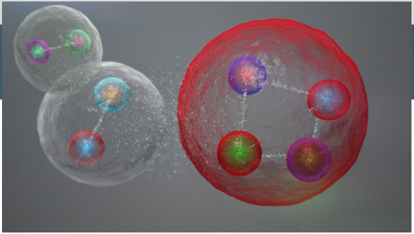
Quark mass dependence of doubly heavy tetraquark binding

William Parrott
parrott@yorku.ca

York University

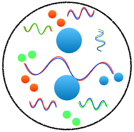
B. Colquhoun, A. Francis, R. J. Hudspith, R. Lewis, K. Maltman

Intro to tetraquarks



- Do tetraquark bound states exist?
- Much interest in $T_{ll\bar{h}\bar{h}}$ ($l \in \{u, d, s\}$ $h \in \{c, b\}$).
- Attractive heavy 3_c colour Coulomb (as $M \to \infty$) and spin dependent 'good-light diquark' $J = 0$ $\bar{3}_c$, not accessible for two separated mesons.
- Generally heavier heavies and lighter lights increases binding.
- 'Deep' binding widely predicted in $I = 0$ ($T_{ud\bar{b}\bar{b}}$) and $I = 1/2$ ($T_{\ell s\bar{b}\bar{b}}$) $J^P = 1^+ \lesssim 200$ MeV.
- For $T_{ud\bar{c}\bar{c}}$ an extremely precise LHCb measurement [LHCb *2109.01038*]of 0.273(62) MeV.

Lattice details



- All of the work discussed uses $N_f = 2 + 1$ Wilson-Clover action [PACS-CS *0911.2561*], with $a^{-1} = 2.194(10)$GeV.
- We use 4 exisiting $32^3 \times 64$ ensembles [PACS-CS *0911.2561*], as well as two $48^3 \times 64$ we generated.
- This gives 6 values for $165 \leq m_\pi \leq 707$ MeV.
- We wish to span a range of heavy quark masses m_h from $m_c < m_h \leq m_b$, as well as $m_h > m_b$.
- Use NRQCD for heavy mass, effective for $m_c \lesssim m_h$. Downside NRQCD is hard to extrapolate to continuum.

Previous work: chiral analysis on $q_1q_2\bar{b}\bar{b}$

Our[1] earliest previous work [*1607.05214*] considered $J^P = 1^+$ $T_{ud\bar{b}\bar{b}}$ and $T_{\ell s\bar{b}\bar{b}}$ ($\ell = u, d$) for three values of the pion mass (165, 299, 415 MeV). Two operators were used:

$$M(x) = \bar{b}_a^\alpha(x)\gamma_5^{\alpha\beta}u_a^\beta(x)\bar{b}_b^\kappa(x)\gamma_i^{\kappa\rho}u_b^\rho(x) - u \leftrightarrow d$$
$$D(x) = (u_a^\alpha(x))^T(C\gamma_5)^{\alpha\beta}d_b^\beta(x)\bar{b}_a^\kappa(x)(C\gamma_i)^{\kappa\rho}(\bar{b}_b^\rho(x))^T.$$

Wall sources were used to generate the propagators. Local sinks were used.

[1] I will say ‘our’ to mean the current group, though I was not involved in previous work.

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Previous work: chiral analysis on $q_1 q_2 \bar{b}\bar{b}$

A GEVP[2] analysis was performed using $G_{\mathcal{O}_1\mathcal{O}_2}(t) = \frac{C_{\mathcal{O}_1\mathcal{O}_2}(t)}{C_{PP}(t)C_{VV}(t)}$.

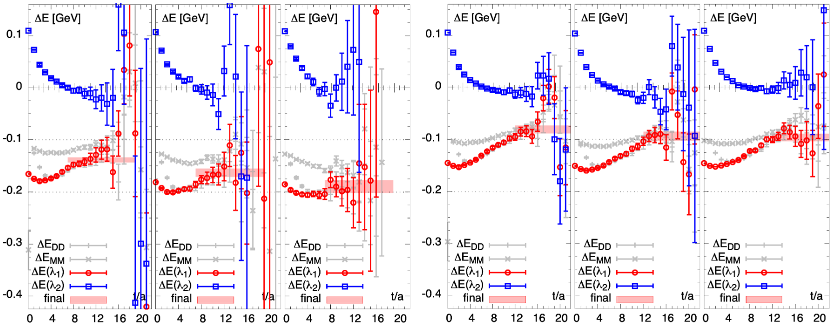
$$F(t) = \begin{pmatrix} G_{DD}(t) & G_{DM}(t) \\ G_{MD}(t) & G_{MM}(t) \end{pmatrix}$$

$$\begin{aligned} F(t)\nu &= \lambda(t)F(t_0)\nu \\ \lambda(t) &= Ae^{-\Delta E(t-t_0)} = (1+\delta)e^{-\Delta E(t-t_0)}. \end{aligned}$$

The 2×2 matrix gives 2 Eigenvalues. The ground state, and a mixture of all excited states. Extract ΔE from a single exponential fit to $\lambda(t)$.

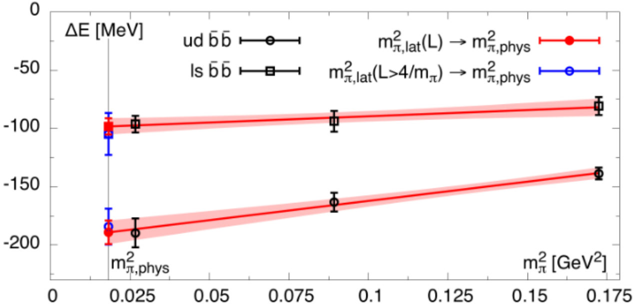
[2] Generalised Eigenvalue problem

Previous work: chiral analysis on $q_1 q_2 \bar{b}\bar{b}$



ΔE for the first and second GEVP Eigenvalues, relative to BB^* ($B_s B^*$) for $ud\bar{b}\bar{b}$ [left] ($\ell s\bar{b}\bar{b}$ [right]). Plateaus are quite short and appear to rise.

Previous work: chiral analysis on $q_1 q_2 \bar{b}\bar{b}$



Ensemble	$\Delta E_{ud\bar{b}\bar{b}}$[MeV]	$\Delta E_{\ell s\bar{b}\bar{b}}$[MeV]
E_H	-139(5)	-81(8)
E_M	-163(8)	-94(9)
E_L	-190(12)	-96(7)
Phys	-189(10)(3)	-98(7)(3)

Bindings were extrapolated in m_π, with ΔE of 189(10)MeV and 98(7)MeV for $T_{ud\bar{b}\bar{b}}$ and $T_{\ell s\bar{b}\bar{b}}$ respectively.

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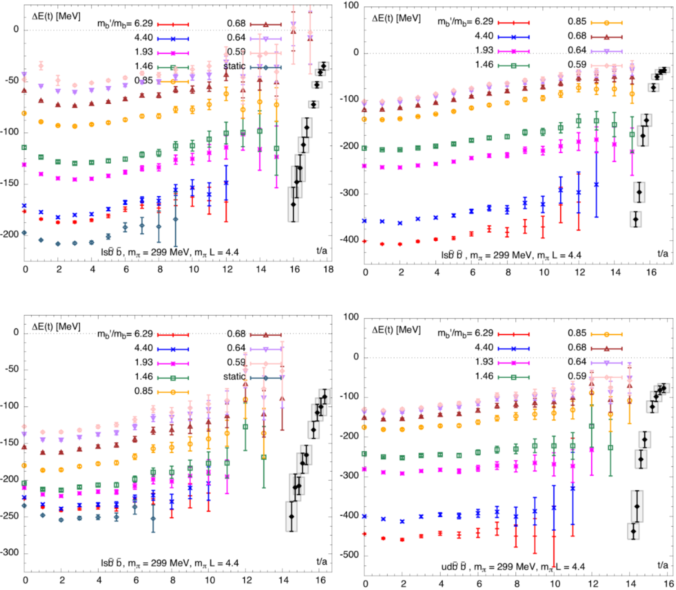
Previous work: m_h analysis on $q_1q_2\bar{h}\bar{h}$ and $q_1q_2\bar{b}\bar{h}$

Next step in [1810.10550] :

- Uses highest statistics previous ensemble with $m_\pi = 299$ MeV, looking for $J^P = 1^+$ tetraquark.
- Variable heavy mass $am_h \in \{0.9, 1.0, 1.2, 1.6, 3.0, 4.0, 8.0, 10.0\}$ $(0.1 \lesssim \frac{m_b}{m_h} \lesssim 1.8)$.
- Four channels: $ud\bar{b}\bar{h}$, $ud\bar{h}\bar{h}$, $\ell s\bar{b}\bar{h}$, $\ell s\bar{h}\bar{h}$.
- Again, use GEVP and wall-local correlators. For $q_1q_2\bar{b}\bar{h}$ case, GEVP is 3×3.

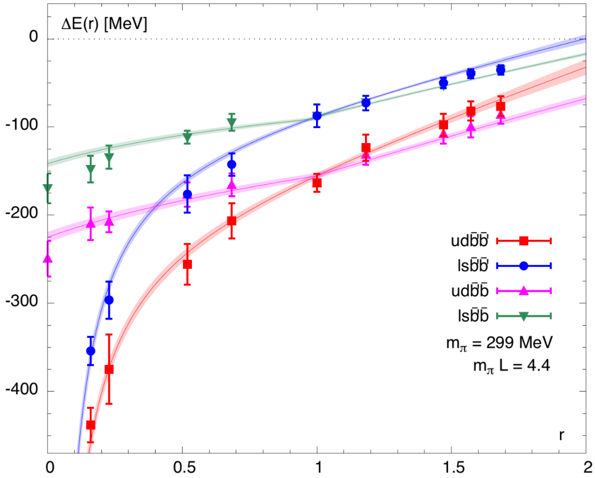
Previous work: m_h analysis on $q_1 q_2 \bar{h}\bar{h}$ and $q_1 q_2 \bar{b}\bar{h}$



Plateaus are better but often appear to be rising.

Note $b' \equiv h$.

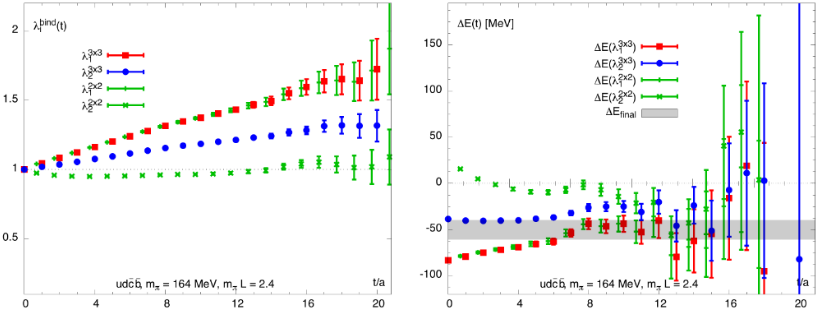
Previous work: m_h analysis on $q_1 q_2 \bar{h}\bar{h}$ and $q_1 q_2 \bar{b}\bar{h}$



$$r = \frac{m_b}{m_h}$$

We get $q_1 q_2 \bar{b}\bar{h}$ point at $m_h = \infty$ from static propagator. Unbounded 3_c colour Coulomb as $m_h \to \infty$.

Previous work: m_h analysis on $q_1 q_2 \bar{h}\bar{h}$ and $q_1 q_2 \bar{b}\bar{h}$



Also look at $ud\bar{b}\bar{c}$ with relativistic heavy quark action for c, with $m_\pi =$ 164, 299 and 415 MeV. Ground state is unaffected by 2×2 or 3×3, but plateaus still short. Find a binding of 38(23)MeV.

Previous work: src snk analysis on $q_1 q_2 \bar{h}_1 \bar{h}_2$

Previous work in [*2006.14294*] :

- Uses meson-meson and diquark-diquark operators.
- Large range of $J^P = 1^+$ possible tetraquark channels: $ud\bar{c}\bar{b}$, $ud\bar{s}\bar{b}$, $ud\bar{s}\bar{c}$, $ud\bar{b}\bar{b}$, $\ell s\bar{b}\bar{b}$, $uc\bar{b}\bar{b}$, $sc\bar{b}\bar{b}$, $us\bar{c}\bar{b}$.
- Looked at $J^P = 0^+$ too.
- Used GEVP method and a single ensemble with $m_\pi = 192\text{MeV}$.
- Key difference - introduction of box sinks.

$$S^{B,R} = \frac{1}{N} \sum_{r^2 \leq R^2} S(x + r, t).$$

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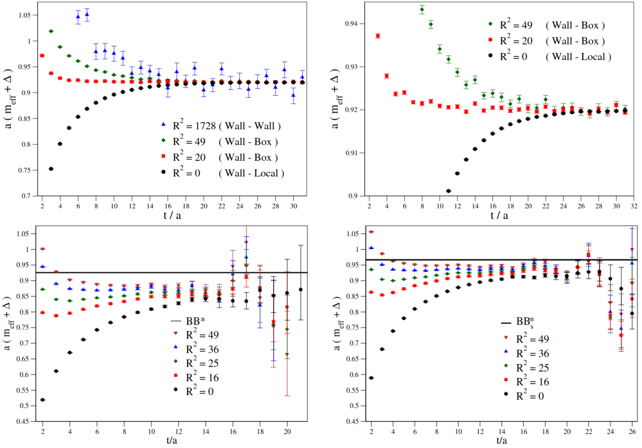
Previous work: src snk analysis on $q_1 q_2 \bar{h}_1 \bar{h}_2$



Box sink for B_c (top), $ud\bar{b}\bar{b}$(bot. l) and $\ell s\bar{b}\bar{b}$(bot. r). Different radii change direction of convergence.

Previous work: src snk analysis on $q_1q_2\bar{h}_1\bar{h}_2$

- Only find deep binding in $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ channels 113 and 36 MeV respectively.
- Did not find measurable binding in $ud\bar{c}\bar{b}$ - not deeply bound.
- Shallow binding can't be ruled out without FV (Luscher) analysis.

Current work on $q_1q_2\bar{h}\bar{h}$ and $q_1q_2\bar{b}\bar{h}$

Current work (see [2407.08816]) builds on [2006.14294] , and applies it to m_l and m_h analyses.

- Uses a large number of (local) operators and four box sink radii.
- Focus on $J^P = 1^+$ $ud\bar{h}\bar{h}$, $ud\bar{b}\bar{h}$, $\ell s\bar{h}\bar{h}$ and $\ell s\bar{b}\bar{h}$ tetraquarks.
- Also look at $J^P = 0^+$ for $ud\bar{b}\bar{h}$ and $\ell s\bar{b}\bar{h}$.
- Carry out heavy mass analysis with nine m_h values.
- Also chiral extrapolation with six m_π values.
- Two newly generated ensembles with $N_x = 48$ give $m_\pi L > 3.6$ in all cases.
- Use multi-exponential fits to extract energies.

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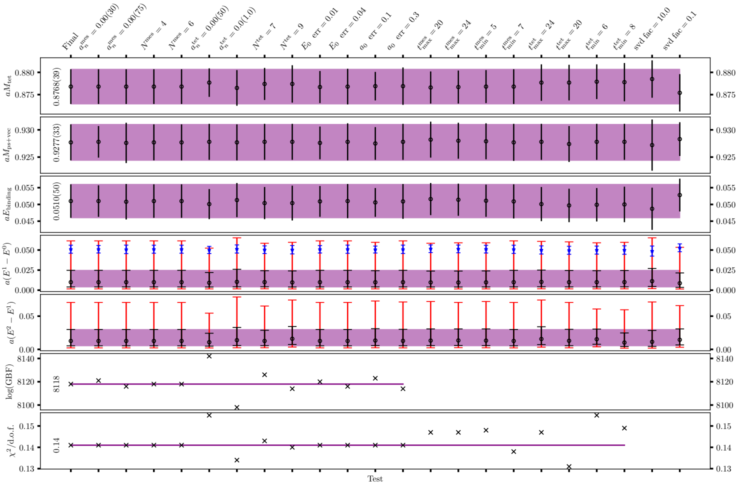
Current work on $q_1 q_2 \bar{h}\bar{h}$ and $q_1 q_2 \bar{b}\bar{h}$

We perform correlated, simultaneous fits to the meson and tetraquark data, across all the source-sink combinations, for all sink radii.

$$C_2^{\mathrm{mes/tet}} = \sum_n^N a_n^{\mathrm{src}} a_n^{\mathrm{snk}} (e^{-E_n^{\mathrm{mes/tet}} t} \pm e^{-E_n^{\mathrm{mes/tet}}(T-t)})$$

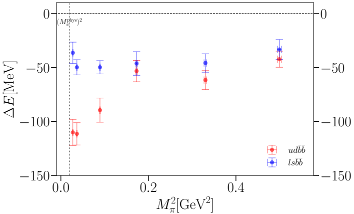
Bayesian fitting approach, with χ^2 and the Gaussian Bayes factor to judge fit quality.

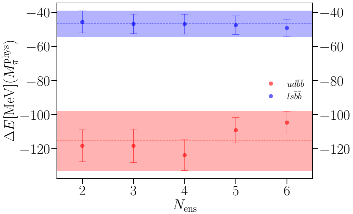
Current work on $q_1 q_2 \bar{h}\bar{h}$ and $q_1 q_2 \bar{b}\bar{h}$

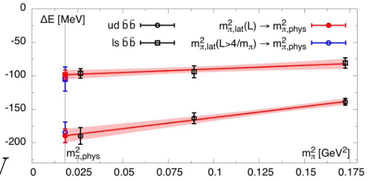


We find our fits to be very stable.

Current work on $q_1 q_2 \bar{h}\bar{h}$ and $q_1 q_2 \bar{b}\bar{h}$







c.f. 2016 results

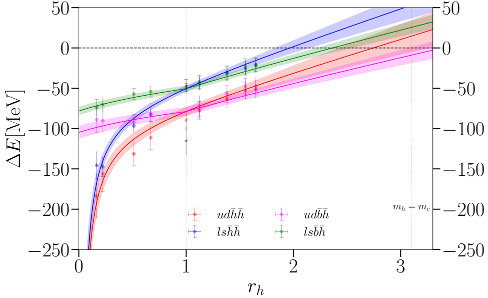
$ud\bar{b}\bar{b} = 115(17)$MeV

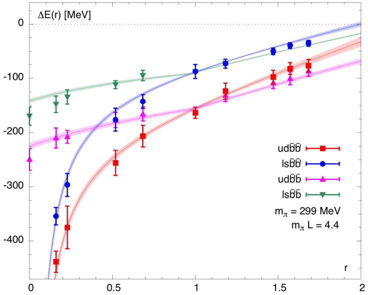
$\ell s\bar{b}\bar{b} = 46.7(7.6)$MeV

$\approx 1\sigma$ of this from FV effects)

Current work on $q_1q_2\bar{h}\bar{h}$ and $q_1q_2\bar{b}\bar{h}$

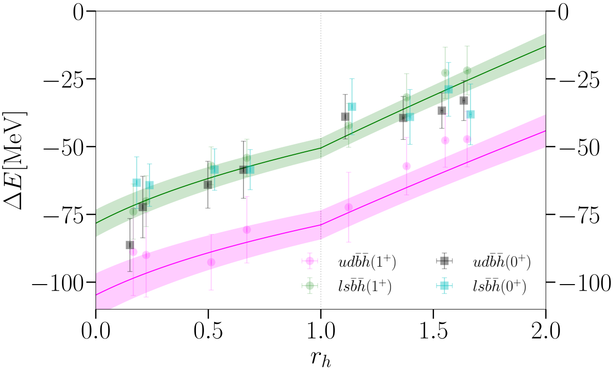
c.f. 2018 results





$J^P = 1^+$ binding shallower than has been found in the past.

Current work on $q_1q_2\bar{h}\bar{h}$ and $q_1q_2\bar{b}\bar{h}$



$udbh$ 0^+ binding ≈ 20 MeV shallower than 1^+ $\implies$ $ud\bar{b}\bar{c}$ decays $1^+ \to 0^+\gamma$ or $1^+ \to B\bar{D}\gamma$. Look for highly collimated $B\bar{D}$.

Conclusion and outlook

Current work [2407.08816] builds on [2006.14294,1810.10550,1607.05214]

- We find deep $J^P = 1^+$ bindings: $ud\bar{b}\bar{b} = 115(17)$MeV and $\ell s\bar{b}\bar{b} = 46.7(7.6)$MeV ($\approx 1\sigma$ of this from FV effects).
- This agrees with less deeply bound emerging consensus [2006.14294, 2404.03588, 2303.17295, 2306.03565] .
- $ud\bar{b}\bar{c}$ not deeply bound - requires Luscher method ([2205.13982, 2205.02925] agrees, as does our 2020 paper [2006.14294]).
- Box sink improve plateaus.
- Difference in $T_{ud\bar{b}\bar{h}}(0^+)$ and $T_{ud\bar{b}\bar{h}}(1^+)$ may aid experimentalists. Look for collimated $B\bar{D}$.
- Future analysis of $ud\bar{b}\bar{c}$ would be useful. Different action?

Back up

Type ($\psi\phi\theta\omega$)	$I(J)^P$	Diquark-Antidiquark	Dimeson
$udcb/udsb/udsc$	$0(1)^+$	$D(\gamma_5,\gamma_i), D(\gamma_t\gamma_5,\gamma_i\gamma_t)$ $E(\gamma_5,\gamma_i), E(\gamma_t\gamma_5,\gamma_i\gamma_t)$	$M(\gamma_5,\gamma_i) - N(\gamma_5,\gamma_i)$ $M(I,\gamma_i\gamma_5) - N(I,\gamma_i\gamma_5)$ $O(\gamma_5,\gamma_i) - P(\gamma_5,\gamma_i)$ $O(I,\gamma_i\gamma_5) - P(I,\gamma_i\gamma_5)$ $\epsilon_{ijk}M(\gamma_j,\gamma_k)$
$udbb$	$0(1)^+$	$D(\gamma_5,\gamma_i),\ D(\gamma_t\gamma_5,\gamma_i\gamma_t)$	$M(\gamma_5,\gamma_i) - N(\gamma_5,\gamma_i)$ $M(I,\gamma_i\gamma_5) - N(I,\gamma_i\gamma_5)$
$lsbb/ucbb/scbb$	$\frac{1}{2}(1)^+$	$D(\gamma_5,\gamma_i),\ D(\gamma_t\gamma_5,\gamma_i\gamma_t)$	$M(\gamma_5,\gamma_i),\ M(I,\gamma_i\gamma_5)$ $N(\gamma_5,\gamma_i),\ N(I,\gamma_i\gamma_5)$ $\epsilon_{ijk}M(\gamma_j,\gamma_k)$
$uscb$	$\frac{1}{2}(1)^+$	$D(\gamma_5,\gamma_i),\ D(\gamma_t\gamma_5,\gamma_i\gamma_t)$ $E(\gamma_5,\gamma_i),\ E(\gamma_t\gamma_5,\gamma_i\gamma_t)$	$M(\gamma_5,\gamma_i),\ M(I,\gamma_i\gamma_5)$ $N(\gamma_5,\gamma_i),\ N(I,\gamma_i\gamma_5)$ $O(\gamma_5,\gamma_i),\ O(I,\gamma_i\gamma_5)$ $\epsilon_{ijk}M(\gamma_j,\gamma_k)$

TABLE I: $J^P = 1^+$ operators used in this work.

Type ($\psi\phi\theta\omega$)	$I(J)^P$	Diquark-Antidiquark	Dimeson
$udcb/udsb/udsc$	$0(0)^+$	$E(\gamma_5,\gamma_5), E(\gamma_t\gamma_5,\gamma_t\gamma_5)$	$M(\gamma_5,\gamma_5) - N(\gamma_5,\gamma_5)$ $M(I,I) - N(I,I)$ $M(\gamma_i,\gamma_i)$
$uscb$	$\frac{1}{2}(0)^+$	$E(\gamma_5,\gamma_5),\ E(\gamma_t\gamma_5,\gamma_t\gamma_5)$	$M(\gamma_5,\gamma_5),\ M(I,I)$ $N(\gamma_5,\gamma_5),\ N(I,I)$ $M(\gamma_i,\gamma_i)$

$$D(\Gamma_1,\Gamma_2) = (\psi_a^T C\Gamma_1\phi_b)(\bar\theta_a C\Gamma_2\bar\omega_b^T),$$
$$E(\Gamma_1,\Gamma_2) = (\psi_a^T C\Gamma_1\phi_b)(\bar\theta_a C\Gamma_2\bar\omega_b^T - \bar\theta_b C\Gamma_2\bar\omega_a^T),$$
$$M(\Gamma_1,\Gamma_2) = (\bar\theta\Gamma_1\psi)(\bar\omega\Gamma_2\phi), \qquad N(\Gamma_1,\Gamma_2) = (\bar\theta\Gamma_1\phi)(\bar\omega\Gamma_2\psi),$$
$$O(\Gamma_1,\Gamma_2) = (\bar\omega\Gamma_1\psi)(\bar\theta\Gamma_2\phi), \qquad P(\Gamma_1,\Gamma_2) = (\bar\omega\Gamma_1\phi)(\bar\theta\Gamma_2\psi).$$

Back up

$$\Delta E = \frac{C_0}{2r} + C_1^{ud} + C_2^{ud}(2r) + 23 \text{ MeV}\, r \tag{1}$$

for the $ud\bar{b}'\bar{b}'$ case, where the first term represents the Coulomb binding contribution, the second the good-ud-diquark attraction, the third the residual heavy-light interactions in the tetraquark state and the fourth the two-meson threshold contribution. The numerical value appearing in the fourth term follows from the observed meson splittings. Similarly, for the $ud\bar{b}'\bar{b}$ case, one expects the form

$$\Delta E = \frac{C_0}{1+r} + C_1^{ud} + C_2^{ud}(1+r) + (34 \text{ MeV} - 11 \text{ MeV}\, r) \tag{2}$$

to provide a good representation for $m_{b'} > m_b$ and the form

$$\Delta E = \frac{C_0}{1+r} + C_1^{ud} + C_2^{ud}(1+r) + (34 \text{ MeV}\, r - 11 \text{ MeV}) \tag{3}$$

to provide a good representation for $m_{b'} < m_b$. The corresponding expectations for the cases involving an ℓs, rather than ud, good-diquark are

$$\Delta E = \frac{C_0}{2r} + C_1^{\ell s} + C_2^{\ell s}(2r) + 24 \text{ MeV}\, r \tag{4}$$

for $\ell s\bar{b}'\bar{b}'$,

$$\Delta E = \frac{C_0}{1+r} + C_1^{\ell s} + C_2^{\ell s}(1+r) + (34 \text{ MeV} - 12 \text{ MeV} r) \tag{5}$$

for $\ell s\bar{b}'\bar{b}$ with $m_{b'} > m_b$ and

$$\Delta E = \frac{C_0}{1+r} + C_1^{\ell s} + C_2^{\ell s}(1+r) + (36 \text{ MeV}\, r - 11 \text{ MeV}) \tag{6}$$

for $\ell s\bar{b}'\bar{b}$ with $m_{b'} < m_b$.

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