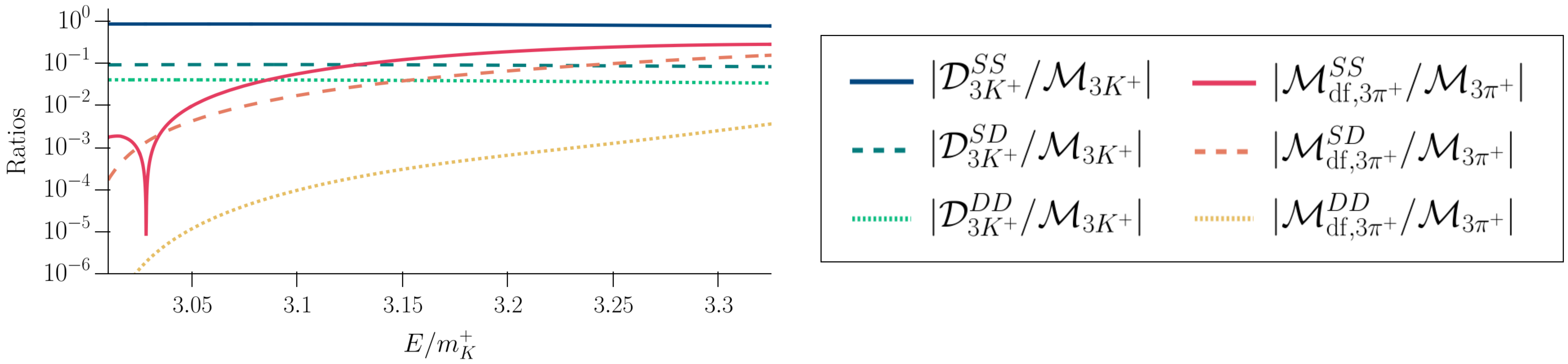
Comparison to ChPT



Three-meson amplitudes

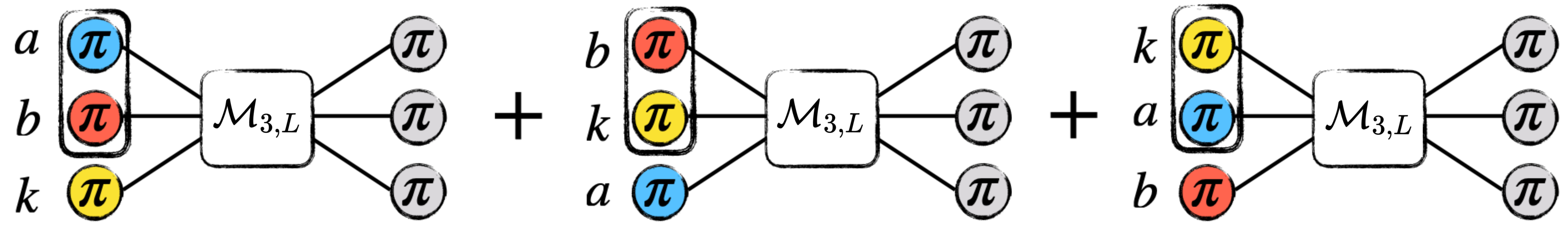




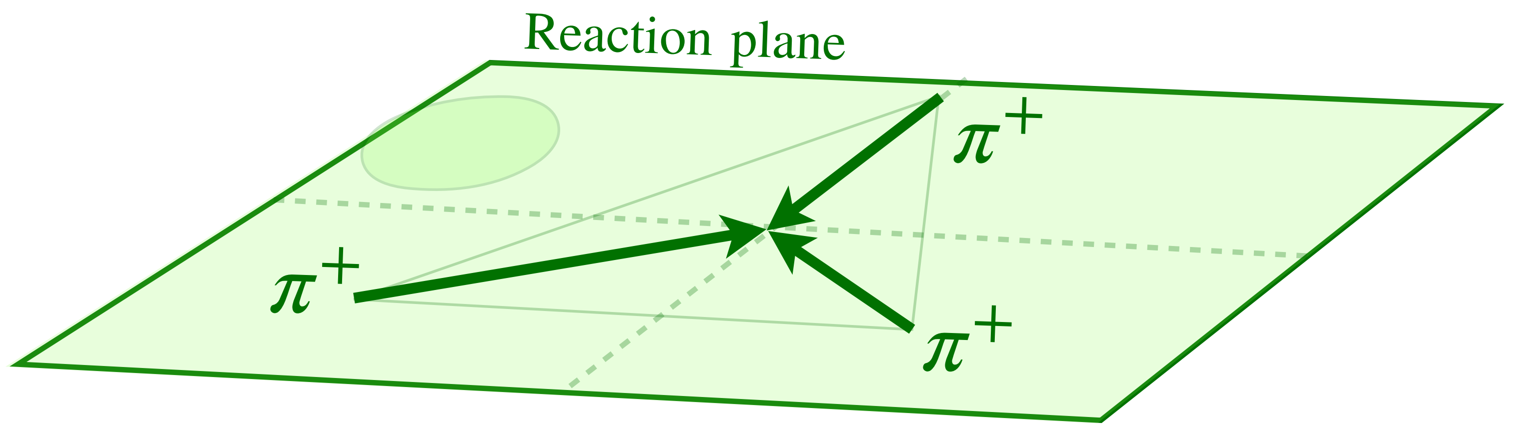
Kinematic configuration

- Symmetrize (each particle gets a turn to be the spectator)

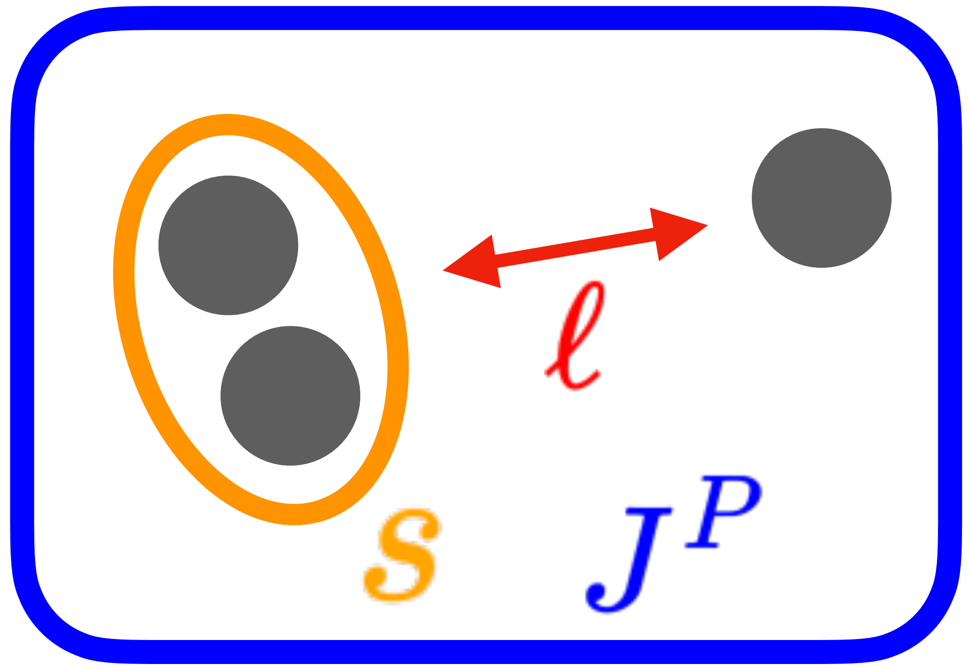
$$\mathcal{M}_{3,L}(P) \equiv \mathcal{S}\left[\mathcal{M}_{3,L}^{(u,u)}(P)\right] =$$



- Equilateral kinematic configuration $E = 3\sqrt{m^2 + p^2}$

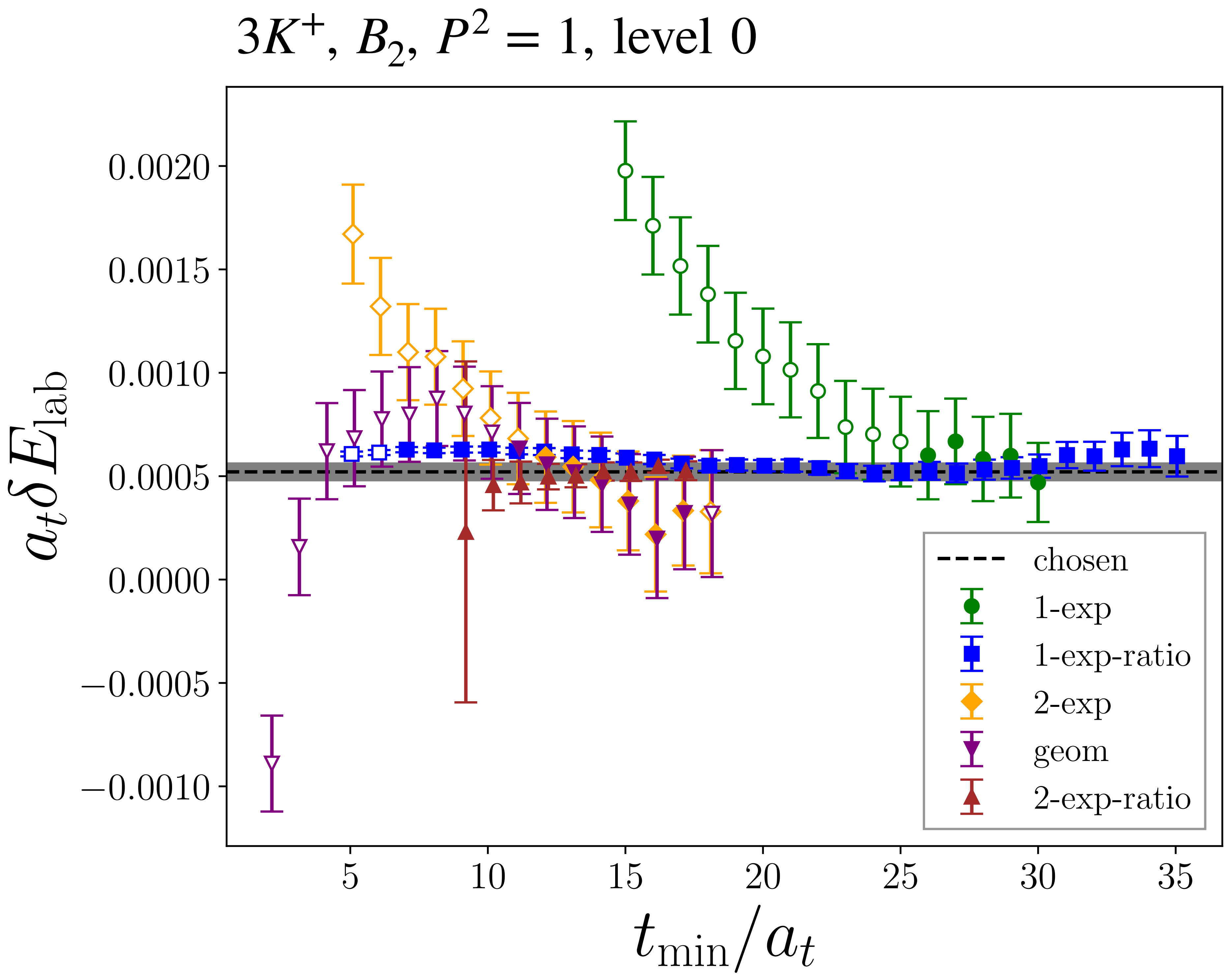


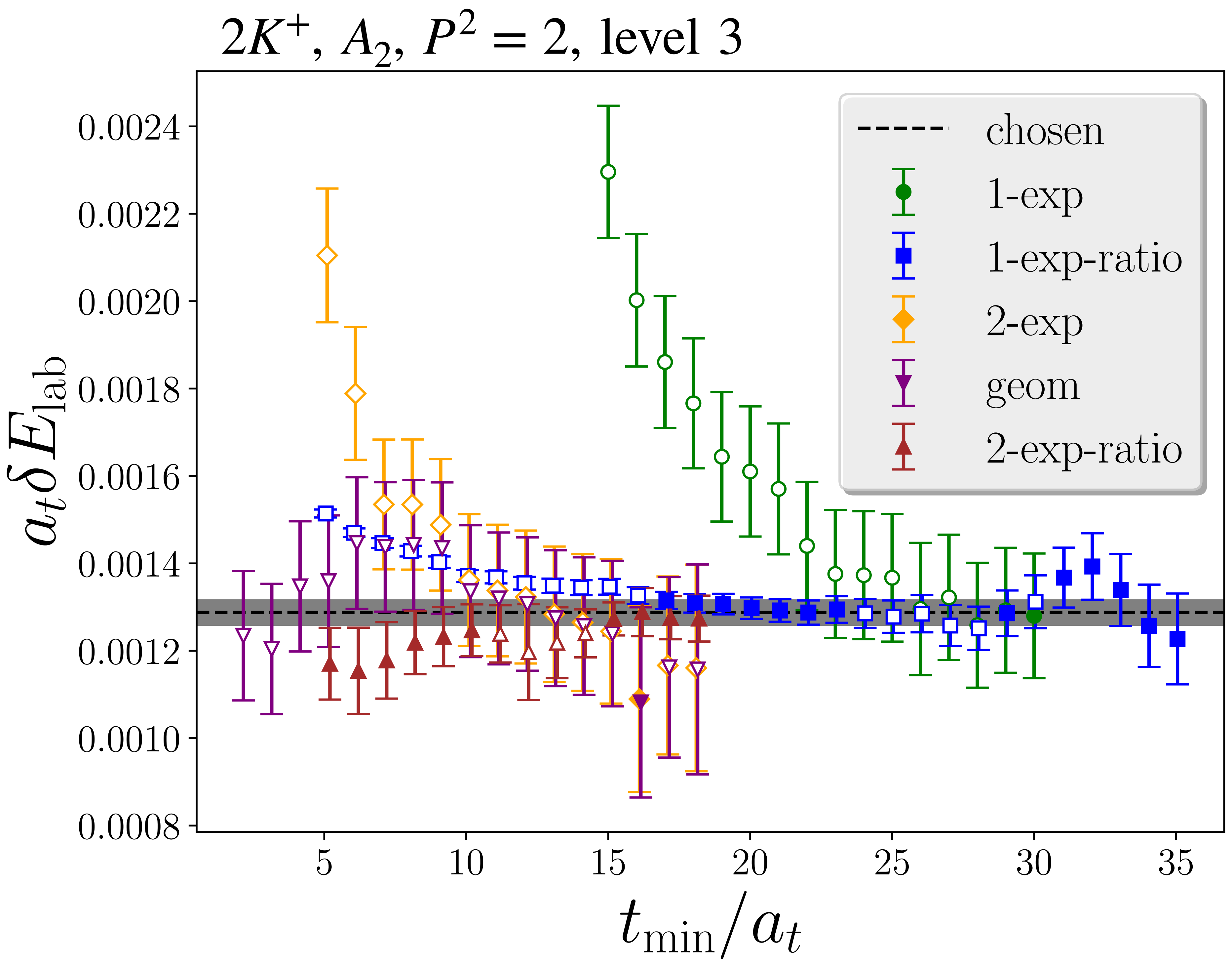
$$\mathcal{M}_3^{J=0}(E) = 9\left[\mathcal{M}_{3,00;00}^{(u,u)J=0}(E) + \frac{5}{4}\mathcal{M}_{3,22;22}^{(u,u)J=0}(E) - \frac{\sqrt{5}}{2}\left(\mathcal{M}_{3,22;00}^{(u,u)J=0}(E) + \mathcal{M}_{3,00;22}^{(u,u)J=0}(E)\right)\right],$$



Partial-wave projection
[Jackura, Briceño, 2312.00625]

Other t-min plot





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