The ω-meson from $\pi\pi\pi$ scattering

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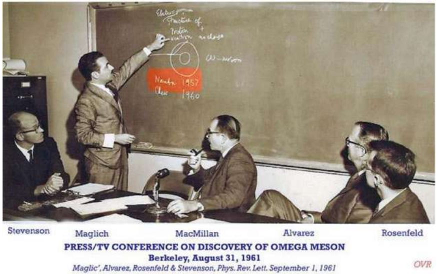
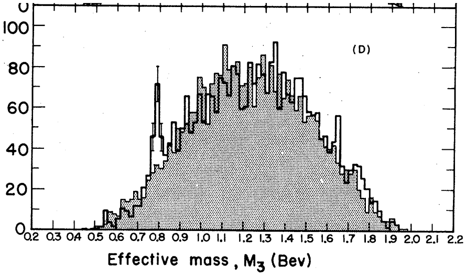
ω: the first neutral vector meson (1961)[1]





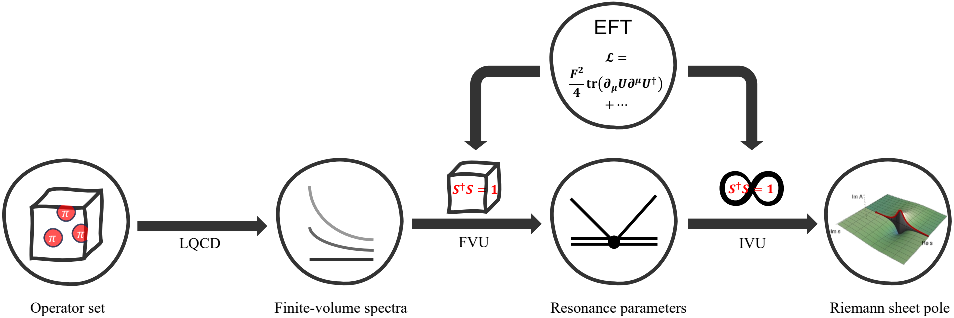
Phenomenologically[2],

- ω is the lightest hadron decaying into three particles: $\omega \to 3\pi$
- ω dominates the isoscalar response within the VMD picture of the photon-nucleon interactions
- ω generates the observed repulsion at $< 1\,\mathrm{fm}$ in the one-boson-exchange picture of the $N - N$ interaction
- ω mixes with the ρ and leads to marked effects in the pion vector form factor
- $\omega - \rho$ mass splitting is phenomenologically interesting, for instance muon $g - 2$ and dark matter

[1] Maglic *et al.* (1961).
[2] Sakurai (1960); Erkelenz (1974); Brown and Jackson (1976); Barkov *et al.*, 1985; Connell *et al.* (1997); Bazavov *et al.* (2021).

- **Three-body problem** with resonances in two-body problem: $\pi\pi\pi \to \omega$; $\pi\pi \to \rho$
- Isoscalar $\Rightarrow$ **most challenging isospin** in the $\pi\pi\pi$ channel

Introduction



- $I = 1$ $\pi\pi$ and $I = 0$ $\pi\pi\pi$ spectra
- Develop the formalism to map finite to infinite volumes
- Establish the pertinent EFT and parametrize the three-body force
- Solve the integral equations and search the poles

Ensemble	Volume	M_π/MeV	N_confs
F32P21	$32^3 \times 64$	$206.8(2.1)$	459
F48P21	$48^3 \times 96$	$207.58(76)$	221
F32P30	$32^3 \times 96$	$303.61(71)$	777
F48P30	$48^3 \times 96$	$304.95(49)$	201

- CLQCD ensembles with $N_\mathrm{f} = 2 + 1$ Clover fermions [CLQCD, 2024]
- Two pion masses with two volumes
- At the same lattice spacing $a = 0.07746(18)\,\mathrm{fm}$
- Distillations [Peardon *et al.*, 2009] for the vast number of annihilation diagrams[2]

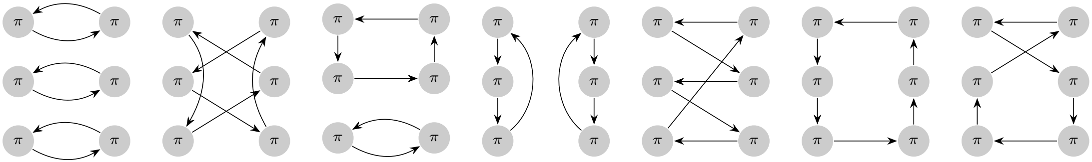
[2] Yan *et al.* $D\pi$ scattering, arXiv:2404.13479.

Contraction topologies

- Construction tool **OpTion**[3] is utilized to generate general N-hadron operators in the T_1^- irrep.

$$\omega \leftrightarrow \rho\pi \leftrightarrow \pi\pi\pi$$

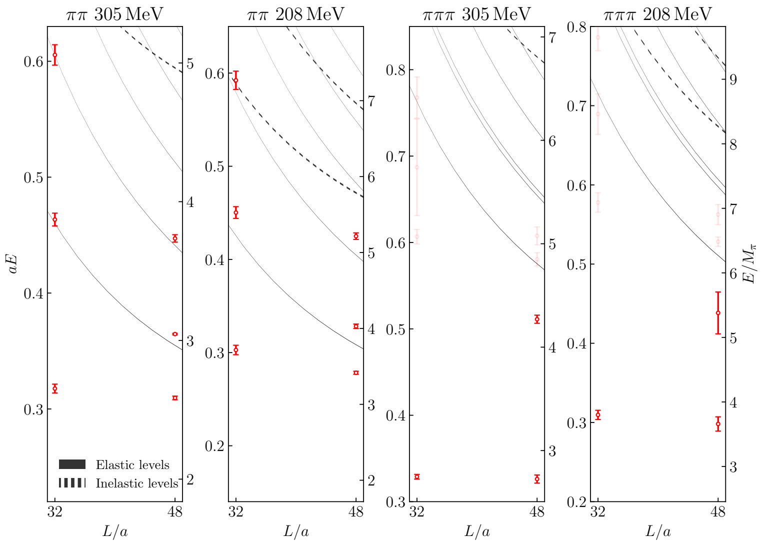
- Necessary to have all kinds of operators for low-lying levels
- **P-wave** between all π
- Insanely many diagrams (202 for only $\pi\pi\pi \to \pi\pi\pi$)
- The topologies for $\pi\pi\pi \to \pi\pi\pi$



- Collect all operators with different momentum configurations and do GEVP
- More non-local operators / thermal pollution / N_v are tested and the spectra are stable

[3]Yan *et al.*, in preparation

Finite-volume spectra

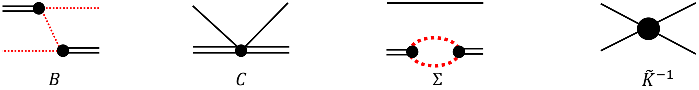


- Strong **attraction** in both the $\pi\pi$ and $\pi\pi\pi$ channels
- In the $\pi\pi\pi$ channel, the ground levels indicate **bound** ω at $M_\pi \approx 305\,\mathrm{MeV}$ and **resonating** ω at $M_\pi \approx 208\,\mathrm{MeV}$
- Restricted to be below the $\omega(1420)$ region

Quantization condition

- Using **FVU** (Finite-Volume Unitarity) of all state-of-art formalisms
  - FVU [Mai and Döring, 2017]
  - RFT [Hansen and Sharpe, 2014]
  - NREFT [Hammer, Pang, and Rusetsky, 2017]

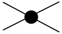
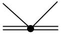
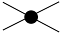
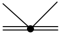
$$\begin{cases} \tilde{K}^{-1}(\sigma) - \Sigma^{FV}(\sigma) = 0 \\ \det[(\tilde{K}^{-1}(s) - \Sigma^{FV}(s))E_L - (\tilde{B}(s) + \tilde{C}(s))] = 0 \end{cases}$$



- Project to T_1^- irrep with the coefficients of the operators
- Spectator momentum cutoff: $\vec{p}_{\max} = [0, 1, 1]$
- Two-body input cutoff: $\tilde{K}^{-1} \to (1 + e^{-(\sigma-\sigma_0)/M_\pi^2})\tilde{K}^{-1}, \sigma_0 = 2$
- Other $\vec{p}_{\max}$ and σ_0 are tested and have **no relevant effect** on the extracted observables

Parametrizations

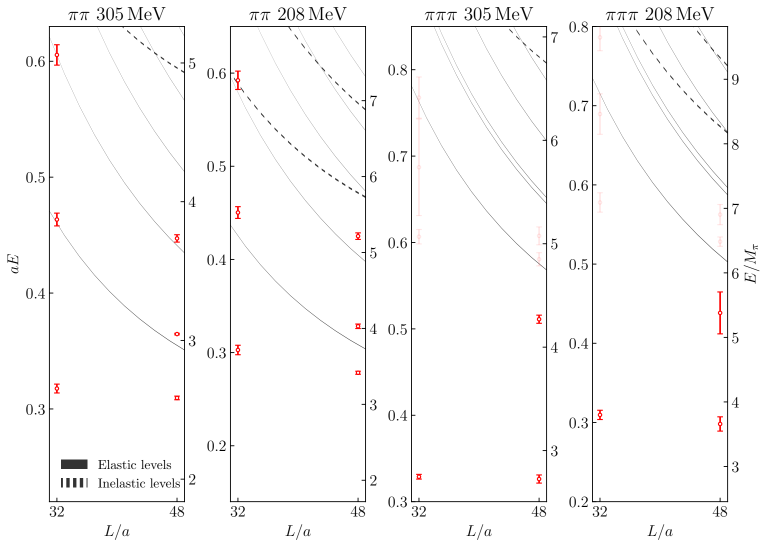
Combined fit for $\pi\pi$ and $\pi\pi\pi$ spectra

1. GEN
   -  $\sim a_0 + a_1\sigma + \cdots$
   -  $\sim \frac{c_0}{s-M_\omega^2} + c_1 + \cdots$
2. EFT[4]
   -  $\sim \frac{\sigma - M_\rho^2}{g^2}$
   -  $\sim \frac{s(M_\rho^2 - \sigma_q + 6g^2 f_\pi^2)(M_\rho^2 - \sigma_p + 6g^2 f_\pi^2)}{g^2 f_\pi^6 (s - M_\omega^2)}$

- Fit each M_π
  - GEN: $[a_0, a_1, c_0, c_1]$
- Fit all M_π's
  - EFT2: $[g, \delta]$: $(M_\rho = \sqrt{2} g f_\pi,\ M_\omega = M_\rho + \delta)$
  - **EFT4** (main): $[g, \delta, M_V, a]$: $(M_\rho = M_V + a\, M_\pi^2,\ M_\omega = M_\rho + \delta)$

[4] Meißner (1988).

Finite-volume spectra revisited



- $\chi^2_{\rm dof}(\text{EFT4}) = 2.3$
- Continuous spectra from FVU
- High-lying energies above the cutoff are also well-predicted
- EFT4 could be improved by including further chiral corrections

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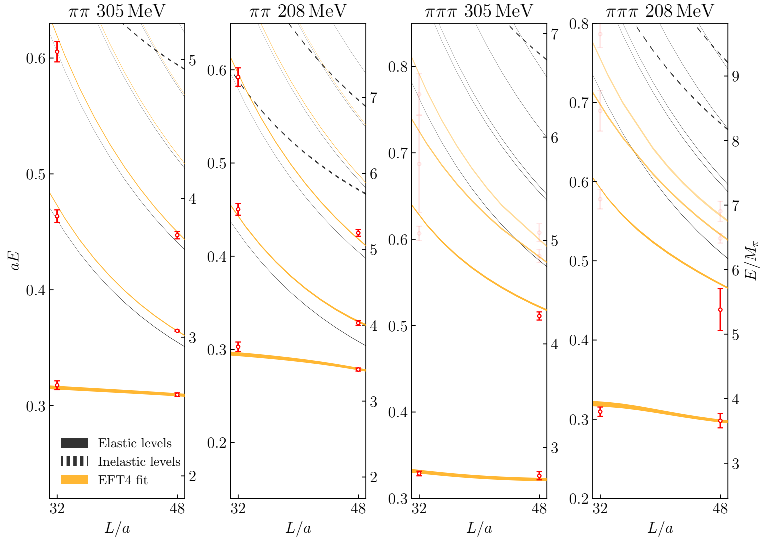

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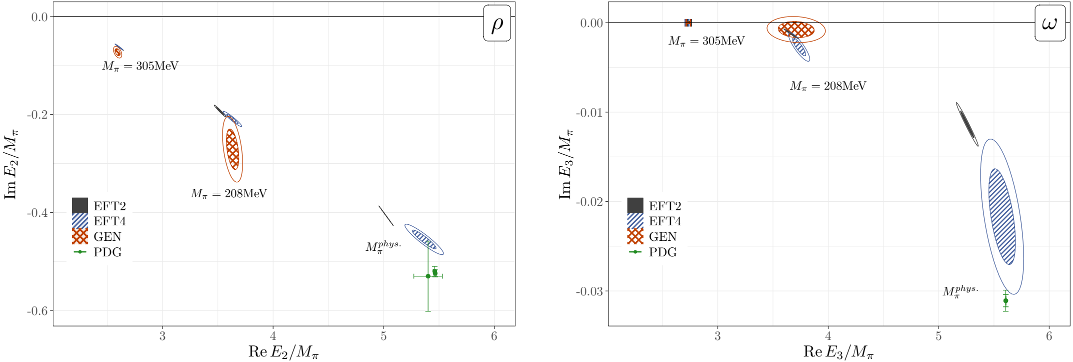
Pole positions



- Solve the integral equation [Mai and Döring, 2017] $T = B + C + \int \frac{d^3 l}{(2\pi)^3} \frac{B+C}{2E_l(\tilde{K}^{-1} - \Sigma^{IV})} T$
- ω is indeed a **bound state** at $M_\pi \approx 305\,\mathrm{MeV}$ and a **resonance** at $M_\pi \approx 208\,\mathrm{MeV}$
- 1σ agreement of $\mathrm{Re}\, M_\omega$ between all three methods
- Extrapolate to the **physical pion mass**, the poles agree astonishingly well with the PDG values

Summary

- First-ever determination of the ω-meson pole from lattice QCD
- Development of the FVU, matching EFT and FVU
- Paved the way to study heavier three-hadron resonances
- The lattice artifacts are to be investigated
- The ρ and ω pole positions at the physical point

$$\sqrt{s_\rho} = (748.9(10.0) - i63.5(1.8))\,\mathrm{MeV}$$
$$\sqrt{s_\omega} = (778.0(11.2) - i3.0(5))\quad\mathrm{MeV}$$

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ω also thanks you!

$(\geqq \omega \leqq)$