Three-body analysis of the tetraquark $T_{cc}^{+}(3875)$

Sebastian M. Dawid

with the honorable
F. Romero-López & S. Sharpe

SUMMARY

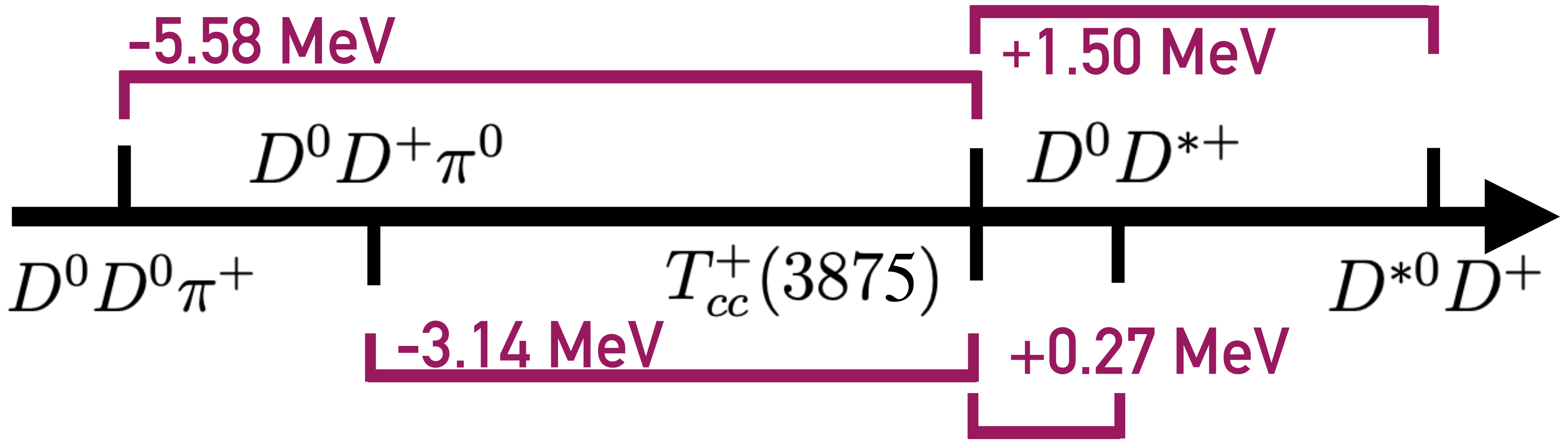
1) We lay out a strategy for a rigorous determination of T_{cc} and related systems from Lattice QCD
2) We propose resolution of the "left-hand cut problem" both in the finite volume and in the continuum
3) We generalize and solve relativistic EFT three-body equations and apply them to existing data

UNIVERSITY of WASHINGTON

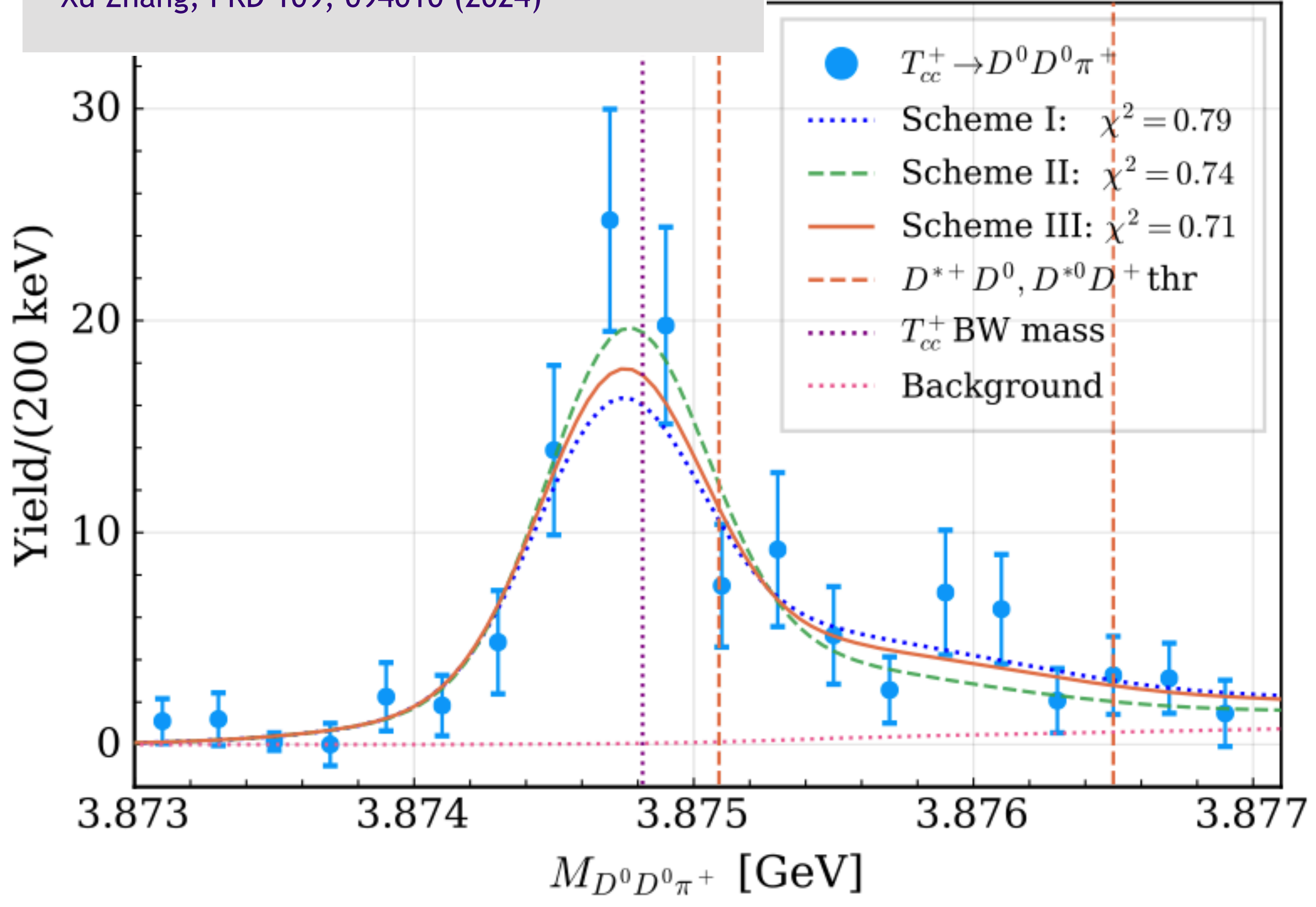
Lattice 2024, Liverpool, 29.07.2024

Infinite Volume

Three-body effects strongly impact properties of the tetraquark due to the proximity of the DDπ thresholds.

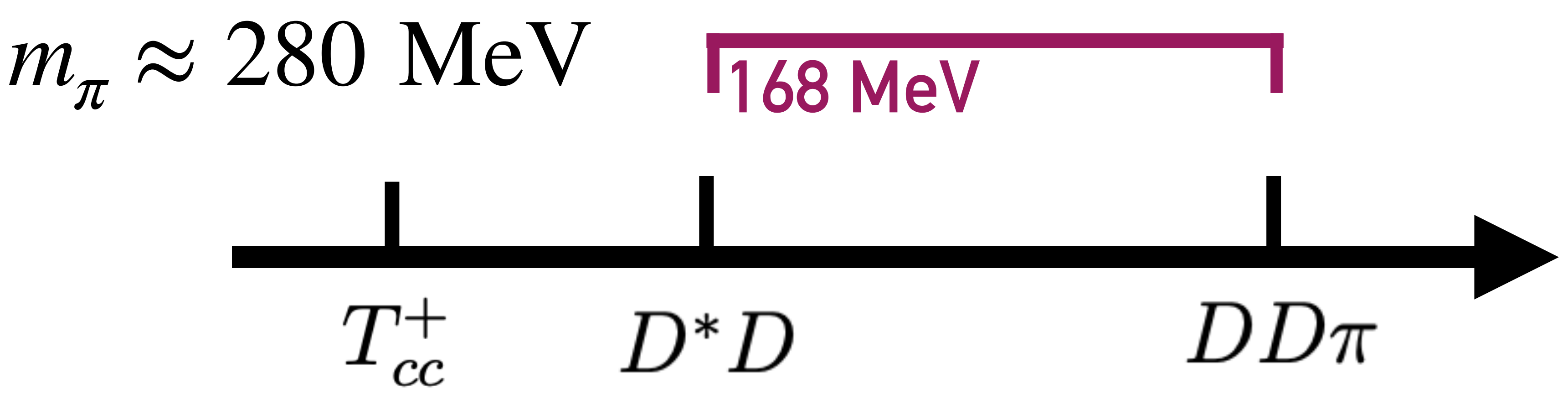


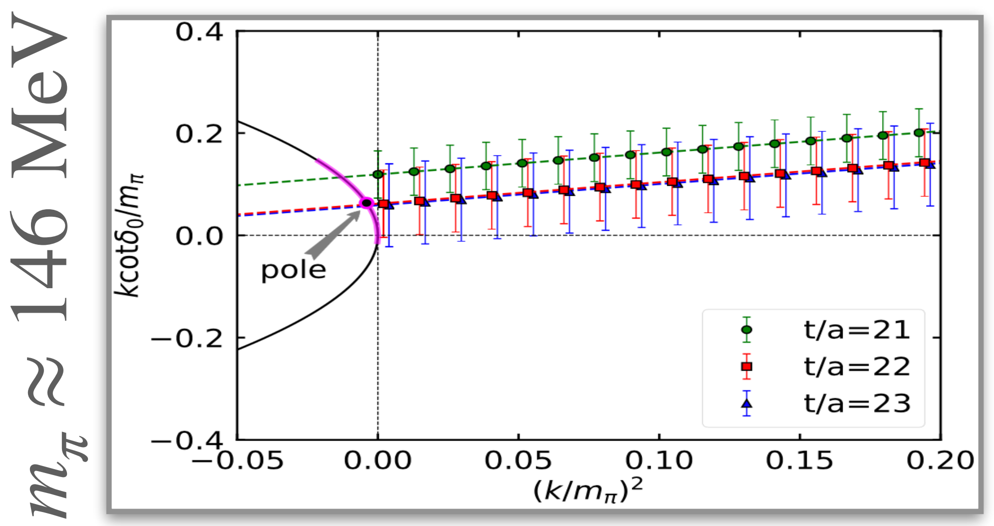
Meng-Ling Du et al., PRD 105, 014024 (2022)
Xu Zhang, PRD 109, 094010 (2024)



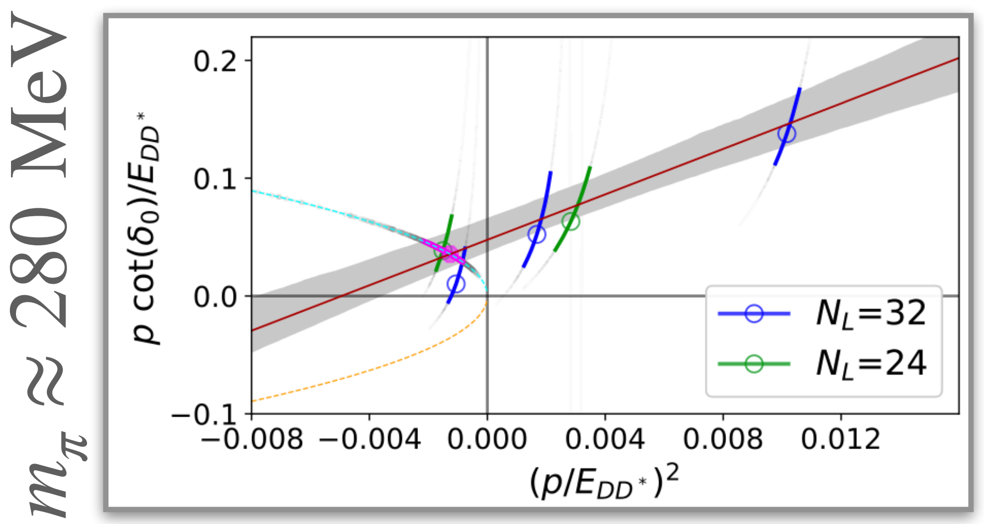
Finite Volume

For heavy pion, thresholds are inverted but three-body effects still play an important role

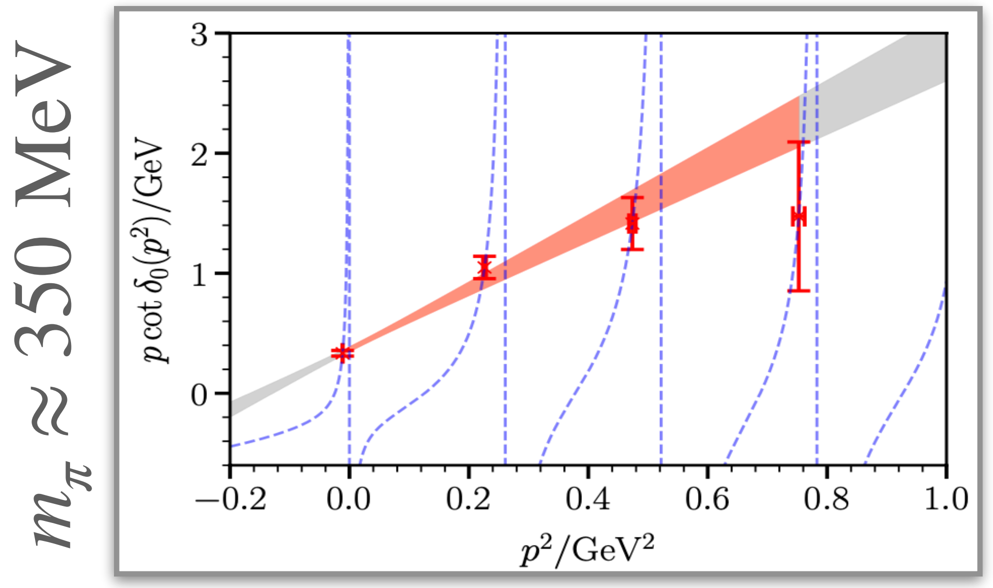




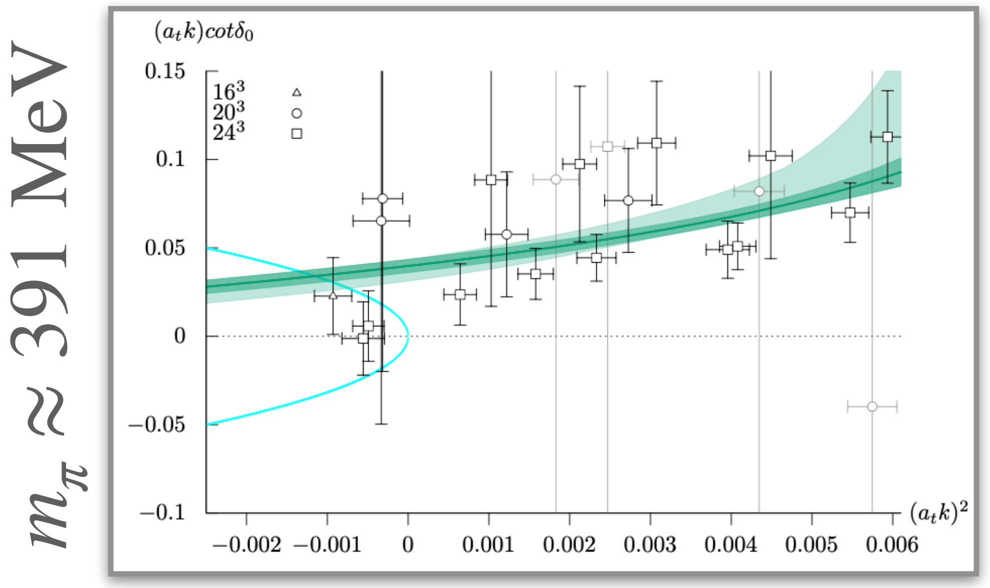
Lyu et al., PRL 131, 161901 (2023)



Padmanath, Prelovsek, PRL 129, 032002 (2022)
Collins et al., PRD 109 (2024) 9, 094509



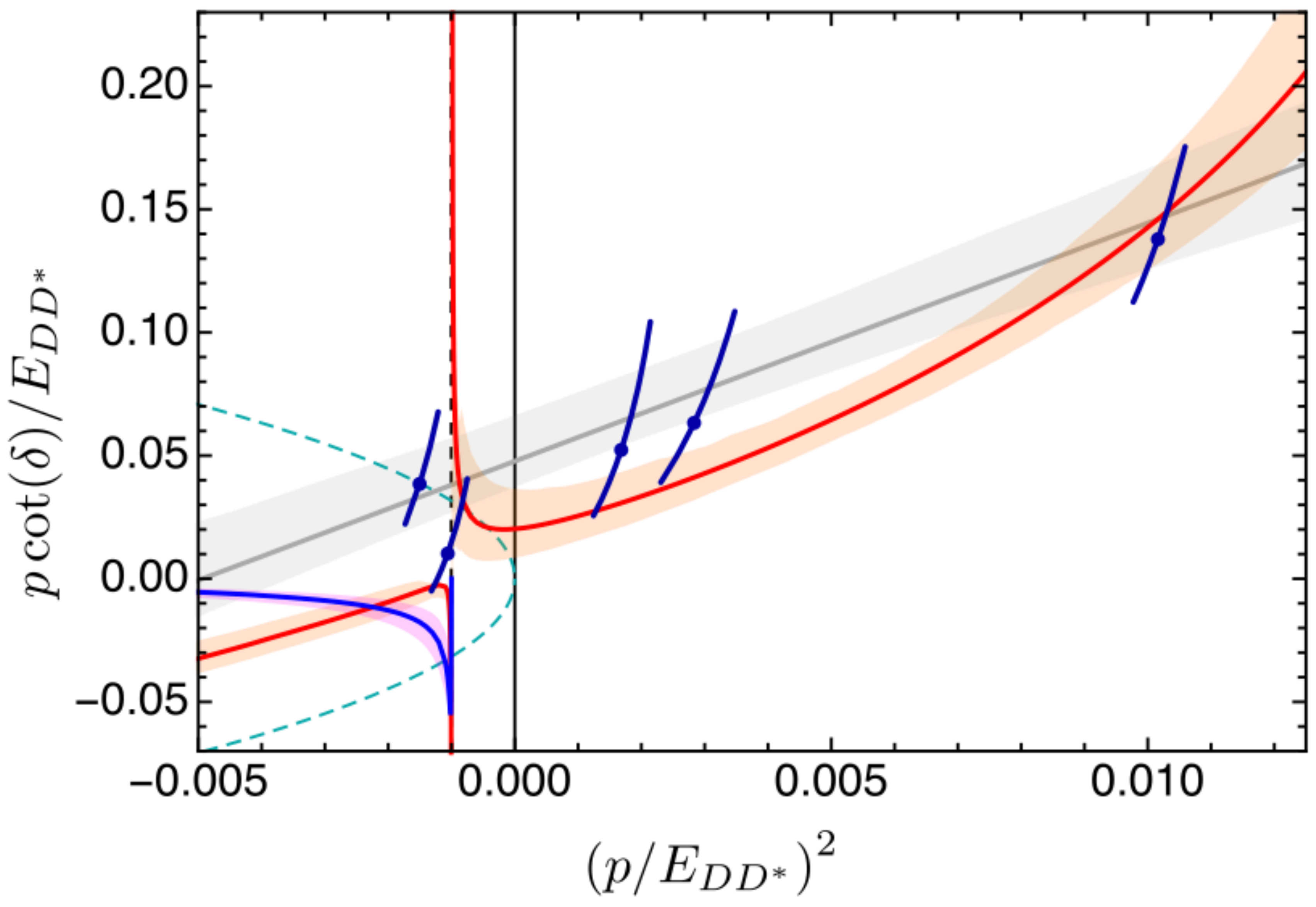
Chen et al. PLB 833, 137391 (2022)



Whyte, Wilson, Thomas, arXiv:2405.15741

The left-hand cut problem

Role of the left-hand cut contributions on pole extractions from lattice data...
Meng-Lin Du et al., PRL 131, 131903 (2023)



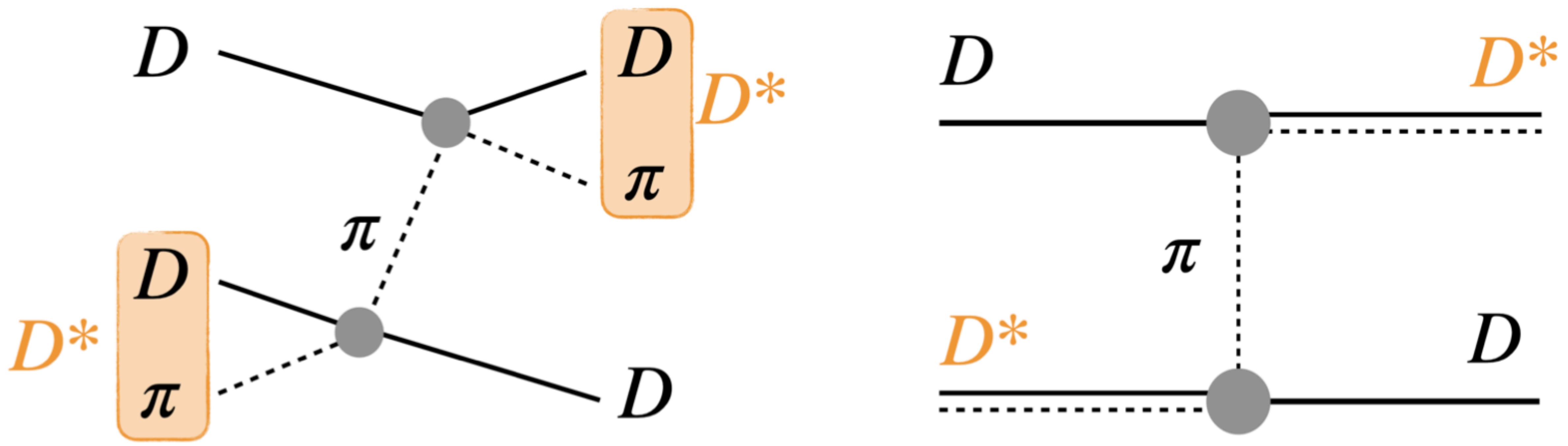
$$s_{\mathrm{lhc}} = s_{\mathrm{thr}} - m_\pi^2 + (m_{D^*} - m_D)^2$$

$$\sqrt{s_{\mathrm{lhc}}} \approx 3966 \text{ MeV} \quad \sqrt{s_{\mathrm{thr}}} \approx 3975 \text{ MeV}$$

Presence of the left-hand cut:
a) invalidates the Lüscher formalism
b) invalidates the effective-range expansion

Incorporating DDπ effects and left-hand cuts in lattice QCD studies of T_{cc}^+
Hansen, Romero-López, Sharpe, arXiv:2401.06609

Raposo, Hansen, arXiv:2311.18793
Lu Meng et al., Phys.Rev.D 109, L071506 (2024)
Bubna et al. JHEP 05 (2024)

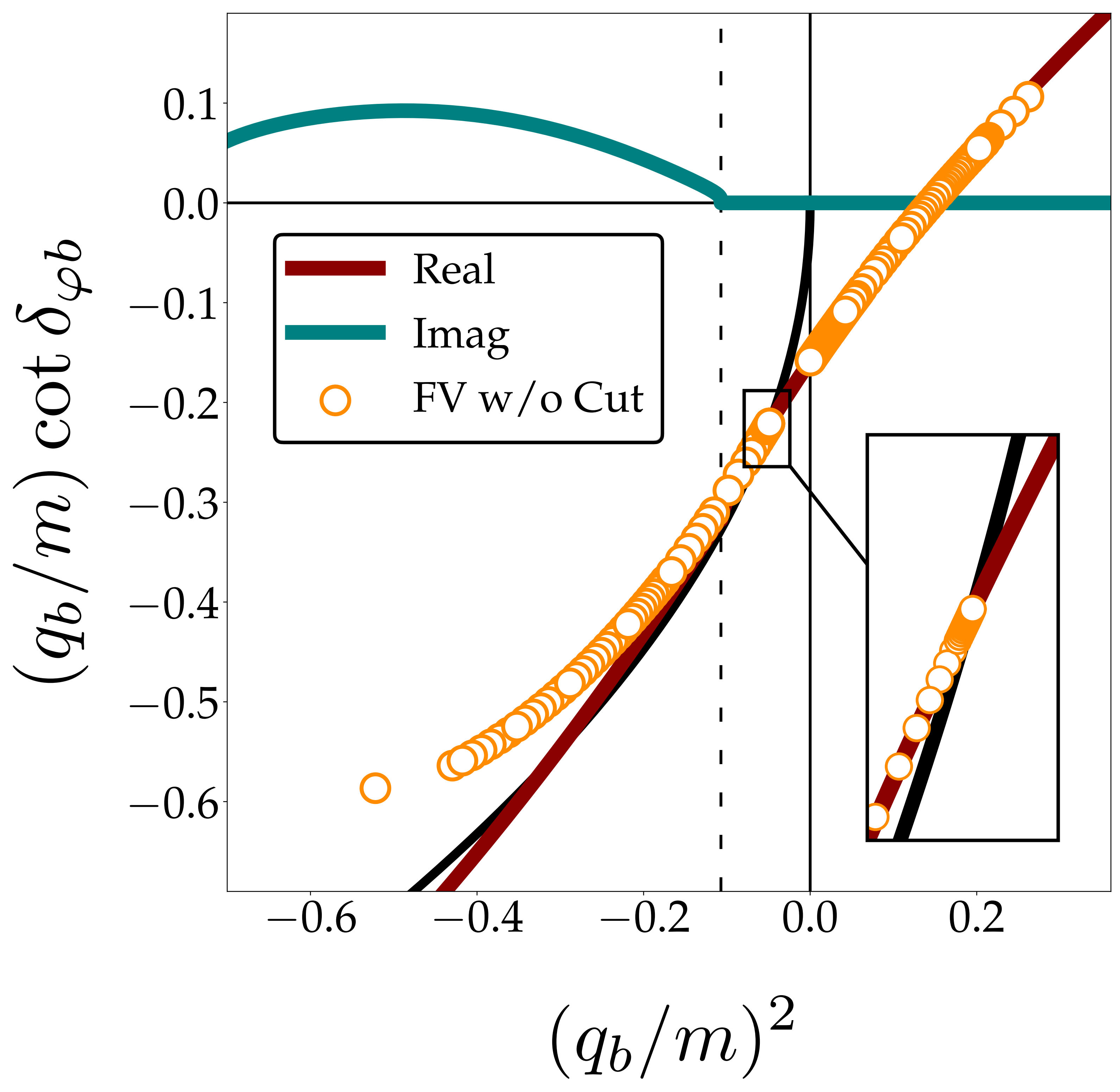


$$s_{\mathrm{lhc},2} = s_{\mathrm{thr}} - 4m_\pi^2 + (m_{D^*} - m_D)^2$$

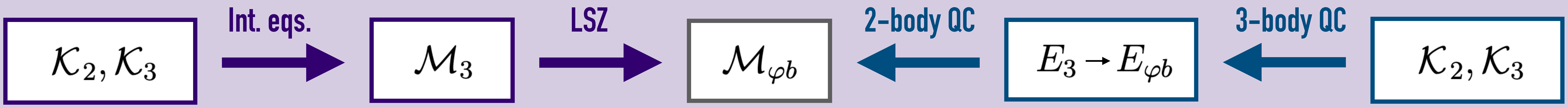
$$\sqrt{s_{\mathrm{lhc},2}} \approx 3937 \text{ MeV} \quad \sqrt{s_{\mathrm{thr}}} \approx 3975 \text{ MeV}$$

Breakdown of the Lüscher formalism

Analytic continuation of the relativistic three-body amplitudes
Dawid, Islam, Briceño, PRD 108 (2023) 3, 034016
Numerical exploration of three relativistic particles...
Romero-Lopez et al. JHEP 10 (2019) 007



$$\lim_{\sigma',\sigma\to m_{D^*}^2} \mathcal{M}_{DD\pi} = \frac{g}{\sigma' - m_{D^*}^2}\,\mathcal{M}_{DD^*}\,\frac{g}{\sigma' - m_{D^*}^2}$$



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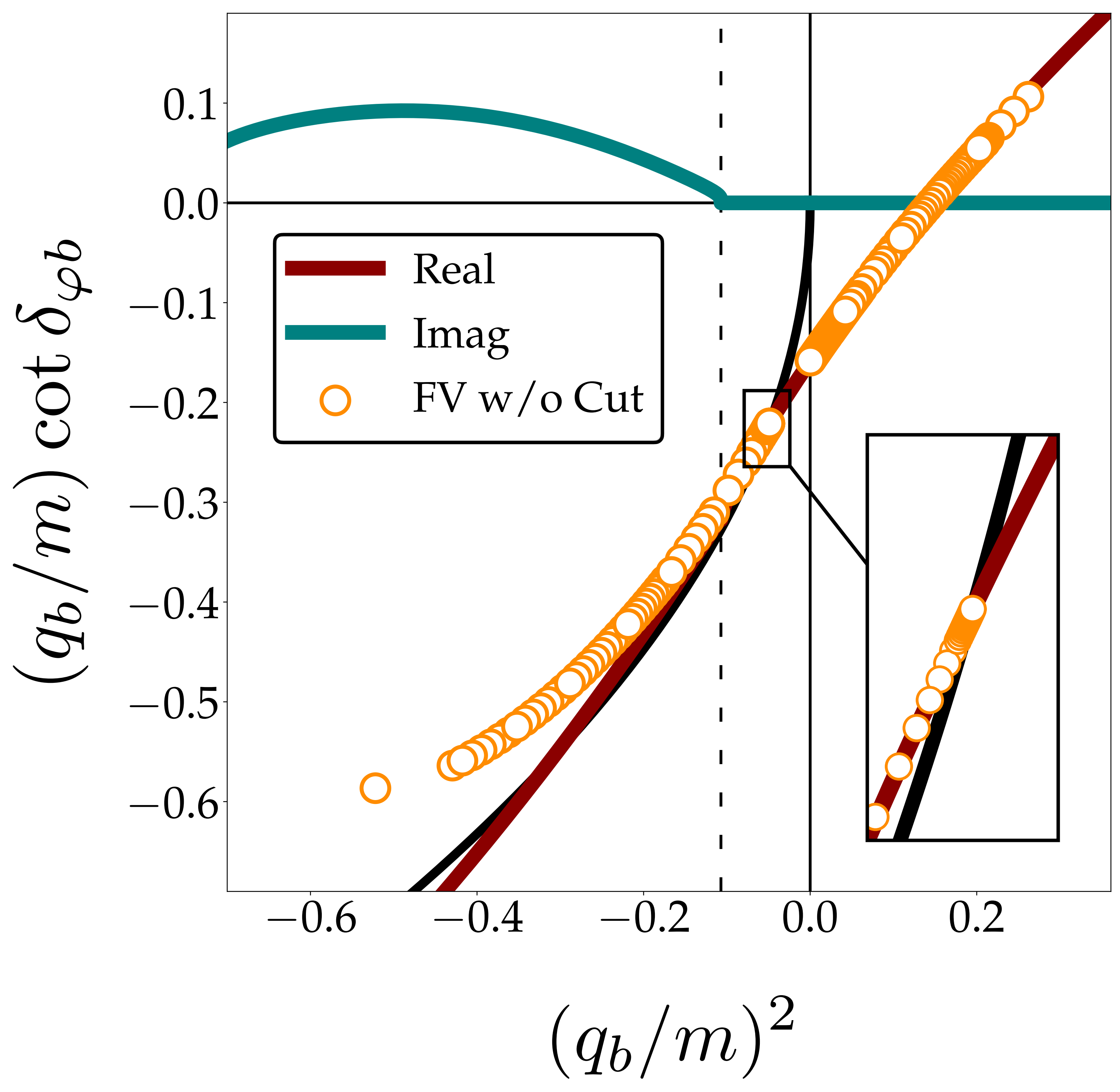
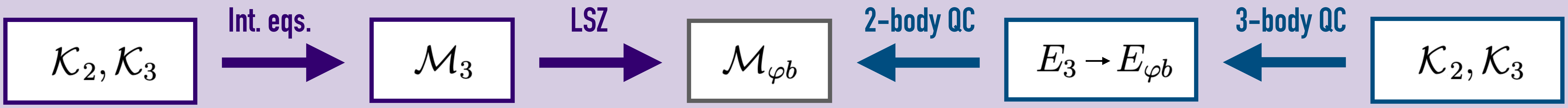


STRATEGY

1. Apply the three-body quantization condition to states with DD* quantum numbers (regardless of the pion mass)
2. Extract the DDπ-relevant two-and three-body K matrices
3. Solve the integral equations relating these objects to the continuum DDπ scattering amplitude
4. Employ the LSZ reduction formula to obtain the DD* amplitude that accounts for the pion exchanges

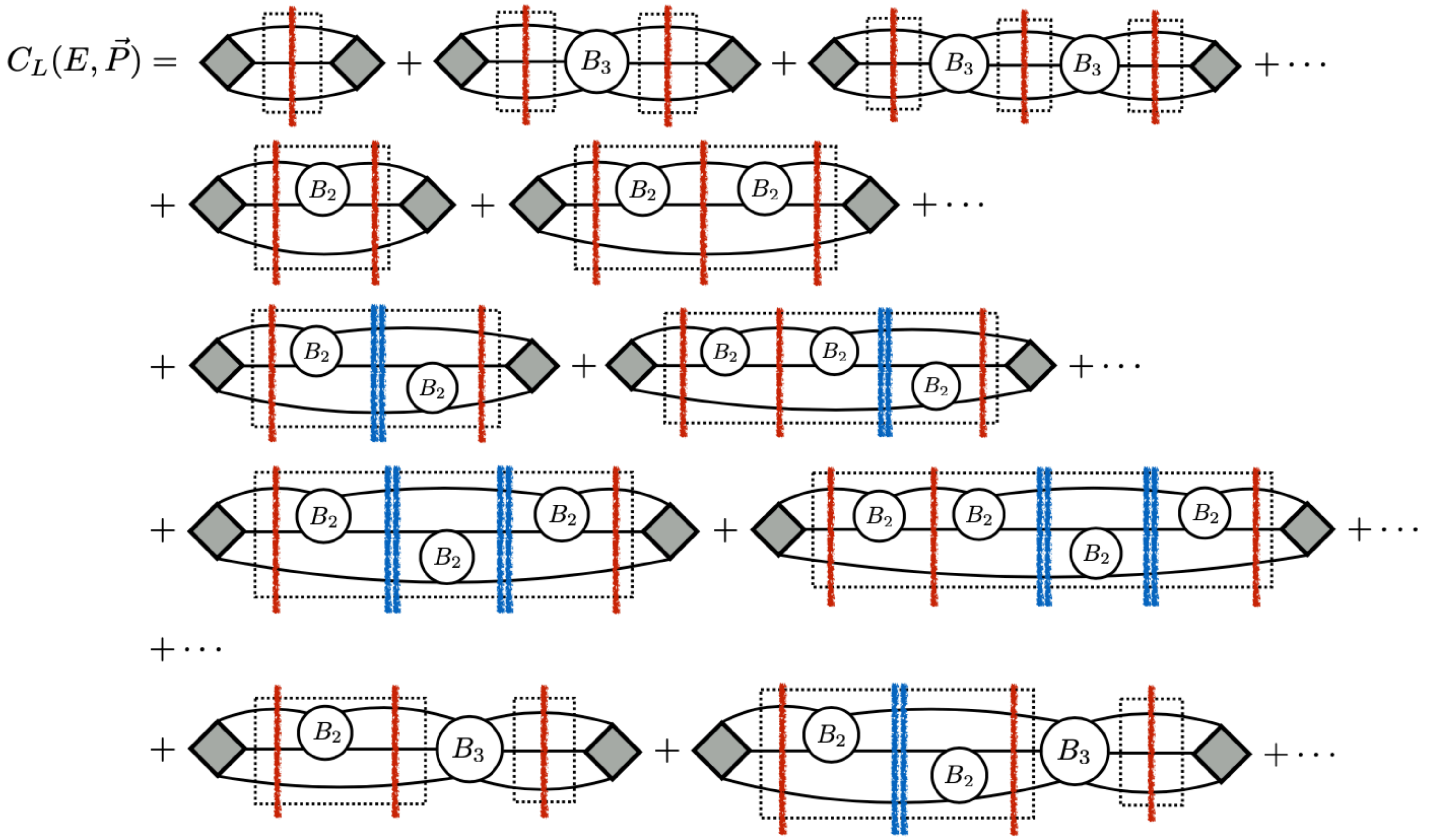


REFT finite-volume quantization

Lattice QCD and three-particle decays of resonances
Hansen, Sharpe, Ann. Rev. Nucl. Part. Sci. 69 (2019) 65-107

Relativistic three-particle quantization condition for non-degenerate scalars
Three-particle finite-volume formalism for $\pi^+\pi^+K^+$ and related systems
Blanton, Sharpe, PRD 103 (2021) 5, 054503 and PRD 104 (2021) 3, 034509

Incorporating DDπ effects and left-hand cuts in lattice QCD studies of T_{cc}^+
Hansen, Romero-López, Sharpe, arXiv:2401.06609



$$\begin{matrix} (D\pi)D & (DD)\pi \end{matrix}$$

$$\begin{pmatrix} \mathcal{K}_3^{(11)} & \mathcal{K}_3^{(12)} \\ \mathcal{K}_3^{(21)} & \mathcal{K}_3^{(22)} \end{pmatrix} \begin{matrix} (D\pi)D \\ (DD)\pi \end{matrix}$$

Generalization to the relevant isospin

$$\frac{\mathbf{1}}{\mathbf{2}} \otimes \frac{\mathbf{1}}{\mathbf{2}} \otimes \mathbf{1} = \mathbf{0} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{2}$$

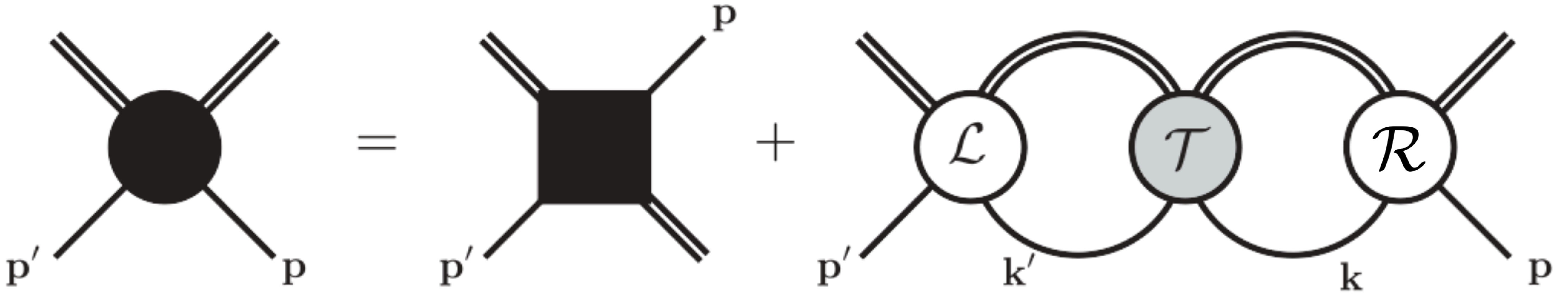
$$\det_{k,\ell,m} \left[\mathbb{1} - \boldsymbol{\mathcal{K}}_3(E^\star)\boldsymbol{F}_3(E,\boldsymbol{P},L)\right] = 0$$

$$\prod_{I\in\{0,1,2\}} \det_{k,\ell,m,f} \left[\mathbb{1} - \boldsymbol{\mathcal{K}}_3^I(E^\star)\boldsymbol{F}_3^I(E,\boldsymbol{P},L)\right] = 0$$

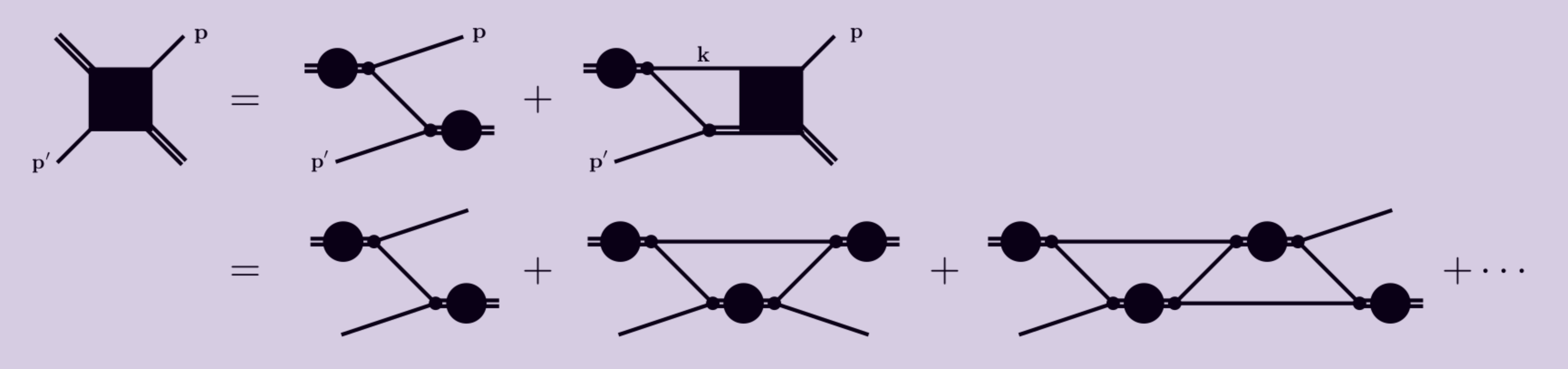
REFT three-body integral equations

diagrams by Andrew Jackura

$$\mathcal{M}_3 = \mathcal{D} + \mathcal{M}_{3,\mathrm{df}}$$



One−particle exchanges

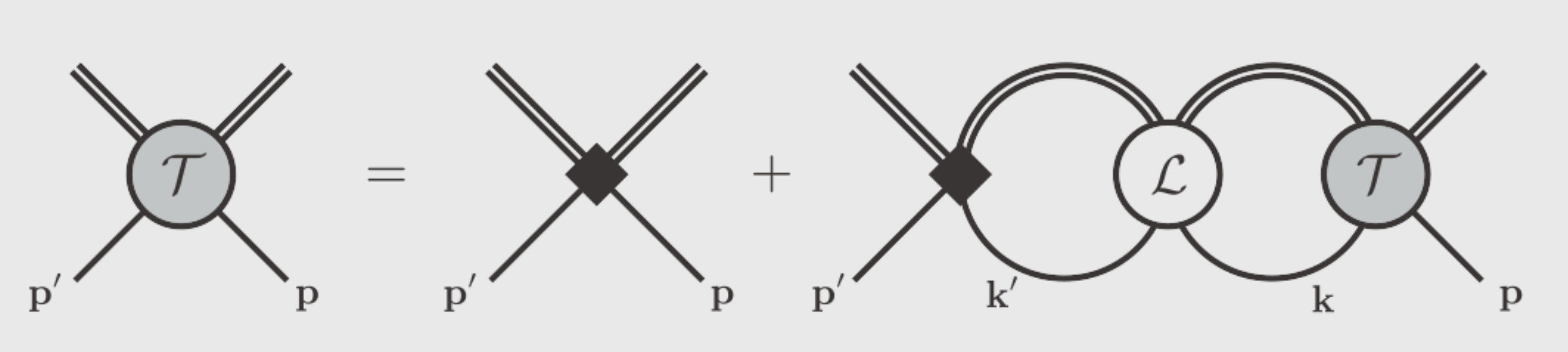


Three-body scattering: Ladders and Resonances
Mikhasenko, Wunderlich, Jackura, et al., *JHEP* 08 (2019) 080

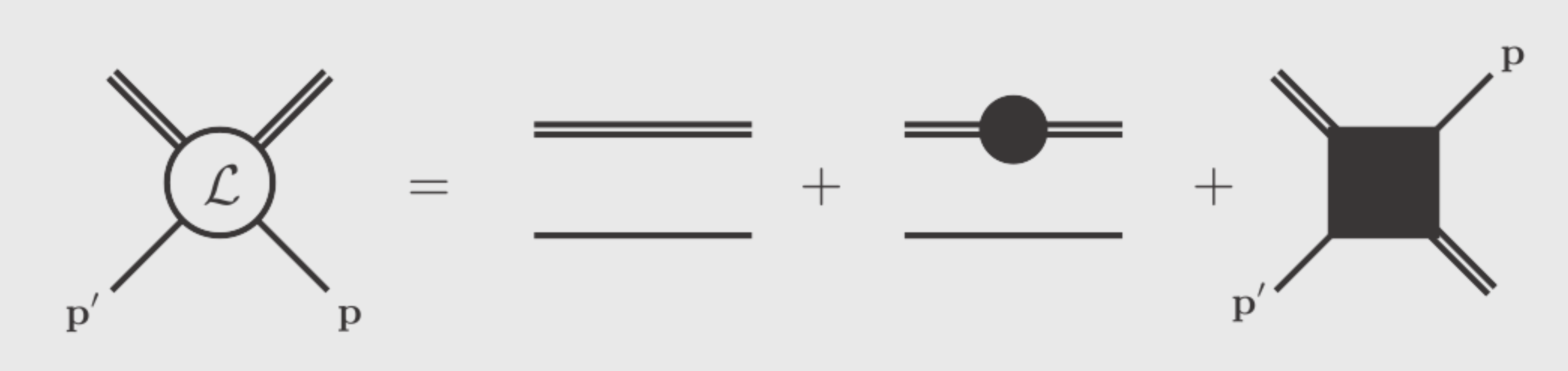
Equivalence of three-particle scattering formalisms
Jackura, Dawid, Fernandez-Ramirez, et al., PRD 100 (2019) 3, 034508

Equivalence of relativistic three-particle quantization conditions
Blanton, Sharpe, PRD 102 (2020) 5, 054515

Short−range amplitude



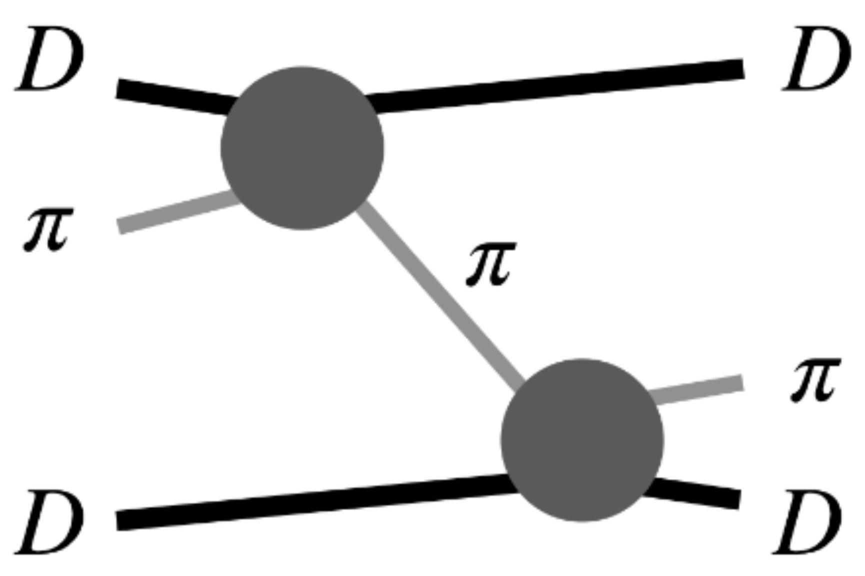
External−state rescatterings

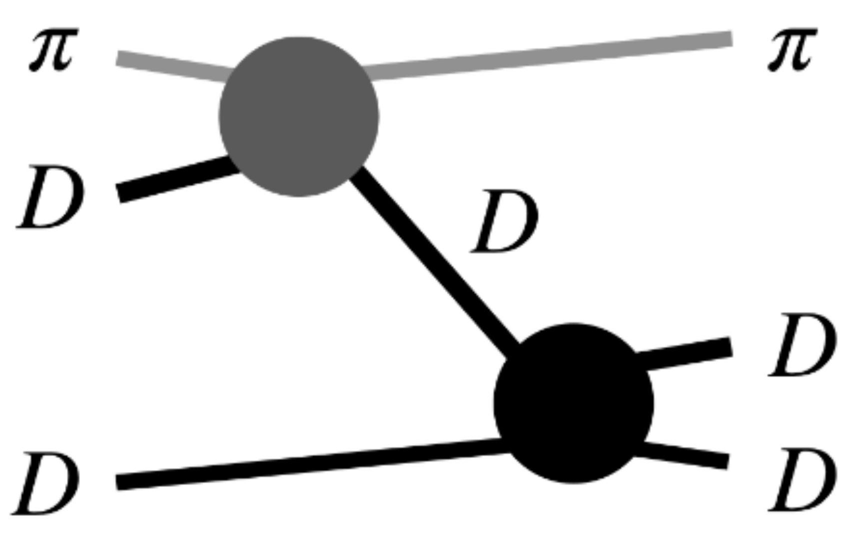


Generalizing to DDπ

$J^P = 1^+$

Dawid, Romero-López, Sharpe, in preparation





$$\mathcal{D} = -\mathcal{M}_2 G \mathcal{M}_2 - \mathcal{M}_2 G \mathcal{D}$$

The amplitude becomes a matrix describing coupled-channel scattering between pairs and spectators of different angular momenta (PW mixing allowed)

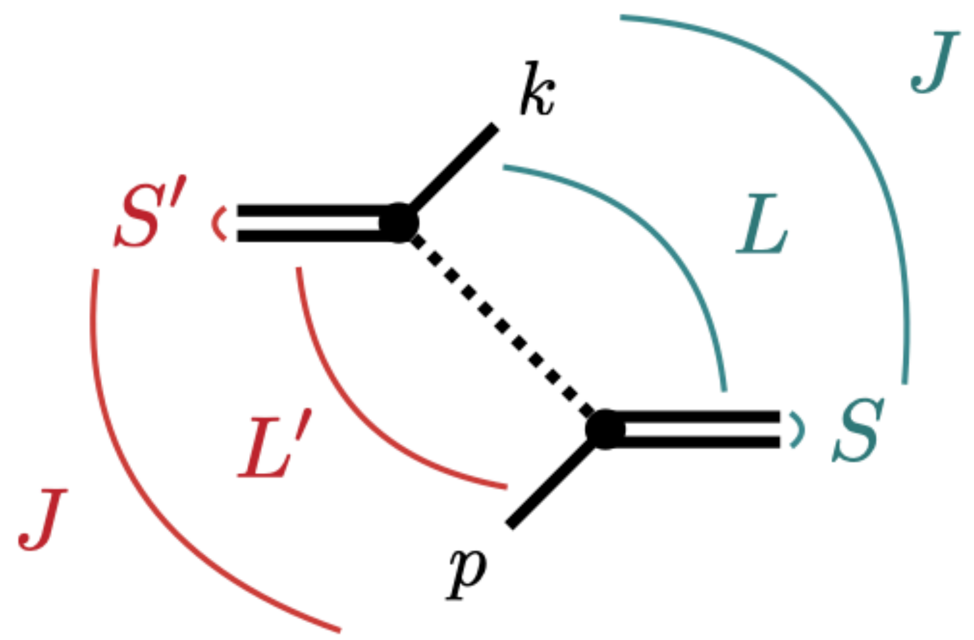
$$\begin{array}{cccc} (D\pi)D & (D\pi)D & (D\pi)D & (DD)\pi \end{array}$$

$$\begin{bmatrix} \mathcal{M}_3({}^1P_1|{}^1P_1) & \mathcal{M}_3({}^1P_1|{}^3S_1) & \mathcal{M}_3({}^3P_1|{}^3D_1) & \mathcal{M}_3({}^1P_1|{}^1P_1) \\ \mathcal{M}_3({}^3S_1|{}^1P_1) & \mathcal{M}_3({}^3S_1|{}^3S_1) & \mathcal{M}_3({}^3S_1|{}^3D_1) & \mathcal{M}_3({}^3S_1|{}^1P_1) \\ \mathcal{M}_3({}^3D_1|{}^1P_1) & \mathcal{M}_3({}^3D_1|{}^3S_1) & \mathcal{M}_3({}^3D_1|{}^3D_1) & \mathcal{M}_3({}^3D_1|{}^1P_1) \\ \mathcal{M}_3({}^1P_1|{}^1P_1) & \mathcal{M}_3({}^1P_1|{}^3S_1) & \mathcal{M}_3({}^1P_1|{}^3D_1) & \mathcal{M}_3({}^1P_1|{}^1P_1) \end{bmatrix} \begin{matrix} (D\pi)D \\ (D\pi)D \\ (D\pi)D \\ (DD)\pi \end{matrix}$$

$$\mathcal{M}_{DD^*}(E) = [\mathcal{M}_{DD^*}({}^3S_1|{}^3S_1)]$$

Partial-wave projection of the one-particle exchange in three-body scattering amplitudes
Jackura, Briceño, PRD 109, 096030 (2024)

$$\mathcal{G}({}^{2S'+1}L'_J|{}^{2S+1}L_J) \; =$$



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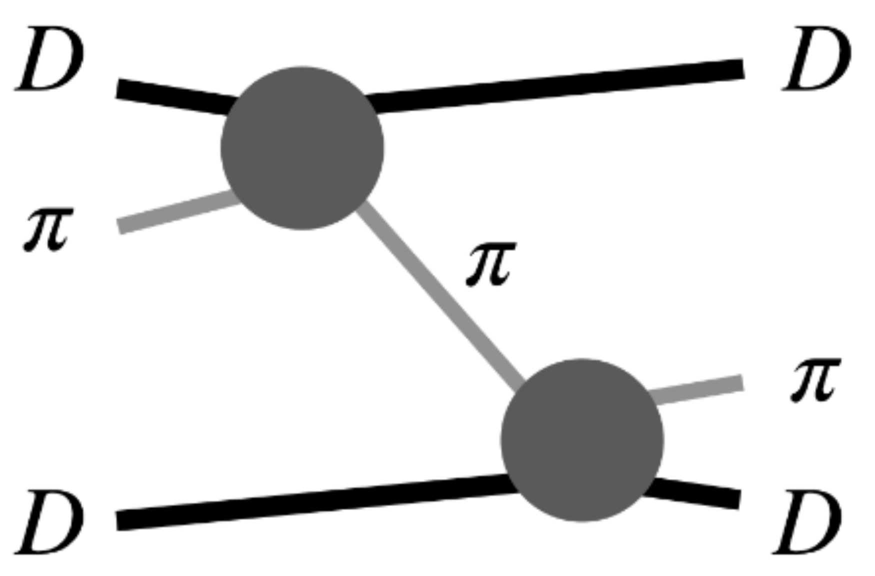

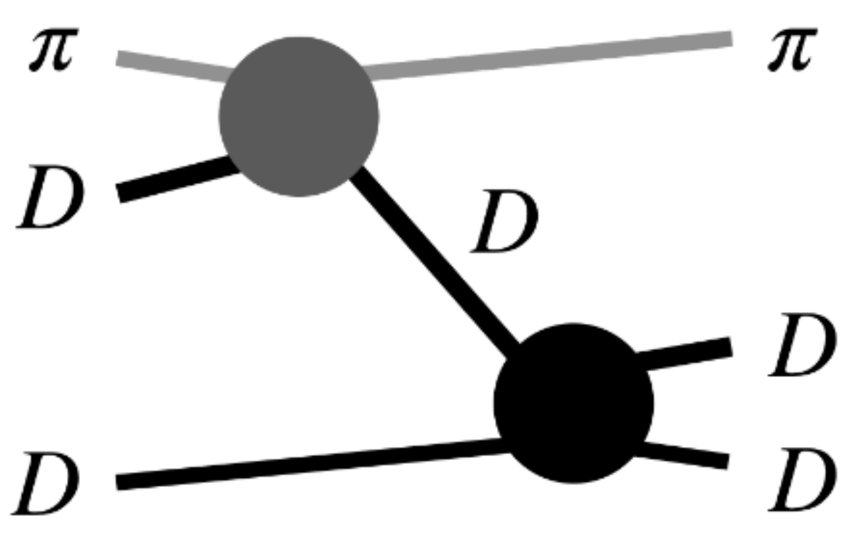

$$\begin{array}{cccc} (D\pi)D & (D\pi)D & (D\pi)D & (DD)\pi \end{array}$$

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$$\mathcal{M}_{DD^*}(E) = \begin{bmatrix} \mathcal{M}_{DD^*}({}^3S_1|{}^3S_1) & \mathcal{M}_{DD^*}({}^3S_1|{}^3D_1) \\ \mathcal{M}_{DD^*}({}^3D_1|{}^3S_1) & \mathcal{M}_{DD^*}({}^3D_1|{}^3D_1) \end{bmatrix}$$

Partial-wave projection of the one-particle exchange in three-body scattering amplitudes
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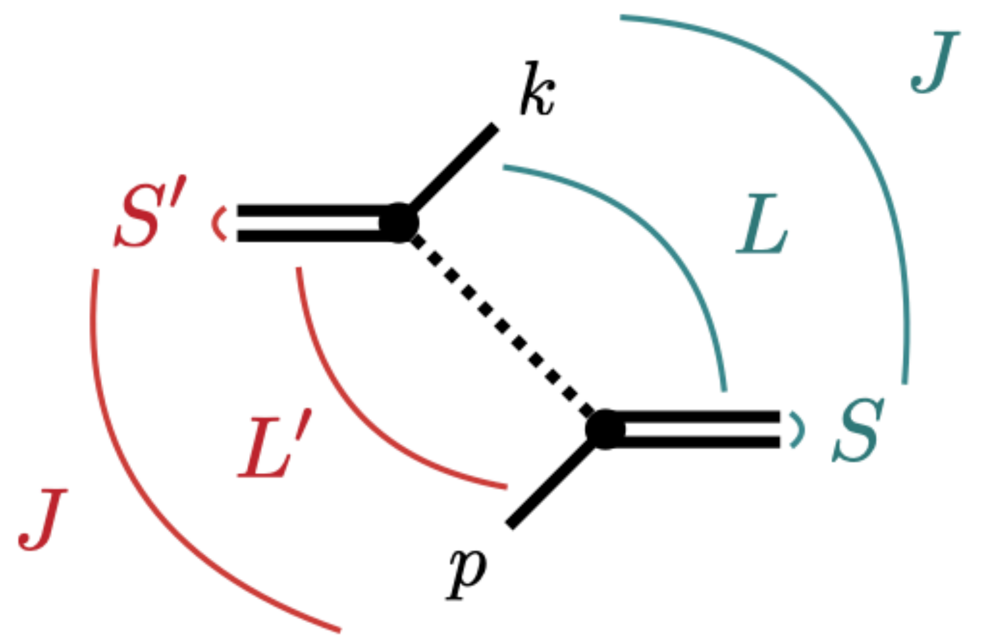


Including three-body forces $J^P = 1^+$

Implementing the three-particle quantization condition for ππK and related systems
Blanton, Romero-López, Sharpe, JHEP 02 (2022) 098

Dawid, Romero-López, Sharpe, in preparation

$$\mathcal{T} = \mathcal{K}_3 - \mathcal{K}_3\, \rho\, \mathcal{L}\, \mathcal{T}$$

Solution of another integral equation is unnecessary for certain models of the three-body K matrix

$$\mathcal{K}_3^{(ij)}(p,k) = \sum_a \mathcal{K}_{L,a}^{(i)}(p)\, \mathcal{K}_{R,a}^{(j)}(k)$$

$$\mathcal{T} = \mathcal{K}_L^T \left[1 + \mathcal{I}\right]^{-1} \mathcal{K}_R$$

Inversion of a 2 x 2 matrix composed of double integrals of the rescattering function.

Threshold expansion

$$\mathcal{K}_3 = \mathcal{K}_3^{\text{iso},0} + \mathcal{K}_3^{\text{iso},1}\Delta + \mathcal{K}_3^B \Delta_2^S + \mathcal{K}_3^E t_{22}'$$

$$\Delta = \frac{s - (2m_D + m_\pi)^2}{(2m_D + m_\pi)^2} \qquad \tilde{t}_{22} = \frac{(p_2 - p_2')^2}{(2m_D + m_\pi)^2}$$

The last term contributes, for instance,

$$\mathcal{K}_3({}^3S_1|{}^3S_1) = \frac{2}{27}\mathcal{K}_3^E\, q_p^\star\, q_k^\star\, (\gamma_p + 2)(\gamma_k + 2)$$

Relative two−body momentum in a pair

Boost to pair's rest frame

Application: two-body interactions

Dawid, Romero-López, Sharpe, in preparation

Padmanath, Prelovsek, PRL 129, 032002 (2022)

Scattering parameters

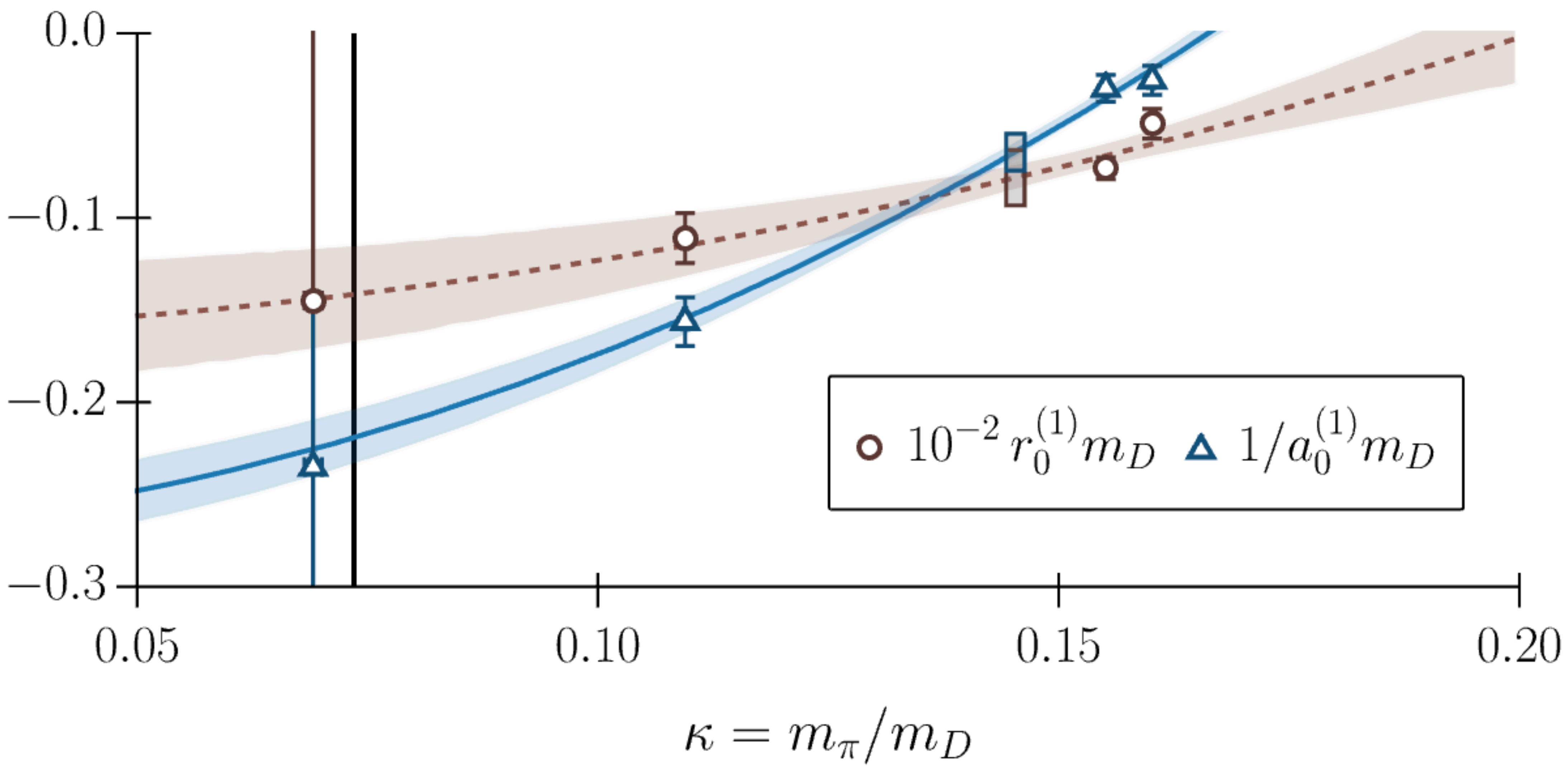
$$a_S^{D\pi},\ r_S^{D\pi},\ a_P^{D\pi},\ r_P^{D\pi},\ a_S^{DD},\ \mathcal{K}_3^E$$

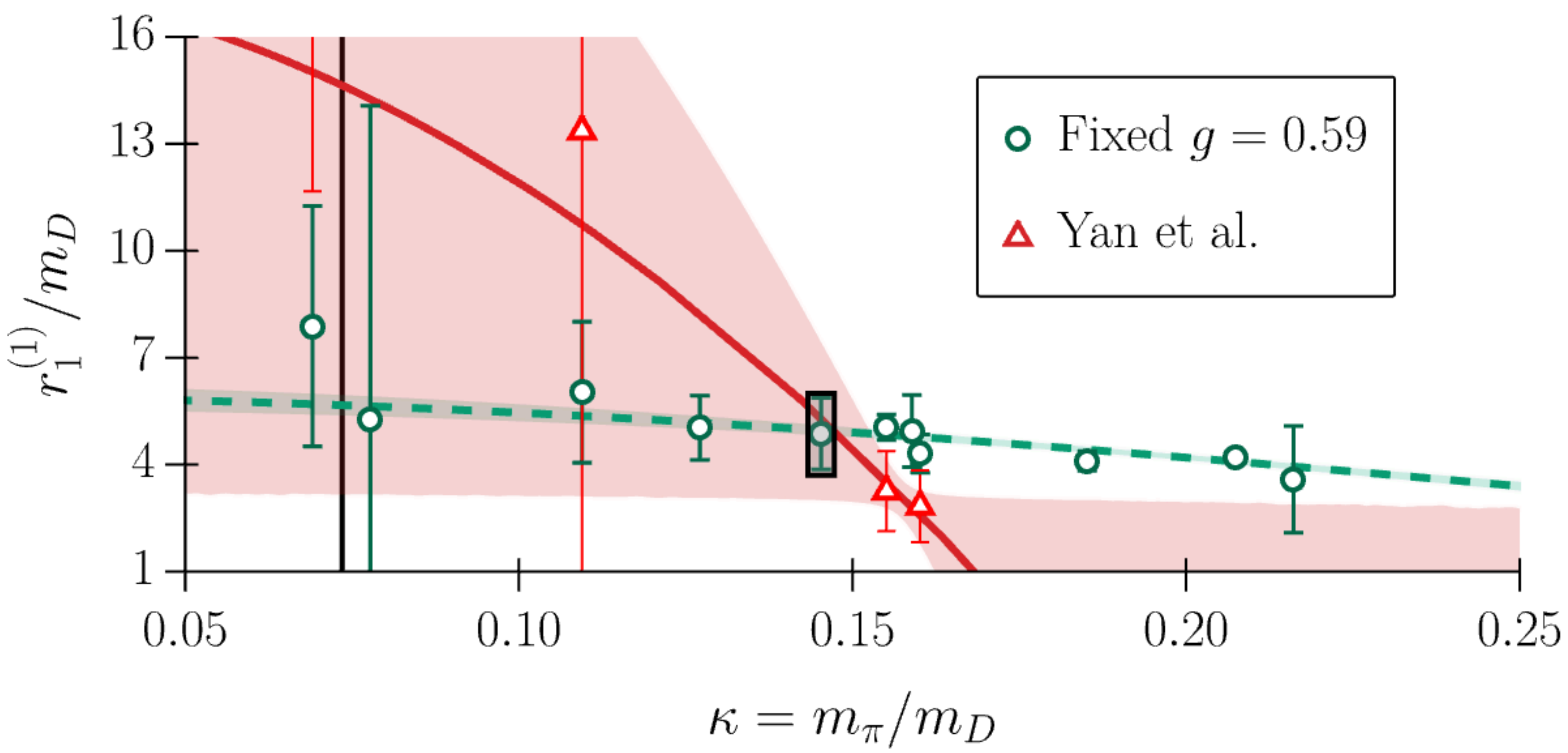
$$q_b^{2s+1}\cot\delta_s^{(n)} = -\frac{1}{a_s^{(n)}} + \frac{1}{2} r_s^{(n)} q_b^2$$

$m_\pi \approx 280$ MeV
$m_D \approx 1927$ MeV
$m_{D^*} \approx 2049$ MeV

$\kappa = m_\pi/m_D \approx 0.145$
$\kappa_{\rm phys} \approx 0.073$

Mohler et al., PRD 87, 034501 (2012)
Becirevic, Sanfilippo, PLB 721 (2013) 94-100
Moir et al. (HadSpec), JHEP 10 (2016) 011 (2016)
Gayer et al. (HadSpec), JHEP 07 (2021) 123
Yan et al., arXiv: 2404.13479 (2024)





Partial-wave mixing amplitude

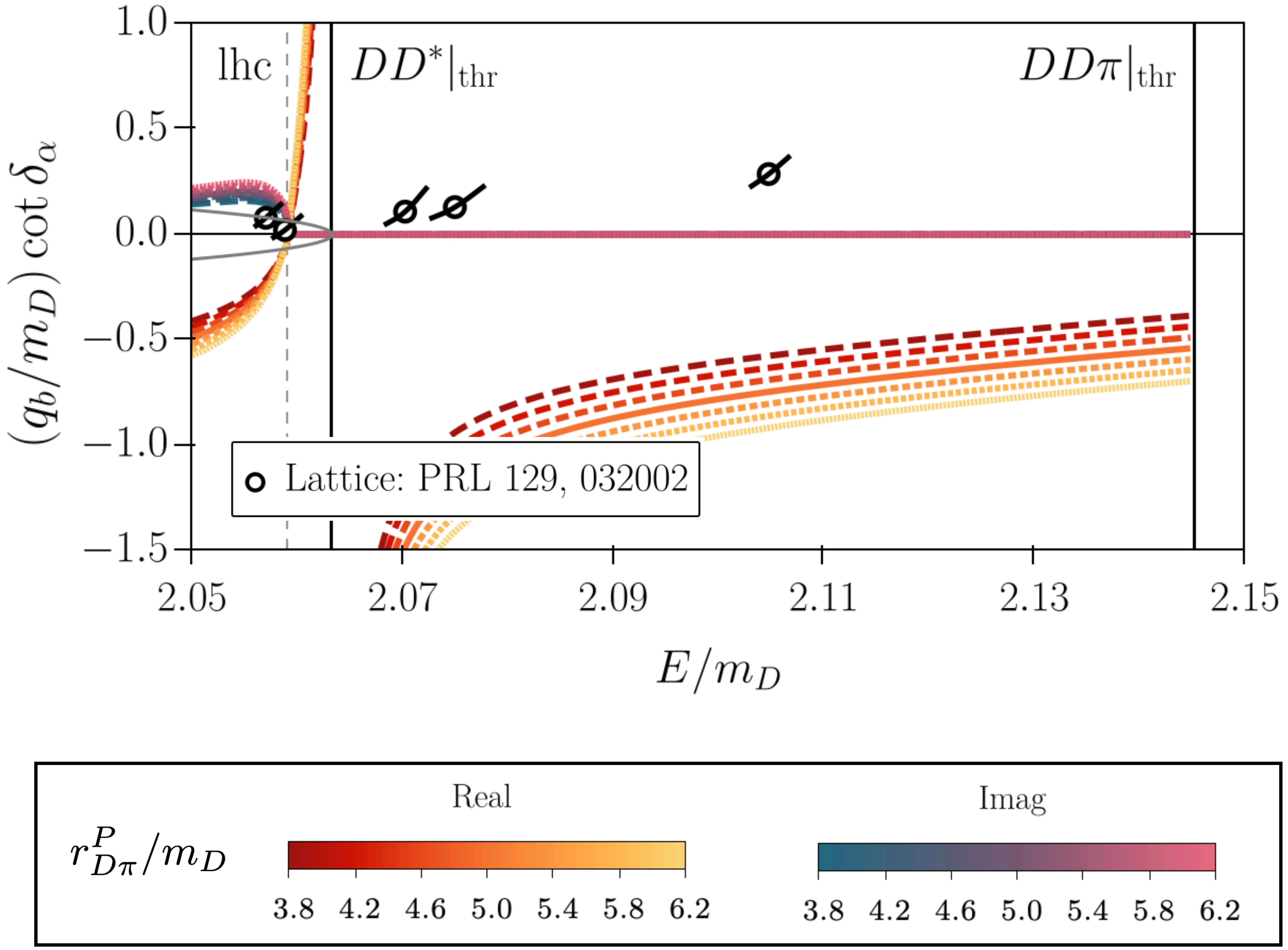
Blatt−Biederharn parametrization

$J^P = 1^+$

$$\begin{bmatrix} \mathcal{M}_{DD^*}(^3S_1|^3S_1) & \mathcal{M}_{DD^*}(^3S_1|^3D_1) \\ \mathcal{M}_{DD^*}(^3D_1|^3S_1) & \mathcal{M}_{DD^*}(^3D_1|^3D_1) \end{bmatrix}$$

$$q_b^{-\ell'} \left[\mathcal{K}_{DD^*}^{-1}\right]_{\ell',\ell} q_b^{-\ell} = \begin{pmatrix} \cos(\epsilon) & -\frac{1}{q_b^2}\sin(\epsilon) \\ q_b^2 \sin(\epsilon) & \cos(\epsilon) \end{pmatrix} \begin{pmatrix} q_b \cot(\delta_\alpha) & 0 \\ 0 & q_b^5 \cot(\delta_\beta) \end{pmatrix} \begin{pmatrix} \cos(\epsilon) & q_b^2 \sin(\epsilon) \\ -\frac{1}{q_b^2}\sin(\epsilon) & \cos(\epsilon) \end{pmatrix}$$

$\mathcal{K}_3^E = 0$



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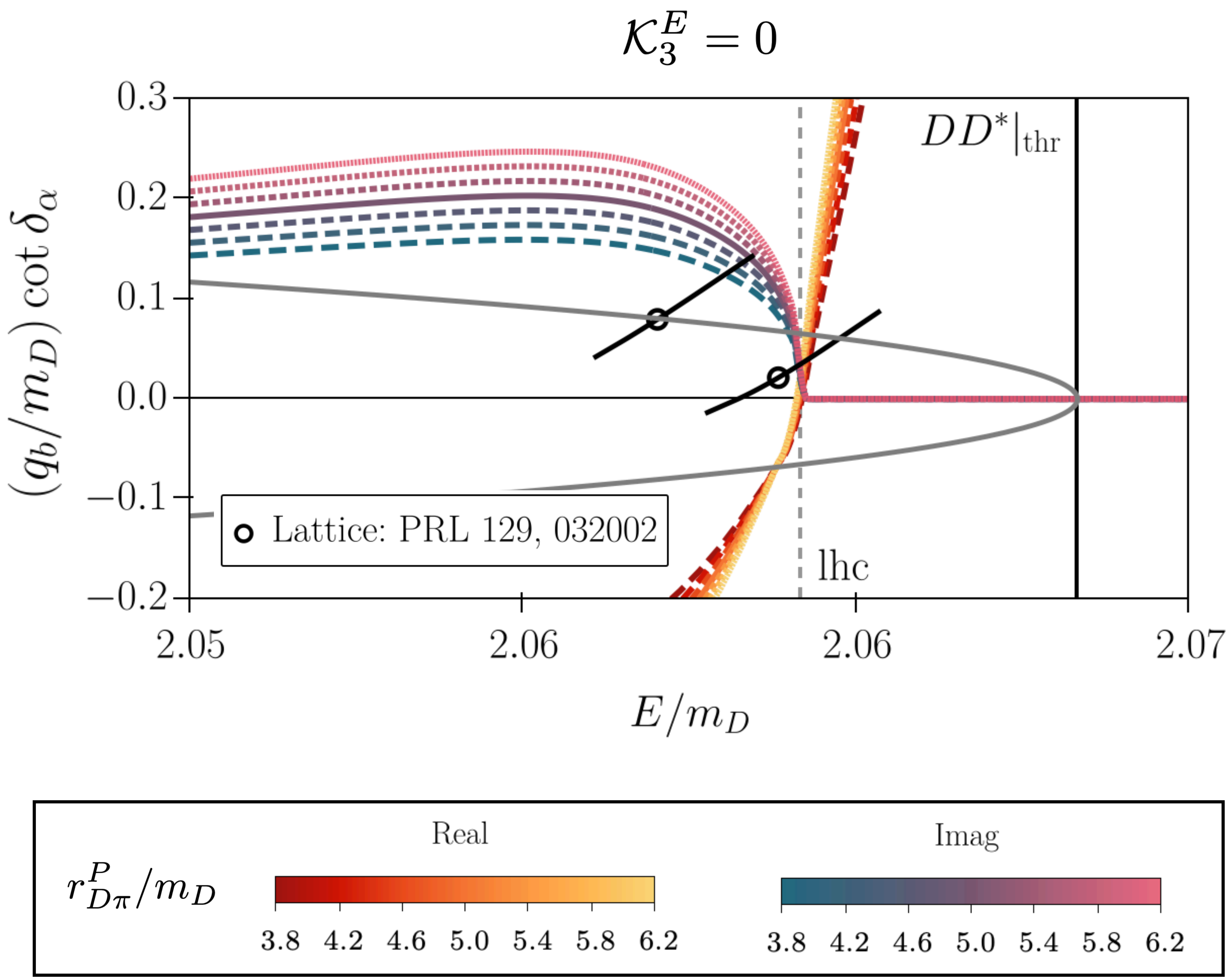
$$q_b^{-\ell'} \left[\mathcal{K}^{-1}_{DD^*}\right]_{\ell',\ell} q_b^{-\ell} = \begin{pmatrix} \cos(\epsilon) & -\frac{1}{q_b^2}\sin(\epsilon) \\ q_b^2 \sin(\epsilon) & \cos(\epsilon) \end{pmatrix} \begin{pmatrix} q_b \cot(\delta_\alpha) & 0 \\ 0 & q_b^5 \cot(\delta_\beta) \end{pmatrix} \begin{pmatrix} \cos(\epsilon) & q_b^2 \sin(\epsilon) \\ -\frac{1}{q_b^2}\sin(\epsilon) & \cos(\epsilon) \end{pmatrix}$$



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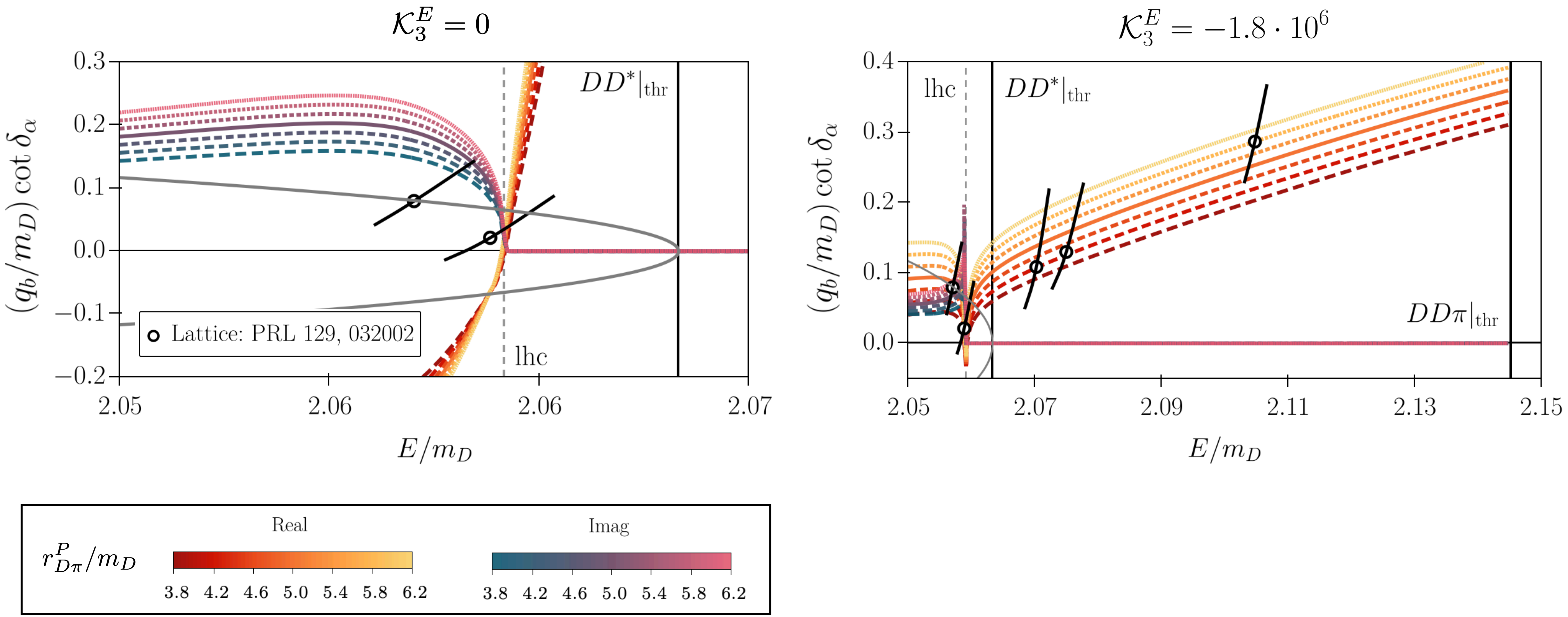

Partial-wave mixing amplitude

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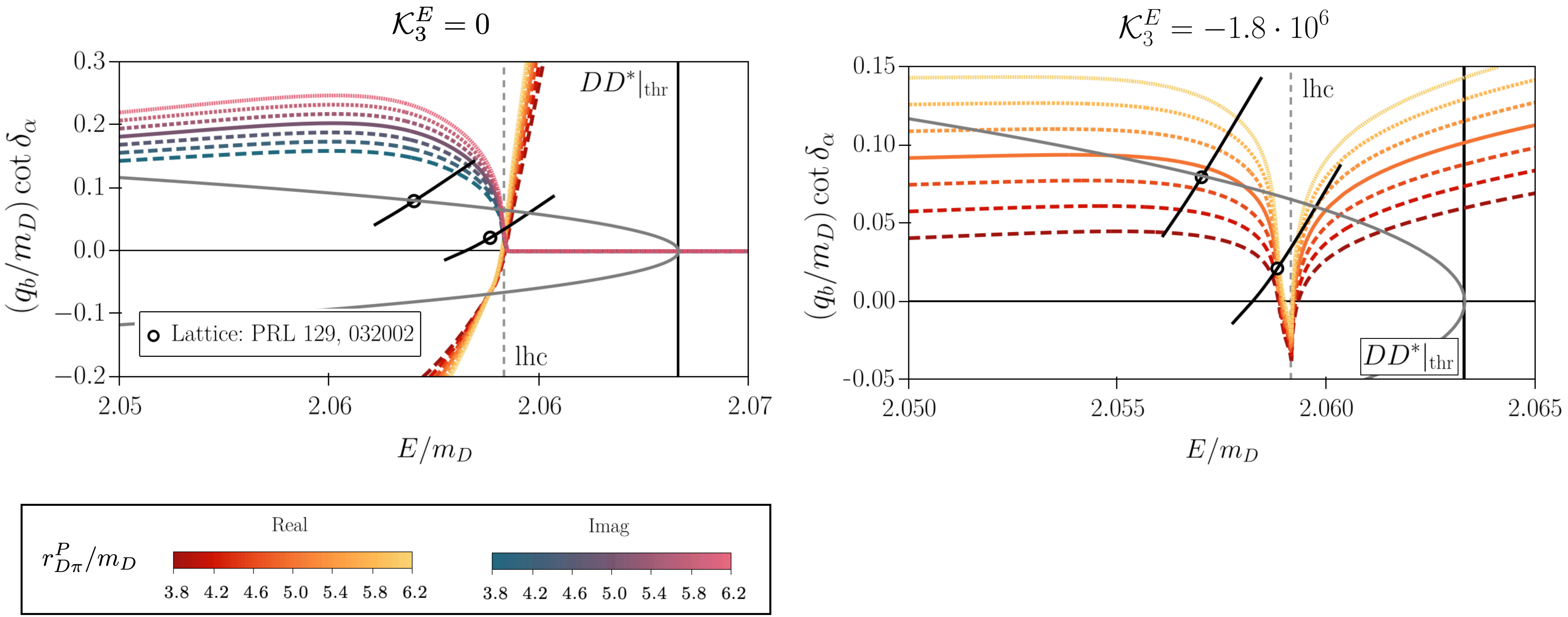

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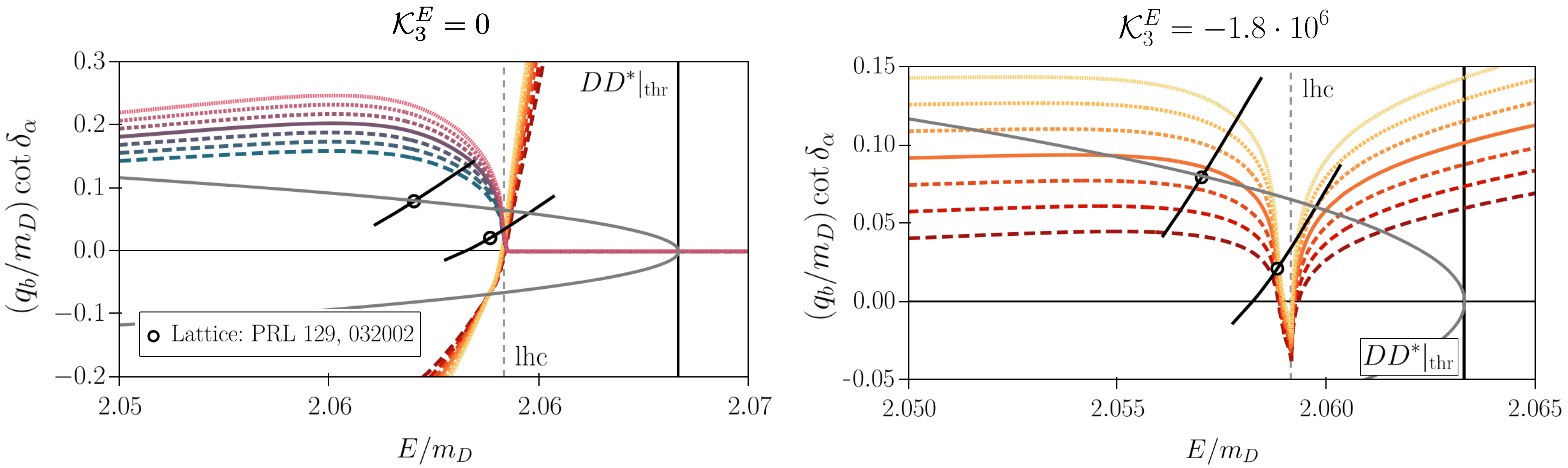
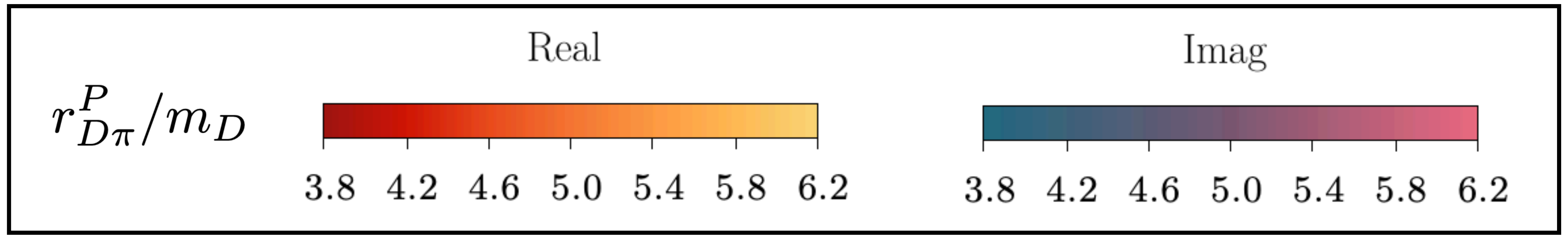
Blatt–Biederharn parametrization

$$\begin{bmatrix} \mathcal{M}_{DD^*}(^3S_1|^3S_1) & \mathcal{M}_{DD^*}(^3S_1|^3D_1) \\ \mathcal{M}_{DD^*}(^3D_1|^3S_1) & \mathcal{M}_{DD^*}(^3D_1|^3D_1) \end{bmatrix}$$

$$q_b^{-\ell'} \left[\mathcal{K}^{-1}_{DD^*}\right]_{\ell',\ell} q_b^{-\ell} = \begin{pmatrix} \cos(\epsilon) & -\frac{1}{q_b^2}\sin(\epsilon) \\ q_b^2 \sin(\epsilon) & \cos(\epsilon) \end{pmatrix} \begin{pmatrix} q_b \cot(\delta_\alpha) & 0 \\ 0 & q_b^5 \cot(\delta_\beta) \end{pmatrix} \begin{pmatrix} \cos(\epsilon) & q_b^2 \sin(\epsilon) \\ -\frac{1}{q_b^2}\sin(\epsilon) & \cos(\epsilon) \end{pmatrix}$$



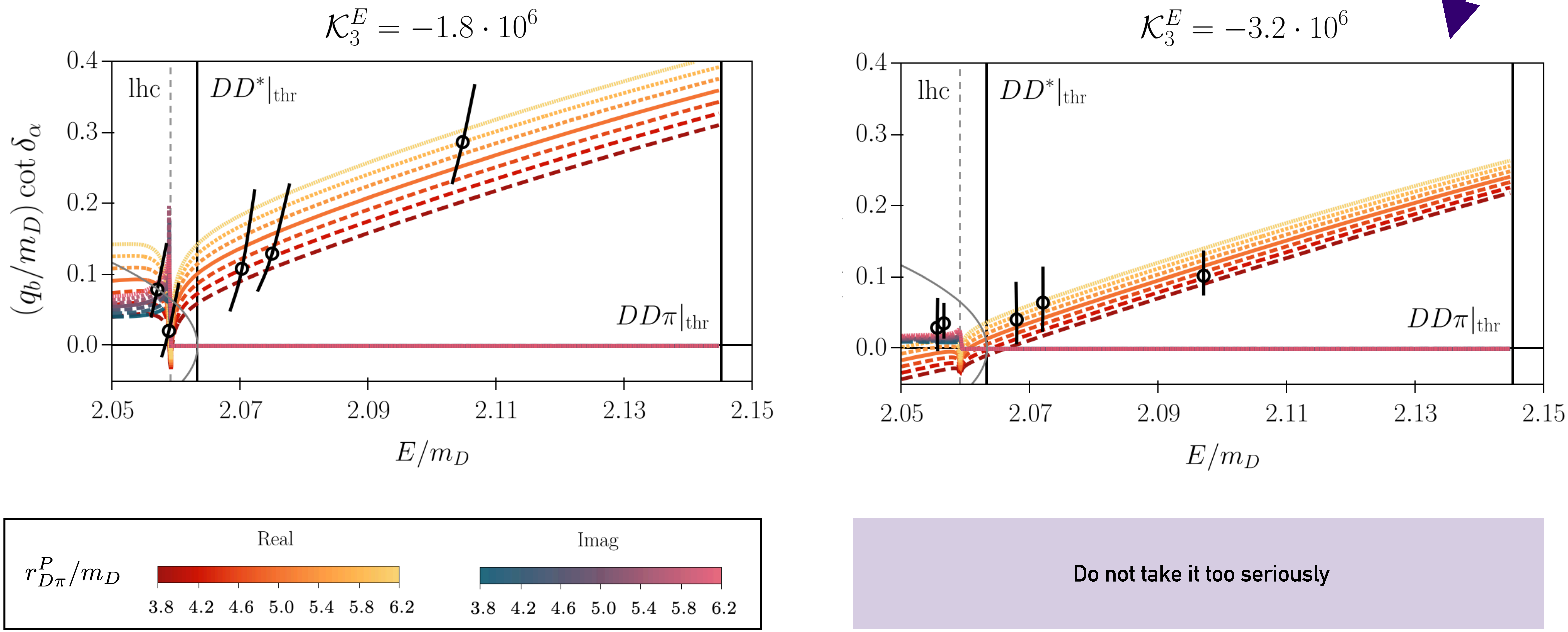


Observations:

(a) we find a virtual state in agreement with other approaches;
(b) simple model of three-body forces is enough to describe data;
(c) status of the effective-range expansion is unclear;

Extra, unplanned slide

Quickly prepared comparison with the data set from the previous talk:
Ivan Vujmilovic "T_{cc} via plane-wave approach and including diquark-antidiquark operators"



Do not take it too seriously

Extra, unplanned slide

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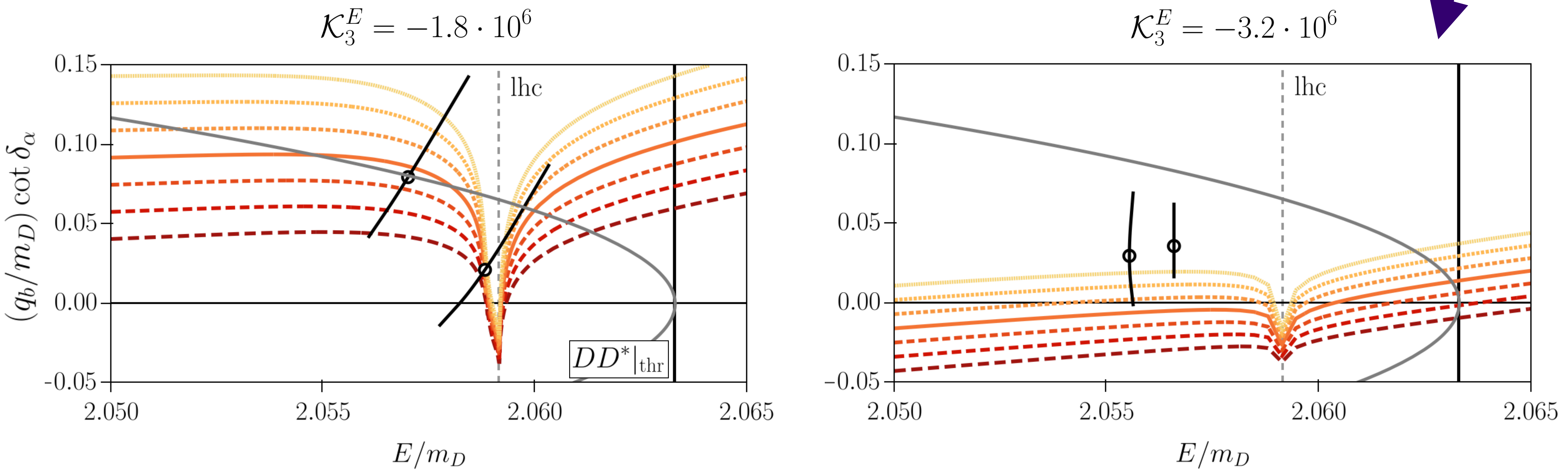
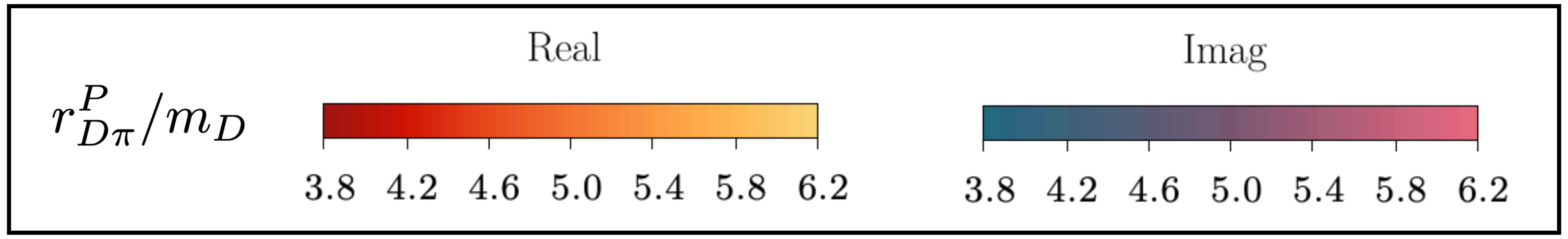
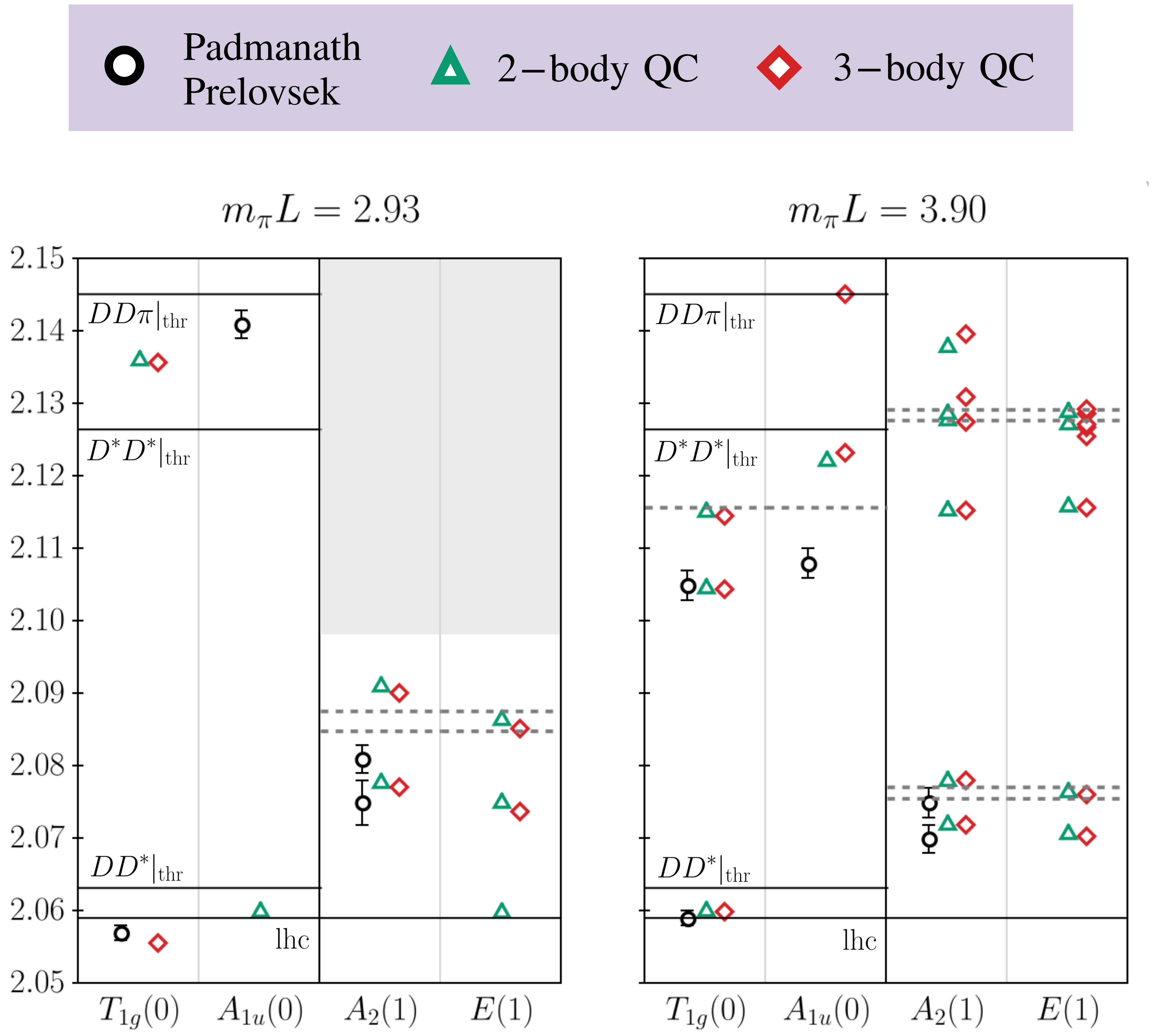




Do not take it too seriously

Comparing the finite-volume spectra

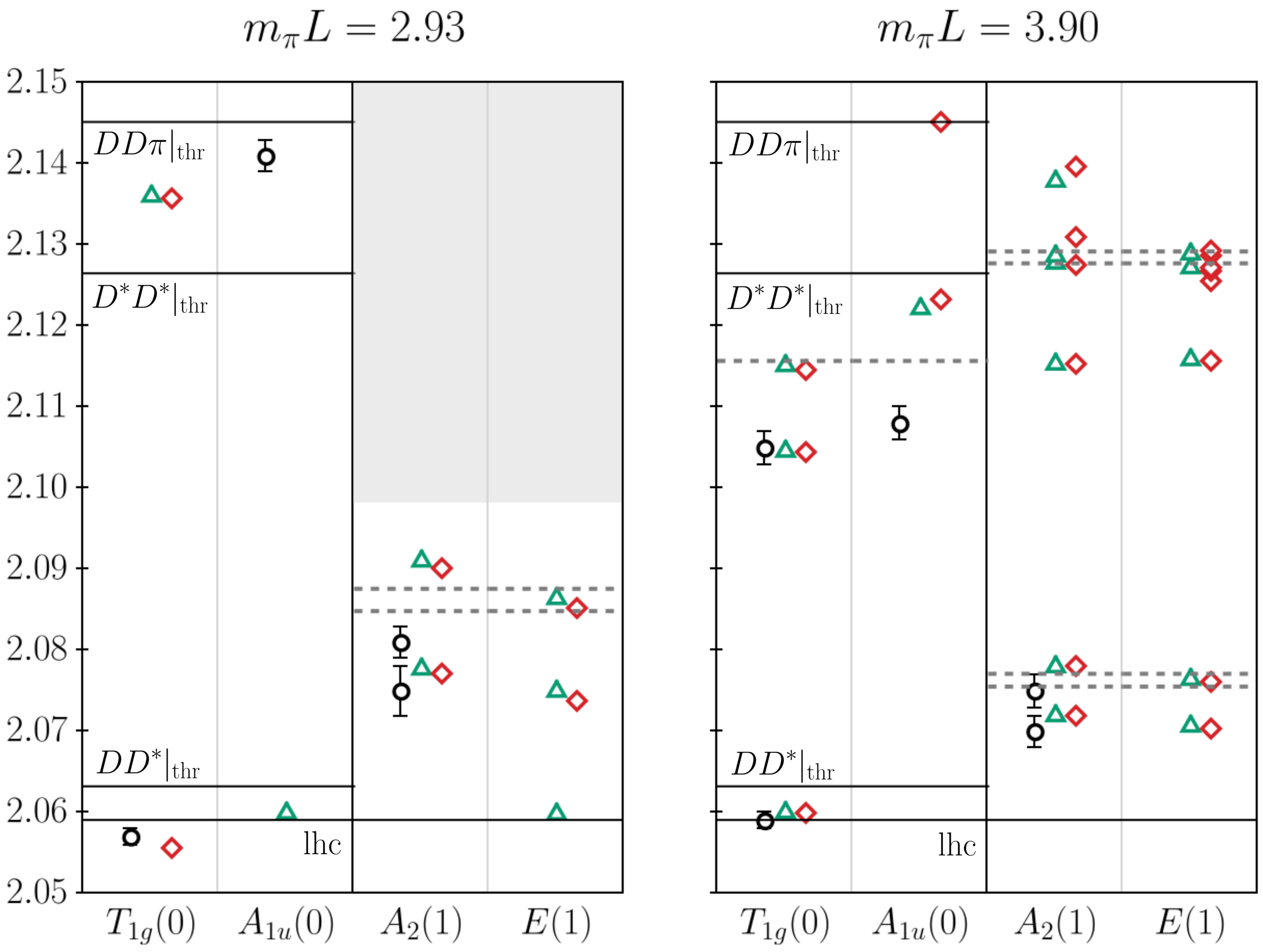


Further observations

- two-body QC clearly breaks down near the lhc for small lattice volumes,
- repulsive interaction in J=0 inconsistent with the lattice; (higher order terms needed)

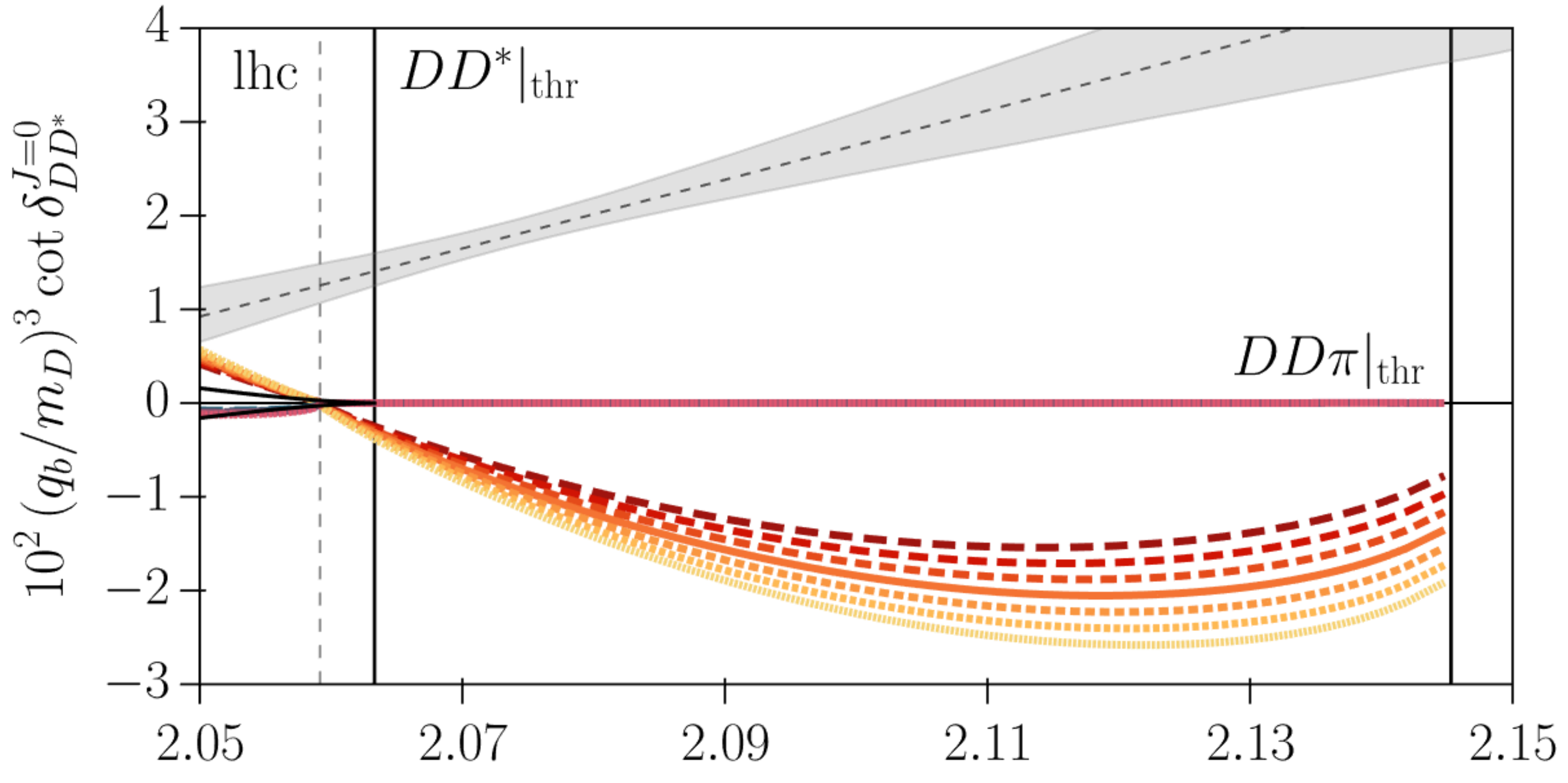
Comparing the finite-volume spectra

○ Padmanath Prelovsek △ 2−body QC ◇ 3−body QC

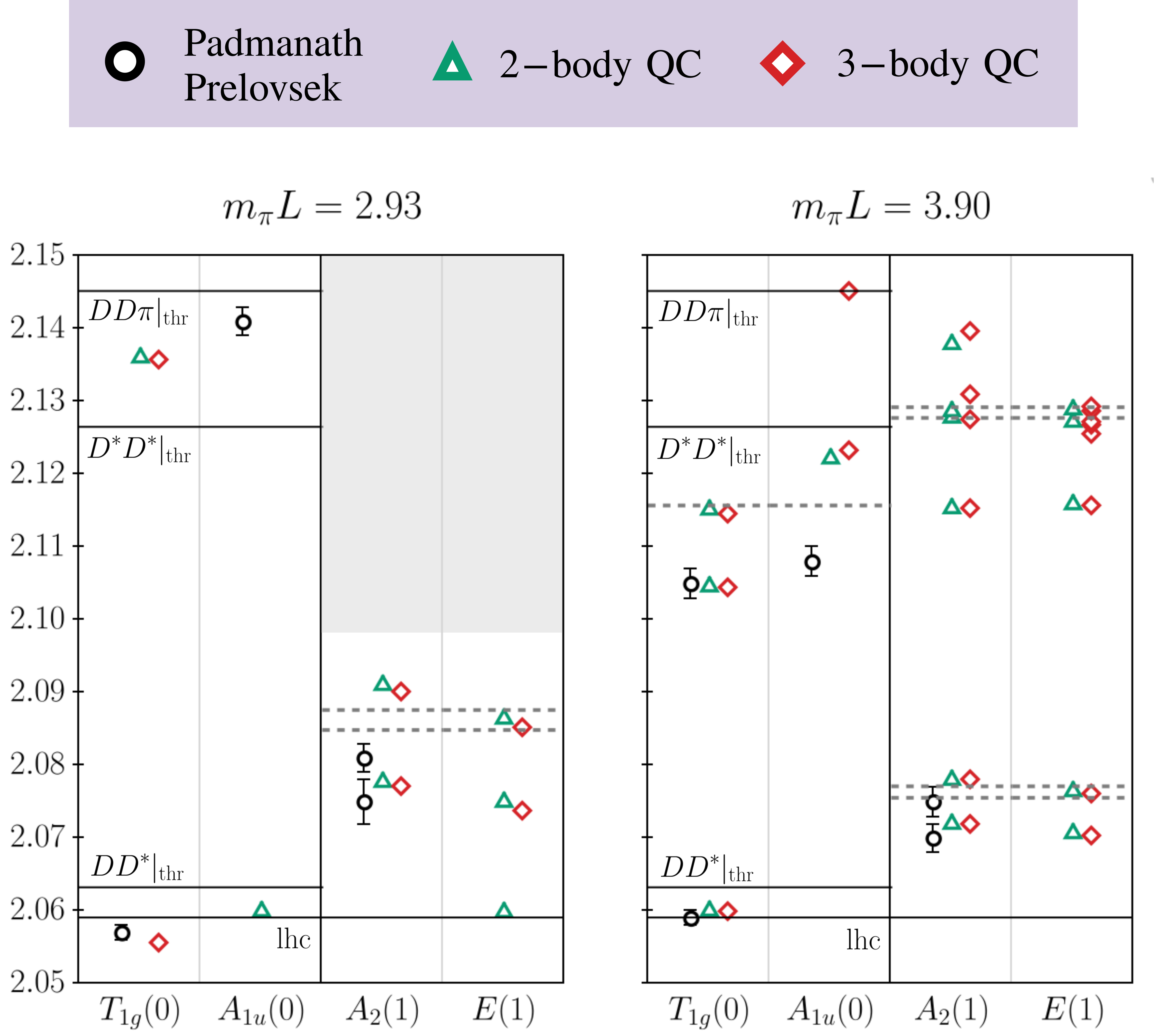


The $J^P = 0^-$ amplitude

$$\begin{bmatrix} \mathcal{M}_3({}^1S_0|{}^1S_0) & \mathcal{M}_3({}^1S_0|{}^3P_0) \\ \mathcal{M}_3({}^3P_0|{}^1S_0) & \mathcal{M}_3({}^3P_0|{}^3P_0) \end{bmatrix}$$



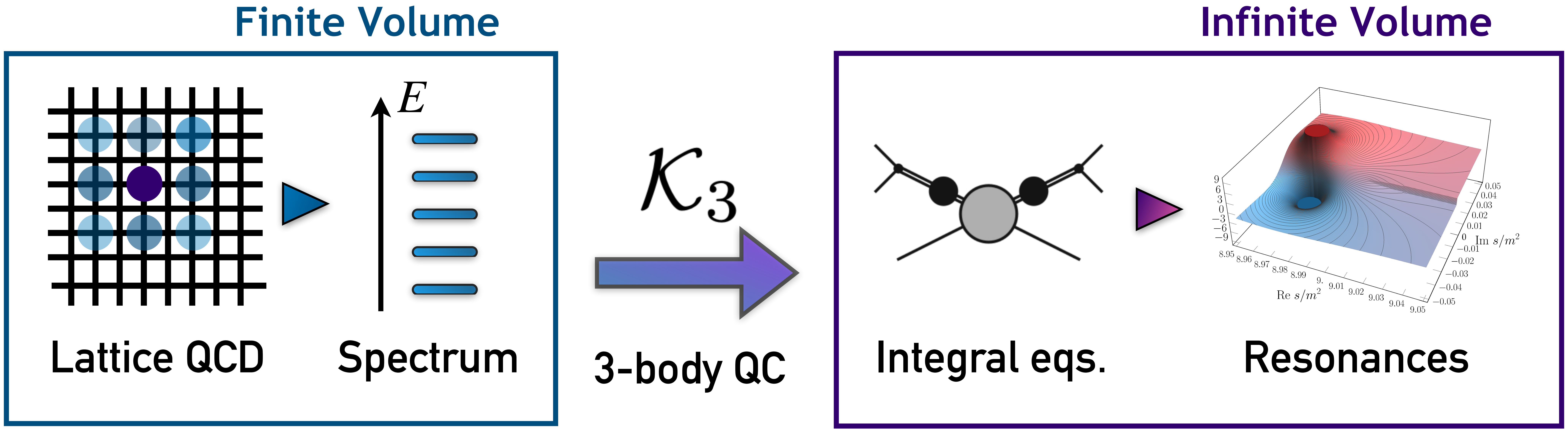
Comparing the finite-volume spectra



Further observations

- two-body QC clearly breaks down near the lhc for small lattice volumes,
- repulsive interaction in J=0 inconsistent with the lattice; (higher order terms needed)

Summary



Towards the tetraquark from Lattice QCD

- [x] proposed resolution of the left-hand cut problem
- [x] generalization of the three-body equations
- [x] comparison with the existing lattice results
- [x] model of T_{cc} = initial condition for LQCD studies

Next steps

- [ ] Systematics of the K matrices
- [ ] Systematic application to other lattice data
- [ ] Three-body computation of T_{cc}
- [ ] Formalism for the Roper resonance

THANK YOU

Relevant talks

Three-body formalism and applications

Alotaibi, Sharpe, Romero-Lopez, and Yan: Monday 11:55 – 13:15

Tetraquarks with various quarks

Parrott, Basak, Prelovsek, and Vujmilovic: Monday 14:15 – 15:35

Bicudo, Hoffman, Radhakrishnan: Tuesday 13:45 – 15:45

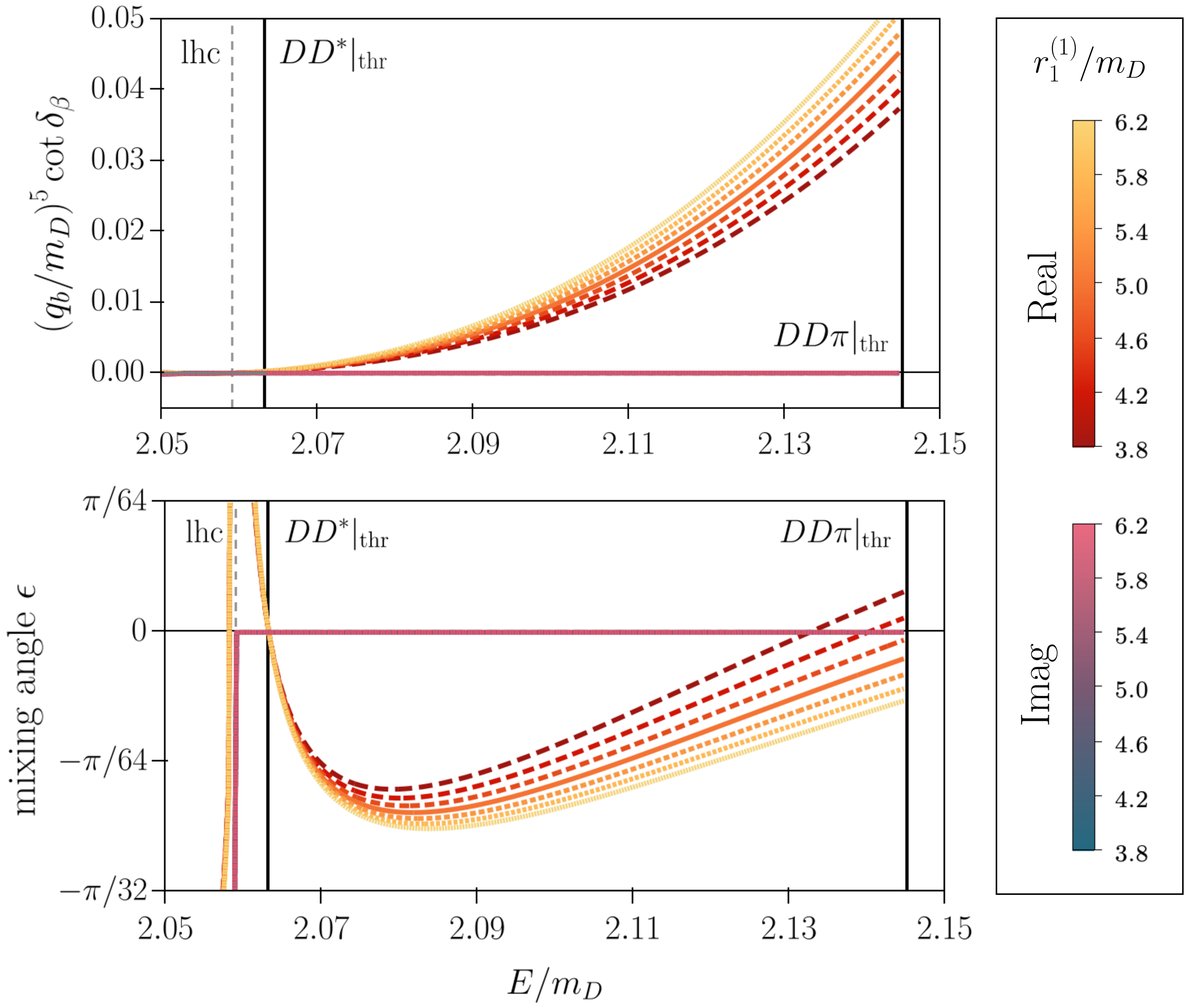
Whyte: Thursday 10:00

Left-hand cuts and such

Aoki, Raposo, Rusetsky: Thursday 11:30 – 12:30

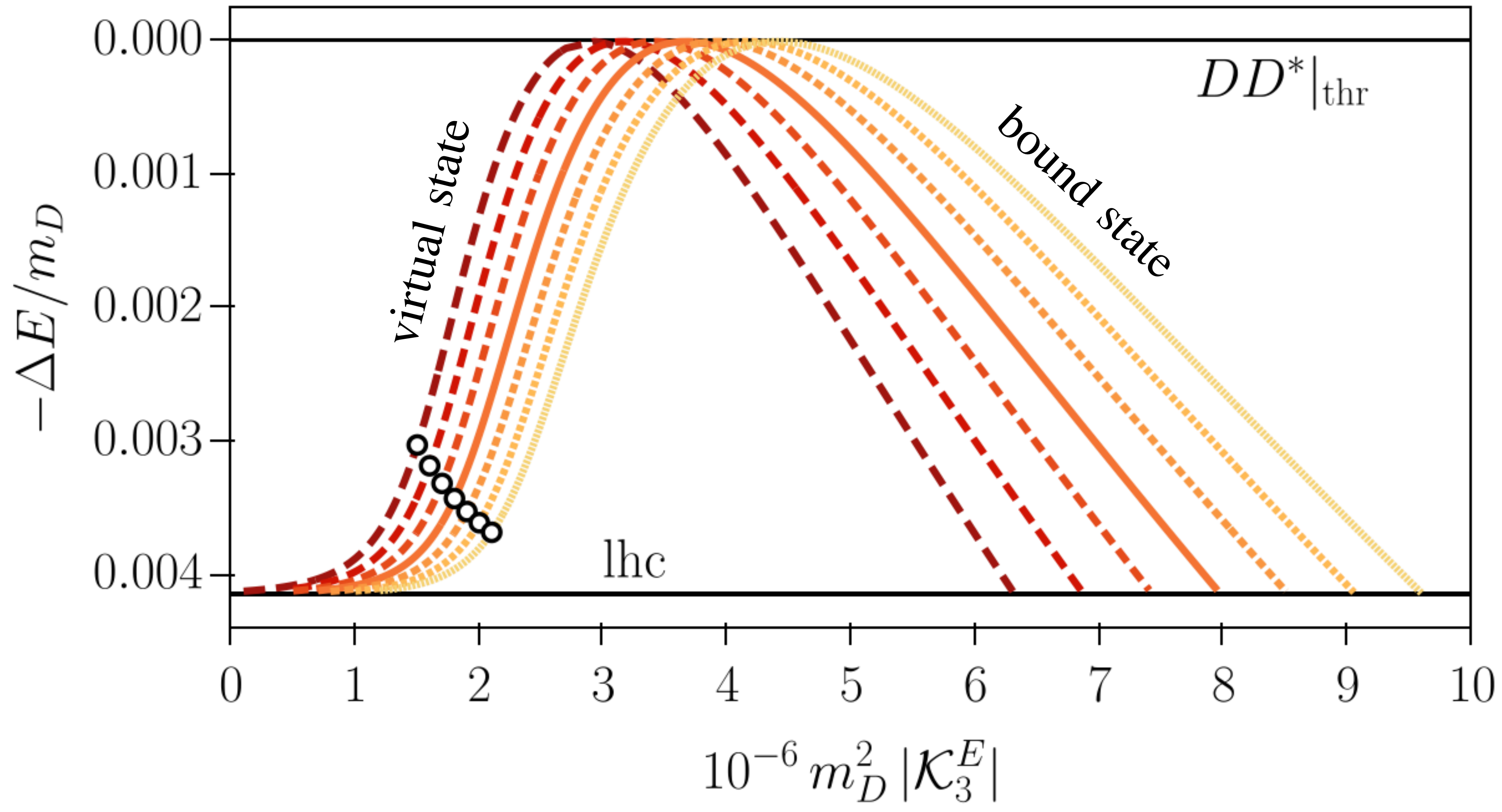
Partial-wave mixing amplitude (continued)

$J^P = 1^+$



More observations

- partial-wave mixing is small
- Dπ S-wave scattering is (almost) negligible
- no additional states appear in the spectrum
- DD S-wave scattering neglected due to cutoff



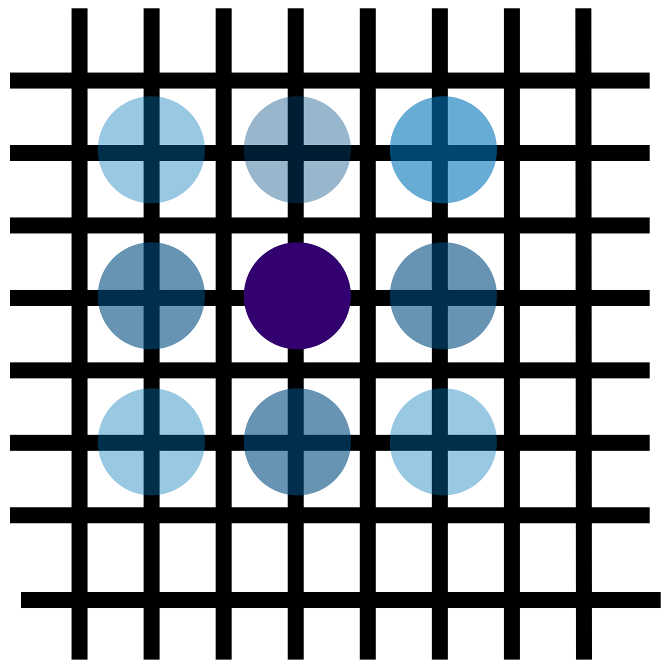
The three-body program

Relativistic, model-independent, three-particle quantization condition
Hansen, Sharpe, PRD 90 (2014) 11, 116003

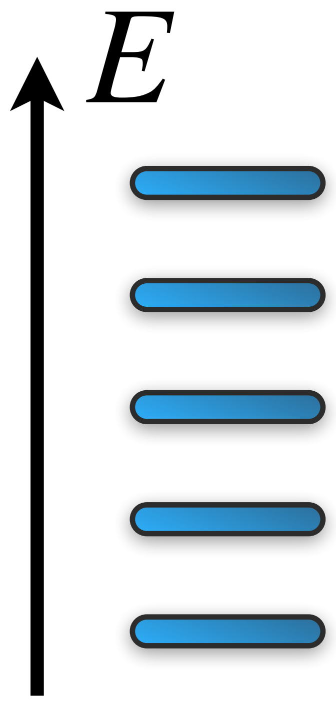
Three-body unitarity in finite volume
Mai, Döring, EPJ A 53 (2017) 12, 240

Relativistic-invariant formulation of the NREFT three-particle quantization condition
Müller, Pang, Rusetsky, Wu, JHEP 02 (2022), 158

Finite Volume



Lattice QCD

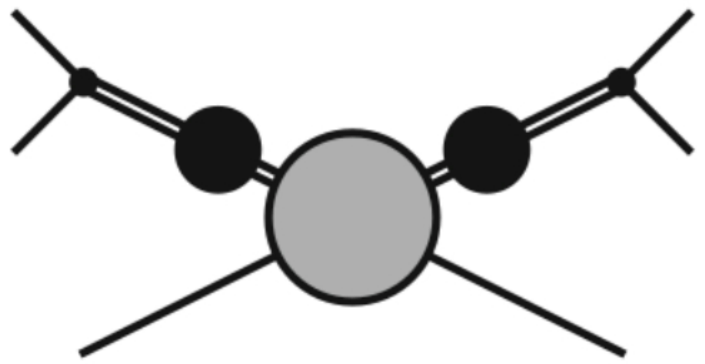


Spectrum

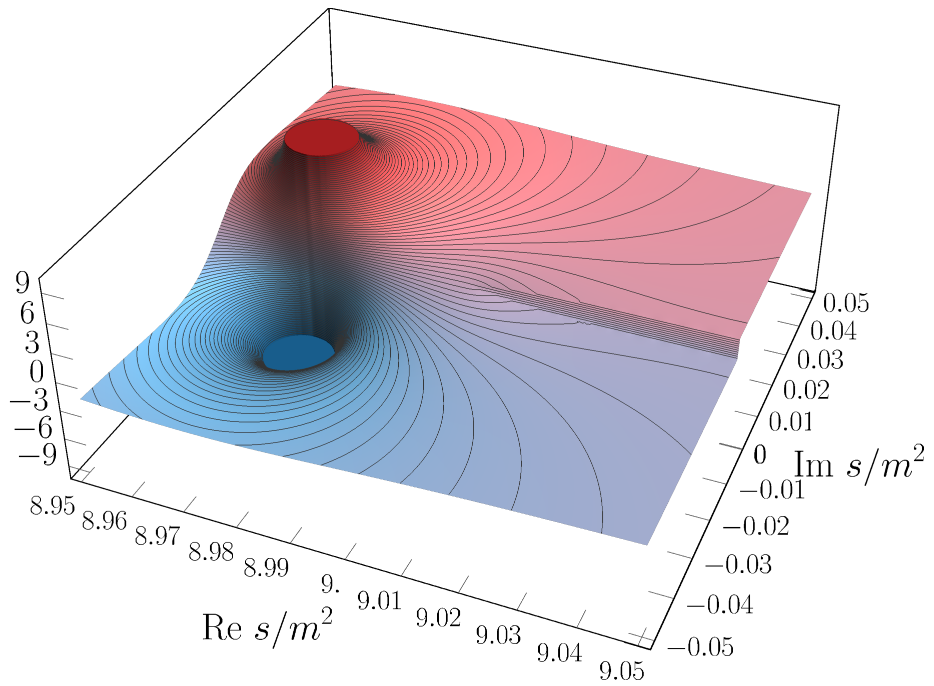
$\mathcal{K}_3$

Quantization Condition

Infinite Volume



Amplitude

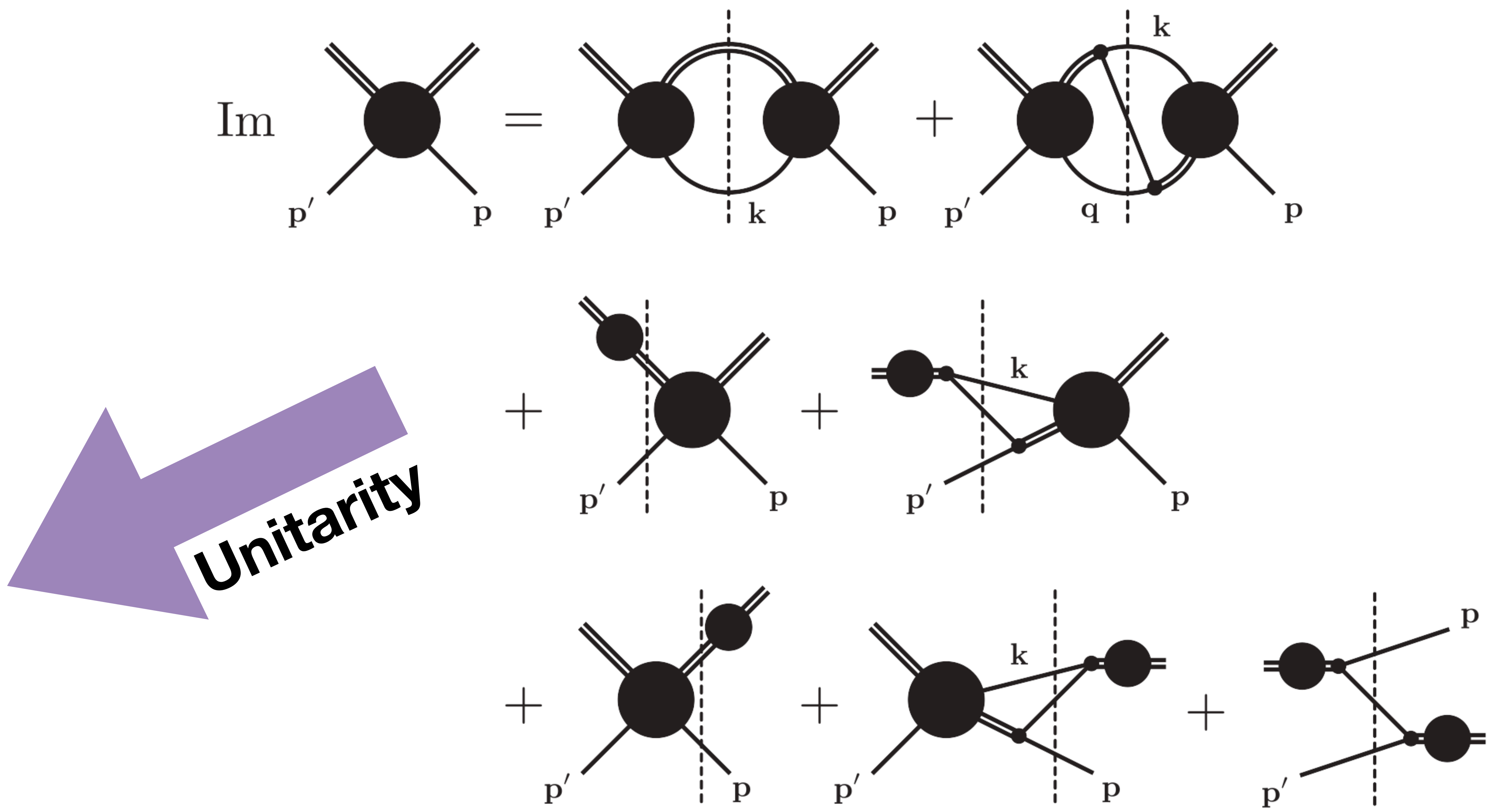


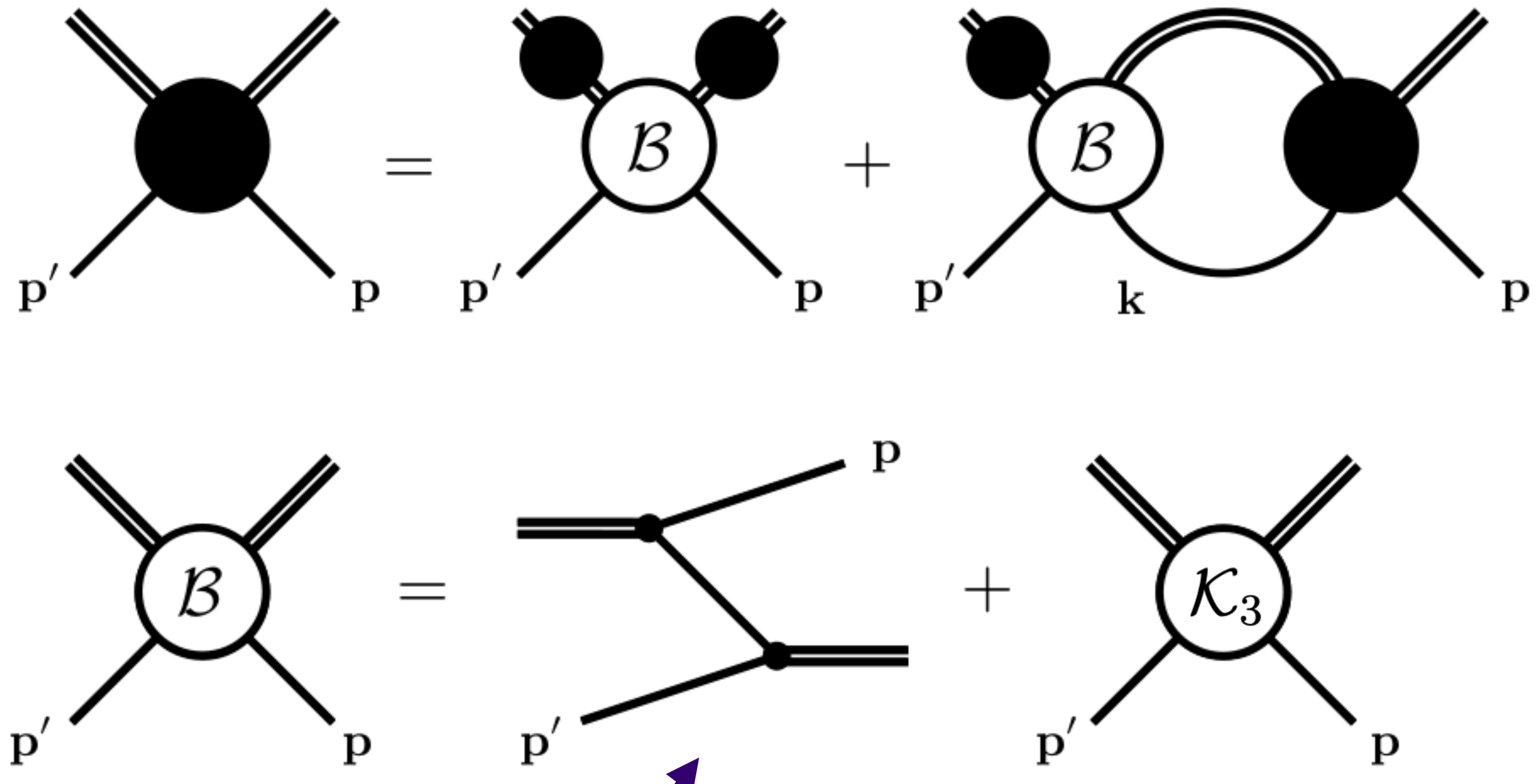
Resonances

$$\det\left[\mathbb{1} - \boldsymbol{\mathcal{K}}_3(E^\star)\boldsymbol{F}_3(E,\boldsymbol{P},L)\right] = 0$$

S-matrix parametrization

Diagrams by Andrew Jackura





One Particle Exchange

Short Range Interactions

Three-body amplitude

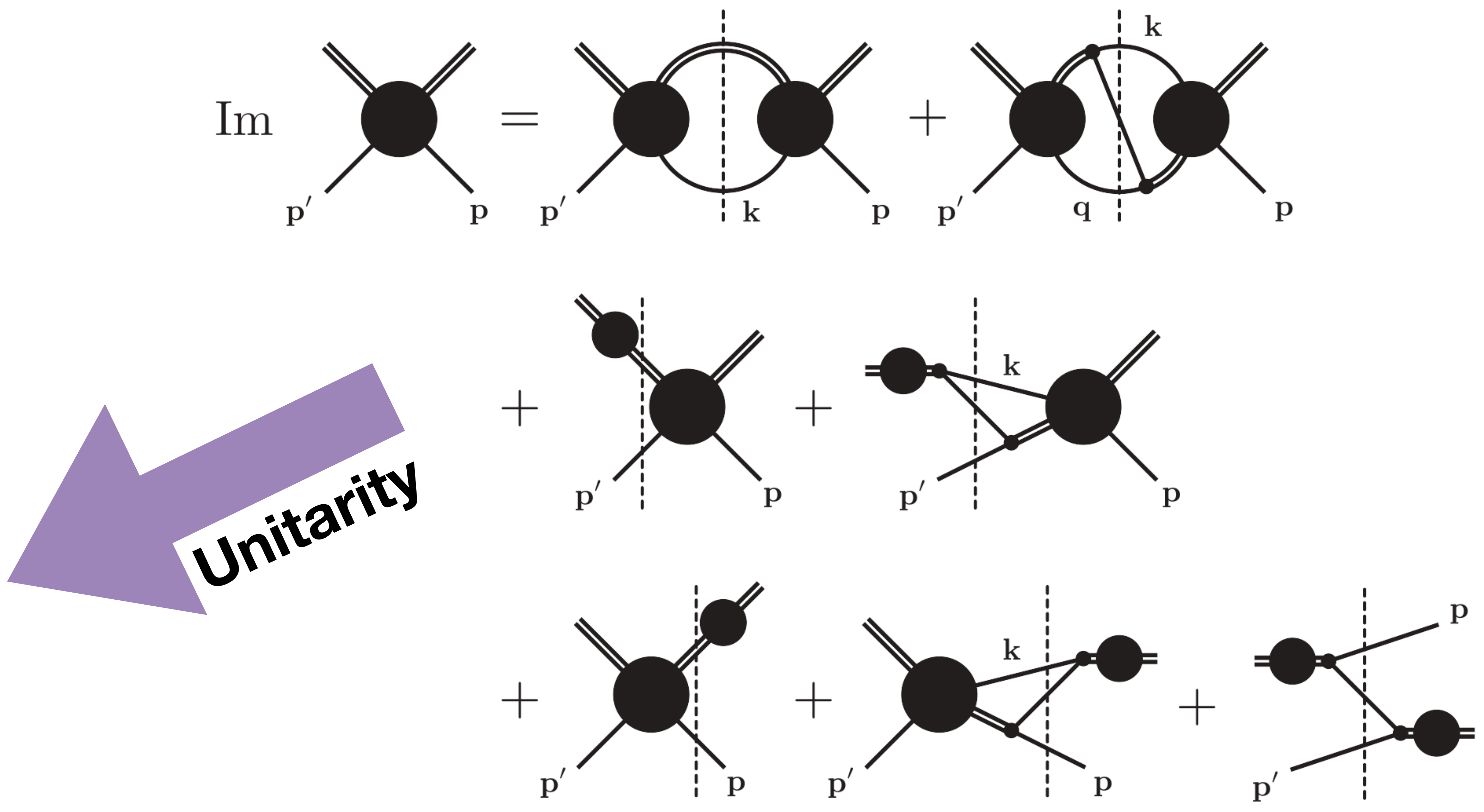
$$[\mathcal{M}_3]^{J}_{\ell' m'_\ell;\ell m_\ell}(p', s, p)$$

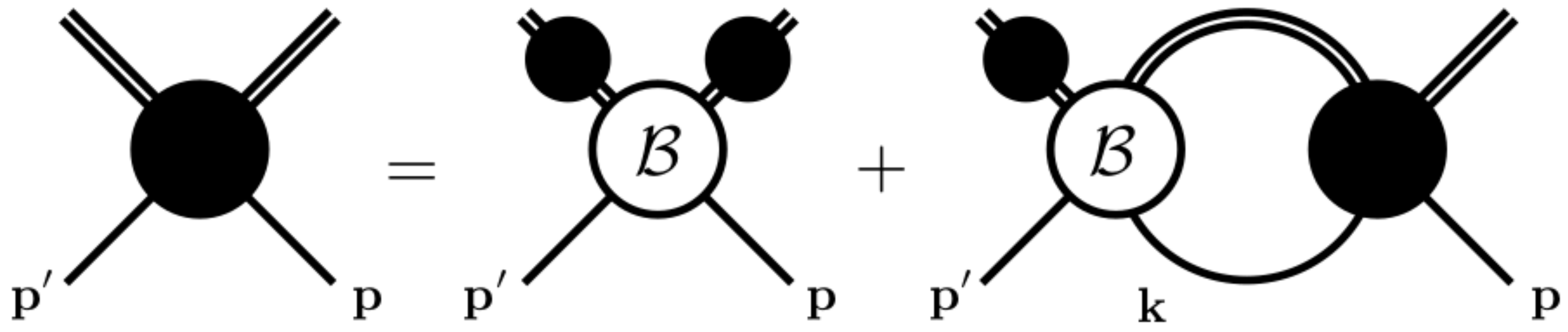
- pair-spectator
- partial waves
- symmetrization

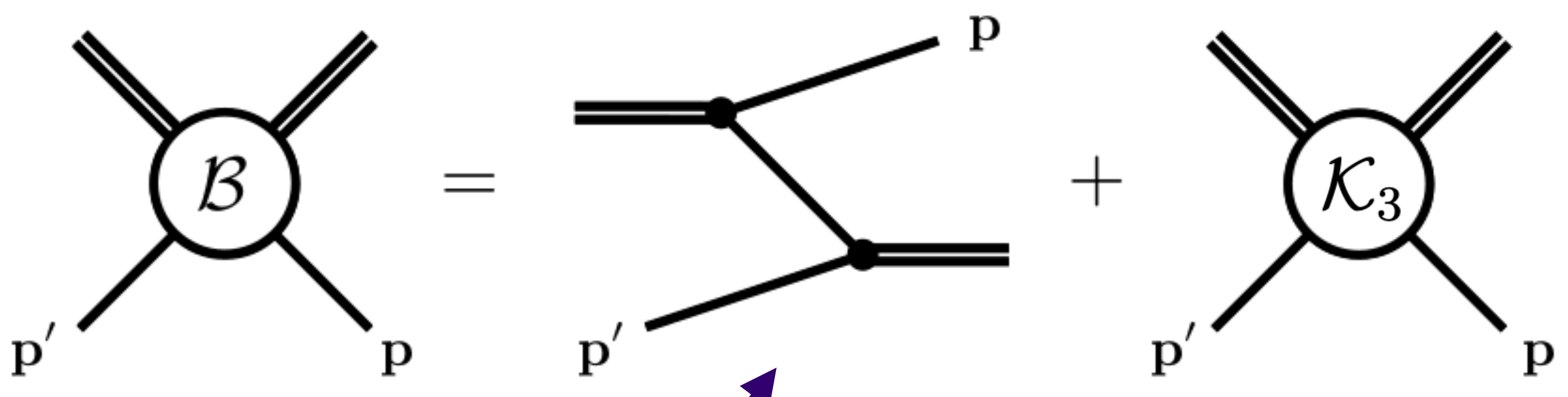
$$\mathcal{M}_3 = \mathcal{M}_2 \mathcal{B} \mathcal{M}_2 + \mathcal{M}_2 \int \mathcal{B}\, \rho_3\, \mathcal{M}_3$$

S-matrix parametrization

Diagrams by Andrew Jackura







One Particle Exchange

Short Range Interactions

Three-body amplitude

$$[\mathcal{M}_3]^J_{\ell' m'_\ell;\ell m_\ell}(p', s, p)$$

- pair-spectator
- partial waves
- symmetrization

$$\widetilde{\mathcal{M}}_3 = \mathcal{B} + \int \mathcal{B}\,\mathcal{M}_2\,\rho_3\,\widetilde{\mathcal{M}}_3$$

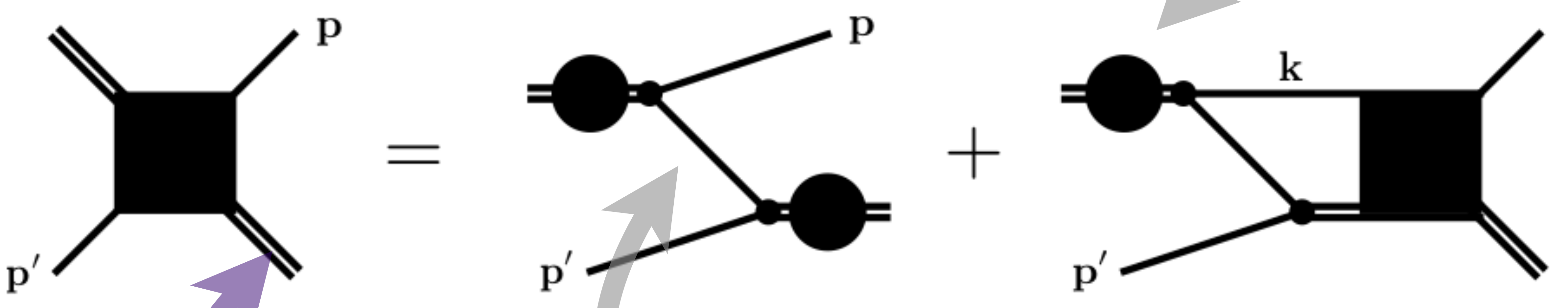
$\mathcal{B}\,\rho_3$

$$\mathcal{M}_2 = \mathcal{K}_2 + \mathcal{K}_2 i\rho_2\mathcal{M}_2$$

Simple example at J=0

diagrams by Andrew Jackura

Solving relativistic three-body integral equations in the presence of bound states
Jackura, Briceño, Dawid, Islam, McCarty, Phys.Rev.D 104 (2021) 1, 014507



Two-body amplitude

$$\mathcal{M}_2^{-1} \sim -\frac{1}{a} - i\rho_2$$

Bound state of mass

$m_b = 1.732\, m \quad (ma = 2)$

$m_b = 1.996\, m \quad (ma = 16)$

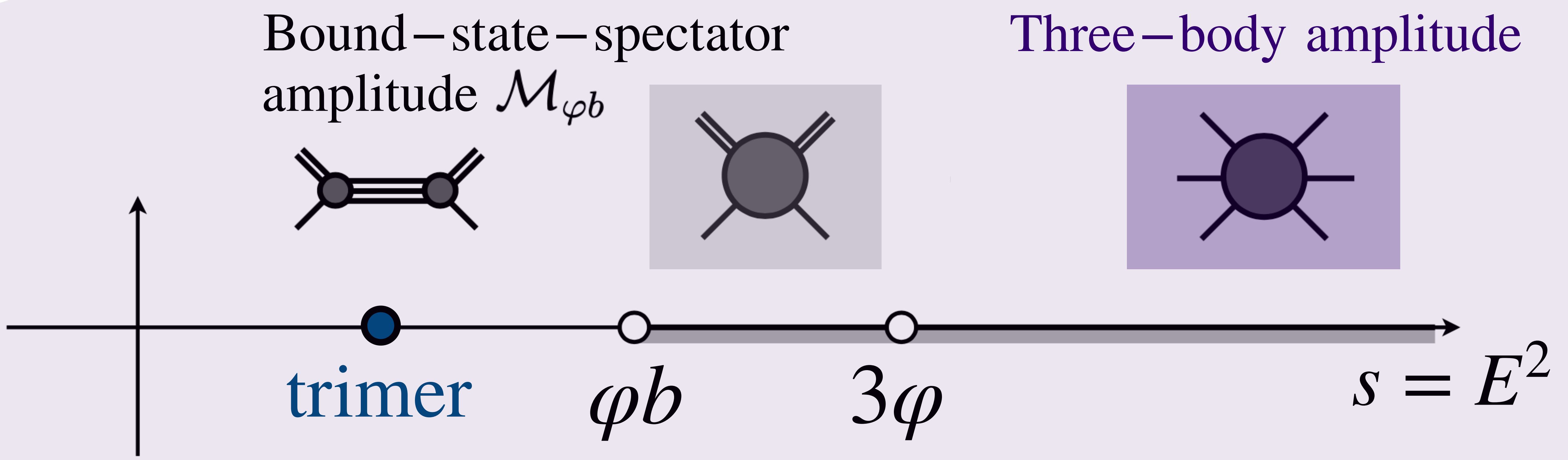
$$\mathcal{M}_2 \approx \frac{-g^2}{\sigma - m_b^2}$$

One-particle exchange

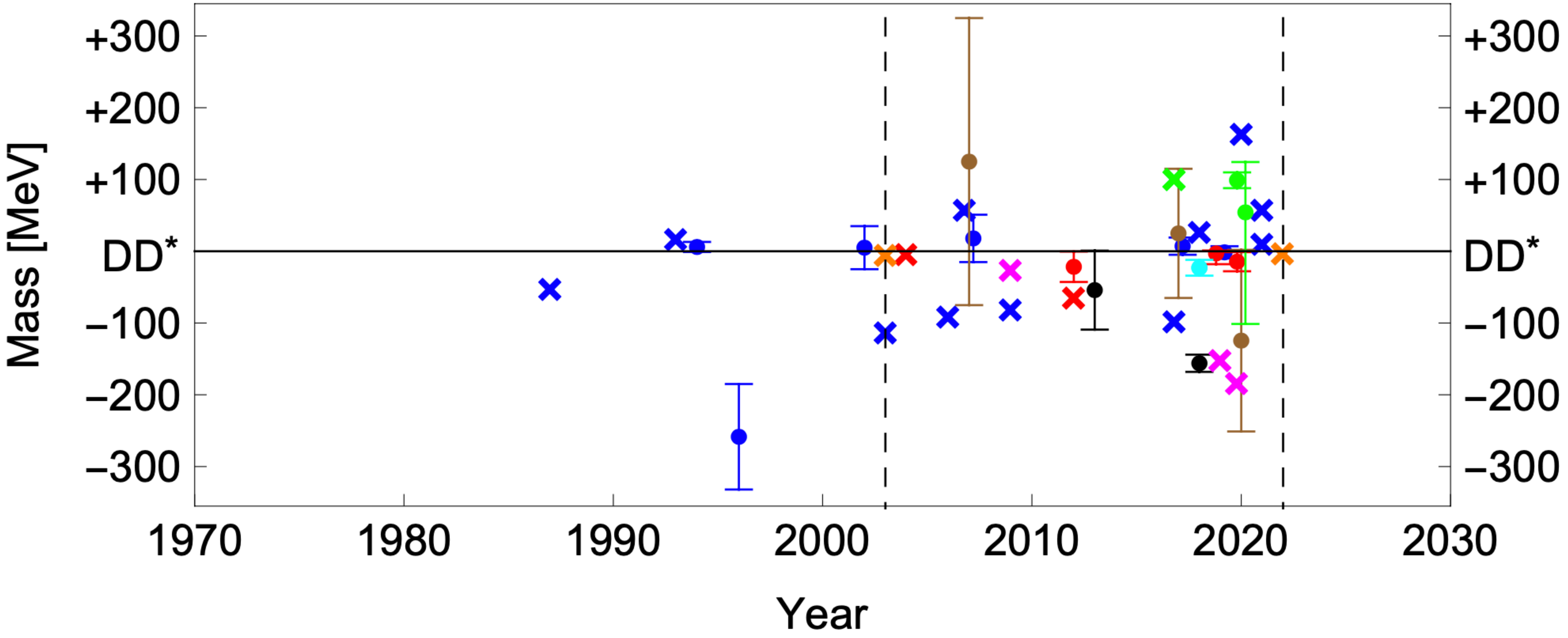
$$G(p', s, p) \propto \log\left(\frac{1 + z(p', s, p)}{1 - z(p', s, p)}\right)$$

LSZ reduction

$$\mathcal{M}_{\varphi b} = \lim_{\sigma', \sigma \to m_b^2} (\sigma' - m_b^2)\, \mathcal{D}\, (\sigma - m_b^2)$$



Problems with interpretations



QCD sum rules
various quark models
meson-meson & diquark-antidiquark
Heavy quark symmetry
Hadronic molecule
Lattice QCD
Other

Chen et al. arXiv:2204.02649v1

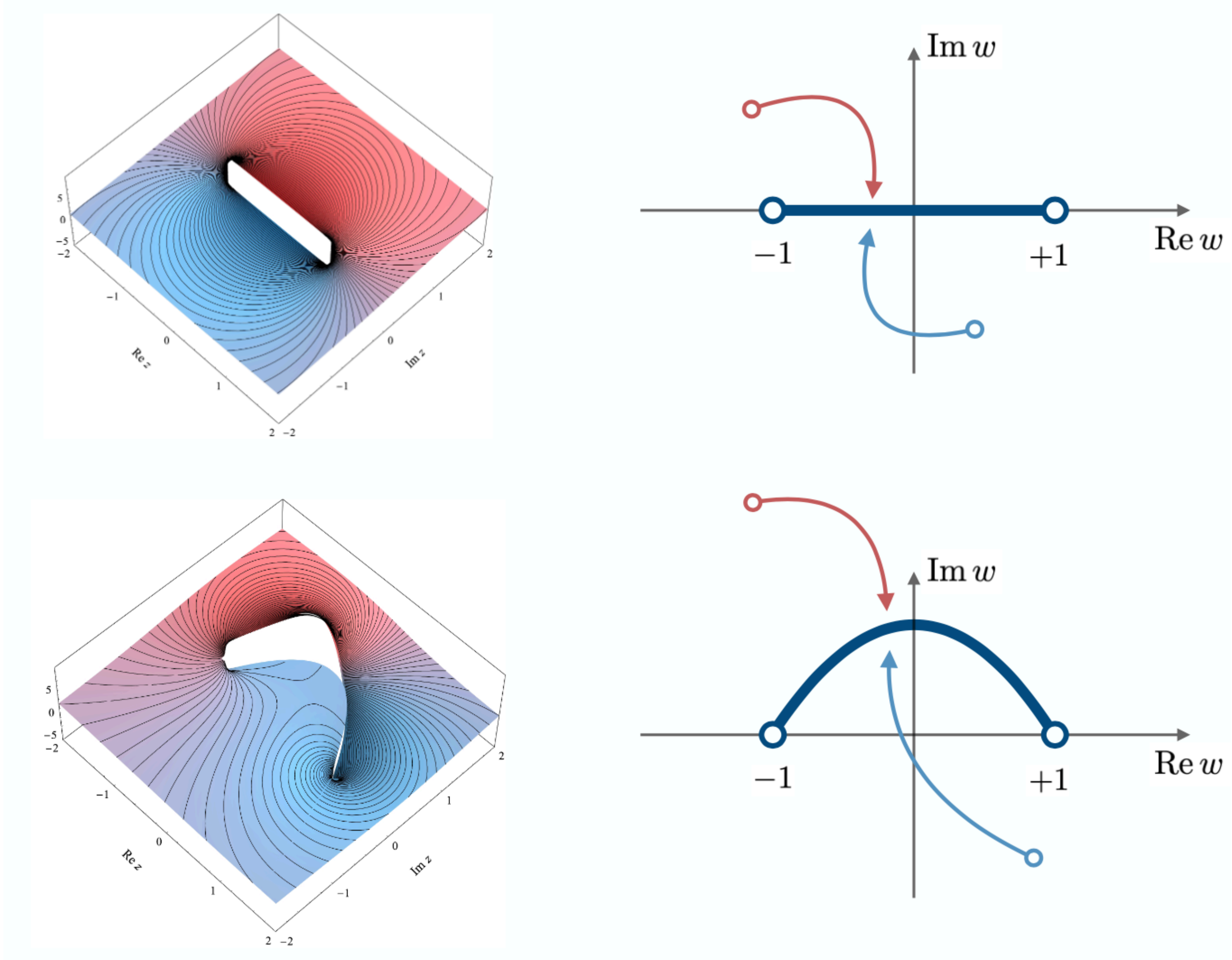
Brief intro to analytic continuation

Analytic continuation of the relativistic three-body amplitudes
Dawid, Islam, Briceño, arXiv:2303.04394

$$I(z) = \int_{\mathcal{C}(w_1, w_2)} f(w, z)\, dw\,,$$

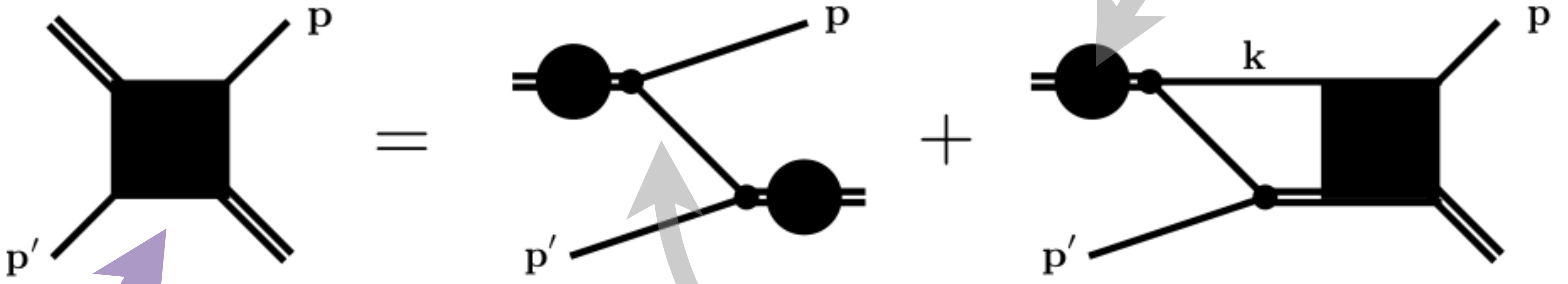
$$I(x) = \int_{-1}^{1} \frac{dw}{w - x} = \log\left(\frac{x-1}{x+1}\right)$$

$$x \in (-\infty, -1) \cup (1, \infty)$$



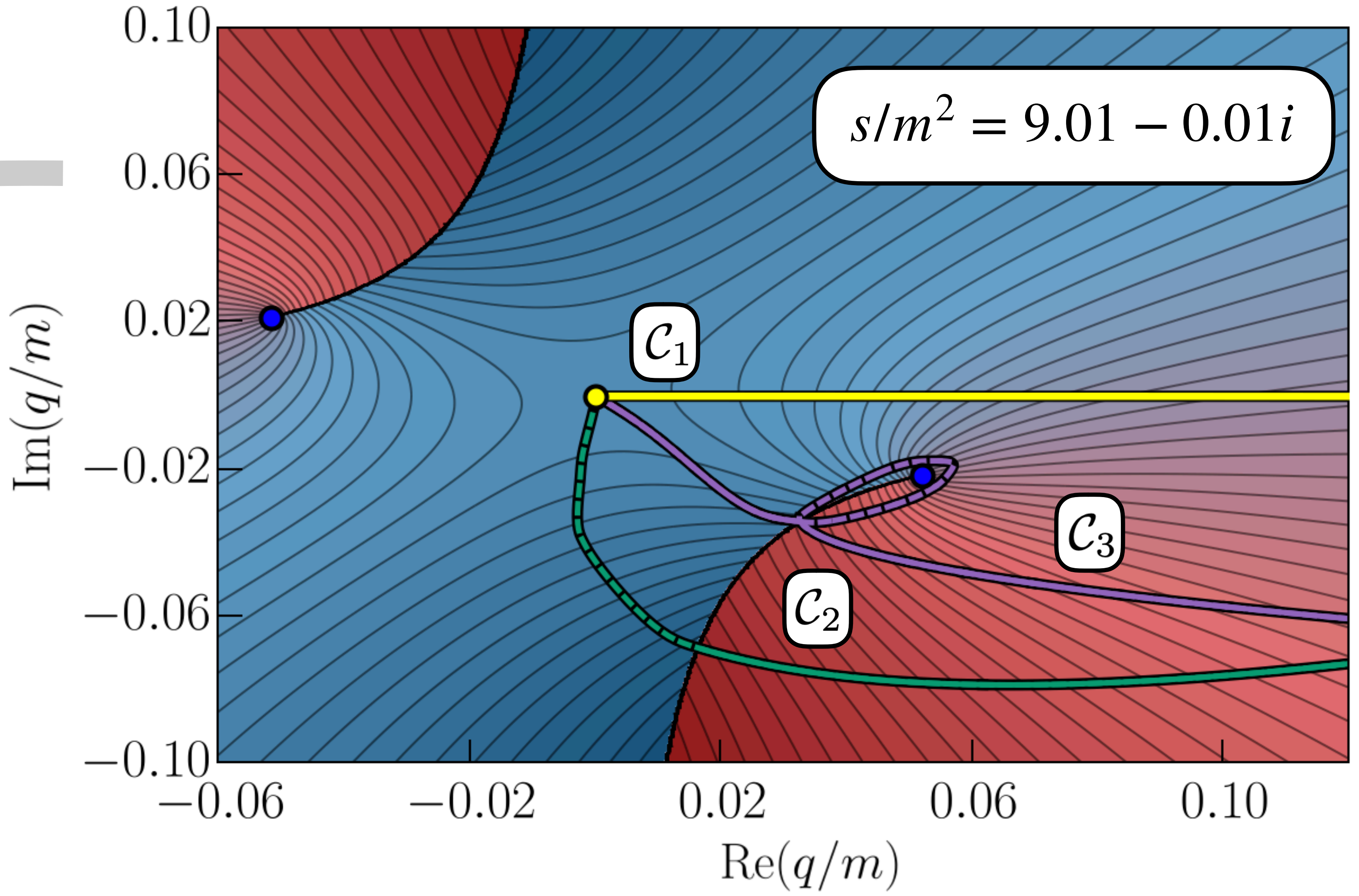
Analytic continuation

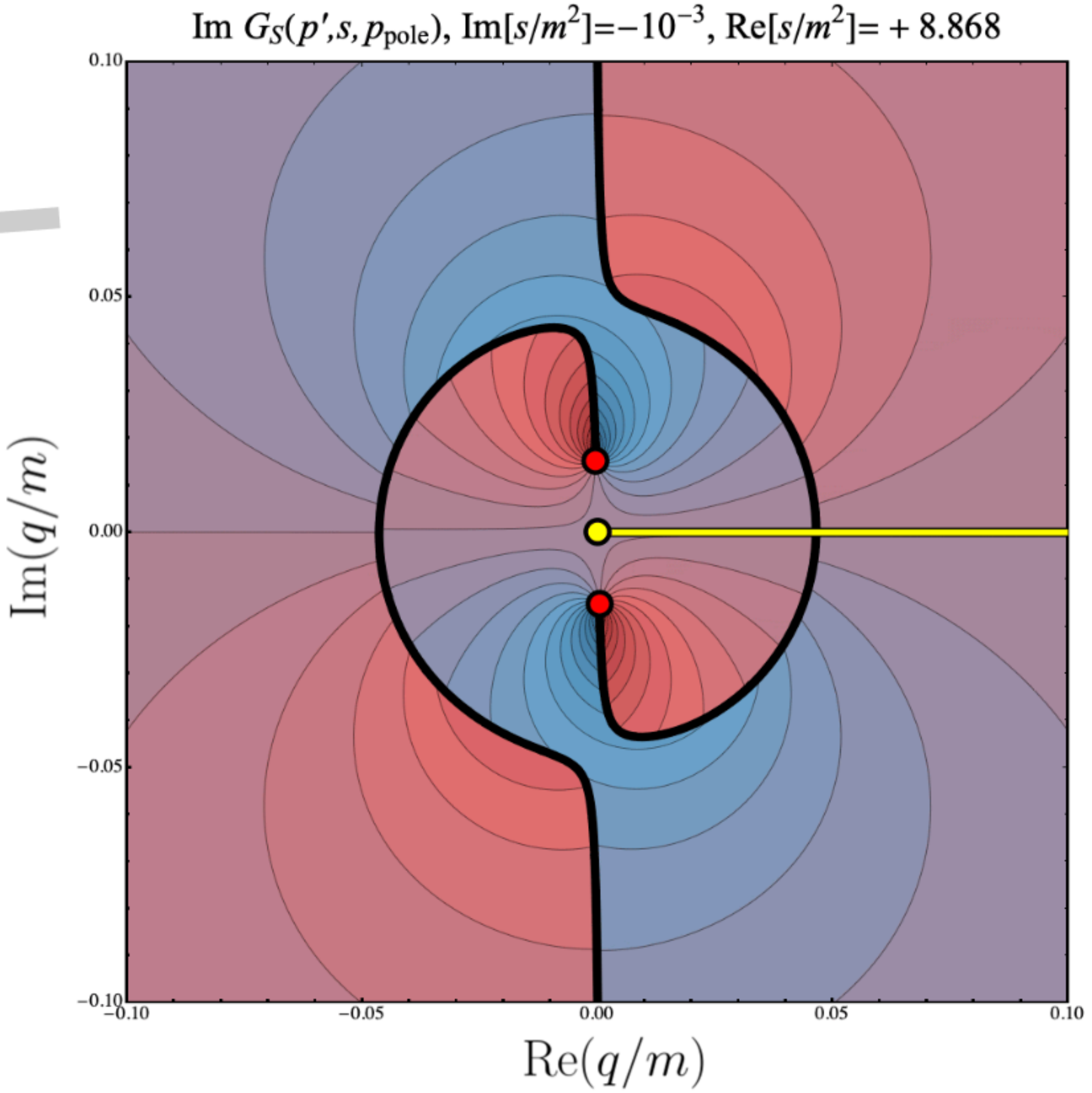
Two-body amplitude



Singularities

One-particle exchange





In a nutshell

- need to avoid crossing the singularities in the integration
- achieved by contour deformations, addition of discontinuities
- Multi-valuedness of the amplitude originates from collisions of the contour with:
  poles (two-body threshold) and branch points (three-body threshold),
- Riemann sheets defined by a monodromy

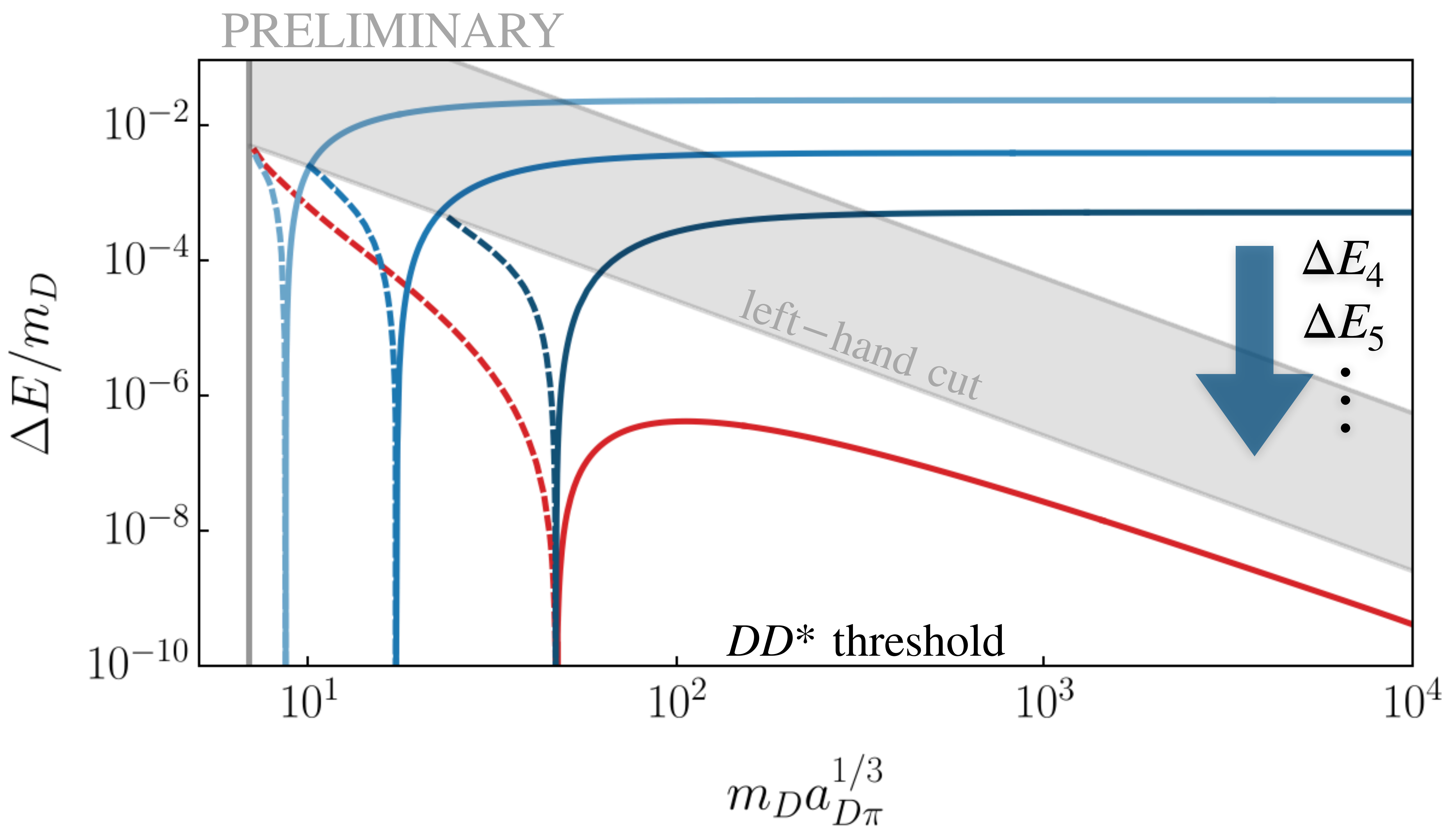
Spectrum of simplified models

$$m_D^3\, a_{D\pi}^P = \text{large}\,, \quad r_{D\pi}^P / m_D = 0.0$$

D^* no longer fixed

Additional lesson:

- "Small" refinement of a model can lead to a significantly different spectrum.
- Efimov physics emerges in the different corners of the parameter space in the three-body problem.



"Single" channel

$\Delta E_1/m_D\ (\ell = 0)$

"Coupled" channels

$\Delta E_1/m_D\ (\ell = 0, 2)$

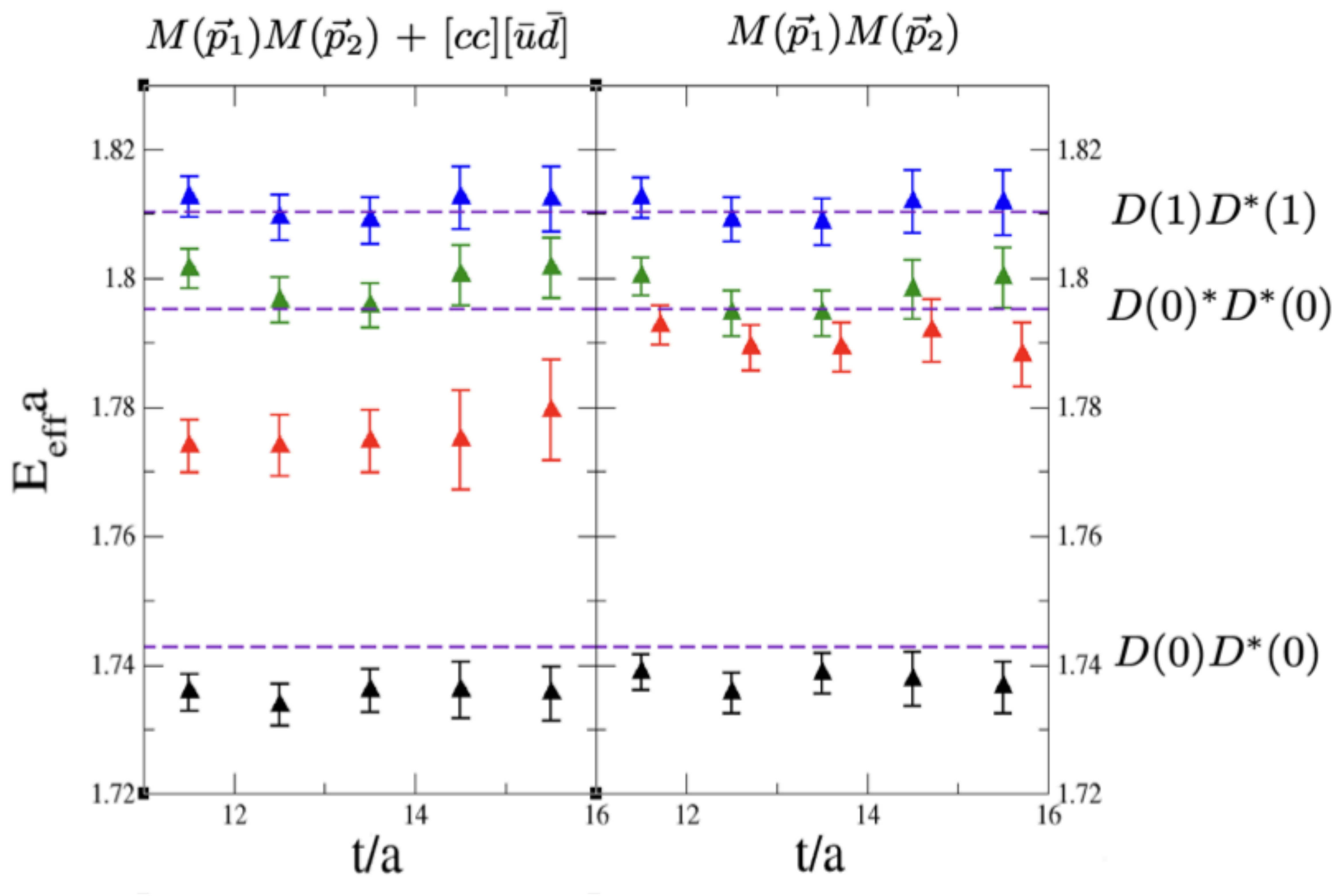
$\Delta E_2/m_D\ (\ell = 0, 2)$

$\Delta E_3/m_D\ (\ell = 0, 2)$

Some other facts

Finite-volume:

- excited-state energy affected by the inclusion of the diquark-antidiquark operators?
- see Ortiz-Pacheco et al. arXiv:2312.13441



Infinite-volume:

- LS equation with three-body effects gives an interesting evolution of singularities, arXiv:2407.04649
- two sub-threshold resonances turn into virtual states