LATTICE 2024

LIVERPOOL

Physical-mass calculation of the $\rho(770)$ and $K^*(892)$ resonance parameters via $\pi\pi$ and $K\pi$ scattering from lattice QCD

[arXiv: 2406.19193 , 2406.19194]

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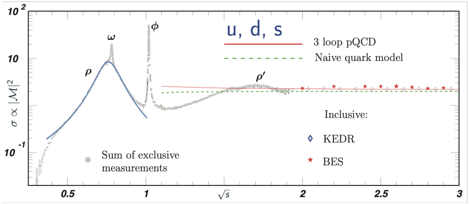
Motivation

Resonances

- QCD-unstable ($\sim 10^{-23} s$)
- process dependent: "bump" on cross-section
- process independent: *scattering amplitude poles*



$$\text{Physics importance \& interests} \begin{cases} \text{QCD Spectroscopy: } \boldsymbol{K^*}, \boldsymbol{\rho}, \kappa, \omega, a_0, f_0, \ldots \\ \text{Standard Model tests: } B_{(s)} \to K^* \ell^+ \ell^-, \ \gamma \to \pi\pi, \ldots \end{cases} \quad (1)$$

Lattice QCD: non-perturbative resonance determinations have advanced [Briceño, Dudek, Young - RevModPhys, 2018]

[Fischer et al - PLB, 2021] [Paul et al - Lattice21, arXiv:2112.07385v1] [Rendon et al - PRD, 2021] [Wilson et al - PRL, 2019] [Bali et al - PRD, 2016] [Brett et al - Nuc.Phys.B, 2018] [Aoki et al - PRD, 2011] [Feng et al - PRD, 2011] [Lang et al - PRD, 2011] [Pelissier et al - PRD, 2013] [Dudek et al - PRD, 2013] [Bulava et al - Nuc.Phys.B, 2016] [Fu et al - PRD, 2016] [Andersen et al - Nuc.Phys.B, 2019] [Erben et al - PRD, 2020] [Alexandrou et al - PRD, 2017]

▶ Towards precision calculations:

physical pion mass, continuum limit, $\geq$ 3-particles formalisms, ...

Overview

Main decay products

- $\boldsymbol{K^*(892) \to K\pi}, K\gamma, K\pi\pi, \ldots$ in $I = 1/2$
- $\boldsymbol{\rho(770) \to \pi\pi}, \pi\gamma, 4\pi, \ldots$ in $I = 1$

(both at $J = \ell = 1$)

In summary:

. Möbius domain-wall $N_f = 2 + 1$ RBC/UKQCD ensemble [Blum, Boyle, Christ et al - PRD, 2016]

volume	a	L	$m_\pi L$	m_π	m_K
$48^3 \times 96$	≈ 0.114 fm	$\approx$ **5.5** fm	≈ 3.8	$\approx$ **139** MeV	≈ 499 MeV

. distillation + GEVP + Lüscher + analytic continuation $\to$ resonance pole positions $M - i\Gamma/2$

. data-driven systematics: fit-range sampling procedure

. see arXiv: 2406.19193 , 2406.19194 for full details

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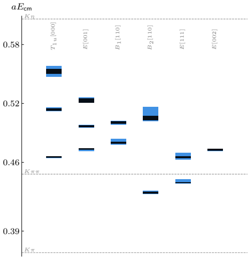

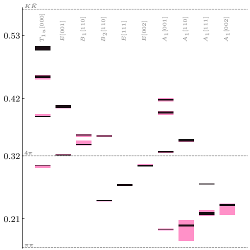

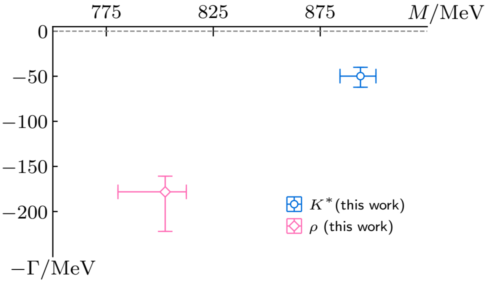

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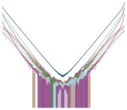
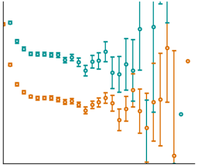
Procedure

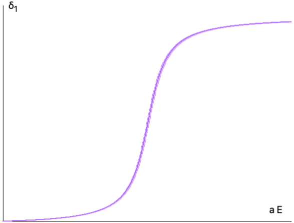


Lattice observables

GEVP



Quantisation Condition (QC)



- correlator matrices via distillation [Peardon et al - PRD, 2009] [Morningstar et al - PRD, 2011]
- GEVP yields spectrum [Lüscher & Wolff - Nucl.Phys.B, 1990] [Blossier et al - JHEP, 2009]
- Lüscher formula (QC) yields phase-shift [Lüscher - Comms.Math.Phys., 1986] [Rummukainen & Gottlieb - Nucl.Phys.B, 1995]
- analytical continuation gives resonance poles [Guo et al - PRD, 2013] [Wilson et al - PRD, 2015]

Lattice Code

Open-source software

GRID

Data-parallel C++ lattice library

[github.com/paboyle/Grid]

Hadrons

Grid-based lattice workflow management system

[Portelli et al, 10.5281/ZENODO.6382460, 2022]

Distillation within Grid and Hadrons [https://aportelli.github.io/Hadrons-doc/#/mdistil]

- any Grid solver – Dirac inversions on CPU and GPU (same code)
- meson field formulation (CPU)

▶ Large-scale calculation using UK DiRAC resources

- exact (non-stochastic) distillation [Peardon et al - PRD, 2009]
- $N_{vec} = 64$ $(R_{smear} \approx 1fm)$ [Lachini et al - Lattice2022, arXiv 2112.09804]

GEVP

Operator basis $\left(|\mathbf{P}|^2, |\mathbf{p}_1|^2, |\mathbf{p}_2|^2 \leq 4\,(2\pi/L)^2\right)$

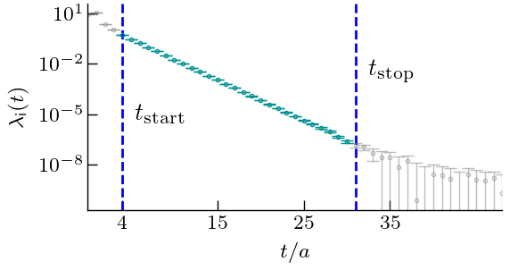
$$\text{bilinear } O_{\bar{q}\gamma_i q}(\mathbf{P}) = \begin{cases} (\bar{s}\gamma_i u)(\mathbf{P}) \\ (\bar{d}\gamma_i u)(\mathbf{P}) \end{cases} , \quad \text{two-bilinear} \begin{cases} O_{K\pi}^{I=1/2}(\mathbf{p}_1, \mathbf{p}_2) \\ O_{\pi\pi}^{I=1}(\mathbf{p}_1, \mathbf{p}_2) \end{cases}$$

Channel	Irreps $\Lambda[\mathbf{d}]$	total number of energies
$K\pi^{I=1/2}$	$T_{1u}[000];\ E[001];\ B_1[110];\ B_2[110];\ E[111];\ E[002]$	13
$\pi\pi^{I=1}$	all above + $A_1[001];\ A_1[110];\ A_1[111];\ A_1[002]$	21

- GEVP eigenvalues : $\lambda_{\Lambda[\mathbf{d}],n} \equiv \lambda_j \xrightarrow{t,t_0 \text{ large}} A_j e^{-tE_j} \left(1 + \mathcal{O}(e^{-t\Delta E_j})\right)$

GEVP eigenvalue fitting

Fit single exponential to each λ_j within $[t_\mathrm{i}, t_\mathrm{f}] \in [t_\mathrm{start}, t_{\mathrm{stop},j}] \;\rightarrow$ "pool" of fits



Example: scan limits of fit ranges for an eigenvalue

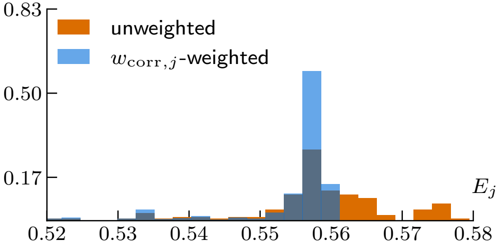
Scan ranges $[t_\mathrm{start}, t_{\mathrm{stop},j}]$:

- exclude large-t data below some conservative signal-to-noise threshold $\mathrm{SNR}_\mathrm{min}$
- perform *all* fits with some minimum consecutive separation $t_\mathrm{f} - t_\mathrm{i} \geq \delta t_\mathrm{min}$

Fit-range weighting

Assign to each fit range: $\mathrm{AIC}_{\mathrm{corr},j} \rightarrow$ weights $w_{corr,j} = \exp -\frac{1}{2}\left(\chi^2_{\mathrm{corr},j} + 2n^{\mathrm{par}}_{corr} - n^{data}_{\mathrm{corr},j}\right)$

[Borsanyi et al - Nature, 2021] [Jay & Neil - PRD, 2021]



Histograms of a single finite-volume energy E_j from all single-exp fit ranges

- **Fit-range sample**: combination formed by *one* fit *per level*

$$\{E_{\mathrm{cm}}\} \equiv \{E_{\mathrm{cm},1},\, E_{\mathrm{cm},2},\, \ldots,\, E_{\mathrm{cm},n^{\mathrm{lev}}}\}$$

($E_j \rightarrow E_{\mathrm{cm},j}$ via continuum dispersion)

Lüscher formula

QC in the pseudophase form

$$\delta_1(E_{\rm cm}) = n\pi - \phi^{\Lambda[\mathbf{d}]}(E_{\rm cm}; L, m_1, m_2)$$

Chi-squared: difference between model $\delta^{\rm mod}{}_1$ and some fit-range sample $\{E_{\rm cm}\}$ $\quad (j \equiv (\Lambda[\mathbf{d}], n))$

$$\chi^2_{\rm PS}(\boldsymbol{\alpha}) \equiv \sum_{jk} \left[E^{\rm mod}_{{\rm cm},j}(\boldsymbol{\alpha}) - E_{{\rm cm},j}\right] \,{\rm Cov}^{-1}_{jk}\, \left[E^{\rm mod}_{{\rm cm},k}(\boldsymbol{\alpha}) - E_{{\rm cm},k}\right]$$ [Guo et al - PRD, 2012]

e.g. mod $\in$ { Breit-Wigner (BW), effective range expansion (ERE) } $\rightarrow$ parameters $\boldsymbol{\alpha}^{\rm mod,s}$

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e.g. mod $\in$ { Breit-Wigner (BW), effective range expansion (ERE) } $\rightarrow$ parameters $\boldsymbol{\alpha}^{\rm mod,s}$

- Assign $\mathrm{AIC}_{\rm PS}(\mathrm{s}, \mathrm{mod})$ to each phase-shift model and fit-range sample $\{E_{\rm cm}\}^{\rm s}$

  $\rightarrow$ weights $w_{\rm PS}(\mathrm{s}, \mathrm{mod}) = \exp -\frac{1}{2}\left(\chi^2_{\rm PS} + 2n^{\rm par}_{\rm mod} - n^{\rm lev}\right)$

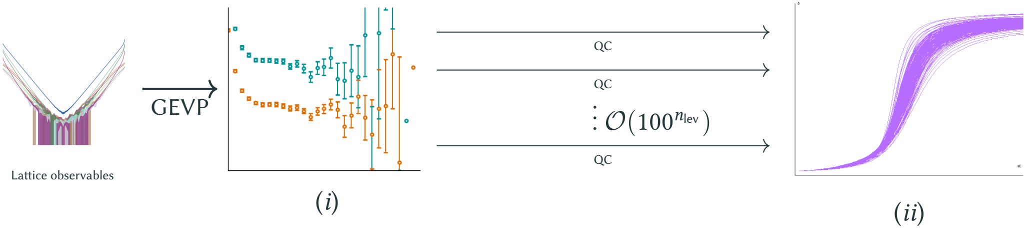
PS : phase-shift

Fit-range systematics : strategy

(i) (ii)

Choice of fit ranges from (i) to (ii) can introduce systematics

Fit-range systematics : strategy



Choice of fit ranges from (i) to (ii) can introduce systematics

- number of combined fit ranges is too high to simply do all the δ_1^{mod} fits

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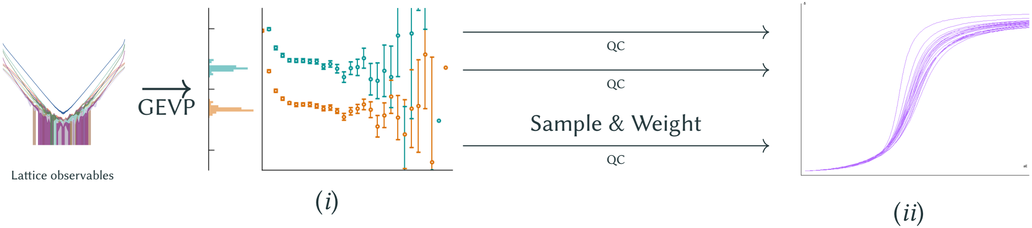


Choice of fit ranges from (i) to (ii) can introduce systematics

- number of combined fit ranges is too high to simply do all the δ_1^{mod} fits

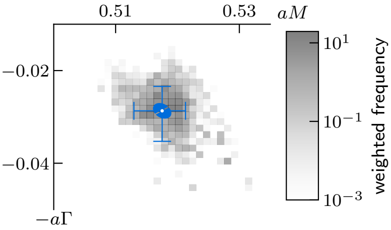
Instead: *sample* FV-energy levels (fit ranges) from *correlator* AIC_{corr}-weighted histogram in (i)

- scattering results weighted by a *total* AIC: $w_t = \exp[-\frac{1}{2}\text{AIC}_{\text{PS}} - \frac{1}{2}\sum_j \text{AIC}_{\text{corr},j}]$

Pole parameters (M, Γ)

Iterate down to the pole positions, $\sqrt{s_{pole}} = (M - i\Gamma/2)$, on II Riemann $(\text{Im}\, q < 0)$ sheet of

$$T_1^{\text{mod}}(\sqrt{s}) = \frac{1}{\cot \delta_1^{\text{mod}}(\sqrt{s})|_{\alpha} - i}, \qquad \sqrt{s} = \sqrt{s}(q) \tag{2}$$



Statistical errors and correlation (ellipse)
Fit-range systematic (cross)

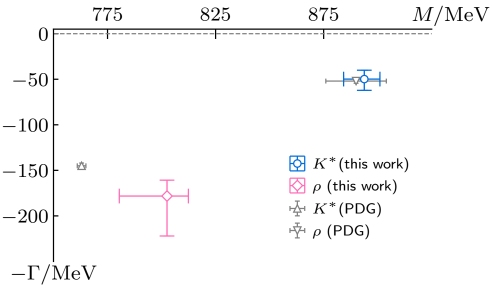
Iterate over:

- N_b bootstrap samples
- N_{scan} fit-range samples $\{E_{cm}\}^s$
- phase-shift models (BW, ERE)
- scan ranges variations $(\delta t_{min}, SNR_{min})$

Uncertainties:

- statistical: deviation of fit-range AIC_{PS}-weighted means over bootstrap samples
- data-driven systematic: AIC_{PS}-weighted 95% confidence interval over central fit-range mean ($\times$ phase-shift model and scan-range variations)

Pole results in physical units



Statistical and data-driven systematic (quadrature in plot)

Symmetrised version (mid of weighted-95%):

$$K^*(892)\begin{cases} M_P = 893(2)(8)\ \text{MeV} \\ \Gamma_P = 51(2)(11)\ \text{MeV} \end{cases}$$

$$\rho(770)\begin{cases} M_P = 796(5)(15)\ \text{MeV} \\ \Gamma_P = 192(10)(28)\ \text{MeV} \end{cases}$$

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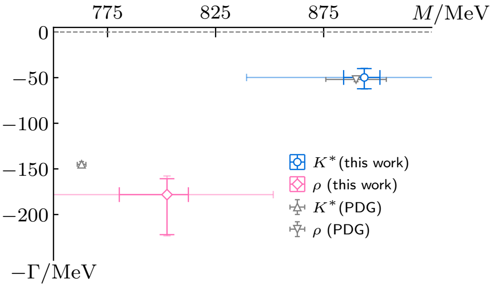

Statistical and data-driven systematic (quadrature in plot), and other

Symmetrised version (mid of weighted-95%):

$$K^*(892) \begin{cases} M_P &= 893(2)(8)(54)\ \text{MeV} \\ \Gamma_P &= 51(2)(11)(3)\ \text{MeV} \end{cases}$$

$$\rho(770) \begin{cases} M_P &= 796(5)(15)(48)\ \text{MeV} \\ \Gamma_P &= 192(10)(28)(12)\ \text{MeV} \end{cases}$$

Other: single lattice spacing and naive power counting :

- assume $(a\Lambda_{QCD})^2 \approx 5\%$ conservative discretisation uncertainty

+ other estimated extra systematics $\sim 6\%$ total

Conclusions

arXiv:
2406.19193
2406.19194

- First-principles $K^*(892)$ and $\rho(770)$ at $\boldsymbol{m_\pi} \approx \mathbf{139\ MeV}$ and $N_f = 2+1$ DWF
- Data-driven systematic estimation via sampling method in Lüscher analysis

Forward considerations:

- call for continuum limit: further impact on SM-resonance phenomenology
- estimate analysis systematics in complicated cases (lower m_π and a, baryons, etc...)
- other systematics: $\geq$ 3-body, higher waves, more operators, IB/EM, etc

Outlook:

- other isospin channels ($\sigma/f_0(500) \to \pi\pi,\ \kappa/K^{*0}(700) \to K\pi$)
- transitions $\pi\gamma \to \pi\pi$, decays $B \to K^*$
- continuum limit of resonance pole positions

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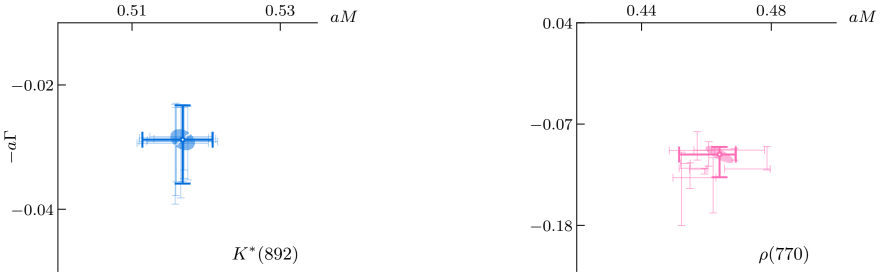
Backup

Different realisations (vary δt_{min}, SNR_{min})

Further iterate over different ways of cutting the initial correlator data

Run	SNR_{min}	δt_{min}
1	8	5
2	5	7
3	7	7
4	6	6

- $t_f - t_i \geq \boldsymbol{\delta t_{min}}$ in fit range $[t_i, t_f] \in [t_{start} = 4, t_{stop,j}]$
- signal to noise: easrliest $t_{stop,j}$ where $\lambda_j(t_{stop,j})/\sigma_j < \mathbf{SNR}_{min}$



Spread over realisations and phase-shift models (fainter crosses). Overall data-driven systematic (full cross) and statistical ellipse.

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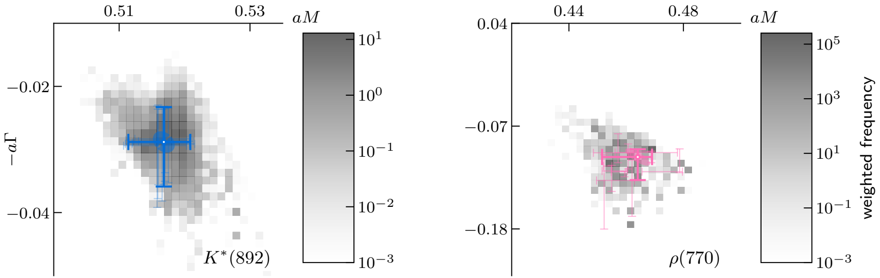

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