Strong decay of double charm tetraquark T_{cc}

Subhasish Basak[1] Protick Mohanta[1] Srijit Paul[2]

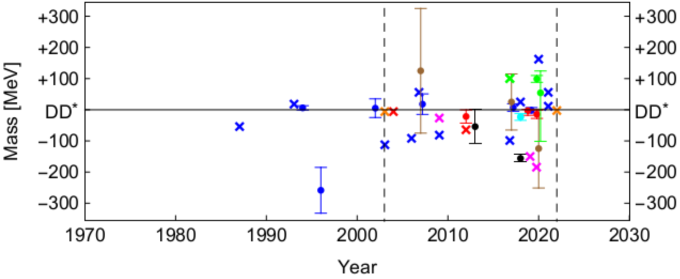
[1]NISER, India

[2]University of Maryland, USA

LATTICE 2024, Liverpool, UK

To begin with ...

- LHCb discovered doubly charmed tetraquark T_{cc} marginally below the $D^0 D^{\star +}$ threshold. It is long lived $ccl_1 l_2 \approx cl_1 + cl_2$ means difficult to decay.
- m_h/m_l large $\Rightarrow$ lattice QCD indicates existence of bound doubly bottom tetraquark $T_{bb}(bb\bar{u}\bar{d})$. Can charm play that role? Any cc state would be around ± 200 MeV about $D^0 D^{\star +}$ threshold.
- Question on T_{cc} bound state is not yet convincingly settled, if the plot from *Chen et al. [2204.02649]* is anything to go by



Our operators . . .

Double bottom tetraquark T_{bb} / Z_{bb} lattice studies, supported bound state and advocated use of diquark-antidiquark operator as it has maximum overlap with ground state and contributes maximally in binding energy.

But . . .

Cheung et al. [JHEP **11***,033]* found diquark operator of form $[\bar{u}\bar{d}] - [cc]$ not to have significant effects on finite volume spectra.

Cheng et al. [Chinese Physics **C45***, 043102]* showed such operator results in an unstable T_{cc}.

Understandably, most of the past lattice T_{cc} investigations didn't consider diquark-antidiquark operator.

In spite of negative press, studies based on heavy quark symmetries *Eichten et al. [PRL* **119***, 202002], Mehen [PRD* **96***, 094028]* showed usefulness of diquark operators in double heavy tetraquarks.

Hints from $\Sigma_Q - \Lambda_Q$

Apart from the usual $D - D^\star$ *Molecular* operator and *Scattering* operator, we included diquark-antidiquark operator in our investigation.

A few other motivations are –

- Role of diquark-antidiquark : $\Sigma_Q,\ \Lambda_Q$ both having same quark content

$$(\Sigma_Q)_\alpha = \epsilon_{abc}({u^a}^T C\gamma_k d^b)Q^c_\alpha$$
$$(\Lambda_Q)_\alpha = \epsilon_{abc}({u^a}^T C\gamma_5 d^b)Q^c_\alpha$$

  For $Q = c$, $\Sigma_c - \Lambda_c \sim 167$ MeV and $Q = b$, $\Sigma_b - \Lambda_b \sim 191$ MeV. With $m_\pi \sim 500$ MeV, these two operators generate a mass difference of ~ 30 MeV. With decreasing m_π the mass splittings gets significant. (*Bowler et al. [PRD* **54***,3619]*)

- Worsening of the $D^\star$ plateau for lighter pion mass indicates the three body effects $T_{cc}(cc\bar{u}\bar{d}) \to D\,D^\star \to D\,D\,\pi$

T_{cc} diquark-antidiquark operator is related to Λ_Q operator by heavy quark-diquark symmetry.

T_{cc} operators

The following operators are used in this project

$$
\begin{aligned}
\mathcal{D}(x) &= \left[c(x)^{a\,T}\, C\gamma_k\, c(x)^b\right] \left[\bar{u}(x)^a\, C\gamma_5\, \bar{d}(x)^{b\,T}\right] \\
\mathcal{M}(x) &= \left[\bar{d}(x)^a\, \gamma_k\, c(x)^a\right] \left[\bar{u}(x)^b\, \gamma_5\, c(x)^b\right] \\
\mathcal{S}(t; \vec{p}_1, \vec{p}_2) &= \sum_{\vec{x}} \left[\bar{d}(x)^a\, \gamma_k\, c(x)^a\right] e^{i\vec{p}_1\cdot\vec{x}} \times \sum_{\vec{y}} \left[\bar{u}(y)^b\, \gamma_5\, c(y)^b\right] e^{i\vec{p}_2\cdot\vec{y}}
\end{aligned}
$$

Spinor indices are summed over within each square brackets. In the center-of-mass frame, D and D^* mesons in the scattering operator $\mathcal{S}$ are given back-to-back momenta $\vec{p}_1 + \vec{p}_2 = 0$.

Subsequently, we calculated the correlator matrix required for GEVP analysis and exploited the time reversal and charge conjugation symmetry for simplifying numerics.

For demonstration, the explicit form of the correlator $\mathcal{C}_{\mathcal{DD}}(t)$ reads

$$\begin{aligned}
\mathcal{C}_{\mathcal{DD}}(t) &= \sum_{\vec{x}} \left\langle \mathcal{D}(x)\, \mathcal{D}(0)^\dagger \right\rangle \\
&= \sum_{\vec{x}} \mathrm{Tr}\left[\left\{ \mathcal{G}_c(t,\vec{x};0) \right\}^{ad\,T} \left(\gamma_k\gamma_4\gamma_2 \mathcal{G}_c(t,\vec{x};0)\gamma_4\gamma_2\gamma_k \right)^{bc} \right] \\
&\qquad \times \mathrm{Tr}\left[\left\{ \gamma_4\gamma_2 \mathcal{G}_u(t,\vec{x};0)^\dagger \gamma_4\gamma_2 \right\}^{da} \left(\gamma_5 \mathcal{G}_d(t,\vec{x};0)^\dagger \gamma_5 \right)^{cb\,T} \right] \\
&\quad - \sum_{\vec{x}} \mathrm{Tr}\left[\left\{ \mathcal{G}_c(t,\vec{x};0)\gamma_4\gamma_2\gamma_k \right\}^{ac} \left(\gamma_k\gamma_4\gamma_2 \mathcal{G}_c(t,\vec{x};0) \right)^{bd\,T} \right] \\
&\qquad \times \mathrm{Tr}\left[\left\{ \gamma_4\gamma_2 \mathcal{G}_u(t,\vec{x};0)^\dagger \gamma_4\gamma_2 \right\}^{da} \left(\gamma_5 \mathcal{G}_d(t,\vec{x};0)^\dagger \gamma_5 \right)^{cb\,T} \right]
\end{aligned}$$

For calculation of the diagonal correlators $\mathcal{C}_{\mathcal{DD}}(t)$, $\mathcal{C}_{\mathcal{MM}}(t)$ and off-diagonal ones $\mathcal{C}_{\mathcal{DM}}(t)$ we need just the point-to-all propagators $\mathcal{G}_u(t,\vec{x};0)$, $\mathcal{G}_c(t,\vec{x};0)$.

Scattering correlators, the diagonal $\mathcal{C}_{\mathcal{SS}}(t;\vec{p}_1,\vec{p}_2)$ and off-diagonals $\mathcal{SM}$, $\mathcal{SD}$ require combination of point-to-all, stochastic time slice-to-all propagators (*Abdel-Rehim et al.* [CPC **220**, 97]).

One end trick for $\mathcal{S}$ operators

Consider the diagonal operator $\mathcal{C}_{\mathcal{S}\mathcal{S}}(t)$

$$\begin{aligned}
\mathcal{C}_{\mathcal{S}\mathcal{S}}(t;\vec{p}_1,\vec{p}_2,\vec{p}_4) &= \left\langle \mathcal{S}(t;\vec{p}_1,\vec{p}_2)\,\mathcal{S}(0;\vec{p}_3,\vec{p}_4)^\dagger \right\rangle \\
&= \sum_{\vec{x},\vec{y},\vec{z}} e^{i(\vec{p}_1\cdot\vec{x}+\vec{p}_2\cdot\vec{y}-\vec{p}_4\cdot\vec{z})}\,\mathrm{Tr}\Big[\Big\{\gamma_5\mathcal{G}_d(t,\vec{x};0)^\dagger\gamma_5\Big\}\Big(\gamma_k\mathcal{G}_c(t,\vec{x};0)\gamma_k\Big)\Big] \\
&\qquad\times\mathrm{Tr}\Big[\Big\{\mathcal{G}_u(0,\vec{z};t,\vec{y})\Big\}\Big(\gamma_5\mathcal{G}_c(t,\vec{y};0,\vec{z})\gamma_5\Big)\Big] \\
&\quad-\sum_{\vec{x},\vec{y},\vec{z}} e^{i(\vec{p}_1\cdot\vec{x}+\vec{p}_2\cdot\vec{y}-\vec{p}_4\cdot\vec{z})}\,\mathrm{Tr}\Big[\Big\{\gamma_5\mathcal{G}_d(t,\vec{x};0)^\dagger\gamma_5\Big\}\Big(\gamma_k\mathcal{G}_c(t,\vec{x};0,\vec{z})\gamma_5\Big) \\
&\qquad\Big\{\mathcal{G}_u(0,\vec{z};t,\vec{y})\Big\}\Big(\gamma_5\mathcal{G}_c(t,\vec{y};0)\gamma_k\Big)\Big]
\end{aligned}$$

The one-end trick is implemented by having complex $\mathbb{Z}(2)\times\mathbb{Z}(2)$ random numbers at $t=0$ and inverting the fermion action. The first term in the above expression can be expressed as

$$\frac{1}{N}\sum_n\sum_{\vec{x}} e^{i(\vec{p}_1\cdot\vec{x})}\Big\{\gamma_k\gamma_5\mathcal{G}_d(t,\vec{x};0)^\dagger\gamma_5\gamma_k\Big\}^{c_1c_2}_{s_1s_2}\Big(\mathcal{G}_c(t,\vec{x};0)\Big)^{c_2c_1}_{s_2s_1}$$
$$\times\sum_{\vec{y}} e^{i(\vec{p}_2\cdot\vec{y})}\Big(\phi^n_c(\vec{y},t)\Big)^{c_3}_{s_3}\Big(\phi^n_u(\vec{y},t)^\dagger\Big)^{c_3}_{s_3}$$

In the last expression, the ϕ's are single column of inverse of Dirac matrix and thus determined as,

$$\Big(D_c(r,x)\Big)^{c_1 c_2}_{s_1 s_2}\Big(\phi^n_c(x)\Big)^{c_2}_{s_2} = \delta_{r_0,0}\Big(\Xi(\vec{r})[n]\Big)^{c_1}_{s_1}$$
$$\Big(D_u(r,x)\Big)^{c_1 c_2}_{s_1 s_2}\Big(\phi^n_u(x)\Big)^{c_2}_{s_2} = \delta_{r_0,0}\Big(\Xi(\vec{r})[n]\Big)^{c_1}_{s_1} e^{i(\vec{p}_4\cdot\vec{r})}$$

where $\Xi[n] \in \mathbb{Z}(2)\times\mathbb{Z}(2)$.

One-end trick is an efficient technique to estimate the product of two propagators stochastically, where the propagators are connected at space-time point, say $(\vec{x}, t)$ with sum over $(\vec{x}$. No additional propagator originating or ending at $(\vec{x}, t)$.

Heavy & light quark actions

For charm quark, we used relativistic heavy quark action (RHQ),

$$S_{\text{charm}} = a^4 \sum_x \bar{\psi}(x) \left[m_Q + \left(\gamma_0 D_0 - \frac{a}{2} D_0^2 \right) + \zeta \left(\gamma_i D_i - \frac{a}{2} D_i^2 \right) - \frac{a}{4} c_P \, \sigma_{\mu\nu} F_{\mu\nu} \right] \psi(x)$$

For the light quarks the standard clover action.

$$S = \sum_n \bar{\psi}(n)\psi(n) - k \sum_{n,\mu} \left[\bar{\psi}(n) \left(1 - \gamma_\mu\right) U_\mu(n)\psi(n + \hat{\mu}) + \bar{\psi}(n) \left(1 + r_\mu\right) U_\mu^{+}(n - \hat{\mu})\psi(n - \hat{\mu}) \right] - \frac{k c_{sw}}{2} \sum_{n,\mu,\nu} \bar{\psi}(n)\sigma_{\mu\nu} F_{\mu\nu}\psi(n)$$

HISQ action that we chose for light quarks in our double bottom tetraquark project (*Protick et al. [PRD* **102**, *094516]*) worked rather well but has a serious drawback. It will make $\mathcal{C}_{\mathcal{SD}}(t)$ and $\mathcal{C}_{\mathcal{SM}}(t)$ identical!

Tuning RHQ action parameters

Tuning of the parameters $\{am_c, c_P, \zeta\}$ is done using (*Aoki et al. [PRD* **86**, *116003], Flynn et al. [PRD* **107**, *114512]*),

- spin-averaged mass $\overline{M}_{c\bar{c}} = (\eta_c + 3J/\psi)/4$
- hyperfine splitting $\Delta M_{c\bar{c}} = M_{J/\psi} - M_{\eta_c}$
- velocity of light $E^2_{\eta_c}(\vec{p}) = c^2\vec{p}^2 + M^2_{\eta_c}(0)$

Assuming linear relation among $\{\overline{M}_{c\bar{c}}, \Delta M_{c\bar{c}}, c\}$ and the parameters $\{am_c, c_P, \zeta\}$ close to their *true* values, we performed multivariate linear regression analysis

$$\begin{pmatrix} \overline{M}_{c\bar{c}} \\ \Delta M_{c\bar{c}} \\ c \end{pmatrix} = \mathbf{J} \cdot \begin{pmatrix} am_Q \\ c_P \\ \zeta \end{pmatrix} + \mathbf{A}$$

Tuned parameters are obtained by matching $a\overline{M}_{c\bar{c}}^{\text{latt}} \approx 3068.5$ MeV, $a\Delta M_{c\bar{c}}^{\text{latt}} \approx 113.5$ MeV and $c^{\text{latt}} \approx 1$ and inverting the above relation,

$$\begin{pmatrix} am_Q \\ c_P \\ \zeta \end{pmatrix}_{\text{tuned}} = \mathbf{J}^{-1} \cdot \left[\begin{pmatrix} \overline{M}_{c\bar{c}} \\ \Delta M_{c\bar{c}} \\ c \end{pmatrix}_{\text{pdg}} - \mathbf{A} \right]$$

Ensemble	m_Q	ζ	C_p	$\overline{M}_{c\bar{c}}$ (MeV)	$\Delta M_{c\bar{c}}$ (MeV)	c^{latt}
$16^3 \times 48$	0.5057	1.4214	2.4665	3069.5	114.2	1.00109
$28^3 \times 96$	0.1141	1.1389	1.9593	3069.3	113.4	1.00788

Lattices used are MILC generated $N_f = 2+1$ asqtad ensembles of $a \approx 0.15$ and 0.09 fm.

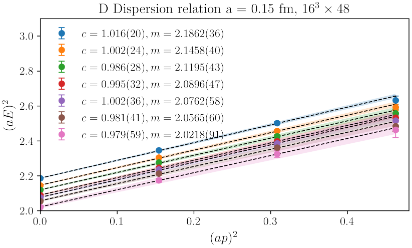
The range of κ values used for light quarks are

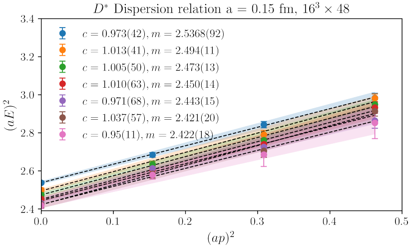
- $16^3 \times 48$: $\kappa = 0.14005, 0.1405, 0.1408, 0.1411, 0.1413, 0.1415, 0.1416, 0.1416, 0.1418$
- $28^3 \times 96$: $\kappa = 0.1379, 0.13815, 0.1383, 0.13845, 0.13855, 0.13865, 0.13875, 0.13880$

For fermionic propagators we used Gaussian smeared point source and the gauge links were APE smeared.

D, D^* dispersion relation

For eventual application of Lüscher quantization condition relating infinite volume $DD^{\star}$ scattering phase shifts to finite volume energy spectrum, we obtained the D and D^* dispersion relation,

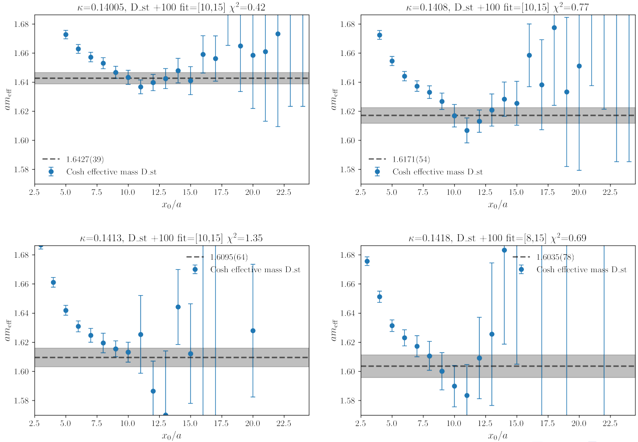




The D and D^* masses and speed of light determined from the dispersion relation are consistent with the relativistic dispersion relation in rest frame.

D^* with reducing quark mass

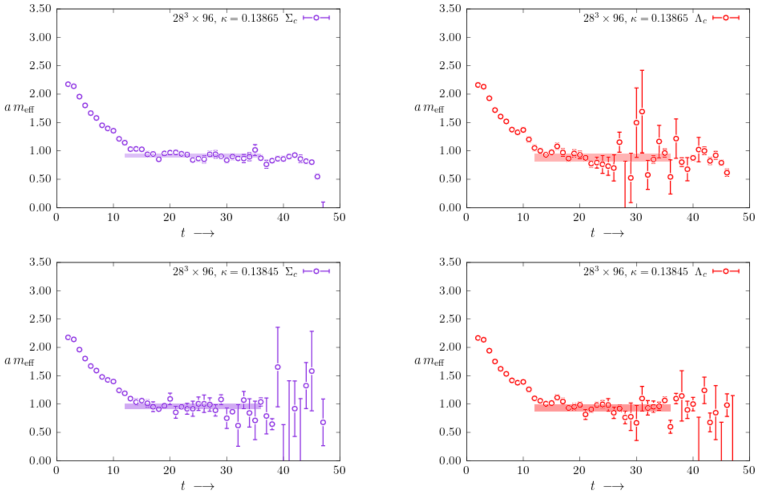
One way inclusion of diquark-antidiquark can be justified is worsening of D^* signal as quark mass is reduced making extraction of the D^* state difficult. Is it because we are getting closer to the left-hand cut. Showing here $16^3 \times 48$ data



$\Sigma_c - \Lambda_c$

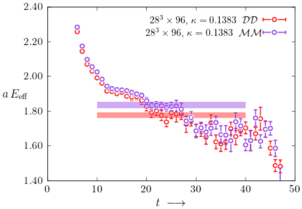
The $\Sigma_c - \Lambda_c$ splitting is opening up for lower quark mass.

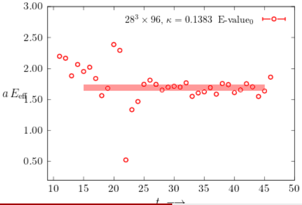
- $\kappa = 0.13865$, $m_\pi = 420$ MeV : 0.033 ($\approx$ 72) MeV
- $\kappa = 0.13845$, $m_\pi = 507$ MeV : 0.026 ($\approx$ 57) MeV

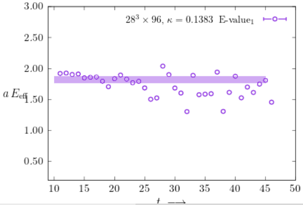


Diagonal $\mathcal{DD}$, $\mathcal{MM}$ and GEVP $28^3 \times 96$

Comparing the diagonal $\mathcal{DD}$, $\mathcal{MM}$ states with the GEVP, we found lowest state ~ 3890 MeV being around $D^0 D^{*+}$ threshold at relatively higher $m_\pi \sim 550$ MeV. At present level of statistics, spectrum at lower m_π are too noisy to extract anything meaningful as of now.







Comparing the diagonals $\mathcal{DD}$, $\mathcal{MM}$, $\mathcal{SS}$ states with the GEVP (preliminary).