Static-light $\bar{Q}\bar{Q}qq$ potentials for $\bar{b}$ and $\bar{c}$ heavy quarks and u, d and s light quarks from lattice QCD

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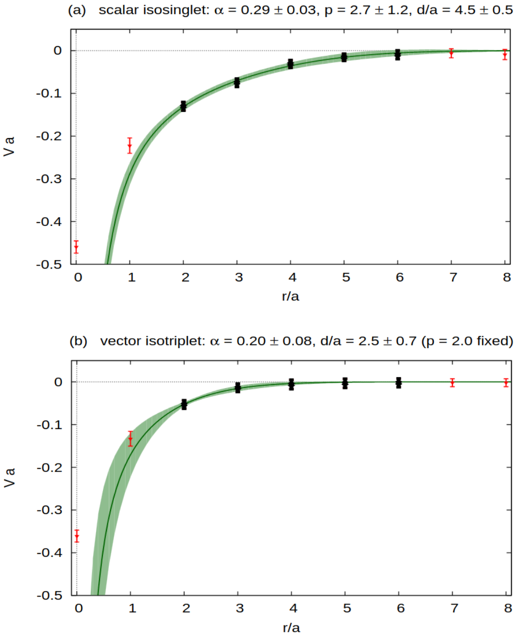
Motivation: previous potentials,Tbb! Tbbs? Tbc? Tcc? tetraquarks



Attractive $\bar{Q}\bar{Q}qq$ potentials from

M. Wagner "Forces between static-light mesons," PoS **LATTICE2010**, 162 (2010) [arXiv:1008.1538 [hep-lat]].

Inspired in T_{cc} tetraquarks proposed since 1981 by J-M. Richard et al, with a Coulomb screened potential fit, with strength α and screening length d

$$V(r) = -\frac{\alpha}{r} \exp\left(-\frac{r^2}{d^2}\right) ,$$

the binding energy from Schrödinger equation, for T_{bb} with $IJ^P = 01^+$ is δmass $= -38 \pm 18$ MeV.

P. Bicudo and M. Wagner, "Lattice QCD signal for a bottom-bottom tetraquark," Phys. Rev. D **87**, no.11, 114511 (2013)

with chiral extrapolation and improved plateau fits δmass $= -90 \pm 43$ MeV.

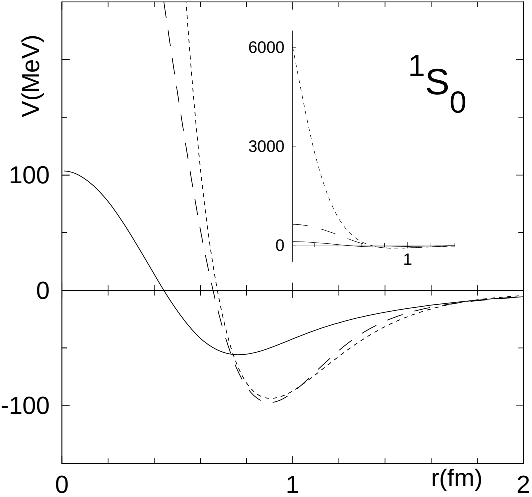
P. Bicudo, K. Cichy, A. Peters, B. Wagenbach and M. Wagner, Phys. Rev. D **92**, no.1, 014507 (2015).

T_{bbs} and T_{bc} are close to binding but T_{cc} is farther.

However spin effects decrease the binding by 30 %.

There is a tension with the lattice computations with NRQCD who find more binding, δmass < -100 MeV.

Motivation: compare with pion exchange and N-N potentials



Example of N − N potentials for n − p

Meson exchange may produce small bumps, with opposite sign to the main part, in our potentials.

Each static light meson has:
I=1/2,
S=1/2,
colour=1,
the same quantum numbers of a nucleon *N*, *p* or *n*,.

Our potentials are qualitatively similar to the nucleon-nucleon (N-N) interaction, fundamental to nuclear physics.

At large distances we expect the OPE pion exchange potential, with

$$(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)[(\boldsymbol{\sigma}_1 \cdot \boldsymbol{r})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{r}) + \cdots] \frac{e^{-m_\pi r}}{r}$$

V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen and J. J. de Swart, "Construction of high quality N N potential models," Phys. Rev. C **49**, 2950-2962 (1994)

Lattice setup: three CLS ensembles + tree level improvement + off axis

ensemble	T/a	L/a	a[fm]	m_π[MeV]	N_{cfg}	α_{APE}	n_{APE}	κ_{G}	n_{G}
A5	64	32	0.0755	331	100	0.5	30	0.5	50
N6	96	48	0.0486	340	50	0.5	50	0.5	120
G8	128	64	0.0658	185	30	0.5	35	0.5	70

Lattice CLS ensembles with two dynamical flavours of improved Wilson quarks and APE smearing

- Our computations use the openQ*D codebase I. Campos *et al.* [RC*], Eur. Phys. J. C **80**, no.3, 195 (2020)
- stochastic timeslice propagators 12 stochastic sources per timeslice on 6(N6)-8(A5, G8) timeslices
- smearing algorithms and notation consistent with K. Jansen *et al.* [ETM], JHEP **12**, 058 (2008) and HYP2 static action

We use a tree-level Gluon propagator G improved separation, replacing the lattice distance,

$$\mathbf{r}_{\mathrm{lat}} \to \mathbf{r}_{\mathrm{impr}} = \frac{4\pi a}{G\left(\frac{\mathbf{r}}{a}\right)}. \tag{1}$$

We use off axis separations, and check for the possible breaking of the rotational symmetry, e.g.

axis	symmetry group	different possible rotations n	$\lvert L \rvert$
$(0, 0, a)$	D_{4h}	4	0,4,8,...
			1,3,5,...
			2,6,10,...
$(0, a, b)$	C_{2h}	1	0,1,2,...

Lattice setup: tetraquark creation operators and correlators

- creation operators

$$\mathcal{O}_{BB}^{I,\Gamma}(\mathbf{r}_1,\mathbf{r}_2) \quad = (\mathcal{C}\Gamma)_{AB}\left(\mathcal{C}\tilde{\Gamma}\right)_{CD}\left(\bar{b}_C^a(\mathbf{r}_1)u_A^a(\mathbf{r}_1)\;\bar{b}_D^b(\mathbf{r}_2)d_B^b(\mathbf{r}_2) \mp (u \leftrightarrow d)\right), \tag{2}$$

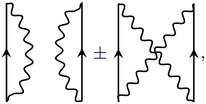
- correlation function

$$\mathcal{C}_{BB}^{I,\Gamma}(\mathbf{r}_2-\mathbf{r}_1,t_2-t_1) = \langle\Omega|\mathcal{O}_{BB}^{I,\Gamma\,\dagger}(\mathbf{r}_1,\mathbf{r}_2;t_2)\mathcal{O}_{BB}^{I,\Gamma}(\mathbf{r}_1,\mathbf{r}_2;t_1)|\Omega\rangle \propto$$

$$\propto \Big\langle \left(\gamma_0\Gamma^\dagger\gamma_0\right)_{BA}\Gamma_{CD}\Big($$

$$\mathrm{Tr}_C\left[U(\mathbf{r}_1,t_2;\mathbf{r}_1,t_1)\left(M_q^{-1}\right)_{CA}(\mathbf{r}_1,t_1;\mathbf{r}_1,t_2)\right]\times\mathrm{Tr}_C\left[U(\mathbf{r}_2,t_2;\mathbf{r}_2,t_1)\left(M_q^{-1}\right)_{DB}(\mathbf{r}_2,t_1;\mathbf{r}_2,t_2)\right]$$

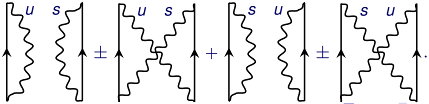
$$\pm\mathrm{Tr}_C\left[U(\mathbf{r}_1,t_2;\mathbf{r}_1,t_1)\left(M_q^{-1}\right)_{CA}(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2)\;U(\mathbf{r}_2,t_2;\mathbf{r}_2,t_1)\left(M_q^{-1}\right)_{DB}(\mathbf{r}_2,t_1;\mathbf{r}_1,t_2)\right]\Big)\Big\rangle \equiv$$

$\equiv$  , (3)

- divide by B-meson correlator

$$\frac{\mathcal{C}_{BB}^{I,\Gamma}(\mathbf{r},t)}{(\mathcal{C}_B(t))^2} \xrightarrow[t\to\infty]{} A\exp\left(-\left(V_{BB}^{I,\Lambda_\eta^\epsilon}(\mathbf{r})-2m_B\right)t\right), \tag{4}$$

- *bbus* correlator: creation operator corresponds to Eqn. 2 with $d \to s$, different light quarks produce 4 diagrams instead of 2:

$\mathcal{C}_{BB_s}^{\Gamma}(\mathbf{r}_2-\mathbf{r}_1,t_2-t_1) \equiv$ 

. (5)

Lattice setup: quantum numbers of antistatic-antistatic potentials

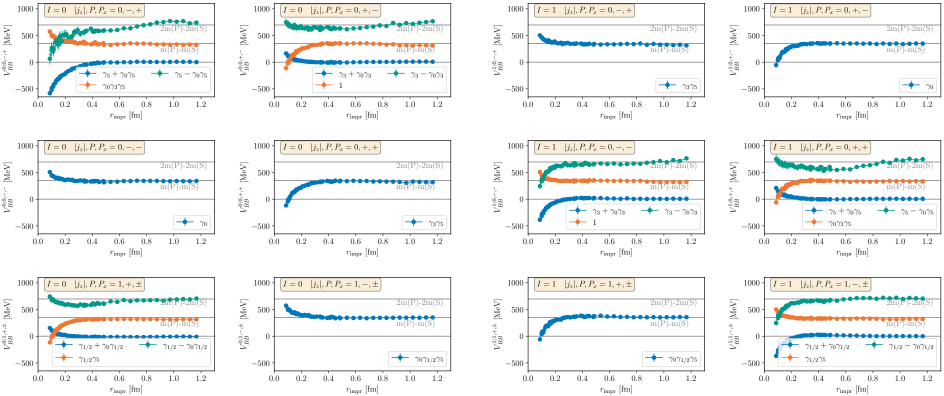
Γ	$I=0$: $\lvert j_z \rvert, \mathcal{P}\mathcal{P}_x$	$I=0$: shape	$I=1$: $\lvert j_z \rvert, \mathcal{P}\mathcal{P}_x$	$I=1$: shape
$\gamma_5 + \gamma_0\gamma_5$	$0, -+$	A,SS	$0, ++$	R,SS
1	$0, +-$	A,SP	$0, --$	R,SP
γ_0	$0, --$	R,SP	$0, +-$	A,SP
$\gamma_5 - \gamma_0\gamma_5$	$0, -+$	A,PP	$0, ++$	R,PP
$\gamma_3 + \gamma_0\gamma_3$	$0, +-$	R,SS	$0, --$	A,SS
$\gamma_3\gamma_5$	$0, ++$	A,SP	$0, -+$	R,SP
$\gamma_0\gamma_3\gamma_5$	$0, -+$	R,SP	$0, ++$	A,SP
$\gamma_3 - \gamma_0\gamma_3$	$0, +-$	R,PP	$0, --$	A,PP
$\gamma_{1/2} + \gamma_0\gamma_{1/2}$	$1, +(+/-)$	R,SS	$1, -(+/-)$	A,SS
$\gamma_{1/2}\gamma_5$	$1, +(-/+)$	A,SP	$1, -(-/+)$	R,SP
$\gamma_0\gamma_{1/2}\gamma_5$	$1, -(-/+)$	R,SP	$1, +(-/+)$	A,SP
$\gamma_{1/2} - \gamma_0\gamma_{1/2}$	$1, +(+/-)$	R,PP	$1, -(+/-)$	A,PP

Quantum numbers and properties of the resulting $\bar{b}\bar{b}ud$ potentials:
A = attractive, R = repulsive; SS, SP ,PP = asymptotic value $2m_B$, $m_B + m_{B_0^*}$, $2m_{B_0^*}$.

Quantum numbers:

- $\lvert j_z \rvert$: angular momentum in separation direction
- $\mathcal{P}$: Behaviour under parity.
- $\mathcal{P}:x$: Behaviour under reflection along an axis perpendicular to the separation axis.

Results: all I=0 and I=1 potentials

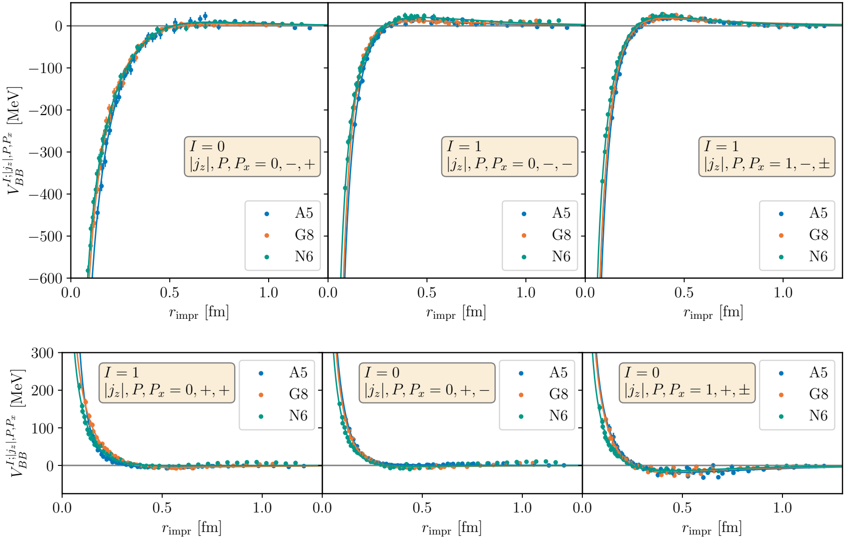


$\bar{b}\bar{b}ud$ potentials for $I = 0$, ensemble N6.

$\bar{b}\bar{b}ud$ potentials for $I = 1$, ensemble N6.

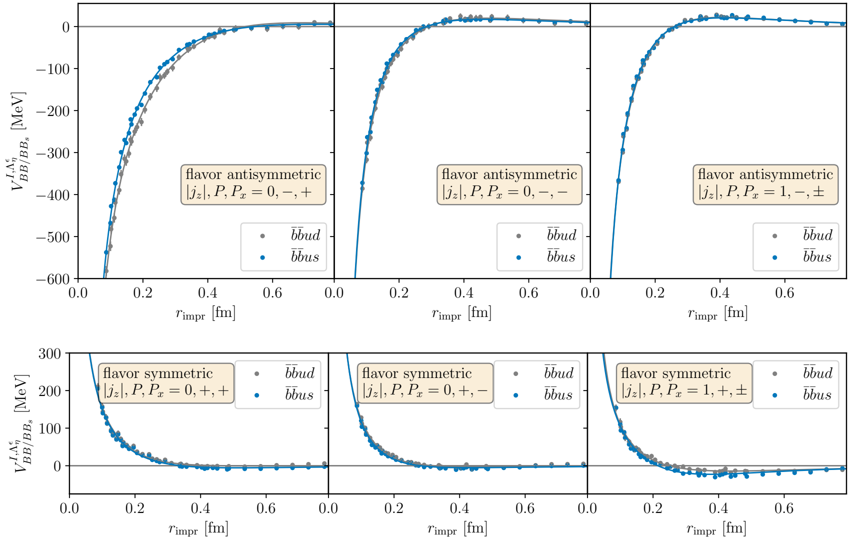
- on-axis separations up to $|\mathbf{r}| = 1.2$ fm
- all possible off-axis separations at shorter distances
- 24 independent creation operators
- no correlation matrices $\rightarrow$ excited potentials might be contaminated by lower potentials.

Results: potentials I=0 and I=1 with lowest asymptotic value 2m(S)



All attractive and repulsive potentials with asymptotic value of $2m(S)$ for ensembles A5, G8 and N6.

Results: potentials strangeness=1 with lowest asymptotic value 2m(S)



All attractive and repulsive $\bar{b}\bar{b}us$ potentials with asymptotic value 2m(S), for ensemble N6. The gray data points and fits indicate corresponding results for $\bar{b}\bar{b}ud$.

Analysis: qualitative expectations

In the quark model, the potential for the two light quarks or the two heavy quarks has the structure:

- leading, colour dependent $\frac{\boldsymbol{\lambda}^1 \cdot \boldsymbol{\lambda}^2}{\boldsymbol{\lambda} \cdot \boldsymbol{\lambda}}$ for $c = 3$ it is $= -\frac{1}{2}$ and attractive whereas it is $= \frac{1}{4}$ repulsive for $c = \bar{6}$,
- next to leading, hyperfine $\frac{\boldsymbol{\lambda}^1 \cdot \boldsymbol{\lambda}^2}{\boldsymbol{\lambda} \cdot \boldsymbol{\lambda}}$ $(-\boldsymbol{\sigma}^1 \cdot \boldsymbol{\sigma}^2)$, extra factor of 3 stronger for $s = 0$ and -1 weaker for $S = 1$,
- the three level for heavy quarks at short distances only has a OGE Coulomb potential $\frac{\boldsymbol{\lambda}^1 \cdot \boldsymbol{\lambda}^2}{\boldsymbol{\lambda} \cdot \boldsymbol{\lambda}} \frac{\alpha}{r}$,

This is consistent with the observation, for the light quarks, that the "good" scalar isoscalar diquark is more attractive than the "bad" vector isovector diquark. They are in $c = \bar{3}$ in the groundstate due to the Pauli principle. The isoscalar vector and isovector scalar are in $c = 6$ and are repulsive.

- Then our heavy quarks have respective colours 3 and $\bar{6}$ and this explains all the potentials at short distance.
- A larger distances we have two static-light mesons, and the potentials are screened with a factor proportional to the static-light wave-function ϕ^2, typically an Airy function or Gaussian with exponent $p = 1.5$ to 2.
- This leads to a simple ansatz, where C_{light} depends on the light quarks and includes medium range effects,
- $$V(r) = \left(\frac{\boldsymbol{\lambda}^1 \cdot \boldsymbol{\lambda}^2}{\boldsymbol{\lambda} \cdot \lambda}_{\text{heavy}} \frac{\alpha}{r} + C_{\text{light}} \right) \exp\left[-\left(\frac{r}{d} \right)^p \right] \quad (6)$$

the ansatz, with quarks, gluons and wavefunction screening already fits most of the potentials.

At large distances we have meson exchanges, in particular the one pion exchange (OPE) potential.

- OPE includes a hyperfine potential $(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1 \cdot \sigma_2) \left[\frac{e^{-m_\pi r}}{r} + \cdots \right]$ which, due to the Pauli principle, is opposed to the dominant quark model term $\frac{\boldsymbol{\lambda}^1 \cdot \boldsymbol{\lambda}^2}{\boldsymbol{\lambda} \cdot \boldsymbol{\lambda}}$, producing an opposite bump at large distances
- but the main signature is the tensor part including $(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)[(\boldsymbol{\sigma}_1 \cdot \boldsymbol{r})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{r})] \left[\frac{e^{-m_\pi r}}{r} + \cdots \right]$
- in our case $(\boldsymbol{\sigma}_1 \cdot \hat{z})(\boldsymbol{\sigma}_2 \cdot \hat{z})$ is $= -1$ for $j_z = 0$ and is $+1$ for $|j_z| = 1$. separates the centre and left plots.

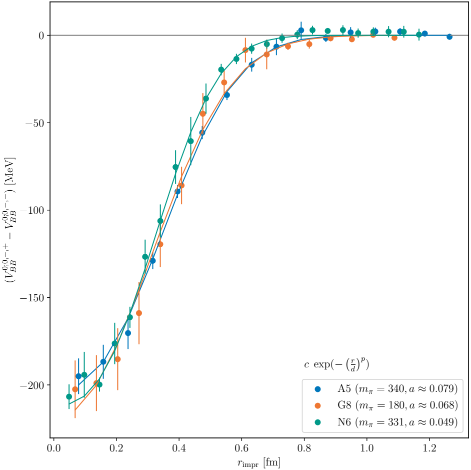
Analysis: potential fits, search for T_{bb} , T_{bbs}, T_{bc}, T_{cc}

$\bar{Q}\bar{Q}qq$	Ens.	V_{fit}	α_1	d[fm]	p	c [MeV]	E_B[MeV]
bbud	A5	V_1	0.356(0.024)	0.307(0.017)	2.21(0.33)		48(11)
		V_2	0.436(0.024)	0.570(0.064)	1.99(0.337)	164(19)	60(12)
	G8	V_1	0.298(0.017)	0.316(0.014)	2.27(0.36)		17(5)
		V_2	0.359(0.012)	0.552(0.067)	2.28(0.74)	130(16)	20(5)
	N6	V_1	0.264(0.010)	0.336(0.015)	2.74(0.29)		11(5)
		V_2	0.271(0.074)	0.349(0.029)	2.93(0.77)	19(53)	12(5)
bbus	A5	V_1	0.335(0.010)	0.282(0.007)	2.05(0.12)		no binding
		V_2	0.414(0.017)	0.468(0.027)	1.70(0.23)	156(6)	no binding
	G8	V_1	0.325(0.009)	0.250(0.005)	1.78(0.08)		3(1)
		V_2	0.404(0.014)	0.409(0.019)	1.28(0.11)	157(6)	5(2)
	N6	V_1	0.251(0.004)	0.310(0.011)	2.11(0.10)		no binding
		V_2	0.290(0.004)	0.612(0.040)	2.14(0.26)	107(5)	no binding

Fitting parameters of our ansatz for the $\bar{Q}\bar{Q}ud$ and $\bar{Q}\bar{Q}us$ potentials, and binding energies $E_B = -\delta$mass for T_{bb} and T_{bbs} energies with the Born-Oppenheimer approximation for $m_{\bar{Q}} = m_b$.

- Notice, with the C parameter, the screening in the isoscalar case is close to a Gaussian.
- We also achieve a larger significance than with the 2010 and 2015 potentials.
- However, the E_B are in general similar to the ones of the 2010 potential, with no binding for T_{bc} and T_{cc}.
- In 2015, Phys. Rev. D **92**, no.1, 014507 (2015) performed chiral extrapolation and improved the plateau fits.
- Besides, the different ensembles show rather different fits, visible in the different binding energies.

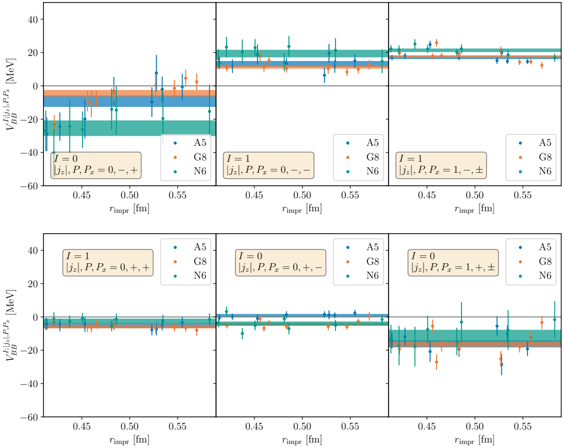
Analysis: the bumps from good and bad diquarks



Difference between the "good" $I = 0$ and $|j_z|, P, P_x = 0, -, +$ and "bad" $I = 1$ and $|j_z|, P, P_x = 0, -, -$ and $|j_z|, P, P_x = 0, -, \pm$ computed with the ratio of the correlators.

Comparing with the study of "good" and "bad" diquarks in lattice QCD, for instance in Ref. A. Francis, P. de Forcrand, R. Lewis and K. Maltman, JHEP **05**, 062 (2022) , we verify this difference to tend to the diquark mass difference $\simeq$ 200 MeV, in the limit of vanishing distances. This explains the mid distance bump.

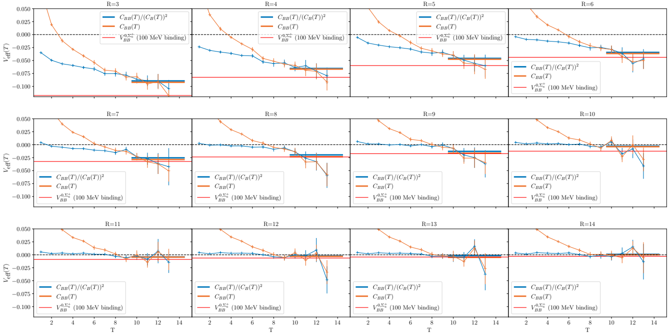
Analysis: the bumps from one pion exchange (OPE)



All attractive (top) and repulsive (bottom) potentials with the lowest asymptotic value of $2m(S)$ *for ensembles A5, G8 and N6 for intermediate separations* $0.4 < r < 0.6$*.*

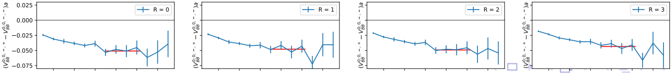
- In this figure, the tensor interaction from OPE is expected shifts the centre panel potentials with $|j_z| = 0$ from the right panel potentials with $|j_z| = 1$ ones. Notice the top and bottom panels have different isospin.
- The horizontal lines indicate a constant fit of the data points in this region for the respective ensemble and the width of the line indicates one standard deviation.
- We expect $(p6 - p5)/(p3 - p2) \simeq -3$ and in the figure it goes in the right direction.

Outlook: improve systematic errors in EMPs



Effective Mass Plots for our most attractive 'good' scalar-isoscalar potential. The red line provided by the next speaker Jakob Hoffmann shows a plateau leading to binding energy $E_B = 100$ MeV.

- Clearly, our EMPs at short distances have not yet finished decreasing before the error bars get too large.
- This systematic error weakens the potential, and creates a tension with the NRQCD results.
- We are considering better smearings
- and aim to compute the correlation matrix to decrease the excited state contamination of the plateaux.
- When dividing the 'good' and 'bad' most attractive potentials, the short range effects are under control:



Outlook: summary

- We compute, more than one decade later, new potentials $\bar{Q}\bar{Q}qq$.
- We improve our computations with with new CLS ensembles including a pion not far from the physical one, the OpenQ*D codebase, tree level and off axis separation improvement.
- We study 24 operators, for $I = 0$, $I = 1$ and strangeness=1, for asymptotic energies static-light masses $2m_S$, $m_S + m_P$ and $2 * m_P$.
- The potentials are compatible, with the short distance One Gluon Exchange (OGE) between the heavy quarks, and at intermediate distances with the static-light wavefunctions screening.
- With our improvements, we are now able to observe bumps in the potentials at intermediate and large separations, opposite in sign to the short range distances.
- Part of these bumps can be explained by the spin-dependent hyperfine interactions between the constituent light quarks, as expected in quark models.
- In the bumps, we also find a qualitative indication for the long distance One Pion Exchange (OPE) potential, with its spin-dependent tensor potential signature.
- However, More statistics and better control of the systematics are still needed.
- Our systematic errors still lead to less attractive potentials, our binding energies are smaller than the ones of with lattice NRQCD bottom quarks.
- One improvement we are certainly planning is to compute the correlation matrix between the different operators, to decrease the plateau contamination from excited states.

THANK YOU !!!